Issues in Replication and Stability of Least-cost Path Calculations

IRMELA HERZOG
The Rhineland Commission for Archaeological Monuments and Sites, Germany

An important and frequently used tool in archaeological spatial analysis is least-cost path (LCP) analysis with the aim of computing routes connecting a set of targets. The outcome depends on the cost model chosen and the topographic data used. A slope-dependent cost model requires a digital elevation model (DEM) that should reflect the landscape in the past. Often, it is not possible to reconstruct the past terrain, and modern high resolution elevation data results in problematic storage requirements and computation times. This article presents a case study that explores issues in replication and stability of LCP calculations for pairs of targets that are close to known old trade routes. These trade routes are used for evaluating a large number of cost models, that are based on two topographic data sets including DEMs of two different resolutions. Six different slope-dependent cost functions for pedestrians are tested as well as a function modeling wheeled transport. The latter generates hairpin curves when steep gradients exceeding a predefined critical value are to be mastered. By introducing additional isotropic cost factors, the initial slope-dependent cost model is refined iteratively in terms of the performance indicators presented. Surprisingly, overall improvement in the performance indicators often leads to worse replication of some of the known trade routes. The overall best model found is optimal for only eight out of 19 trade route sections considered. This is probably due to different functions of the routes. The performance of the best cost model based on the DEM with a cell size of 25 m is not significantly higher than that based on a larger cell size of 50 m. It is shown that the best-performing cost model parameters for the two topographic data sets are closely related but not identical.

Keywords:
Landscape Analysis, Least-cost paths, Digital Elevation Models, Replicability, Replication

SDH Reference:
Irmela Herzog. 2021. Issues in Replication and Stability of Least-cost Path Calculations, SDH, 5, 2, 131-155.

https://doi.org/10.14434/sdh.v5i2.33796

1. INTRODUCTION

In the past two decades, least-cost path (LCP) approaches have become a popular method in archaeological spatial analysis. Their aim is often to predict or reconstruct transport routes [e.g., Verhagen and Jeneson 2012] or to compare the computed paths to known routes to understand the principles governing route construction in the period of time considered [e.g., Bell and Lock 2000].

Author's address: Irmela Herzog, The Rhineland Commission for Archaeological Monuments and Sites, Endenicher Str. 133, 53115 Bonn, Germany; email: i.herzog@lvr.de

LCP methods have been applied for past societies in many parts of the world and in different periods of time (for references see [Herzog 2020]).

Ten years ago, most archaeological LCP studies relied merely on one cost model, sometimes the outcomes of two or three slope-dependent cost functions were discussed qualitatively. Most of the studies applying at least two cost models come to the conclusion that the choice of the cost function matters [e.g., Gietl et al. 2008; Kantner 2012; Rademaker et al. 2012]. Several issues of these case studies may invalidate the results. Firstly, they often rely on inappropriate implementations of LCP algorithms that support only moves in eight directions and do not take the direction of movement on a slope into account. Secondly, ground truth for the routes to be modelled is sparse or not available at all. Thirdly, the impact of the resolution and accuracy of the digital elevation model (DEM) that is the basis of a slope-dependent cost component is not considered. Finally, the limited number of cost models and cost parameters considered probably does not include the most appropriate cost model. More systematic tests of cost models were introduced in the publication by Güimil-Fariña and Parcero-Oubiña [2015], that compared the LCPs derived from four different slope-dependent cost functions combined with a constant high-cost factor for movement in riverbeds. They used a buffer criterion as a performance indicator, which will be used in this new case study as well.

The aim of the case study is to investigate the stability and replicability of LCP results taking the issues mentioned above into account. The study addresses the basic LCP task of connecting pairs of targets rather than a more complex route network reconstruction. The hilly study region with a large number of mostly small water courses is located in Germany east of the River Rhine (Fig. 1). It covers 2107 km², with altitudes ranging from 42 to 660 m above sea level.

The ground truth for the LCP computations is provided by a set of old trade route courses published in books by Nicke [2000; 2001a; 2001b]. Nicke was a geography teacher who explored the study region by walking and also used travel accounts as well as old maps for his reconstructions of the routes. Archaeologists at the Rhineland Commission for Archaeological Sites and Monuments who are working in the study area believe that he described the routes adequately but that the dates attributed by him are not as reliable (personal communication, e.g., by Michael Gechter). The evidence for dating the old roads described in the books by Nicke is limited. For instance, the first known document (1464 AD) mentioning the road name Brüderstraße (in Fig. 1 the east-west connection traversing Overath) indicates by the adjective "old" that this road had been in use for a long time [Nicke 2000: 15]. Sometimes, Nicke describes several alternative routes between two targets, for instance concerning the Zeitstraße south and north of Much (Fig. 1 in the south). He assumes that they were used in different time periods.

For the research presented in this article, a Geographic Information System (GIS) was used to digitize the routes based on the descriptions in the books by Nicke and old maps that are available in two WMS services provided by the Ordnance Survey Institute of North Rhine-Westphalia in Germany (Geobasis NRW). The WMS services show rectified map sheets of map sets that cover the study area, the first dating back to about 1845, the second was created approximately 50 years later.

In nearly all LCP studies, a DEM is the basis of a slope-dependent cost component. Therefore, it is important to investigate the impact of the DEM on any slope-dependent LCP with respect to stability of the result. Nowadays, high resolution elevation data, mostly acquired by airborne laser scanning

(ALS), is available in many countries. Due to lack of access to high resolution DEM data, various LCP studies in the past were based on DEM resolutions of 90 m or more. For instance, the case study by Rademaker et al. [2012] applied the 90-m-SRTM 3" DEM after discussing DEM accuracy and resolution.

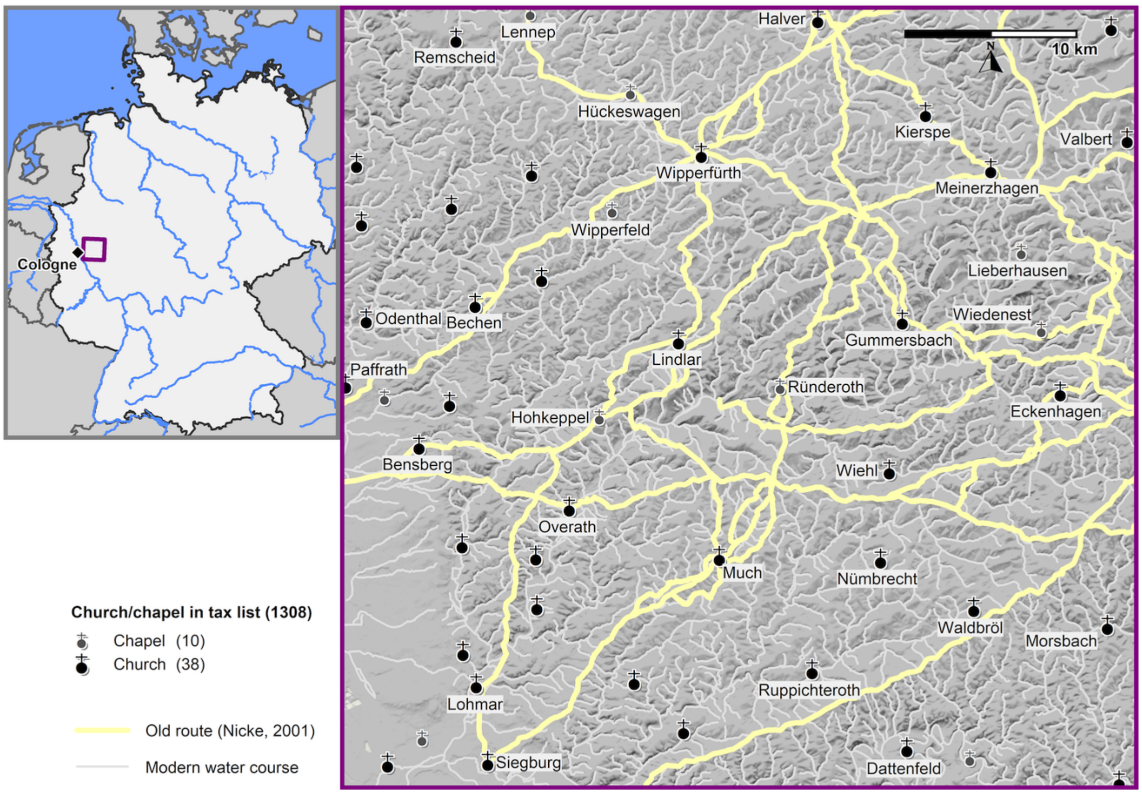

Figure 1. Left: location of the study region in Germany. Right: study region with digitized old trade routes as well as medieval church and chapel locations; background: DEM and water courses provided by Geobasis NRW.

If the DEM-derived slope costs are to be combined with several other cost factors in search for the best performing cost parameters, many LCP computations are required. In this context it is important that LCP computation times and storage requirements increase quadratically with resolution. Therefore, some studies use lower resolution data upscaled from a high resolution DEM for LCP computations [e.g., Rom et al. 2020]. Verhagen and Jeneson [2012] found that the LCP based on a DEM derived from ALS data (resolution: 5 m) ran on a highway. In order to reduce the influence of man-made structures on the LCPs, they used the ASTER DEM with a coarser resolution of approximately 35 m and a lower vertical accuracy. In general, elevation data does not necessarily reflect the landscape in the past but includes more recent man-made features. Reconstructing the past surface is beyond the scope of most archaeological LCP studies. In this situation, low resolution DEMs that smooth out features such as roads or railway lines seem to be more appropriate. However, natural features such as steeply incised creek valleys may be smoothed out as well although they are relevant for past movement. Several archaeological publications of slope-dependent LCP results mention the impact of DEM resolution, most of them show that resolution matters [e.g., Harris 2000; Herzog and Posluschny 2011; Kantner 2012]. But a systematic investigation of the stability of the LCP results depending on the resolution of the DEM is missing in these studies.

The case study set in a hilly region in Germany investigates the reliability and reproducibility of LCP computations based on two highly accurate DEMs that differ in resolution (25 and 50 m) as well as in projection. Moreover, two different sets of data of water courses and wet areas are used. The known Nicke routes provide the ground truth for a systematic search for the best fitting cost model. The cost models combine slope-dependent costs derived from a DEM with costs of traversing water courses and wet areas. The LCP outcomes are assessed by computing appropriate performance indicators. Probable reasons for unsuccessful LCP reconstructions of some old route sections are also discussed.

