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The Effect of Problem-Based Learning on Middle School Students' Problem-Solving Skills: The Content Area of Probability

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ABSTRACT

This study investigated the effect of the problem-based learning (PBL) approach on the problem-solving skills of middle school students as they learned about probability, a content area of eighth-grade curriculum. The research was carried out in the 2018–2019 academic year. The study group consisted of 39 eighth-grade students (14 female and 25 male students) studying at a middle school located in a province in the northern part of the Black Sea Region of Türkiye. Data collection tools included the equivalence test (ET), the problem-solving test (PST), the Semi-Structured Interview Form (SSIF), and scenarios and activities prepared for the experimental group (EG). Statistical Package for the Social Sciences (SPSS) 22.0 statistical software was used for data analysis. Analyses were performed using the Wilcoxon signed-rank test for paired samples and the Mann-Whitney U Test. The results showed that the problem-solving skills of the EG students, who were taught using the PBL approach, were better than those of the control group (CG) students, who were taught using traditional expository teaching. Therefore, in mathematics teaching, PBL is the recommended approach to support students' problem-solving process in the mathematics curriculum.

Keywords: *probability, problem-based learning (PBL), problem-solving, problem-solving skills, scenario, middle school students*

One theory that holds significant importance in mathematics education is constructivism, wherein the student is central and the teacher acts as a facilitator and guide. Constructivist theory, which posits that knowledge is not passively acquired but is obtained through the active participation of the individual and the comprehension of the subject (von Glasersfeld, 1995), has become increasingly significant in recent years. According to the theory, learning is defined as "the process of constructing meaning through the relationship between previous and new knowledge and experience" (Alesandrini & Larson, 2002, p. 118). At the core of the theory are the concepts of learning to learn, transferring gained knowledge to different situations, creating solutions, and socializing. In a classroom setting that employs this theory, students become "active constructors of their own knowledge" (Wood et al., 1991, p. 590). Contrary to the traditional theories in which the teacher teaches in front of the class and the students are predominantly passive listeners, constructivist classroom settings engage students in activities that involve solving problems designed to develop the required knowledge and skills (Brooks & Brooks, 1993). One of the prominent approaches in constructivist theory is the problem-based learning (PBL) approach.

Literature Review

About PBL

PBL is a teaching approach based on the principle of using problems as the starting point for acquiring new knowledge (Lambros, 2004). The instructional design of PBL focuses on "problem-solving" and "contextual learning" (Şimşek, 2011, as cited in Uygun & Tertemiz, 2014, p. 76). The constructivist theory emphasizes that learning by doing is at the core of PBL (Schank et al., 1999). According to Barrows (1984), as cited in Aryan & Shettar, 2023), the PBL approach emerged from the need to develop reasoning and problem-solving process skills. PBL is an experiential learning method that focuses on investigating, explaining, and solving existing complexity through experimental learning organized around research to solve complex problems based on real-life scenarios (Barrows, 2002; Torp & Sage, 2002). In PBL, students focus on a complex problem that does not have a single right answer. They work in collaborative groups to find out what they need to learn to solve the problem, apply the acquired knowledge to the problem, and reflect on the solution strategies used (Hmelo-Silver, 2004). In the problem-solving process, newly acquired knowledge becomes more important than the problem itself (Hassan Majeed et al., 2021).

According to Barrows and Tamblyn (1980), in PBL, students learn by solving problems and reflecting on their experiences. This method places learning topics in real-life problems and encourages students to take responsibility for their own learning, which helps them become active learners. Therefore, PBL provides a learning environment in which problems drive the learning process (Roh, 2003). More specifically, the PBL process begins with a problem that needs to be solved, which is presented in a way that requires students to acquire new knowledge to find a solution (Hung et al., 2007). Furthermore, Roh (2003) states that, rather than searching for a single correct answer, students interpret the problem, gather necessary information through research, identify possible solutions, evaluate alternatives, and present their findings. The PBL learning process can be carried out in several sessions (Barrows, 2002). Following the presentation of the problem, students can gather new information by conducting research or experiments (Torp & Sage, 2002).

The goals of PBL include developing effective problem-solving skills (Hmelo-Silver, 2004). During the PBL problem-solving process, students typically reflect on the information they have gathered and generate hypotheses about possible explanations for the problem. The students identify new learning topics and investigate them independently. Afterward, they regroup to share what they have learned, reconsider their hypotheses, and generate new hypotheses in light of the new information. The problem-solving process concludes with a presentation of the best solution, followed by a review of the PBL process and an evaluation of the learning outcomes (Hmelo-Silver, 2004). One of the most important components in achieving the goals of PBL consists of problem scenarios based on complex real-life problems that do not have a single correct answer. (Delisle, 1997). PBL posits that students construct knowledge through their experiences, and the method appreciates their interpretations. Accordingly, PBL claims that students learn by placing themselves in the protagonist role of a scenario designed around a learning outcome and solving the problems they encounter (Schank, 1999). Barrows (1996, 2002) states that in this approach, learning occurs in small, student-centered groups. The problems are presented through scenarios serving as stimuli that initiate learning and organize its focus. In this process, students are expected to address the problem context presented in the scenario using their prior knowledge (Delisle, 1997).

PBL Environment vs. Traditional Environment

PBL provides students with a genuine professional learning environment in which ill-structured problems requiring multiple solutions (Ronis, 2008) are presented through scenarios, and students engage in searching for possible solutions (Delisle, 1997). As real-life problems encountered in daily life are incorporated into the classroom, learning takes place in solving these problems (Schank, 1999). Thus, students participate in meaningful learning experiences and prepare for their future (Koh & Chapman, 2019). The problems serve as tools for developing students' problem-solving skills (Barrows, 1996, 2002; Delisle, 1997).

According to Collins (1994), presenting content in a narrative format through scenarios assigns students responsibilities and tasks, positioning them in expert roles that enhance their self-confidence. This strategy, in turn, encourages more active participation in the learning process and facilitates the retention of scenario-based content. In classrooms where traditional instructional approaches are implemented, teachers assume a leadership role, making decisions regarding the course flow as well as assessment and evaluation methods. Such environments are predominantly based on whole-class learning, assuming that all individuals possess equal qualifications, identical learning durations, and similar personal characteristics (Duruhan, 2004). One of the traditional teaching approaches that remains relevant today is Ausubel's (1962, 1963, 2000) meaningful learning theory. According to Ausubel (2000), meaningful learning occurs when the learner constructs meaning from the material presented, necessitating that teachers organize and present course content in a way that makes it significant for learners. The outcome of this process is the creation of new meanings (Ausubel & Robinson, 1969). In contrast, the PBL approach requires students to identify the knowledge they need and determine where to begin their learning process.

According to Kindley (2002), in the traditional approach, learning typically occurs at the level of knowledge and comprehension; however, PBL aims to facilitate student achievement at the levels of analysis and synthesis (as cited in Sorin, 2013). Schettino (2016) asserts that in the traditional approach, knowledge is derived from the teacher, with students primarily listening to, learning from, and subsequently applying what the teacher conveys. In such settings, students spend most of their class time engaged in individual exercises (Boaler, 2008). However, education inherently possesses a relational character, and learning is fundamentally shaped by the interactions among individuals in settings that provide opportunities for engagement (Biesta, 2004; Lee et al., 2023). In the PBL process, group work allows each member's learning needs to be addressed through the contributions of others (Lambros, 2004). Consequently, students develop a broader and deeper understanding by establishing connections among materials, concepts, and contexts, leading to meaningful learning (Thayer-Bacon, 2004).

In the PBL process, rote learning, which emerges from the teacher-centered instruction of traditional approaches, is replaced by lasting learning (Allen et al., 2011; Drake & Long, 2009; Leuchter et al., 2014; Martin & Jamieson-Proctor, 2022; Wong & Day, 2009; Yahya & Zaman, 2008). This shift leads to increased academic achievement (Merritt et al., 2017; Zamir et al., 2022) and enhanced mathematical motivation (Zamir et al., 2022), while also improving students' ability to apply their knowledge to new contexts (Lambros, 2004). Moreover, PBL fosters the development of students' collaborative skills (Dahl, 2018; Schettino, 2016), their ability to engage in discussions and critiques (Allen et al., 2011; Merritt et al., 2017), and their problem-solving abilities (Dorimana et al., 2022; Lambros, 2004; Yahya & Zaman, 2008; Westwood, 2011; Zamir et al., 2022). The literature also includes studies examining the challenges and benefits of PBL for teachers and pre-service teachers (Nariman & Chrispeels, 2016), its impact on reflective thinking skills (Koh & Tan, 2016), teachers' perceptions of assessment (Hemker et al., 2017), the development of assessment literacy and competencies, and the necessity of integrating the PBL approach into teacher instructional programs (Rillero et al., 2017). Additionally, research has explored PBL's role in enhancing teachers' beliefs about mathematics teaching (Bosica et al., 2021) and its positive impact on the development of educational websites (Wan Hassan et al., 2020).

Teachers' and Students' Roles in the PBL Environment

In the PBL process, the role of the teacher is to guide students and prompt them to reflect on their learning processes by asking questions that activate their thinking skills (Martin & Jamieson-Proctor, 2022). Through this approach, students are given opportunities to direct their learning, enabling them to become self-regulated learners (Delisle, 1997). During this process, students should acquire new knowledge through their own experiences and strive to develop expertise (Barrows, 1996, 2002).

Teachers in the PBL process need to ensure that the problem is appropriate for the students' level, aligned with the course objectives, and based on the curriculum (Tan, 2003). They should determine whether students possess the foundational knowledge necessary to analyze and comprehend the problem and establish a structured framework when designing the problem scenario. Within this framework, various factors should be considered, ranging from the complexity of the problem scenario to its constraints. Specifically, teachers should reflect on whether the problem situation is too challenging for students to understand, whether it fosters problem-solving skills, and what kind of outcome might emerge by the end of the process (Tan, 2003).

Lambros (2004, p. 9) suggests that students in the PBL process should prepare lists, including a Facts List, Need-to-Know List, Learning Issue List, New Learning Issues List, and Possible Solutions List. These lists assist students in understanding the problem scenario, identifying what they know and what they need to learn, determining the topics that require further research, recognizing emerging issues throughout the process, and ultimately deciding on a solution. The lists can be individualized for each student or designed to reflect the study plan of the entire group. Each list includes the questions students need to ask and the tasks they need to complete.

Problem Solving and PBL

Engaging in problem-solving activities—often referred to as "doing mathematics"—is considered essential for acquiring and effectively utilizing 21st-century skills. Polya (1962) defines problem solving as the process of solving situations where the solution is not known. In this context, mathematics itself can be viewed as a problem-solving activity. Consequently, due to its significance across various fields, problem-solving is integrated into educational curricula, particularly as a fundamental method in mathematics teaching (Jitendra et al., 2007; Ministry of National Education [MoNE], 2013; National Council of Teachers of Mathematics, 1989, 2000; Polya, 1973). The experiences gained during the problem-solving process and the conclusions drawn from these experiences contribute to the development of problem-solving skills (Schoenfeld, 1983). This process provides clear insights into how students think, how they understand mathematical concepts, how they establish relationships between concepts, and how they propose solutions (Chinnappan, 1998). A well-known fact is that individuals with problem-solving skills can think independently and creatively, have developed social skills, possess self-confidence, and can tolerate uncertainty (Dow & Mayer, 2004).

The problem-solving process, originally rooted in John Dewey's philosophy, was reinterpreted by George Polya and structured into a four-stage model: (a) *understanding the problem*; (b) *devising a plan*; (c) *carrying out the plan*; and (d) *looking back*. According to Polya (1973), this four-stage problem-solving process specifically involves understanding the problem; developing an appropriate solution plan; modifying strategies when necessary; monitoring methods; evaluating the data obtained during the process; and interpreting the meaningfulness of the solution. In Stage 1 (understanding the problem) of Polya's model, students summarize the given problem, rephrase it in simpler terms using their own words, and identify both the given information and what needs to be determined. During Stage 2 (devising a plan), students select the strategy to be used to solve the problem. According to Baki (2018), the first two stages of the process play a crucial role in developing mathematical literacy. Reapplying these two stages to each problem requires students to employ new models and expressions each time, thereby enhancing their ability to comprehend and use mathematical language effectively. In Stage 3 (carrying out the plan), students apply the selected strategy and modify it if necessary. Finally, during Stage 4 (looking back), they examine the correctness and logical soundness of the solution. At this stage, students explore alternative solutions and attempt to solve the problem using different approaches. This stage is particularly significant in fostering students' critical thinking skills (Baki, 2018).

One of the core objectives of PBL is to develop problem-solving skills, and the results of research (e.g., Gallagher et al., 1992) largely support the validity of this goal (Hung et al., 2007). In addition, a study by Lohman and Finkelstein (2000) has shown that PBL has a positive impact on students' ability to transfer problem-solving skills to real-world problems or personal situations (as cited in Hung et al., 2007). However, Norman and Schmidt (1992) argue that no conclusive evidence exists to confirm the advantages of PBL in fostering general problem-solving skills independent of content. They suggest this finding reinforces the effectiveness of contextualized learning within the PBL framework (as cited in Hung et al., 2007).

"PBL is a learning method that initiates students' learning by creating a need to solve a real problem" (Hung et al., 2007, p. 486). In PBL, students develop problem-solving skills while building content knowledge throughout the problem-solving process. Problem solving is "the process of understanding the inconsistency between the current and goal states of a problem, generating and testing hypotheses for the causes of the problem, designing solutions to the problem, and implementing the solution in a way that satisfies the goal state of the problem" (Hung et al., 2007, p. 486).

PBL is a classroom strategy that organizes mathematics education around problem-solving activities, providing students with greater opportunities to think critically, present their creative ideas, and engage in mathematical communication with their peers (Carpenter et al., 1993; Erickson, 1999; Hung et al., 2007). Students in PBL environments often have more opportunities to learn mathematical processes related to communication, representation, modeling, and reasoning than those provided in traditional mathematics education settings (Erickson, 1999; Schmidt, 1983).

Probability

The development of independent creative thinking skills and probabilistic thinking, a fundamental mode of reasoning, is a key objective of mathematics education. In this regard, the topic of probability holds a significant place in mathematics teaching (Fischbein, 1975). Probability is defined as "predicting the outcome of an event whose result is not certain in advance" (Güzel & Ünal, 2013, p. 710), "as a measure of the likelihood of an event occurring" (Franklin, et al., 2005, as cited in

Van de Walle et al., 2014, p. 460), or "as a measure of the belief in certainty or uncertainty based on specific interpretations and predictions" (Popper, 1998, as cited in Erdem, 2011, p. 10). Probability involves making logical inferences by considering all possible outcomes of an event or situation (Erdem, 2011). Reasoning, on the other hand, means evaluating all aspects of a problem or situation and plays a crucial role in problem-solving and decision-making (Lavigne & Lajoie, 2007). In everyday life, situations often arise in which reasoning about all possible outcomes is necessary, and decisions must be made under conditions of uncertainty. In such circumstances, probabilistic thinking serves as a vital component in facilitating effective reasoning (Schlottmann, 2001). Specifically, probabilistic thinking aids an individual to make logical predictions and reason through intuition about events with multiple potential outcomes (Gürbüz et al., 2010). Fischbein (1975) shares a similar perspective, asserting that intuition about chance and probability serves as a guiding tool for individuals when faced with uncertainty.