2. HISTORY OF TRANSPORT IN THE STUDY REGION

The hilly part of the study region does not provide favorable conditions for farming compared to other regions of the Rhineland: soil quality is mostly less than average, steep slopes and wet weather complicate agriculture. It is generally assumed that hardly any people lived in the hilly and forested parts of the study area before AD 700, but that long-distance trade routes mostly running on ridges existed before that time [Andernach 1984; Nicke 2000: 209].

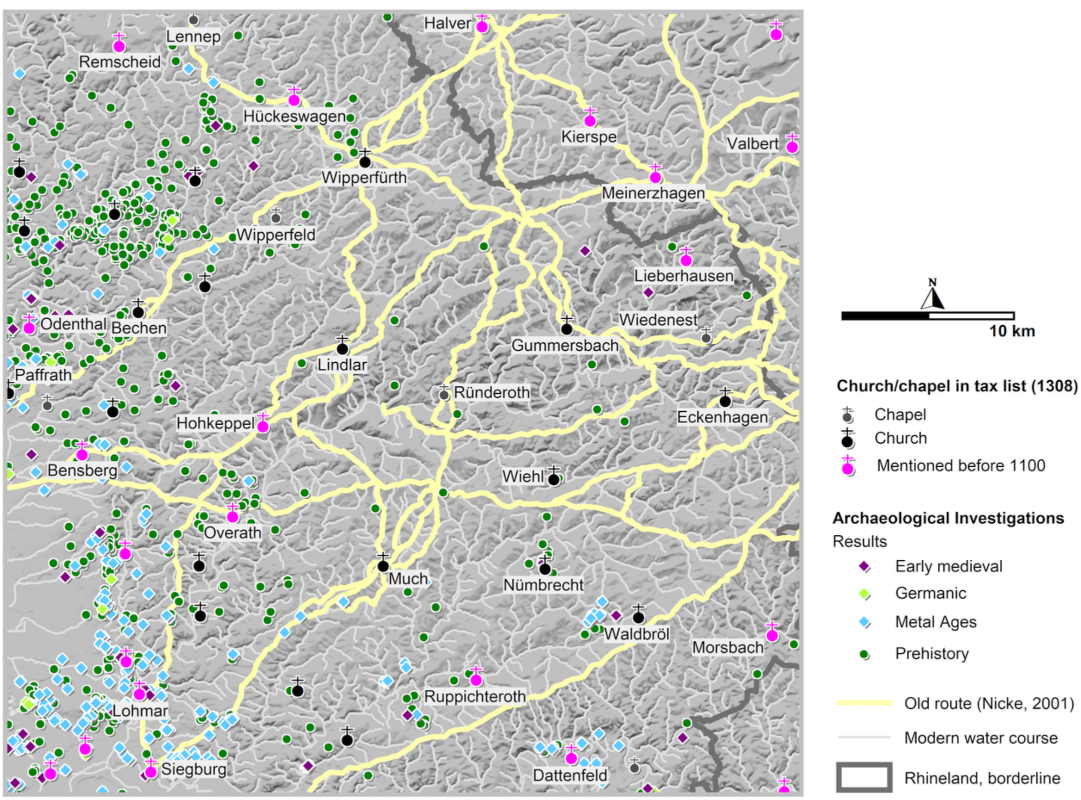

Figure 2. Distribution of known archaeological find spots in the Rhineland part of the study area.

Janssen [2014] points out that historical sources concerning the central part of the Bergisches Land in high medieval times are very sparse and that historical research mostly must rely on archaeological data.

Fig. 2 shows the outcomes of archaeological investigations recorded in the sites and monuments data base of the Rhineland (status in January 2020; [Herzog and Weber 2021]). Only sites within the Rhineland are depicted (grey borderline), and the map is restricted to investigations with features or finds predating 920 AD. Fig. 2 suggests a low site density in the central part of the study area and therefore a largely uninhabited territory in pre-medieval times. However, the density of archaeological investigations in this area is lower than in the western part of the study area. A close relationship between trade routes and the site locations is not evident from this map. Based on the assumption that the old roads predate most of the settlements, it is expected that important medieval settlements (i.e., settlements with a church) and the trade route network are closely related. Statistical tests support this hypothesis [Herzog 2014]. Therefore, Figs. 1 and 2 depict churches and chapels that are included in a tax list set up in 1308 [Oediger 1967].

Initially, several of the roads described by Nicke were used for transporting lime, wood, iron ore and other metals [Nicke 2001a: 24, 106–108]. In late medieval times, the town Wipperfürth became a member of the trade association known as Hanseatic League. Due to these trade relationships as well as new settlements and an increasing number of metal workshops in the hilly part of the study area, the roads were travelled more frequently and new regional roads were created.

Nicke [2000: 198–201; 2001a: 21] outlines conditions and means of travelling and transport in the study area in former times: nearly all medieval travelers walked, better-off people rode on horses. The steep routes in the study area hardly allowed the use of stage coaches first introduced in the late 17th century. In the early 19th century, transport was still largely by carrying loads, often on the back. Oxen carts were mostly used for short distance transport, long distance transport was mainly by pack animals. Two-wheeled horse carts became the most popular means of transport on the roads of the 17th and 18th century. Wheelbarrows played an important role for transport of coal and in the mining industries. A traveler's report from 1766 indicates that walkers moved faster than the mail coaches at that time [Nicke 2000: 206].

The initial aim of this article was to find the cost model that reconstructs the known old Nicke routes in the study area most successfully for each of two topographic data sets. For each old road section considered, two medieval settlements serve as destinations that are on or very close to the road.

3. COST MODELS FOR THE STUDY REGION

The descriptions by Nicke suggest combining two cost factors: (1) attributing appropriate costs to ascending or descending slopes and (2) isotropic costs, i.e., traversing water courses as well as wet areas. Previous publications [e.g., Herzog 2013a; 2013c], that focused on merely one road or a smaller study area, replicated most sections the Nicke routes quite successfully by applying these cost factors. The best cost models identified in these studies varied somewhat. LCP studies by several other authors applied similar cost components in different parts of the world [table 18.1 in Herzog, 2020]. The aim of this and the next two sections is to find the best cost model for all Nicke routes in the study region by performing systematic tests for the cost parameters of two different modern topographic data sets.

Usually, slope is computed from a DEM. In this study, two DEMs are used: (1) DTM50 with a cell size of 50 m (2) DTM25 with a cell size of 25 m. Both DEMs were provided by Geobasis NRW. DTM50 was created around 2005 using the Gauss-Krüger projection and is a subset of a 10 m elevation grid created by Geobasis NRW. The elevation data available at that time was mainly acquired by laser scanning and traditional photogrammetry, only in some areas contour lines were digitized. The newer DTM25 was provided in UTM-based ETRS89 projection and was derived from laser scanning data. About 1% of the study area in the south-east is not covered by this DEM. Bilinear interpolation from the DTM50 was applied to fill this gap.

Modern water course data is also available from Geobasis NRW. Two width classes (less than and exceeding 3 m) can be found in the study area. The DTM50-based cost models do not take the width of the rivers into account due to the expectation that the wet area zones surrounding the rivers would be larger anyway. In a later phase of the project, the DTM25-based cost models were set up, and for these models, the impact of different weights for the two water course width classes is investigated. The water course data is supplemented by the areas with wet soils recorded in a digital soil map of the study region provided by the Geological Service Institution in North Rhine-Westphalia (Geologischer Dienst Nordrhein-Westfalen). The main reason for including the wet soils in the cost model are missing minor water courses in the water course layer provided by Geobasis NRW (Fig. 3).

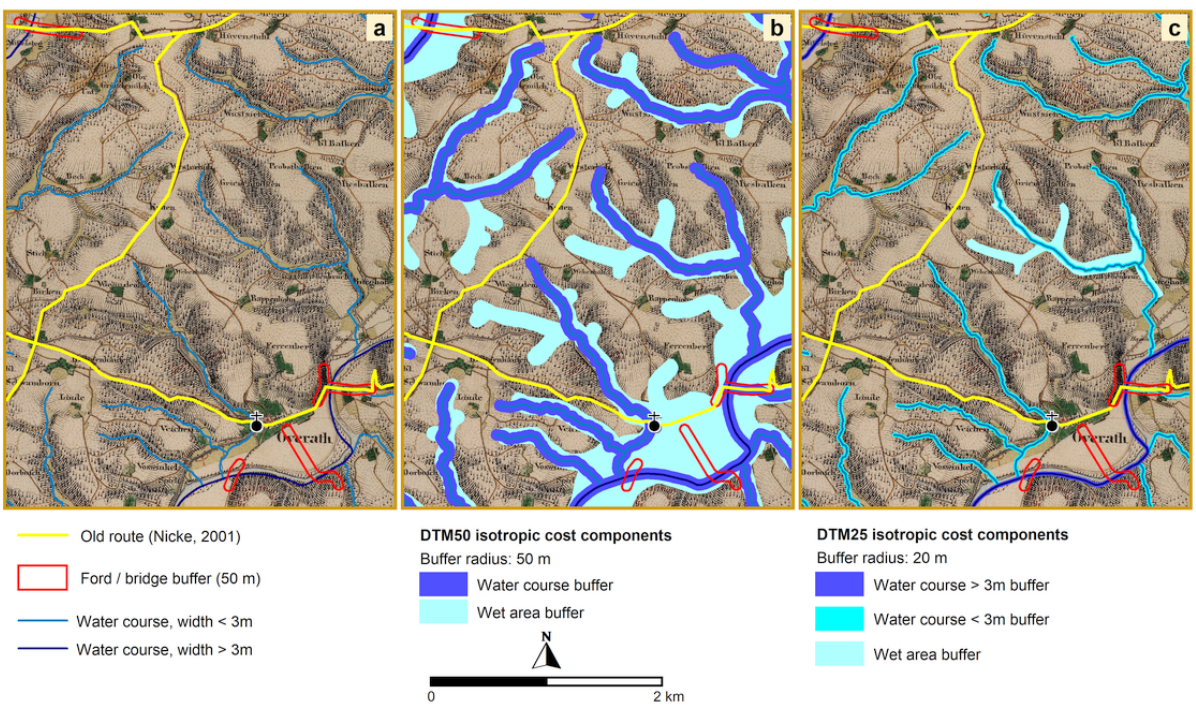

Figure 3. a: Water courses and ford or bridge locations. b: DTM50 – areas of increased isotropic costs. c: DTM25 – areas of increased isotropic costs. Background map: WMS based on rectified map sheets created around 1845 (© Geobasis NRW).