According to Kazak (2010), students find learning probability to be challenging when their concepts about probability—developed by their intuitions and personal experiences—are combined with the counter-intuitive nature of fundamental concepts and principles of probability theory. Underdeveloped probabilistic intuitions are often insufficient for advanced and complex reasoning tasks (Fischbein, 1975). O'Connell (1999) found that students' errors typically stem from a lack of complete and accurate understanding of the text (e.g., misinterpreting equality; misjudging probability values; thinking of probability values as negative; considering probability values greater than 1; and failing to grasp the relationship between the probability of an event occurring and not occurring). Additionally, foundational concepts such as set theory, percentage calculations, fractions, fraction comparison, decimals, and sample space are essential for understanding probability (Fischbein & Schnarch, 1997). Comprehending probability requires more nuanced skills than many other areas of mathematics. Thinking more carefully, critically, intuitively, and deeply; making logical predictions; possessing a strong mathematical language; and engaging in logical reasoning are necessary for understanding (Carter, 2005). Furthermore, difficulties arising from the epistemological structure of the subject can prevent students from achieving a conceptual understanding of probability. Students who are accustomed to dealing with conventional problems in the classroom may struggle with probabilistic situations that require reasoning. A strong understanding of probability provides students with the ability to verify the correctness of information obtained through reasoning (Korkmaz, 2005).

Contribution of Research on Literature and Its Rationale

The subject of probability is more intertwined with daily life compared to other mathematical topics, as people often use it in making decisions in real-life situations (Van de Walle et al., 2014). For instance, simulations of complex real-world situations are often based on probability, and these simulations are used in the design of processes for planning responses to disaster scenarios (Van de Walle et al., 2014). For instance, Konold (1994) designed a simulation in which students were directed to read newspaper articles featuring real-life situations related to probability and asked to interpret these reports. Through this approach, students were able to test the accuracy of their predictions with the assistance of a computer.

The integration of probability into daily life has led to using various problem scenarios in teaching. Probability plays a crucial role in various aspects of daily life, including weather forecasting; sports and game strategies; insurance; online gaming and shopping; blood group analysis; and political strategy assessment. Probability is also essential in evaluating the likelihood of experiencing adverse effects from certain medications, predicting profit-and-loss probabilities in business, and making decisions in high-risk situations (Manikyamba & Ramalakshmi, 2024). For instance, meteorologists calculate the probability of weather events and assess the associated risks. They create various graphs to predict temperature fluctuations and potential weather patterns. Another example involves waiting periods at traffic lights. The problem of whether waiting time at traffic lights changes according to traffic density is related to the subject of probability. (Manikyamba & Ramalakshmi, 2024). Integrating real-life problem scenarios like these into the classroom through the PBL approach can help students understand the relevance of probability in daily life and extend their knowledge to similar situations. Learning activities conducted in an environment in which students are both physically and mentally engaged contribute positively to their problem-solving skills. (Lambros, 2004), Therefore, this research examines the impact of the PBL approach on students' problem-solving skills.

According to Nickerson (2004), the development of probabilistic reasoning skills is essential for making logical judgments. Therefore, the subject of probability plays a fundamental role in learning. Significantly, the subject leads students to explain their thoughts and question the reasons for their solutions in learning environments, enabling them to engage in quality reasoning. The PBL approach creates a learning environment that provides students with this opportunity. In this context, this research, which examines the impact of using the PBL approach in learning probability on students' problem-solving skills, contributes to the relevant field of mathematics education.

The distinctive aspect of this study from similar research in the literature lies in the integration of Barrows's (1986) closed-loop PBL model—one of the six taxonomies he outlined based on various definitions of PBL—together with Lambros's (2004) suggested PBL implementation process. Another key distinguishing feature of this study is the implementation of the closed-loop PBL model to eighth graders in the context of probability and the subsequent detailed examination of the impact of this model on students' problem-solving skills at each phase of the process.

The rationale for selecting Barrows' (1986) closed-loop PBL model can be explained as follows: among the six models in Barrows' PBL taxonomy, the closed-loop PBL model is the version in which the student is most central and instructional scenarios play a significant role. Moreover, in this model, the facilitator-teacher assists students in stimulating their prior knowledge through inquiry and presents real or highly realistic scenarios that enable students to discover and analyze new knowledge (Martin & Jamieson-Proctor, 2022). This model, which emphasizes self-directed learning, provides students with opportunities to present their research, reflect on what they have learned, and make assessments regarding how to improve their reasoning processes. In this model, students return to the original problem to complete the process and respond to a series of reflective questions. These questions prompt them to revisit the original problem, make inferences, and reflect on the solution both individually and as a group. For instance, questions such as, "If you were to address the problem again, what adjustments would you make to improve your reasoning processes?" help students close the loop by re-engaging with the original problem. This model offers a framework that aids students in structuring their knowledge and enhancing their autonomous learning skills (Martin & Jamieson-Proctor, 2022, p. 8). Based on these explanations, this research provides a detailed examination of the impact of the PBL approach on students' problem-solving skills in the content area of probability by each stage of the process.

Aim of the Study

The main purpose of this research is to examine the effect of PBL on middle school students' problem-solving skills in the content area of probability. This study aims to examine the impact of the closed-loop PBL approach on students' problem-solving skills in teaching probability, one of the sub-learning areas of the eighth grade. The research problem is to determine whether a statistically significant difference exists between the problem-solving skills of the experimental group (EG) students, who were taught using the PBL approach, and the control group (CG) students, who were taught using the traditional expository teaching method, as shown in their scores obtained before and after the intervention.

Method

Research Design

In this study, the experimental method—a quantitative research method—was employed. The research design was a quasi-experimental model with a pre-test and post-test CG. "In a quasi-experimental model, two of the pre-existing groups are matched based on certain variables" (Karadeniz et al., 2020, p. 216). The independent variables of the study are the instructional methods used—the PBL approach for the EG and the traditional expository teaching method for the CG. The dependent variable, on the other hand, is the students' problem-solving skills. The research model is presented in Table 1, which shows the tests administered to the EG and the CG throughout the study. Accordingly, the same problem-solving test (PST) was administered to both the EG and CG twice, once before and once after the intervention, as a pre-test and post-test.

Groups	Pre-test				Post-test
	Number of Students (N)	ET	PST	Implementation	PST
EG	18	✓	✓	Constructivist theory – PBL approach	✓
CG	21	✓	✓	Traditional theory/Meaningful learning approach – Expository teaching	✓

Table 1. Model of Research

Study Group

The study was conducted during the 2018-2019 academic year. The research group consisted of 39 students, including 14 females and 25 males. These participants were enrolled in the eighth grade at a public middle school in a province located in the Western Black Sea region of Türkiye, and all students were from the same socio-economic background. The EG included 18 students, with seven females and 11 males, while the CG consisted of 21 students, with seven females and 14 males. Initially, the EG consisted of 20 students, but two students did not regularly attend classes during the process. Hence, their data were excluded from the analyses performed after the equivalence test (ET). The analysis of the ET results indicated that the two groups were equivalent in terms of mathematics achievement. Consequently, one group was assigned as the EG and the other as the CG through a random draw. Before the intervention, both the EG and CG had been taught mathematics by the same teacher. However, once the study commenced, a researcher assumed the role of teacher over the EG, while the existing mathematics teacher taught the CG. The decision to retain the existing teacher as instructor of the CG was based on the rationale that the teacher's familiarity with the students and their effectiveness in implementing the traditional expository teaching method would be advantageous (Ausubel, 2000).

According to this method, the teacher, who is familiar with the students, moves to the next step only after ensuring the students have understood each stage of the topic. Therefore, to avoid disrupting the ongoing relationship between the student and the teacher by introducing a different instructor, the researchers did not teach the CG. The students in the CG were accustomed to their teacher and could easily ask questions. As a result, the researcher participated in the lessons given in the CG by acting as an observer and evaluated the teacher's use of the expository teaching method. Before starting the intervention, the researcher who would teach the EG spent five lesson periods with the EG students to help them become familiar with the researcher. In the PBL approach, which is based on constructivist theory, the student is active, and the teacher plays a facilitating role. Due to the features of the PBL approach, EG students were able to easily interact among themselves and with the researcher conducting the lessons. In both groups, the implementation was carried out by the learning outcomes related to the content area of probability as outlined in the 2018 Mathematics Curriculum.

Data Collection Tools

The data collection tools used in the research were the ET: the PST designed to measure the impact of the PBL approach; the scenarios and activities prepared by the researchers for the EG; and the Semi-Structured Interview Form (SSIF) used to collect the EG students' opinions.

Administering the ET

Before the intervention, the ET, which covered basic mathematics knowledge, was administered to both the EG and the CG to determine whether their initial achievement levels were equal. The ET was used to ensure that the students in both groups were equivalent in terms of overall mathematics achievement levels, preventing any potential bias for the results of the research. This process helped identify and control any variables that might influence the research outcomes before the intervention.

Before determining the groups, a 20-question multiple-choice ET consisting of basic mathematics knowledge was prepared by the researchers upon receiving experts' views (see Appendix A). The item analyses of the test were performed using the Test Analysis Program (TAP) program. The test was designed according to the learning outcomes identified in the mathematics curriculum (MoNE, 2018). A pilot study was conducted with 104 eighth graders at a public school and a private school in a city in the Western Black Sea region of Türkiye. Since the discrimination level of the questions was above 0.15, the 20-question test was used as the ET. Cronbach's alpha reliability coefficient for the ET was found to be .873, and the test was administered to both groups within one lesson period.

Administering the PST

A PST was prepared to assess students' problem-solving skills in probability and included open-ended questions found in the literature (Ata, 2013; Bulut, 1994; Çakmak & Durmuş, 2015; Karaaslan & Ay, 2017; Koparan & Güven, 2014; Tunç, 2006) with already-performed validity and reliability analyses. The PST was developed using the key components necessary for an assessment tool: validity, reliability, and practicality (Baykul, 2014; Özçelik, 2013; Yıldırım & Şimşek, 2008). Before the PST was administered, the content validity of the open-ended problems in the test was reviewed by two experts and a middle school mathematics teacher, who evaluated it in terms of content, level, and language. The PST was then finalized, and a pilot study was conducted with 60 students to assess the reliability of the study and the practicality of the data collection instrument. Following the pilot study, one lesson period was found to be sufficient for students to answer the questions. As a result, the final form of PST, consisting of 14 problems, was identified as the data collection tool (see Appendix B). The PST was administered to both the EG and CG within their respective classrooms during a designated lesson period under the supervision of the researchers. In analyzing the PST, each phase of the problem-solving process—understanding the problem, devising a plan, carrying out the plan, and looking back—was regarded as a separate sub-problem. Table 2 shows the learning outcomes about probability in the mathematics curriculum in Türkiye (MoNE, 2018) and the corresponding question numbers in the PST regarding each learning outcome.

Learning Outcomes of Probability	PST Question Number
Can determine the probability related to an event	1, 2, 5
Can distinguish between "more", "equally", and "less" probable events and gives examples	9, 10, 12, 13, 14
Can explain that the value of each output is equal in events with equal chances, and this value is 1/n	7, 8
Can comprehend that the probability of an event is between 0 and 1 (including 0 and 1)	4
Can calculate the probability of a simple event	3, 6, 11

Table 2. Learning Outcomes of Probability and Numbers of Questions in PST for Each Outcome

In the PST, different numbers of questions were provided for each learning outcome about probability. According to Özçelik (2013), a test prepared for a subject should include at least one question for each learning outcome. The questions in the PST were selected from existing questions in literature; therefore, this research included a limited number of questions for each learning outcome about probability. During the implementation of the tests, the researchers gave detailed explanations to the students. They asked the students to solve the problems given by taking each stage into account according to the following instructions:

1. State the problem in your own words.
2. What is the problem given in this problem? What is asked?
3. Explain how you would solve this problem.
4. Solve this problem.
5. Check your result. Is your solution correct? Where could you have gone wrong? Explain your answer.

In this study, Polya's problem-solving process was used since it was considered suitable for mathematics teaching (Baki, 2018). The students' responses obtained from the PST were evaluated using the Progressive Scoring Scale (PSS) developed by Baki (2014), in which each stage of the problem-solving process was scored. PST was coded using the Progressive Scoring Scale (PSS; see Appendix C). The PSS used in the study consists of five categories. Since *problem-posing skills* were not examined in this study, the fifth category, *problem-posing*, was excluded. The four categories included are listed in Table 3. Under each of these four categories, four points (3, 2, 1, and 0) represent the highest and lowest performance levels for each criterion. For clarity on the scoring, Table 3 shows the maximum and minimum scores a student can receive from a problem. The students' answers to the PST questions were scored according to the stages of problem solving using the PSS. For each category in the grading key provided by the researchers, scores ranging from 3 to 0 were assigned. Each stage of the students' responses was evaluated separately based on these scores.

Categories	Max.	Min.
Understanding the problem	3	0
Devising a plan	3	0
Carrying out the plan	3	0
Looking back	3	0
Total score a student can receive	12	0

Table 3. Categories Related to Progressive Scoring Scale

The PSS in Appendix C justifies the scores given for each stage. According to the PSS, if a student has made no effort to answer, the resulting score is 0. An answer to a problem is scored as 1 if the problem is not understood, an inappropriate strategy is chosen, an inappropriate or incorrect solution is provided, and results for it are only partially verified. A score of 2 is given if only part of the problem is understood, only part of the solution strategy is chosen, a partially correct solution is provided, and results are logically verified. A score of 3 is given if the problem is fully understood, an appropriate strategy is chosen, a suitable and correct solution is found, and the result is logically explained and verified. Excerpts from student responses are provided in Figure 1 and Figure 2.