The soil type names were used as indicators of wet soils for the DTM50-based cost models. Updated soil geodata was combined with the DTM25, and wet areas were defined based on the wetness field available in this dataset.

The LCP algorithm requires converting the vector data of the water courses to raster format. To avoid corner-connected cells with high costs surrounded by cells of low friction [Conolly and Lake 2006: 216], buffers for linear features like the water courses were computed before converting them to raster format [Herzog 2013b]. The minimum buffer radius is 0.6 times the cell resolution. Buffers were also constructed for the wet zones derived from the soil map because these zones are often quite narrow.

Fig. 3 shows an area in the west of the study region traversed by several old trade routes. Comparing Fig. 3b with Fig. 3c suggests that the wet zones derived from soil type names in Fig. 3b cover too much ground whereas the wet zones surrounding some small creeks are not included in the wet zone buffers (defined using the wetness field in the newer digital soil map) shown in Fig. 3c. The water courses depicted on the 19th century map sometimes deviate from the modern water courses, but deviations rarely exceed 50 m.

Fig. 3b suggests that Overath is located in a wet area. A cost model assigning increased costs to the wet area buffer shown in Fig. 3b will most probably generate LCPs avoiding Overath. Some of the issues with the wet zones layer were overcome by recording possible ford or bridge corridors that are assigned lower costs. Evidence of the importance of fords for transport in past times are place names such as Wipperfürth that refers to a ford on the River Wipper. Merchants had to wait at fords in times of flooding [Nicke 2001a: 108–109]. According to Nicke [2001a: 15], the water course must run quickly at the ford position so that no sandbanks or gravel deposits are formed. The stream should be not too deep and crossing at meanders should be avoided as the riverbed might be deeper in some of these areas. In addition, the route should leave the muddy areas near the stream as soon as possible, ascending to the ridgeway again. Based on these criteria, predicting ford locations with the help of modern DEM data requires high resolution data. As most of the riverbeds were changed by natural and human forces in the course of time, it appeared more advisable to spot fords on old maps.

Therefore, 227 ford or bridge locations in the study region were digitized from the rectified maps that were created around 1845. To reduce the work load, digitalization focused on the major water courses with a width exceeding 3 m. To be more precise, short road sections traversing the water courses were digitized. As explained above, buffers for these lines were constructed and are labelled “Ford / bridge buffer” in Fig. 3. Due to inaccuracies of the rectified old maps and changes of the water courses, the digitized road sections sometimes were adjusted to fit with the modern topography.

Movement costs are computed by multiplying slope-dependent costs with isotropic cost factors, i.e., costs for traversing water courses as well as wet zones. Obviously, the isotropic cost factor for dry zones is 1, different sets of cost factors were investigated for raster cells within water course buffers, in wet areas, and in ford/bridge areas. For the DTM50, isotropic cost models are identified by names indicating the three cost factors used, e.g., Iso 4-2-1.5 means: factor 4 for water course areas, 2 for wet areas and 1.5 for ford/bridge areas. The DTM25 was combined with four isotropic cost factors, the water courses were subdivided into two width classes (greater than and less than 3 m). Consequently, the isotropic cost models are defined by four numbers, for instance 6-5-3-2. If ford or bridge locations are disregarded by a cost model, this is indicated by the letter x instead of the cost value.

Several functions estimating slope-dependent movement costs were selected for the tests that aim to identify the most appropriate cost models for reconstructing the Nicke routes. Costs of walking are mostly estimated in terms of energy consumption [Llobera and Sluckin 2007; Minetti et al. 2002] or time [Ericson and Goldstein 1980; Irmischer and Clarke 2017; Langmuir 2004; Tobler 1993]. Moreover, quadratic cost functions with a given critical slope (i.e., the least slope requiring hairpin turns for effectively moving up or down a gradient) were tested, that can be applied for modelling movement by wheeled vehicles (for details see Herzog [2020]). The latter cost functions are designated by Q(s) with s the critical slope. The best performing LCPs in previous articles dealing with trade routes in the study region were generated using Q(s), with s in the range of 10 to 13%, combined with isotropic costs for wet soils [Herzog 2013a; 2013b; 2013c].

The LCP algorithm used supports moves in 48 directions and takes the direction of movement on a slope into account [Herzog 2020]. The calculations assume that the trade flow intensity was similar in both directions. Consequently, new symmetric cost functions were generated that average the costs of uphill and downhill movement [Herzog 2013a; 2020].

4. TARGET SELECTION AND FIRST LCP RESULTS

For computing the LCPs, medieval settlements on or very close to a Nicke route were selected as targets. Preferably, settlements mentioned in the tax list for churches and chapels created in 1308 were chosen [Oediger 1967]. Consequently, the list of targets consists of nine medieval church or chapel locations, supplemented by another six medieval settlements. Locations where a Nicke route changes its general direction were also inserted in the list of targets. This applies for Lindlar, Hufenstuhl, and Gummersbach. The target pairs selected and the corresponding trade route names according to Nicke are listed in Table 1.

The target locations are marked by a star symbol in the map shown in Fig. 4. This map also depicts LCPs connecting the target pairs listed in Table 1. Each LCP was generated based on a cost model combining isotropic factors (Iso 5-3-2) with a slope-dependent cost function (slope derived from DTM50). Eight different LCP sets are shown, their colors correspond to the different normed, symmetric slope-dependent cost functions shown on the left in Fig. 4. The only exception is that instead of the cost function Q(12) (i.e., the function with a critical slope of 12%) two different vehicle functions (Q(10) – thick dark violet lines; Q(16) – thin dark violet lines) form the basis for the LCPs depicted in the map.

For long stretches, the LCPs based on different slope-dependent cost functions combined with the isotropic cost component Iso 5-3-2 agree. Exceptions are the LCPs generated from the cost function for male on-road walkers by Irmischer and Clarke [2017], these are often more direct than the other LCPs. For instance, this is the only LCP linking Halver with Lindlar that does not roughly coincide with the Polizeiweg north of Lindlar. The LCPs connecting Frankenforst in the west with Eckenhagen in the east of the study region run on the Brüderstraße until they meet the Zeitstraße north of Much. Unexpectedly, they continue partly on the Homburgische Eisenstraße. This is also an area where two different routes are suggested by the LCPs. Three of them (including Q(10) and Q(16)) continue in northern direction close to the Zeitstraße before turning and moving in eastern direction; all the other LCPs take a more direct route to the Homburgische Eisenstraße.

Some Nicke routes are reconstructed successfully by nearly all LCPs shown in Fig. 4, for instance the Nutscheid connecting Driesch with Erdingen in the south-east of the study region. Another example is the section of the Heerweg between Paffrath (at the western border of the study area) and Wipperfürth. But the Bergische Eisenstraße between Wipperfürth and Lennep in the northwest of the study region does not coincide with the LCPs for a long stretch west of Wipperfürth. The map section in Fig. 5a illustrates the reason for this issue: the old route traverses several water courses, whereas the black LCPs – based on different slope-dependent cost functions combined with the isotropic cost component Iso 5-3-2 – cross only one creek close to Wipperfürth. Nicke [2001a: 107] points out that this stretch of the route in the valley is an exception. The map created around 1845 also shows roads close to the LCPs, so the computed routes are realistic. But the more difficult route of the Bergische Eisenstraße was probably chosen because this allows an intermediate stop in the town of Hückeswagen. Hückeswagen, first mentioned in historical documents in 1085, achieved privileges comparable to towns around 1300 [Pampus 1998: 146]. Including a target in the town center of Hückeswagen generates improved LCPs for cost models combining Iso 5-3-2 with Q(12) or Q(14) (red LCPs in Fig. 5a). This exemplifies the importance of selecting all relevant intermediate stops for successful route reconstruction.

Table 1. Pairs of targets selected for LCP computations

Target 1	**Target 2**	**Name of the road in [Nicke 2001]**
Gummersbach	Wipperfürth	Bergische Eisenstraße
Lennep	Wipperfürth	Bergische Eisenstraße
Gummersbach	Belmicke	Bergische Eisenstraße, Alternativr. III
Erdingen	Frankenforst	Brüderstraße
Frankenforst	Eckenhagen	Brüderstraße + Oberberg. Diagonale
Halver	Handstrasse	Heerweg
Handstrasse	Wipperfürth	Heerweg
Frankenforst	Lindlar	Heidenstraße
Meinerzhagen	Lindlar	Heidenstraße
Meinerzhagen	Belmicke	Hileweg
Meinerzhagen	Halver	Hileweg
Erdingen	Driesch	Nutscheid
Halver	Lindlar	Polizeiweg
Lindlar	Wipperfürth	Polizeiweg
Lindlar	Hufenstuhl	Polizeiweg
Siegburg	Hufenstuhl	Polizeiweg
Halver	Much	Zeitstraße
Siegburg	Much	Zeitstraße
Halver	Gummersbach	Zeitstraße + Bergische Eisenstraße