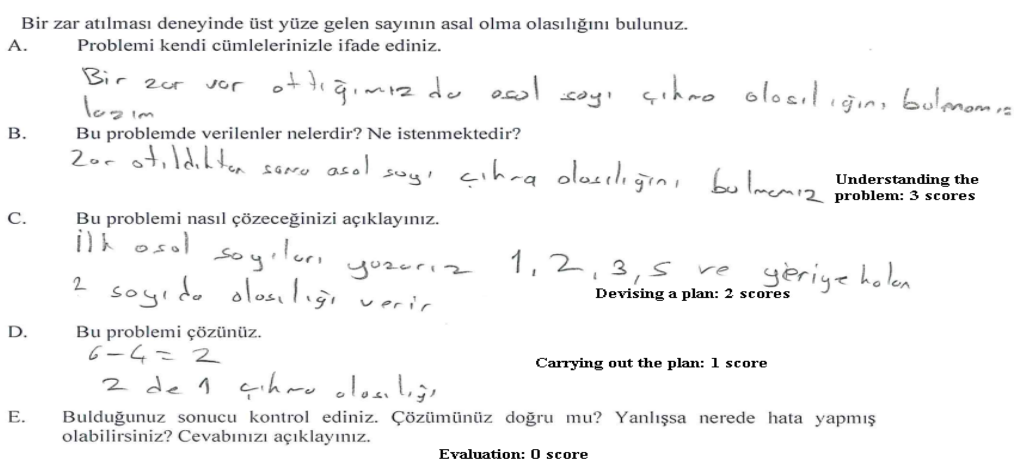
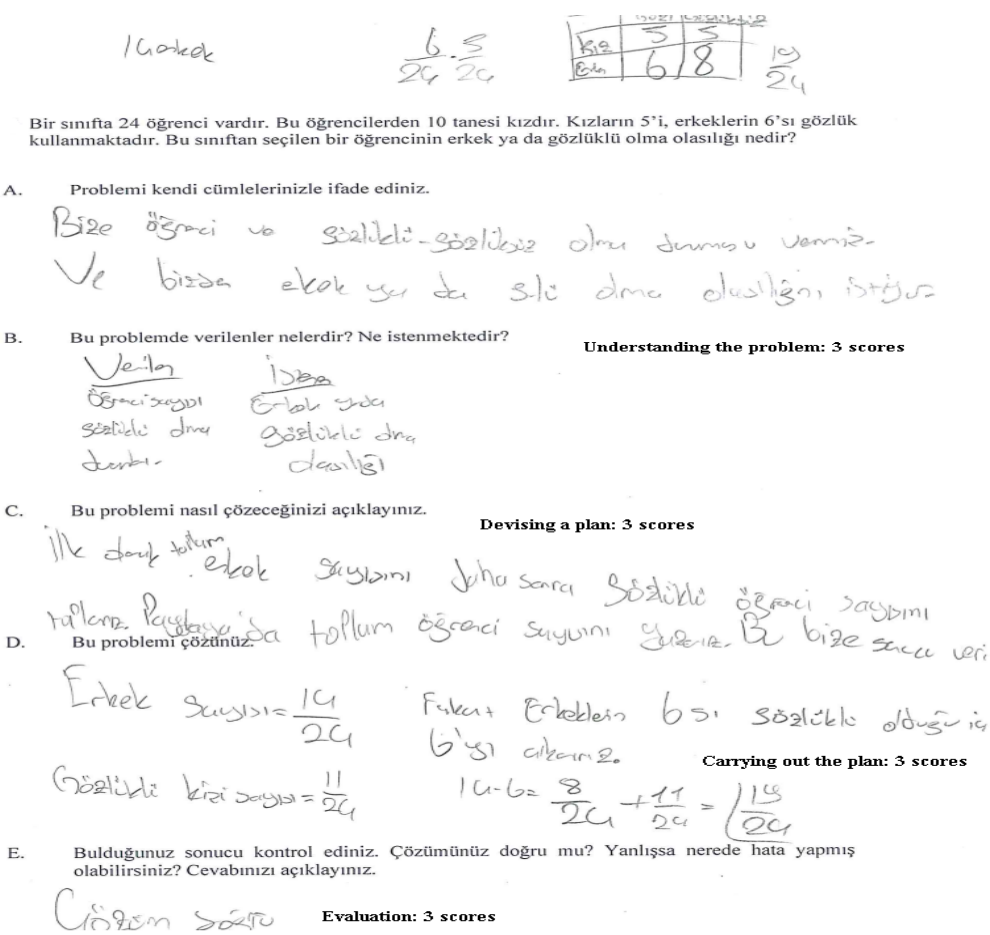


Figure 1. An Example Student Response (Scored)

Figure 1 shows a student's response to the problem of finding the probability that a rolled die shows a prime number on the upper face. The students answered each question as follows:

- Question A: "Finding the probability of getting a prime number by rolling a die."
- Question B: "Finding the probability of getting a prime number after the die is rolled."
- Question C: "First, we write down the prime numbers: 1, 2, 3, 5, and the remaining 2 numbers give the probability."
- Question D: "6-4=2, 2 is the probability of getting a 1."
- Question E: No response.

The students accurately expressed the problem in their own words and clearly stated the information given and what was required. Thus, the student in Figure 1 received 3 points for this stage. However, it was noted that this student did not consider that 1 is not a prime number in the stage of devising a plan. As a result, the student did not fully explain how to solve the problem and received 2 points for devising a plan. In the stage of carrying out the plan, the student attempted an irrelevant solution and received 1 point for this stage. In the stage of looking back, the student made no effort to assess their solution, resulting in 0 points for this stage.


Bir sınıfta 24 öğrenci vardır. Bu öğrencilerden 10 tanesi kızdır. Kızların 5'i, erkeklerin 6'sı gözlük kullanmaktadır. Bu sınıftan seçilen bir öğrencinin erkek ya da gözlüklü olma olasılığı nedir?

A. Problemi kendi cümlelerinizle ifade ediniz.

B. Bu problemde verilenler nelerdir? Ne istenmektedir?

Understanding the problem: 3 scores

C. Bu problemi nasıl çözeceğinizi açıklayınız.

Devising a plan: 3 scores

D. Bu problemi çözünüz.

Carrying out the plan: 3 scores

E. Bulduğunuz sonucu kontrol ediniz. Çözümünüz doğru mu? Yanlışsa nerede hata yapmış olabilirsiniz? Cevabınızı açıklayınız.

Evaluation: 3 scores

Figure 2. An Example Student Response (Scored) – 2

In Figure 2, a student's response to a question asking about the probability of a selected student being male or wearing glasses is shown. The students provided the following answers:

- Question A: "We are given the student count and the situation of being with or without glasses. We are asked to find the probability of being male or wearing glasses."
- Question B: "Given; the number of students, situation of wearing glasses, wanted; probability of being male or wearing glasses."
- Question C: "First, we add the total number of males and then the number of students wearing glasses. We write the total number of students in the denominator. This gives us the result."

- Question D: "The number of males = 14/24, and the number of students with glasses = 11/24, but since 6 of the males wear glasses, we subtract 6. The result is 19/24."
- Question E: "The solution is correct."

Since the student proceeded in each stage of the problem-solving process correctly and as required, they were given 3 points for each stage. For the stage of "looking back", the student received 3 points because it was observed that they checked the information using a table at the top of the page.

Scenarios and Activities

In this study, three scenarios and three activities about probability were implemented with the EG students. An example scenario is provided in Appendix D. During the development of the scenarios, Brock's (2003) eight-step scenario development framework was used. This framework consists of the following eight steps: purpose; integration; authenticity; prevalence and relevance; resources and format for delivery; impact and interest; sequence; brevity and complexity; and modality. Brock's framework, which is specifically designed to plan the demonstration of certain skills (Kocadağ, 2010), was chosen for this study because it offers a detailed explanation for scenario development. Tables 4 and 5 explain the first scenario, "Weekend at The Amusement Park," which was developed by the researchers according to Brock's (2003) scenario-development framework. An example of this scenario is provided in Appendix D.