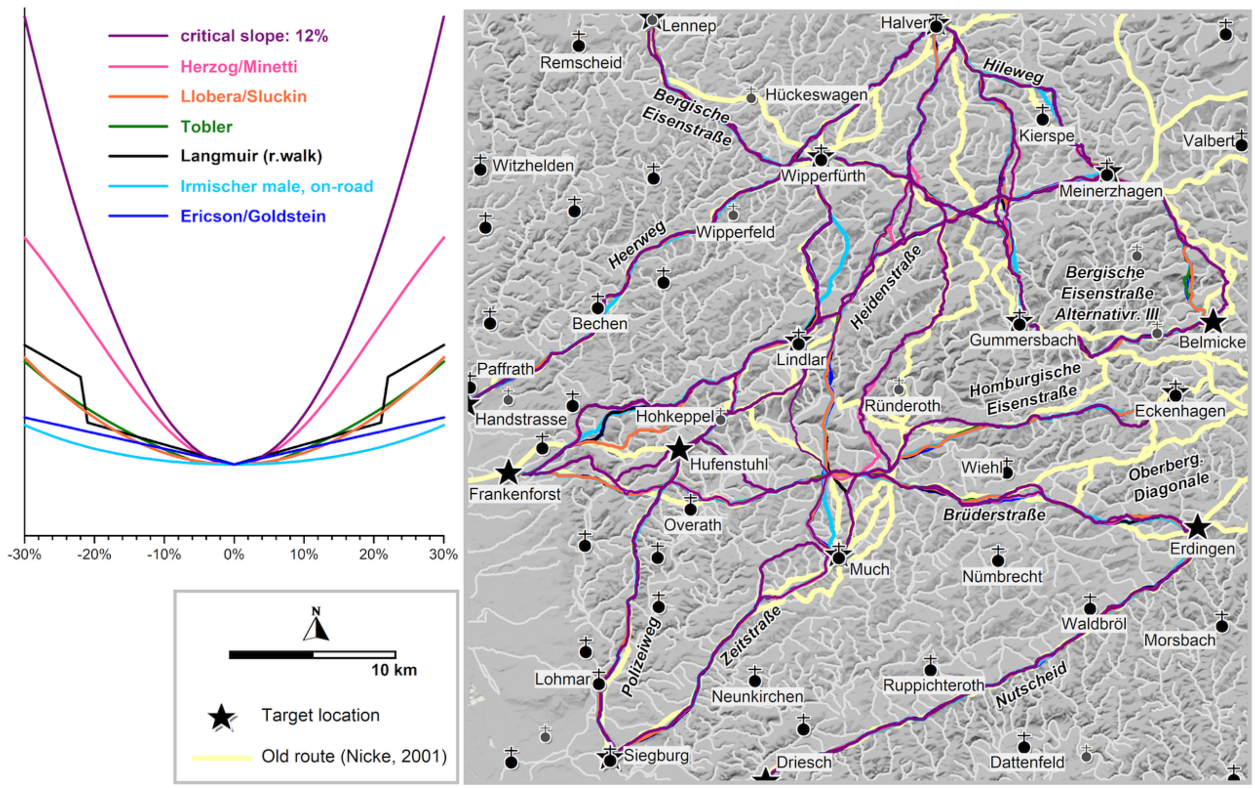

Figure 4. Left: selected normed slope-dependent cost functions. Right: LCPs connecting the target pairs listed in table 1; the cost model combines slope-dependent costs with penalties for traversing water courses, wet areas, and fords or bridges (Iso 5-3-2).

Unexpectedly, the LCPs connecting Handstrasse with Halver traverse the River Wupper west of Wipperfürth (Fig. 4), although no ford or bridge location is depicted in this area on the map created around 1845. The southern part of Fig. 5a shows that the old route Heerweg traversed two minor water courses before reaching the Wipper ford, whereas the LCPs avoid these creeks. This is the reason why an additional target pair connecting Handstraße with Wipperfürth was inserted in Table 1 and intermediate targets at Wipperfürth were introduced for the Bergische Eisenstraße and the Polizeiweg.

In Fig. 5b, the DTM25-based LCP set Q(14) Iso 1-1-1-x also reconstructs the old road section between Hückeswagen und Wipperfürth quite successfully although Hückeswagen was not selected as target. The cost model applied for these LCPs does not include any penalties for traversing water courses or wet areas, therefore, the LCPs run on water for some stretches. According to Nicke [2001a: 11], the necessity to construct roads in the wet areas arose when metalworking in the valleys became increasingly important requiring delivery by vehicles with heavy loads. Precursors of this industrial revolution in the 19th century have been recorded in historical documents. In the 15th century, a collection of bills mentions about 200 kg of copper that were provided by a citizen of Wipperfürth and processed in Hückeswagen [Jacobi 2010: 48]. This historical document suggests that metalworking started very early in this area, and this might be the reason why a different cost model for the road connecting Wipperfürth with Lennep is adequate.

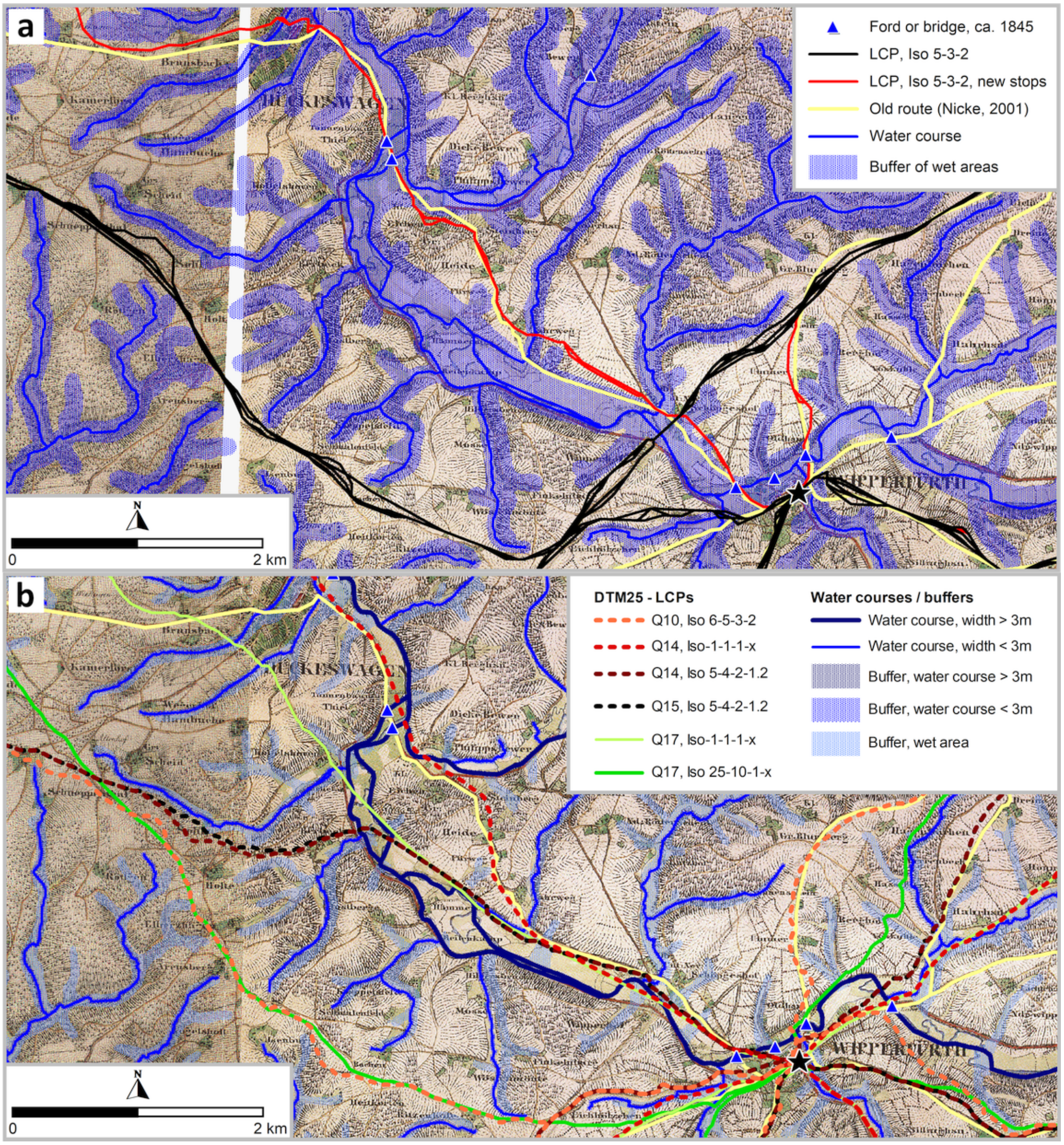

Figure 5. Comparison of LCPs and old routes in the Wipperfürth area. (a) LCPs based on the DTM50, (b) LCPs based on the DTM25. Background map: WMS based on rectified maps created around 1845, (a) in January 2020, (b) in October 2020 (© Geobasis NRW).

.

5. PERFORMANCE INDICATORS FOR COST MODELS

Beyond the comparison by visual inspection, the digitized old trade routes described in the books by Nicke can be used for evaluating the cost models quantitatively. If the aim of an LCP study is the reconstruction of a known road, the similarity between the LCP to the known route can be assessed by determining the proportion of the LCP that lies within a buffer distance from the known road [Goodchild and Hunter 2001]. This is applied to the trade route network by generating a buffer around all Nicke routes. Buffer radius values of 200 and 500 m were selected. A buffer radius of 200 m is suggested by the limited accuracy of the rectified map sheets created around 1845 used as a basis for road digitization (e.g., gaps between rectified sheets in Fig. 5a). Moreover, bundles of sunken roads found in forested parts of the study region indicate that many trade route sections were broad corridors rather than narrow tracks [Herzog 2017; Nicke 2000: 23, 34–36].

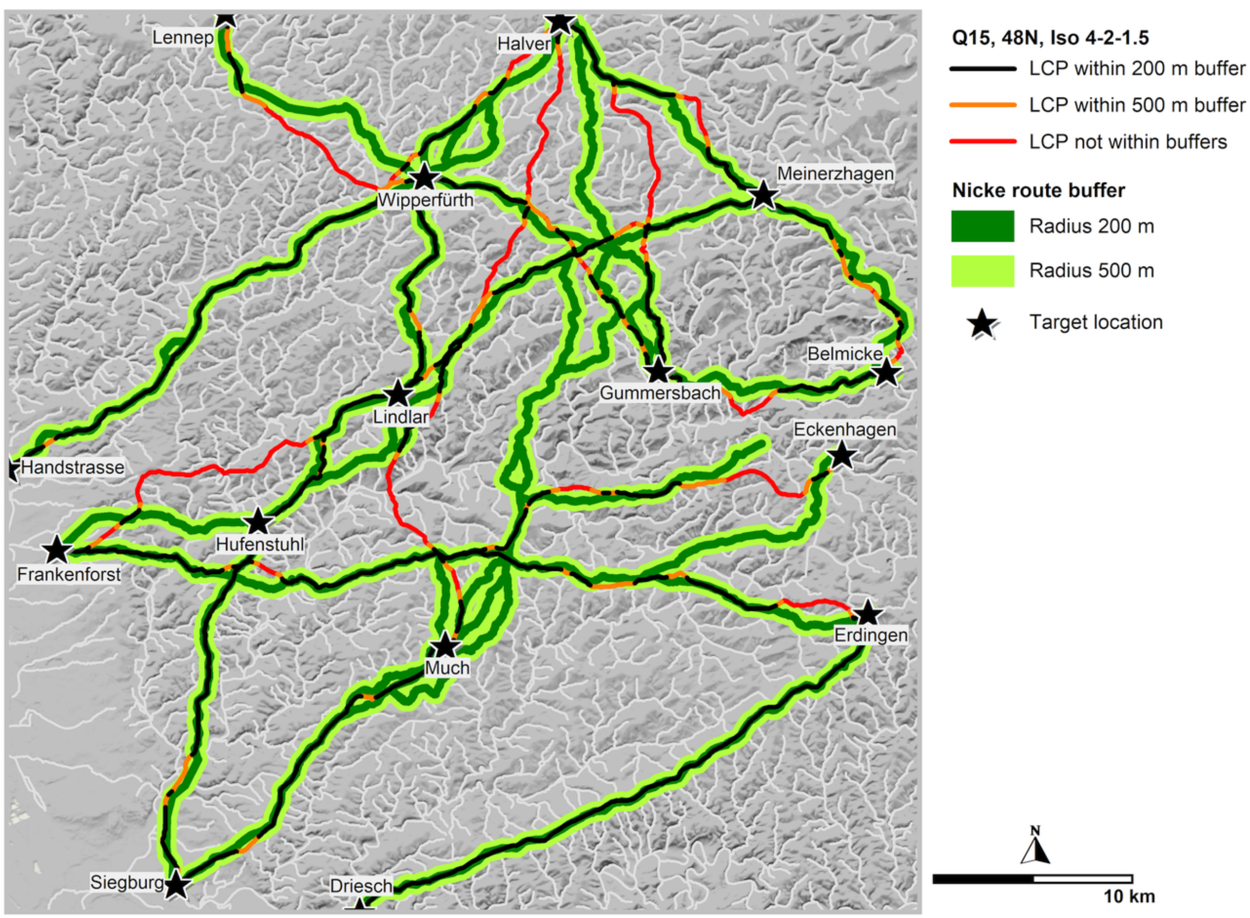

Figure 6. Assessing the performance of a set of LCPs generated by the cost model combining the DTM50-based slope-dependent cost function Q(15) with the isotropic cost component Iso 4-2-1.5.

For each cost model, the set of derived LCPs is analyzed, i.e., the accumulated length of the LCP stretches within the buffers is computed and used as performance indicator. This is illustrated in Fig. 6. This performance indicator might fail to provide an intuitive result if the total length of the LCPs within a set substantially deviates from that of the known route network considered. In general, the total length of the known routes digitized on the basis of historical maps at a scale of about 1:25,000 tends to be less than that of the true routes because of cartographic generalization. However, LCP networks with a total length considerably exceeding that of the digitized network, for instance by paths mainly consisting of small-scale turns, are not realistic in flat terrain. Therefore, the proportion of the accumulated length within each buffer is a more reliable performance indicator. The difference between the total length of the LCPs within a set and the corresponding routes in the known route network might be considered as secondary performance indicator. In the case study presented, this value is not computed because of the alternative routes described by Nicke (e.g., the Zeitstraße north of Much). The total length of the known network depends on the alternative route chosen.

5.1 Results for DTM50

Table 2 lists the DTM50-based LCP sets computed for the target pairs shown in Table 1 and their performance indicators. Four groups of LCP sets are evaluated in Table 2: (i) cost models combining different slope-dependent cost functions with isotropic cost component Iso 4-2-1.5, (ii) slope-dependent cost functions combined with Iso 5-3-2 (depicted in Fig. 4), (iii) cost models attributing higher costs to traversing water courses (Iso 8-3-2 and Iso 12-3-2), and (iv) best performing cost models without isotropic cost component. The cost models in the third group were tested because the initially computed LCPs (based on Iso 4-2-1.5 or Iso 5-3-2) connecting Wipperfürth with Lennep do not coincide with the Bergische Eisenstraße (Fig. 5). But the two LCP sets in group (iii) are clearly outperformed by the other LCP sets.

For the groups (i) and (ii), the best (yellow) and second-best (orange) performing cost models are highlighted in Table 2. The difference in performance between most of the models in groups (i) and (ii) is quite small. The ranking might change when including another old trade route section or omitting one of the target pairs. For the computations presented in the next section, the combination of the slope-dependent cost function Q(14) with Iso 5-3-2 is considered the best performing cost model.

By increasing the number of intermediate stops, the performance indicators of the cost models increase as well (as expected). For instance, by introducing four additional target locations and considering 23 instead of the 19 target pairs listed in table 1, the "buffer 200m %" performance indicator for the cost model combining Q(14) with Iso 5-3-2 reaches 72.55%, and the "buffer 500m %" value is 82.83%.

The last group in Table 2 (lines 25 zo 27) was added for better comparison with the DTM25-based results presented in the next section. Moreover, this group allows investigating the impact of the isotropic costs on the performance. Many archaeological LCP studies rely merely on slope-based costs [e.g., Bell and Lock 2000; Kantner 2012; Rademaker et al. 2012; Seifried and Gardner 2019;

Verhagen and Jeneson 2012], but in this case study, a substantial improvement in replication of Nicke routes is achieved by introducing costs for traversing water courses and wet areas.

Table 2. Performance indicators of the DTM50-based cost models investigated

	Iso 4-2-1.5	Buffer 200 m	Buffer 500 m	Length (m)	200 m %	500 m %
1	Ericson & Goldstein	287747	334006	435570	66.06	76.68
2	Langmuir	288325	333297	439554	65.59	75.83
3	Tobler	287394	319354	438655	65.52	72.80
4	Irmischer & Clarke	272999	324290	431260	63.30	75.20
5	Herzog/Minetti	298341	346197	455224	65.54	76.05
6	LLobera & Sluckin	286104	330698	436986	65.47	75.68
7	Q(10)	294943	338842	471001	62.62	71.94
8	Q(12)	291869	336669	462616	63.09	72.78
9	Q(13)	298656	341645	459949	64.93	74.28
10	Q(14)	289862	335473	455501	63.64	73.65
11	Q(15)	300911	345349	452622	66.48	76.30
	Iso 5-3-2	**Buffer 200 m**	**Buffer 500 m**	**Length (m)**	**200 m %**	**500 m %**
12	Ericson & Goldstein	286457	352256	441560	64.87	79.78
13	Langmuir	291307	357111	444079	65.60	80.42
14	Tobler	292903	354782	444429	65.91	79.83
15	Irmischer & Clarke	277227	344882	436024	63.58	79.10
16	Herzog/Minetti	307412	353064	460947	66.69	76.60
17	LLobera & Sluckin	290373	353057	442641	65.60	79.76
18	Q(10)	302402	371635	478185	63.24	77.72
19	Q(13)	300510	365021	465976	64.49	78.33
20	Q(14)	308373	353048	461102	66.88	76.57
21	Q(15)	305705	349490	459071	66.59	76.13
22	Q(16)	298116	364173	457571	65.15	79.59
23	Q(14), Iso 8-3-2	291387	357640	466346	62.48	76.69
24	Q(14), Iso 12-3-2	282481	345777	472491	59.79	73.18
25	Q(15), Iso 1-1-1	225184	299374	432204	52.10	69.27
26	Q(16), Iso 1-1-1	223142	297236	429129	52.00	69.26
27	Q(17), Iso 1-1-1	219080	296686	426595	51.36	69.55

5.2 Results for DTM25

As expected, the LCPs generated from the DTM25 perform better than those based on the DTM50 when ignoring isotropic costs (Table 2: lines 25 to 27; Table 3: lines 6 to 8). The best "buffer 200 m %" performance indicator found increases from 52.10 (DTM50, cost function: Q(15)) to 56.97 (DTM25, cost function: Q(18)).

Initially, it was assumed that the experiences from the DTM50 computations with respect to isotropic costs could be transferred to the DTM25-based LCP calculations. Therefore, the isotropic costs in the rows 1 to 5 of Table 3 were selected considering the best-performing cost models found in the experiments presented in the rows 1 to 22 of Table 2. But none of these initially investigated DTM25 cost models outperforms the best DTM50 cost models in terms of any of the two buffer proportions.

Table 3. Performance indicators of the DTM25-based cost models investigated

	DTM25, ETRS89	Buffer 200 m	Buffer 500 m	Length (m)	200 m %	500 m %
1	Iso 5-4-2-1.2, Tobler	256897	315862	401608	63.97	78.65
2	Iso 5-4-2-1.2, Margaria	254817	314481	400132	63.68	78.59
3	Iso 5-4-2-1.2, Q(14)	264796	333913	421983	62.75	79.13
4	Iso 6-5-4-2, Tobler	260790	324751	410443	63.54	79.12
5	Iso 8-5-3-2, Tobler	257806	323688	409208	63.00	79.10
6	Iso 1-1-1-x, Q(16)	229394	289021	407884	56.24	70.86
7	Iso 1-1-1-x, Q(17)	229886	289798	406111	56.61	71.36
8	Iso 1-1-1-x, Q(18)	229745	285169	403250	56.97	70.72
9	Iso 19-1-1-x, Q(17)	255648	313715	406739	62.85	77.13
10	Iso 20-1-1-x, Q(17)	255648	313715	406739	62.85	77.13
11	Iso 25-1-1-x, Q(17)	255686	313753	406777	62.86	77.13
12	Iso 25-10-1-x, Q(17)	272370	336574	414890	65.65	81.12
13	Iso 25-11-1-x, Q(17)	269556	332681	416147	64.77	79.94
14	Iso 25-10-3-1.7, Q(17)	271760	336376	422802	64.28	79.56
15	Iso 25-10-4-1.7, Q(17)	290672	351523	424743	68.43	82.76
16	Iso 25-10-5-1.7, Q(17)	285011	348127	425389	67.00	81.84

Due to this disappointing result, it was decided to proceed more systematically by refining the cost model iteratively. This systematic approach required the computation of LCP sets for 77 cost models with computation times ranging from 17 to 54 minutes. For each step, the results of the best performing LCP sets are presented in rows 6 to 16 of Table 3. In a first step, the best performing slope-dependent cost function was chosen, with no isotropic costs considered (rows 6 to 8 in Table 3). In the next steps, the performance of cost models using this slope-dependent cost function are investigated that also include isotropic costs; each step identifies the best performing isotropic cost component (i.e., major water courses, minor water courses, wet areas and bridge/ford locations), relying on the best performing cost model found in the previous step.