Purpose

Learning Objective: "The student will be able to explain that in events with equal chances, the probability value of each outcome is equal, and this value is 1/n." *Learning Outcome:* "The student should distinguish between events with equal and unequal chances."

Integration

Information to be Used: "The student determines the possible outcomes of an event," "Distinguishes and provides examples of events with 'more,' 'equal,' and 'less' probability" and utilizes knowledge of fractions, simplification, and expansion of fractions, as well as ratios and proportions. *Expected from Students:* To identify problem-solving strategies, engage in group discussions, and use reasoning to obtain accurate information. *Expected from the Teacher:* To guide students in using the internet, smart board, and relevant books, and to assist them in linking prior and new knowledge to facilitate their reasoning during activities.

Authenticity

Domain-Specific Qualities: An instruction was provided in the scenario for this purpose. Being able to identify all possible events within the context of the problem. Having the ability of proportional reasoning is needed to calculate the probability of a specific event. Understanding correlations that are crucial for comprehending risk.

Prevalence and relevance

Connection to Real Life: The scenario does not include any unrealistic elements. One of the games played at amusement parks is the ring toss game. The scenario involves comparing the relationship between the number of rings tossed and the number of rings that successfully land on the target. Two friends are asked to reflect on the results, where one successfully lands nine out of 36 rings, and the other lands 16 out of 64 rings.

Table 4. Adaptation of Brock's (2003) Scenario Development Framework – Part 1

Resources and format for delivery

Access to Resources: Scenarios and activity worksheets, the internet, relevant websites, smartboards, and various textbooks were employed. The teacher, acting as a facilitator, guided students toward these resources.

Impact and interest

Attractiveness of Scenario Content: Students in this age group often enjoy playing games and going to amusement parks. The scenario incorporates an amusement park as the learning environment. The ring toss is a type of activity likely to capture students' interest.

Sequence, brevity and complexity

Structure of the scenario, level of complexity, alignment with objectives, and duration of implementation: The scenario was written concisely to accommodate students who might be reluctant to read lengthy questions. This approach ensured that the attention of all students in the class was captured, reaching every student effectively. The scenario and the related activity were completed within four lesson periods. The instructions provided in the scenario were designed to guide the learning process in a way that encourages students to take responsibility for their learning.

Modality

Provision of visual, auditory, and video films, activity worksheets, and other learning aids to facilitate student learning, and what these aids included: The scenario emphasizes the use of visual stimuli. Smartboards and computers were also utilized during this process to enhance student engagement and understanding.

Table 5. Adaptation of Brock's (2003) Scenario Development Framework – Part 2

Administering the SSIF

Following the implementation of the PBL process, SSIF was administered to the EG students to gather their opinions on the implementation. This form was developed by researchers and used open-ended questions. The primary areas of SSIF in the PBL implementation process include the difficulties students encounter; their opinions on scenario design; the most favored scenario and the reasons behind their preference; the impact of the problem situation on learning, socialization, and group work; their perspectives on the continuation of the implementation; and a comparison between traditional instruction based on the current curriculum and instruction using the PBL approach. Before the intervention, the questions in the SSIF were reviewed by two faculty members and a teacher to ensure their appropriateness in terms of language and expression. Based on their feedback, the SSIF was revised and finalized. The SSIF included questions about what the students experienced during the PBL process regarding the learning of probability, the challenges they faced, how they managed the process, their thoughts on PBL, and the benefits they perceived. Interviews with EG students were voluntary and lasted 15-20 minutes. The interviews were recorded and later transcribed by the researchers for analysis. Responses of the students who did not want their interviews to be recorded or declined to participate were either received in written form or noted by the researchers during the interviews.

Experimental Work Process

The implementations for both the EG and CG were completed in 12 lesson periods each. The experimental process of the study is illustrated in Figure 3.

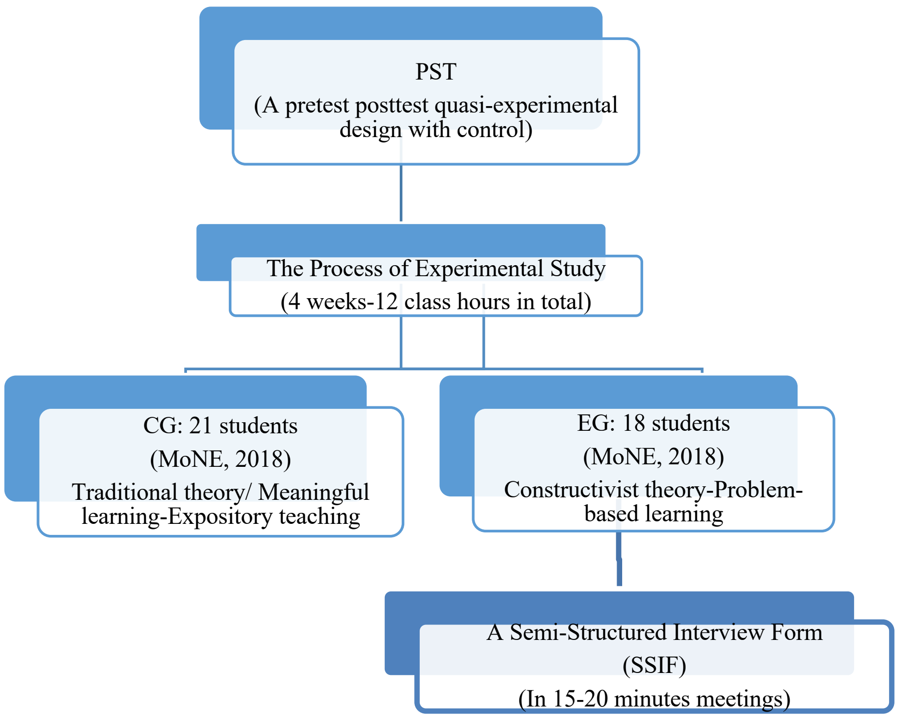


Figure 3. Experimental Process of the Study

Both the EG and CG had prior knowledge of problem-solving stages from their previous learning experiences. Therefore, no specific instruction on problem-solving stages was provided to the groups. The PST is a test that requires detailed responses for each of the problem-solving stages. Thus, the lack of a significant difference between the groups in the pre-test analysis of the PST indicates that the groups were equivalent in terms of their knowledge of the problem-solving stages.

Intervention Process

Table 6 provides information on the activities conducted with both groups throughout the research process, including what was done, when it was done, and how long it lasted. Detailed explanations of the activities carried out with each group are presented under subheadings.

Group	Activities and Their Duration	Before Intervention	1st week	2nd week	3rd week	4th week	After Intervention
EG	ET (1 lesson period/40 minutes)	✓					
	PST (Pre-test) (1 lesson period/40 minutes)	✓					

	Providing information about the PBL (Problem-Based Learning) implementation process / Formation of groups (2 lesson periods / 80 minutes)		✓				
	Implementation of PBL scenarios and activities / Execution of the PBL process (12 lesson periods)		✓	✓	✓	✓	
	PST (Post-test) (1 lesson period/40 minutes)						✓
	Implementation of the SSIF (in 15–20-minute interviews / 2 hours)						✓
	ET (1 lesson period/40 minutes)	✓					
	PST (Pre-test) (1 lesson period/40 minutes)	✓					
CG	Teaching by using the expository teaching method (12 lesson periods)		✓	✓	✓	✓	
	PST (Post-test) (1 lesson period /40 minutes)						✓

Table 6. Activities Performed with EG and CG Students and Duration of Activities

The Implementation Process for the EG

Before the lesson, students were divided into heterogeneous groups of four to five members. These groups were called heterogeneous groups since they combined students with low, medium, and high achievement levels. In such groups, each member's learning needs are complemented by other group members. Within the scope of PBL, group members learn to engage in cooperative discussions, communication, mutual respect, and self-management. The classroom setting for the EG was reorganized to help the groups work comfortably. Before the lesson started, EG students were informed about the implementation process, and student and teacher responsibilities were explained.