Row 15 of Table 3 presents the results for the best performing cost model found in this systematic approach. In terms of the "buffer 200 m %" performance indicator, this cost model outperforms the best model found in the initial phase (row 1) by 4.46 %. The difference in "buffer 200 m %" performance to the best performing DTM50-based cost model is 1.55 %, i.e., the best performance indicator is 68.43 compared to 66.88 achieved previously.

Differences between the DTM50 and DTM25 and the DTM-based best performing cost model parameters are shown in Table 4. The disparity in the critical slope values of the quadratic slope-dependent cost functions is reflected in the differences between the mean slope values. It is well-known that DTMs with low resolution are smoother than those with a smaller cell size [de Smith et al. 2007; Herzog and Posluschny 2011]. Consequently, the mean slope of DTM50 is lower than that of DTM25.

Table 4. Comparison of the DTMs and the best performing cost model parameters

	Mean slope	**LCP critical slope**	**Water course weight**	**Buffer width**
DTM25	15.3%	17%	25 / 10	20
DTM50	11.4%	14%	5	50
Difference	3.9%	3%	20 / 5	x 2.5

With hindsight, it is quite obvious that water course and wet area weights depend on the size of the buffer width chosen to avoid corner-connected cells. Consequently, for the known old routes in the study area, the results in Table 4 suggest that the outcomes derived from the DTM50 data set are replicable using the DTM25 data base when considering the impact of the change of average slope and buffer width.

Due to the fairly small increase in performance by applying the newer topographic data but substantial increase in computation times, the next section will use the results based on the DTM50.

6. ASSESSING THE STABILITY OF THE LCP RESULTS

The aim of Fig. 7 is to explore the stability of the LCP outcomes by a heat map highlighting coinciding LCPs. Similar approaches were proposed for identifying corridors of movement [Verhagen 2013]. A grid with a cell size of 200 m was created and for each cell, the number of LCPs passing this cell was counted for all LCPs generated by the cost models listed in rows 12 to 22 in Table 2.

The visualization of the counts quantitatively confirms the observations with respect to coinciding LCPs mentioned when discussing Fig. 4. However, the LCPs also coincide nearly perfectly south of Halver, but a corresponding old route is neither found in the descriptions by Nicke nor on the WMS map set created in about 1845 or on more recent maps. This observation shows that coinciding LCPs are not a reliable indicator of successful route reconstruction.

Marienheide (cross symbol in Fig. 7) south of Halver is an important hub in the Nicke route network. This settlement was first mentioned in 1417 (Pampus 1998: 183) in a document referring to the construction of a new monastery at this place. Fig. 7 suggests that Marienheide is situated close to a natural location for a trade route hub.

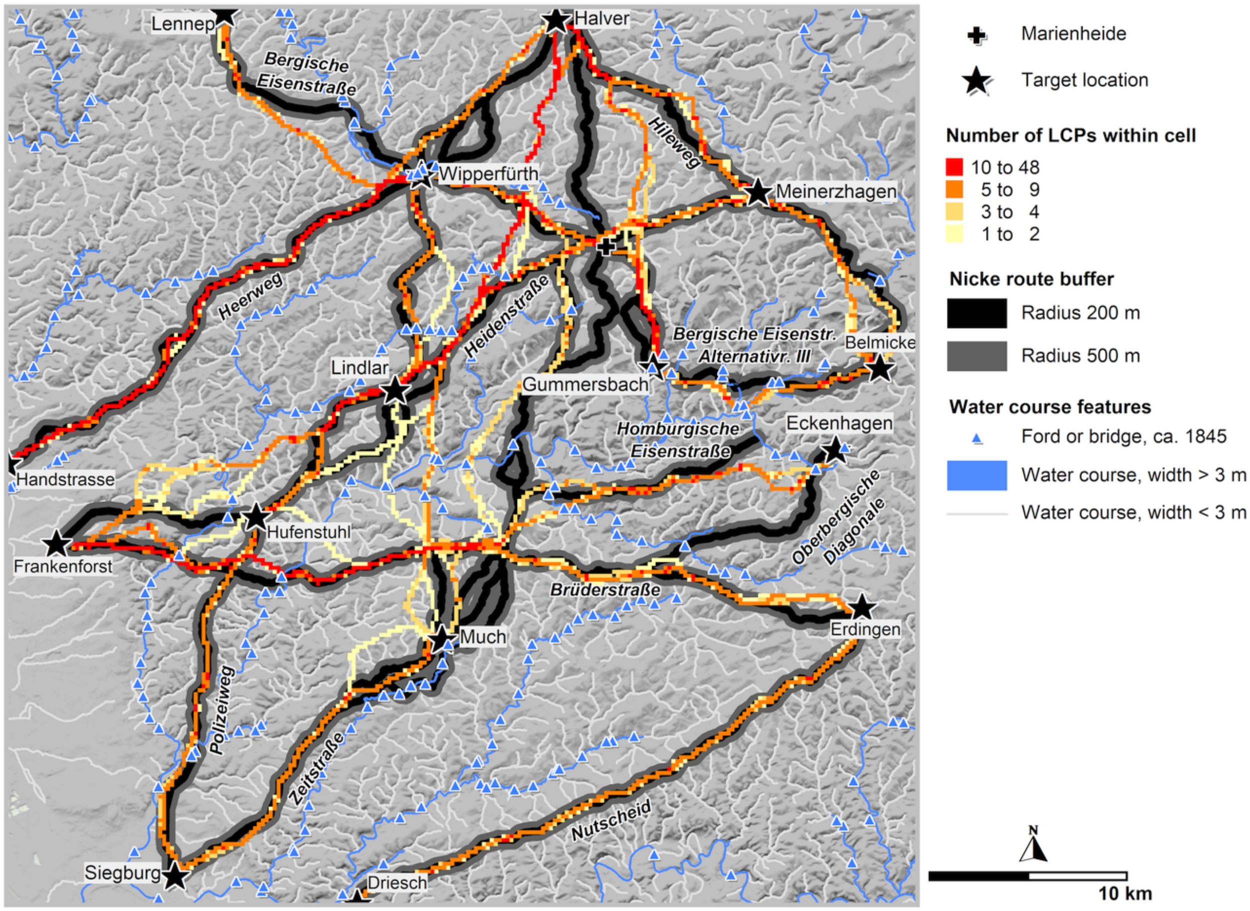

Figure 7. Coinciding LCPs generated from a set of cost models based on DTM50 combining different slope-dependent cost functions with the Iso 5-3-2 weight assignments for isotropic costs.

Moreover, Fig. 7 illustrates that a good model fit for one part of a known route does not necessarily mean that the model will provide reliable LCP results for another route section or another old road in the same region. This is also illustrated by column M5 in Table 5.

The values of the "200 m % buffer" performance for the individual road reconstructions, when applying the best-performing DTM25-based cost model, vary between 25 and 100%. Table 5 was created to analyze the observation that the improvements in performance reached by successively introducing refinements in isotropic costs were not as substantial as expected. Visual inspection of the optimal LCP sets identified in two successive steps suggested larger differences than those recorded in the "200 m %" column in Table 3. Table 5 shows the performance for each step and each target pair considered, starting with the simplest cost model M1. The table illustrates that in each step, the impact of improvements for some LCPs is reduced by poorer performance of some other

LCPs. For instance, optimizing the "200 m %" performance by refining the cost model Iso 25-1-1-X, Q(17) (M2) with respect to costs for small water courses (M3) results in a gain of more than 5 % for eight LCPs, but also in a loss of more than 5 % for another six LCPs. In Table 5, the largest improvement (39 %) is recorded for the LCP linking Lindlar and Wipperfürth (row 14). The biggest losers are the LCPs connecting Meinerzhagen and Belmicke (28 %) as well as Halver and Handstrasse (27%).

Table 5. "200 m % buffer" performance of the individual road reconstructions, based on DTM25: M1 = Iso 1-1-1-1, Q(17); M2 = Iso 25-1-1-x, Q(17); M3 = Iso 25-10-1-x, Q(17); M4 = Iso 25-10-1-1.7, Q(17); M5 = Iso 25-10-4-1.7, Q(17)

	Target 1	Target 2	km	M1	M2	M3	M4	M5
1	Gummersbach	Wipperfürth	15.2	40	40	75	75	81
2	Lennep	Wipperfürth	12.9	29	13	38	58	25
3	Gummersbach	Belmicke	11.4	80	90	92	60	57
4	Erdingen	Frankenforst	40.8	68	68	59	69	71
5	Frankenforst	Eckenhagen	39.7	34	34	55	61	67
6	Halver	Handstrasse	35.7	43	61	34	49	50
7	Handstrasse	Wipperfürth	25.5	92	92	91	91	91
8	Frankenforst	Lindlar	18.8	17	21	34	41	38
9	Meinerzhagen	Lindlar	20.9	26	93	88	88	82
10	Meinerzhagen	Belmicke	10.8	79	79	51	51	51
11	Meinerzhagen	Halver	13.4	58	58	90	90	73
12	Erdingen	Driesch	29.6	100	100	100	100	100
13	Halver	Lindlar	20.4	25	46	29	29	70
14	Lindlar	Wipperfürth	11.0	46	47	86	65	89
15	Lindlar	Hufenstuhl	9.5	54	54	51	51	87
16	Siegburg	Hufenstuhl	18.7	84	84	91	97	94
17	Halver	Much	31.9	45	56	45	23	26
18	Siegburg	Much	18.0	98	98	90	90	93
19	Halver	Gummersbach	18.3	45	45	53	53	53
Weighted sum of best performing LCPs				2.2	4.7	3.0	4.0	6.5

For each pair of targets, the best performing cost model found in the systematic approach is highlighted by a yellow background in Table 5. In case of ties, all relevant cost models are highlighted. For instance, the connection in row 12 reaches optimal performance for all cost models considered, resulting in yellow highlights for models M1 to M5. Surprisingly, M5 is highlighted for only eight out of the 19 routes to be reconstructed. The weighted sum in the last row of Table 5 shows another key figure for assessing the performance of models M1 to M5. The number n of optimal models for a given route induces a weight of 1/n. For instance, the contribution (weight) of the route no. 7 to the weighted sums of M1 and M2 is 0.5 and 0.0 for M3 to M5. Contrary to intuition, M2 is the second-best cost model in terms of this key figure, and M4 ranks third. The performance of four of the M5 LCPs does not exceed 50 % (light blue cells in Table 5). These are four out of eleven connections that can be reconstructed more successfully by another cost model.