The scenarios were distributed to each student and projected on a smart board. Upon the distribution of the scenarios, students were asked to look at the visuals and generate different ideas about the scenarios. The goal was to motivate the students to engage in the lesson. By discussing the visual elements of the scenario, students were able to share their experiences and thoughts. Students were then asked to work individually. During this time, students noted their thoughts, identified the problem presented in the scenario, and found the information needed to solve it. They searched the internet, visited relevant websites, read various books about the topic, and sought help from the facilitator when they had difficulty accessing resources. Acting as a facilitator, the researcher guided the students by asking questions that encouraged them to reason and think critically, keeping them focused on the context.

Each student within the groups then prepared a report containing the results they found while solving the problem. The information and solutions obtained through individual work were shared within the group and discussed, and a consensus on a common solution was reached. During this process, each student critically evaluated their learning process, themselves, and their peers. Afterward, the groups' presentations were shared with the other students. In their presentations, the groups defended their solutions with logical reasoning, explained their approach, and attempted to convince their peers of their conclusions. Subsequently, each group received feedback from the other groups and the researcher regarding the correctness and effectiveness of their solutions. Based on the feedback, groups revised and discussed their solutions, aiming to refine them. They also attempted to write a scenario in which they could apply what they had learned. These types of discussions and persuasion efforts help students synthesize their prior and new knowledge, aiding them in achieving the intended learning outcomes.

To deepen the understanding and reinforce what was learned, worksheets and activities (titled "I am Applying What I Learned, What Did I Learn") related to each learning outcome were distributed to the students. They were first asked to work individually and then collaboratively within their groups. The class then discussed the results of each group's activities and the solutions to the questions, thereby concluding the process. The EG students conducted their work following a checklist suggested by Lambros (2004) for use in the PBL process. Table 7 and Table 8 present an example checklist reflecting the students' responses during the work process related to the scenario of "Weekend at the Amusement Park".

Facts	Need to Know	Learning Issues
I went to the amusement park, but I never played / A few times, I played the "Ring toss game". Arda, Sinan, Melis, and Umut went to the amusement park over the weekend. They rode bumper cars and played the ring toss game. Sinan, who had a broken arm, watched his friends. Umut tossed rings 36 times and succeeded in getting 9 rings onto the target. Melis tossed rings 95 times and got 19 rings onto the target. Arda tossed rings 64 times and successfully placed 16 rings onto the target.	What is the "ring toss game" and how is it played? What is the problem in the scenario? Among Arda, Umut, and Melis, who tosses better? Who has a higher chance? Whom should Sinan support? Who should I support? What does "making an equal number of throws" mean? By making an equal number of throws, who succeeds in getting the most rings through? Who gets the fewest rings through? What mathematics topic is this scenario about? Where and how can I learn about this topic? What resources can I use? How can I access the information most quickly?	Possible outcomes of an event. What are examples of these outcomes? The topic of probability. What does probability mean as a mathematical concept? How can I calculate the probabilities of events? What is the relationship between the concepts of "more, equal, less" and probabilistic events? What is the concept of "equal probability"? How should I decide which of Umut, Arda, or Melis should be supported? What kind of reasoning should I use?

Table 7. The Starting Process of the "Weekend at the Amusement Park" Scenario

Possible Solutions

To assist Sinan, the following tables were prepared. Accordingly, some students drew the table based solely on the information provided in the scenario. At the same time, the others included the number of rings that were not successfully tossed around the peg in the table based on the given information.

	Number of Tosses	Number of Rings Landed on the Peg
Umut	36	9
Melis	95	19
Arda	64	16

	Number of Tosses	Number of Rings Landed on the Peg	Number of Rings that were not Landed on the Peg
Umut	36	9	27
Melis	95	19	76
Arda	64	16	48

The students' responses and comments regarding their peers who landed the most and the fewest rings on the target were as follows:

Umut tossed rings 36 times and 9 rings were successfully thrown. Melis tossed 95 times and landed 19 rings on the target successfully. Arda made 64 throws and 16 of them were successful. Although Melis appears to have transferred the most rings (19), this is not completely true because Melis had a higher number of tosses. If Umut and Arda had made as many tosses as Melis, they might have been able to land a higher number of rings on the target as well.

By using the formula for probability, which is the Number of Favorable Outcomes / Total Number of Outcomes, we can compare these friends' probability of successful throws.

$$Arda = \frac{16}{64} = \frac{1}{4} \quad Melis = \frac{19}{95} = \frac{1}{5} \quad Umut = \frac{9}{36} = \frac{1}{4}$$

Based on these ratios, we understand that Arda and Umut have equal probabilities. The highest probability of successful throws belongs to Arda and Umut, while Melis has a lower chance.

Whom should Sinan support? Why? *Sinan can support either Arda or Umut in this game. The probability values of Arda and Umut are equal, while Melis's probability is lower.* ($\frac{1}{4} > \frac{1}{5}$)

If an equal number of throws are made, who will be the winner? *If an equal number of throws are made, Arda and Umut will be able to land the most rings on the target. They have equal probabilities.*

Defendable Solution(s)

What conclusion have you drawn from what you have done so far? Express the conclusion mathematically. *Even if the numbers given in the problem (number of tosses and number of successful rings) differ, we should first calculate the probability values. After that, we can make a comparison. We can make a ranking as follows: Arda = Umut > Melis. Arda and Umut's probability of successful tosses is equal, while Melis's probability is lower* ($\frac{1}{4} = \frac{1}{4} > \frac{1}{5}$).

New Learning Issues and Applying What is Learnt

I need to review my knowledge on fractions, simplifying and expanding fractions, percentages, and ratios.

Class	Male	Female
1-A	9	17
1-B	12	15
1-C	14	18
1-D	16	20

The table beside shows the number of male and female students in four different classes. Which two classes have the same probability of selecting a male student at random? To answer this question, what do we need to know? What information do we need and how should we use this information?

To answer this question, we need to calculate the probability of selecting a male student from each class separately. We should know the formula for probability, which is "the number of favorable outcomes/total number of outcomes. We also need to know that the total number of outcomes represents the total number of students.

1/A, number of male students: 9; class size: 26 $\frac{9}{26}$

1/B, number of male students: 12; class size: 27 $\frac{12}{27} = \frac{4}{9}$

1/C, number of male students: 14; class size: 32 $\frac{14}{32} = \frac{7}{16}$

1/D, number of male students: 16; class size: 36 $\frac{16}{36} = \frac{4}{9}$

Even though the number of students in classes 1/B and 1/D is different, the probability of a randomly selected student being male is the same for both classes. We should choose classes 1/B and 1/D.

Table 8. Finalization Process for the "Weekend at the Amusement Park" Scenario

Implementation Process for the CG

In the CG, the lessons based on the learning outcomes of the probability content area in the curriculum (MoNE, 2018) were taught by the current mathematics teacher using the expository teaching method, one of the traditional, theory-based, meaningful learning approaches. In the mathematics curriculum, the use of the expository teaching method and techniques based on this approach is recommended for lesson implementation (MoNE, 2018). In this context, the expository teaching method (Ausubel & Robinson, 1969) was used in the CG without any additional intervention. This method is related to learners creating meaning from the material presented to them (Ausubel, 2000). Throughout this process, the teacher is responsible for selecting and organizing the lesson content, making it meaningful for the learner, and presenting and explaining it using various instructional materials.