The straight-line distance between the target pair to be connected is given in the "km" column of Table 5. It seems that this distance has only minor impact on the success of the reconstruction by LCPs. For instance, the performance of the M5-derived LCP connecting Erdingen with Frankenforst (the longest straight-line distance; no. 4 in Table 5) is 71; the performance of the LCP connecting Halver with Lindlar (no. 13 in Table 5) is nearly the same, though only half of the straight-line distance between Erdingen and Frankenforst is covered. This is an issue that needs further investigation.

Different cost models may be appropriate for different purposes of the road, for instance roads for transporting heavy loads such as iron ore may differ from those used for driving cattle to the market in Cologne. Moreover, misinterpretations of Nicke's descriptions and errors by Nicke cannot be ruled out. This issue needs further investigation, as the subsequent discussion of the connection no. 8 in Table 5 (Heidenstraße between Frankenforst and Lindlar, with waypoints in Hufenstuhl and Hohkeppel, cf. Fig. 4) shows.

This route is reconstructed least successfully, i.e., the performance of the best reconstruction in terms of "200 m %" is merely 41 %. Some evidence is available that the description of the route by Nicke refers to a local connection or to one out of several possibilities. The maps by Ploennies [1715, edited by Dietz in 1988] covering this area show an alternative road linking Frankenforst with Hufenstuhl. The route depicted on a map from 1805 attributed to Eversmann [Nicke 2001b: 9] seems to coincide quite well with the Ploennies route. These old maps and most LCPs suggest that the best way from Frankenforst to Hufenstuhl is to use the Brüderstraße for the first part until the river Sülz is traversed. Only if the settlement of Immekeppel (first mentioned in historical sources in 1166) about 2.6 km west of Hufenstuhl is included as an additional target, the LCPs run closer to the route described by Nicke. According to Nicke [2001b: 8], the earliest historical map depicting a road between Bensberg (2.6 km north-east of Frankenforst) and Immekeppel was created by Wiebeking in 1790/1792. This is a secondary road on the Wiebeking map compared to the Brüderstraße also shown on this map (no. 4 in Table 5). The Heerweg running parallel north of the Heidenstraße (no. 7 in Table 5; cf. Fig. 4) is reconstructed more successfully. Both the Heidenstraße and the Heerweg originate in Cologne west of the study area. In 1663 a group of people with 12 horses and two carts travelled from Cologne to Valbert east of Meinerzhagen and rested for the night in Wipperfürth. This indicates that they first proceeded on the Heerweg, switching to the Bergische Eisenstraße in Wipperfürth (no. 1 in Table 5) until this route joins the Heidenstraße [Nicke 2001b: 15-16]. The western part of the Heidenstraße (Cologne to Hufenstuhl) was not considered a main route by these travelers. For the reasons outlined above, the road between Frankenforst and Lindlar described by Nicke might have been of local importance only. This might be the reason for the unsatisfying LCP results for this part of the Heidenstraße.

Some of the unsuccessful reconstructions are due to modern landscape changes. For instance, east of Frankenforst, some stretches of the LCPs to Eckenhagen take the motorway A4.

7. DISCUSSION, CONCLUSIONS AND FUTURE WORK

This article investigated systematically the impact on LCP results of choosing different slope-dependent cost-functions combined with different weights for isotropic costs related to the movement on wet areas and traversing water courses. Some of the known road sections considered are reconstructed by most of the cost models presented in this article. This applies for instance to the Nutscheid and sections of the Brüderstraße. Both roads are depicted on a map created in 1575 [Mercator 1575]. It seems that most Nicke routes that are also found on the maps predating the 19th century can be reconstructed more successfully than the other roads. The only exception are parts of the Polizeiweg.

A more systematic comparison of the digitized Nicke routes with routes derived from old maps is a task for the future. Moreover, errors in interpreting Nicke's descriptions cannot be ruled out. Another digitization of the Nicke routes by a colleague is available; currently the differences exceeding 200 m are checked.

The investigations presented are based on the hypothesis that small changes in the cost model will give rise to small changes in the performance. The outcomes presented in Table 3 support this hypothesis. This result does not agree with the observation by Kantner [2012] that "cost-path studies are extremely sensitive to small changes in variables".

These lessons have been learnt, some of them need further investigation:

- Target selection is an issue.
- In the study area, the distance between two targets has only minor impact on the success of the trade route replications by LCPs.
- Tables 2 and 3 clearly illustrate that appropriate cost selection is important for successfully reconstructing routes in the past.
- A good model fit for one part of a known route does not necessarily imply that the model will provide reliable LCP results for another route section or another old road in the same region. This is exemplified in Table 5: For cost model M2, the replicability of the known trade routes in terms of the "200 m % buffer" criterion varies between 13 and 100%.
- Different cost models might be appropriate for roads of different purposes. Therefore, a cost model improving the overall performance for a set of roads may result in less similar LCPs for some roads in the set.
- Coinciding LCPs with different slope-dependent cost functions are not a reliable indicator of successful route reconstruction.
- Using a DEM with a different resolution changes the average slope value and therefore adjusting the slope-dependent cost function might be required for optimal cost model performance. Similarly, the buffer width for water courses is related to the optimal weight for the corresponding isotropic cost component (Table 4).
- By using a DEM with higher resolution, the performance did not improve as much as expected. Introducing different weights for water courses with width exceeding 3 m and searching

systematically for the best performing cost model mainly resulted in a substantial increase in computational load, whereas only minor improvements were achieved.

High resolution DEMs clearly show modern landscape modifications due to natural and anthropogenic processes such as modern roads or World War II bomb explosion craters. This may have a major impact on the LCP results. Larger landscape modifications by quarries, mining activities or large dams can also be found in the study region. Ideally, paleogeographic reconstruction of the landscape should be included in any study using LCP approaches.

An algorithm for investigating the impact of the DEM error on the LCP outcome has been proposed by Lewis [2021]. The DEM used in his case study has a resolution of 50 m and 4 m RMSE. The error of the DEMs used in the case study presented is lower, when considering the modern surface. However, the landscape modifications in the past often exceeded 4 m. Of course, the modern man-made landscape features are not as nicely distributed as measurement errors. Therefore, the approach by Lewis might underestimate the variation of the LCP outcomes. Moreover, this approach involves a high computational load. Alternatively but with similar requirements of computation times, a normally distributed random error might be added to each cost value computed, with the standard deviation parameter depending on slope because steep slope measurements tend to be less reliable.

In the case study, another issue are the wet soil areas derived from the geological data. The DTM50 data set includes wet areas that extend into medieval settlements, which does not seem realistic. Some of the creeks shown in the 19th century maps are surrounded by wet areas according to the DTM50 data set, but corresponding wet areas are missing quite often in the DTM25 data set. A new layer combining the two wet area layers might produce improved LCP results. Flow accumulation algorithms implemented in the Sextante toolbox of the GIS software gvSIG have been applied alternatively in the past for identifying wet areas. This did not work reliably, but sometimes, useful results were achieved that seemed more consistent than the geological layers when considering small water courses. Moreover, only the water courses with a width exceeding 3 m were considered when digitizing possible ford or bridge locations, which seems reasonable at first sight. But in the study area with several huge water reservoirs, the modern width of a water course might differ substantially from the width in medieval times. Modifications of the water courses can be deduced from the 19th century maps, but this is complicated by map inaccuracies. For part of the study area investigated, the water courses depicted on the maps dating back to about 1845 were digitized. It is planned to check if cost models on this basis outperform those listed in Tables 2 and 3.

A drawback of LCP analysis is the fact that only one solution is presented although several LCPs may accumulate identical or nearly identical costs. This disadvantage is overcome by the method applied by Seifried and Gardner [2019]. They use corridor analysis that adds the accumulative cost grids for starting and target locations, resulting in a probability map with branches of the corridors highlighting alternative path options. The maximum width of single branch corridors may be used for assessing the stability of the LCP results as well as the number of branches.

This article has shown that assessing the stability of LCP results has many facets, such as choosing targets and cost models. Focusing on only one aspect such as DEM accuracy does not provide the full picture.

8. ACKNOWLEDGEMENTS

Part of this article was presented at the Landscape Archaeology Conference in Newcastle upon Tyne in September, 2018. Several aspects of this article were also included in an invited talk given at the "Graphdatenbanken, GIS und 3D-Modelle in der Bauforschung des Mittelalters" conference in Heidelberg, in December 2019 and an invited online talk at the Computational and Digital Archaeology Lab (CDAL) at the University of Cambridge in May 2021. Sincere thanks to two anonymous reviewers whose suggestions have been very helpful.

9. REFERENCES

Norbert Andernach. 1984. Entwicklung der Grafschaft Berg. In Guido de Wird, ed. *Land im Mittelpunkt der Mächte: Die Herzogtümer Jülich, Kleve, Berg*. Kleve: Boss-Verlag, 63–73.

Tyler Bell and Gary Lock. 2000. Topographic and cultural influences on walking the Ridgeway in later prehistoric times. In Gary Lock, ed. *Beyond the Map: Archaeology and Spatial Technologies*. Amsterdam: IOS Press, 85–100.

James Conolly and Mark Lake. 2006. *Geographical Information Systems in Archaeology*, Cambridge Manuals in Archaeology, Cambridge: Cambridge University Press.

Burkhard Dietz ed. 1988. Erich Philipp Ploennies, Topographia Ducatus Montani (1715), Bergische Forschungen XX, Neustadt/Aisch: Schmidt.

Jonathon E. Ericson and R. Goldstein. 1980. Work Space: A New Approach to the Analysis of Energy Expenditure within Site Catchments. *Anthropology UCLA 10*, 1&2, 21–30.

Rupert Gietl, Michael Doneus, and Martin Fera. 2008. Cost Distance Analysis in an Alpine Environment. Comparison of Different Cost-surface Modules. In Axel Posluschny, Karsten Lambers & Irmela Herzog, eds. *Layers of Perception. Proceedings of the 35th International Conference on Computer Applications and Quantitative Methods in Archaeology (CAA). Berlin, Germany, April 2-6, 2007*. Kolloquien zur Vor- und Frühgeschichte, 10, (p. 342, full paper on CD). Bonn: Rudolf Habelt.

Michael F. Goodchild and Gary J. Hunter. 2001. A simple positional accuracy measure for linear features. *Int. J. Geographical Information Science* 11, 3 (1997, received 01 Sep 2001, published online 06 Aug 2010), 299–306. https://doi.org/10.1080/136588197242419

Alejandro Güimil-Fariña and César Parcero-Oubiña. 2015. "Dotting the joins": a non-reconstructive use of Least Cost Paths to approach ancient roads. The case of the Roman roads in the NW Iberian Peninsula. *J. Arch. Science* 54, 31–44. https://doi.org/10.1016/j.jas.2014.11.030

Trevor Harris. 2000. Moving GIS: exploring movements within prehistoric cultural landscapes using GIS. In Gary Lock ed. *Beyond the Map: Archaeology and Spatial Technologies*. NATO Science Series, Series A 321. Amsterdam: IOS Press, 116–123.

Irmela Herzog. 2013a. Theory and Practice of Cost Functions. In Francisco Contreras, Mercedes Farjas & Francisco Javier Melero, eds. Fusion of Cultures: Proc. 38th Annual Conference on CAA, Granada, Spain, April 2010. Oxford: Archaeopress, 375–382.

Irmela Herzog. 2013b. The Potential and Limits of Optimal Path Analysis. In Andrew Bevan & Mark Lake, eds. *Computational Approaches to Archaeological Spaces*. Walnut Creek: Left Coast Press, 179–211.

Irmela Herzog. 2013c. Least-cost networks. In Graeme Earl, Timothy Sly, Angeliki Chrysanthi, Patricia Murrieta Flores, Constantinos Papadopoulos, Iza Romanowska & David Wheatley, eds. *Archaeology in the Digital Era. Papers from the 40th Annual Conference of Computer Applications and Quantitative Methods in Archaeology (CAA), Southampton, 26-29 March 2012*, Vol. I. Amsterdam: Amsterdam University Press, 237–248.

Irmela Herzog. 2014. Noch einmal: Mittelalterliche Orte und Altstraßen im Bergischen Land. *Archäologie im Rheinland 2013*, 37–39.

Irmela Herzog. 2017. Reconstructing Pre-Industrial Long Distance Roads in a Hilly Region in Germany, Based on Historical and Archaeological Data. *Studies in Digital Heritage* 1(2), 642–660. https://doi.org/10.14434/sdh.v1i2.23283

Irmela Herzog. 2020. Spatial analysis based on cost functions. In Mark Gillings, Piraye Hacigüzeller, & Gary Lock, eds. *Archaeological Spatial Analysis: A Methodological Guide to GIS* . London and New York: Routledge, Taylor & Francis Group, 333–358.

Irmela Herzog and Axel Posluschny. 2011. Tilt – Slope-Dependent Least Cost Path Calculations Revisited. In Erzsebet Jerem, Ferenc Redő & Vajk Szeverényi, eds. *On the Road to Reconstructing the Past. Computer Applications and Quantitative Methods in Archaeology (CAA). Proceedings of the 36th International Conference. Budapest, April 2-6, 2008*. Archeaeolingua: Budapest, 212–218.

Irmela Herzog and Claus Weber. 2021. Data Structures for Major Archaeological Projects in the Rhineland Area, Germany. In Museen der Stadt Wien, Stadtarchäologie, eds. *Monumental Computations: Digital archaeology of large urban and underground infrastructures*, Heidelberg: Propylaeum, 2021 (Proceedings of the International Conference on Cultural Heritage and New Technologies, Vienna, Band 24), 29–32.
https://doi.org/10.11588/propylaeum.747.c11739

Ian J. Irmischer and Keith C. Clarke. 2017. Measuring and modeling the speed of human navigation. *Cartography and Geographic Information Science* 45(2), 177–186. https://doi.org/10.1080/15230406.2017.1292150

Günter Jacobi. 2010. 356 Rechnungsbelege des 15. Jahrhunderts aus der Kellnerei des Amtes Steinbach. *Schriftenreihe des Bergischen Geschichtsvereins Rhein-Berg e.V.* 62. Bergisch Gladbach.

Wilhelm Janssen. 2014. Das Bergische Land im Mittelalter. In Stefan Gorißen, Horst Sassin & Kurt Wesoly, eds., *Geschichte des Bergischen Landes 1: Bis zum Ende des alten Herzogtums 1806*. Bielefeld: Verlag für Regionalgeschichte, 24–139.

John Kantner. 2012. Realism, Reality, and Routes. In Devin A. White & Surface-Evans White, D. & Sarah L. Surface-Evans, eds*. Least Cost Analysis of Social Landscapes: Archaeological Case Studies*. Salt Lake City: The University of Utah Press, 225–238.

Eric Langmuir. 2004. *Mountaincraft and Leadership*, Revised Third Edition. Cordee, UK: Mountain Leader Training England & Mountain Leader Training Scotland.

Joseph Lewis. 2021. Probabilistic Modelling for Incorporating Uncertainty in Least Cost Path Results: a Postdictive Roman Road Case Study. *J. Arch. Method and Theory* 28, 911–924. https://doi.org/10.1007/s10816-021-09522-w

Marco Llobera and T. J. Sluckin. 2007. Zigzagging: Theoretical insights on climbing strategies, *J. Theoretical Biology* 249, 206–217.

Arnold Mercator 1575. Grundtliche Beschreibungh vnd Gelegenheit etlicher warer Grenntzen dem Bergischen Ampt Windeck vnd Herschafft Hombergh betreffend. Map copied by Hans Weirich in 1995. Published in 1999, along with 2 pages describing the history of the map by Hans Weirich, Lothar Wirths, and Klaus Pampus: Grenzen des Bergischen Amtes Windeck und der Herrschaft Homburg. Bergischer Geschichtsverein, Abt. Oberberg e.V.

Alberto E. Minetti, Christian Moia, Giulio S. Roi, Davide Susta & Guido Ferreti. 2002. Energy cost of walking and running at extreme uphill and downhill slopes. *J. Applied Physiology* 93, 1039–1046.

Herbert Nicke. 2000. Die Brüderstraße: Aus der Geschichte der alten Landstraße von Köln nach Siegen, Wiehl: Martina Galunder-Verlag.

Herbert Nicke. 2001a. Vergessene Wege: Das historische Fernwegenetz zwischen Rhein, Weser, Hellweg und Westerwald, seine Schutzanlagen und Knotenpunkte, Nümbrecht: Martina Galunder-Verlag.

Herbert Nicke. 2001b. Die Heidenstraße: Geschichte und Landschaft entlang der historischen Landstraße von Köln nach Kassel, Wiehl: Martina Galunder-Verlag.

Friedrich.Wilhelm Oediger. 1967. *Die Erzdiözese Köln um 1300. Erstes Heft: Der Liber Valoris*. Publikationen der Gesellschaft für rheinische Geschichtskunde, XII, 9(1), Bonn: Peter Hanstein-Verlag.

Klaus Pampus 1998. *Urkundliche Erstnennungen oberbergischer Orte*. Beiträge zur Oberbergischen Geschichte. Sonderband 1, Gummersbach: Oberbergische Abt. 1924 e.V. des Bergischen Geschichtsvereins.

Kurt Rademaker, David A. Reid, and Gordon R. M. Bromley. 2012. Connecting the Dots. In Devin A. White & Surface-Evans White, D. & Sarah L. Surface-Evans, eds. *Least Cost Analysis of Social Landscapes: Archaeological Case Studies*. Salt Lake City: The University of Utah Press, 32–45.

Jakob Rom, Florian Haas, Manuel Stark, Fabian Dremel, Michael Becht, Karin Kopetzky, Christoph Schwall, Michael Wimmer, Norbert Pfeifer, Mahmoud Mardini, and Hermann Genz. 2020. Between Land and Sea: An Airborne LiDAR Field Survey to Detect Ancient Sites in the Chekka Region/Lebanon Using Spatial Analyses. *De Gruyter Open Access* | Online published: October 15, 2020. https://doi.org/10.1515/opar-2020-0113

Rebecca M. Seifried and Chelsea A. M. Gardner. 2019. Reconstructing historical journeys with least-cost analysis: Colonel William Leake in the Mani Peninsula, Greece. *J. Arch. Sc. Reports* 24, 391–411.

Michael John De Smith, Michael F. Goodchild, and Paul Longley. 2007. *Geospatial Analysis. A Comprehensive Guide to Principles, Techniques and Software Tools*, Leicester: Troubadour Publishing.

Waldo Tobler. 1993. Non-isotropic geographic modeling. Technical Report No. 93-1. Retrieved October 30, 2021 from https://cloudfront.escholarship.org/dist/prd/content/qt05r820mz/qt05r820mz.pdf

Philip Verhagen. 2013. On the Road to Nowhere? Least Cost Paths, Accessibility and the Predictive Modelling Perspective. In Francisco Contreras, Mercedes Farjas & Francisco Javier Melero, eds. *Fusion of Cultures. Proc. 38th Annual Conference on CAA, Granada, Spain, April 2010*. Oxford: Archaeopress, 383–390.

Philip Verhagen and Karen Jeneson. 2012. A Roman Puzzle: Trying to Find the Via Belgica with GIS. In Angeliki Chrysanthi, Patricia Murrieta Flores & Constantinos Papadopoulus, eds. *Thinking*

Beyond the Tool: Archaeological Computing and the Interpretive Process. Oxford: Archaeopress, 123–130.

Received November 2021; revised March 2022; accepted March 2022.