Teaching probability in the CG lasted 12 lesson periods. The seating arrangement in the classroom was designed with individual desks placed in rows and columns, allowing each student to work independently. The lessons proceeded with the teacher's explanations, discussions, and feedback on students' understanding of the topic, while the students took notes. The class finished with the teacher providing feedback, summarizing the topic, and assigning homework.

Data Analysis

To reveal whether students' problem-solving skills differed by the approaches used, quantitative data analysis techniques were employed. First, open-ended questions in the PST were scored by two researchers using a PSS and the agreement percentage was calculated using the Miles and Huberman (1994) formula of $\frac{\text{Agreement}}{\text{Agreement+Disagreement}}$ x 100. The agreement percentage was 96%. For the remaining 4% difference, the two researchers discussed and reached a consensus, resulting in a full agreement (100%). Second, the scores analyzed according to the PSS were transferred to an SPSS file. Subsequently, an analysis was performed to determine if there was a statistically significant difference in the scores between the EG and CG, both within and between groups. Non-parametric techniques were used due to the small number of students in the groups and the non-normal distribution of the data. The Wilcoxon signed-rank test was used for dependent groups, and the Mann-Whitney U Test was used for independent groups. Additionally, the PST data of two students in the EG, who did not regularly attend the classes, were excluded from the analysis.

Significant clarification is essential to understanding the data analysis in this study. The scores obtained from each stage of the problem-solving process were separately analyzed using quantitative data analysis methods. Subsequently, an analysis based on the total scores obtained from the four stages was performed. Thus, determining the impact of PBL on each stage independently became possible. This analysis was useful in identifying the stage or stages of the problem-solving process with which the EG and CG students struggled.

Identifying the stages where students had difficulties was crucial. If a student does not fully understand a problem, they cannot propose a solution, nor can they carry out or interpret the solution. On the other hand, fully understanding the problem does not guarantee a solution will be generated. However, if a student cannot find a solution, understanding whether this result is from a lack of comprehension or a deficiency in the procedural or conceptual knowledge required for the solution can be helpful. Thus, gaining insights into the approach appropriate for students becomes possible. A student who struggles during the phase of carrying out the plan may have deficiencies in subject knowledge or may not fully understand the solution algorithm. Even if a student achieves the correct solution, they may face difficulties in the phase of looking back to see whether the solution is logical. In such cases, linking problems with real-life situations is crucial. This connection enables them to assess whether the solution has practical relevance in real life, thereby helping them to consider the logical validity of the solution. Difficulties encountered in any of these stages can prevent the problem-solving process from successfully completing. Therefore, identifying at which stage or stages the process is hindered is important. Consequently, this study examined the impact of the PBL approach on each stage of the problem-solving process separately.

A content analysis—a qualitative data analysis technique—was performed to analyze the data obtained through the SSIF, which gathered the views of the EG students on the PBL process. In content analysis, certain parts of a text are summarized using fewer and more concise words within a systematic framework and set rules (Karadeniz et al., 2020, p. 288). In this study, the content analysis involved transcription of the recorded interviews. Subsequently, the written transcripts and audio recordings were independently coded by the researchers. The agreement rate between categories and codes created through independent coding by the researchers was calculated using Miles and Huberman's (1994) agreement percentage formula, resulting in an agreement percentage of 93%. The researchers then met to discuss and reconcile differing codes, achieving full agreement and thus ensuring coding reliability.

Results and Interpretation

The findings and interpretations derived from the data collected throughout the research are presented in this section of the study. For this study, the problem-solving skills score was obtained by considering the sum of the scores from each phase of problem-solving. Therefore, problem-solving skills were referred to as *problem-solving skills* (general).

Results of the ET

The normality test results performed to show the equivalence of the EG and CG are given in Table 9.

Shapiro-Wilk			
Groups	Statistic	df	Sig
CG	.959	21	.503
EG	.926	20	.130

Table 9. Normality Test Results Based on ET Scores of the EG and CG

Table 9 shows that EG's and CG's ET scores showed a normal distribution (p_{EG} = .130, p_{CG} = .503). The results of the independent samples *t* test are presented in Table 10.

Groups	*N*	*M*	*SD*	*t*	*p*
CG	21	15.38	3.138	1.046	.302
EG	20	14.15	4.332		

Table 10. T-test results Based on ET Scores of the EG and CG

The *p* value of the independent samples *t* test performed between the EG's and CG's ET scores was found as .302. Since the value was greater than .05, the conclusion was that no significant difference existed between the EG and CG before the intervention [$t_{(39)}$ = 1.046, p = .403 > .05]. In this case, it can be suggested that the EG and CG were equivalent before the implementation of the PBL approach.

Based on the ET results, students in both groups were found to be equivalent in terms of their overall mathematics achievement levels. This conclusion indicates that the research findings were not influenced by any differences in the initial achievement levels of the students. This result also means that the research findings are limited to the implemented approaches and the small sample size.

Findings and Interpretations from PST Pre-Test and Post-Test Scores of EG and CG

PST Pre-test Scores

A Mann-Whitney U test was performed to reveal if a statistically significant difference existed between the PST pre-test scores of the students in the EG and the CG. The results of this test are shown in Table 11.

Problem-Solving Phases	Group	*N*	*M*	*SD*	Mean Rank	Rank Sum	*U*	*Z*	*p*
Understanding the Problem	CG	21	13.33	7.16	18.83	395.50	164.500	-.691	.489
	EG	18	14.67	8.64	21.36	384.50			
Devising a Plan	CG	21	10.95	6.47	21.38	449.00	160.000	-.819	.413
	EG	18	10.22	8.38	18.39	331.00			
Carrying Out the Plan	CG	21	10.57	6.30	20.79	436.50	172.500	-.467	.641
	EG	18	9.66	7.32	19.08	343.50			
Looking Back	CG	21	8.28	4.48	20.90	439.00	170.000	-.537	.591
	EG	18	7.55	5.97	18.94	341.00			
Problem-Solving Skills (General)	CG	21	43.14	23.47	20.02	420.50	188.500	-.014	.989
	EG	18	42.11	28.07	19.97	359.50			

Table 11. Mann-Whitney U Pre-Test Analysis Regarding Problem-Solving Skills of EG and CG

The pre-test analysis results regarding the problem-solving skills of the EG and CG students can be seen in Table 11. This table presents the Mann-Whitney U test results for each phase of problem solving based on the pre-test scores obtained from the PSS by the EG and CG. According to the results, no significant difference was found between the two groups in any of the problem-solving phases: understanding the problem (U = 164.500, p > .05), devising a plan (U = 160.00, p > .05), carrying out the plan (U = 172.500, $p > .05$), and looking back (U = 170.000, $p > .05$). The U test results regarding the sum of the pre-test scores for all phases of problem-solving for both groups also indicated no significant difference between the two groups (U = 188.500, $p > .05$). This finding suggests the EG and CG were equivalent in terms of problem-solving skills before the intervention.

PST Post-Test Scores

The Mann-Whitney U test was performed to reveal any statistically significant difference between the PST post-test scores of the EG and CG students. The results showing whether a significant difference existed are shown in Table 12.

Problem-Solving Phases	Group	*N*	*M*	*SD*	Mean Rank	Rank Sum	*U*	*Z*	*p*
Understanding the Problem	CG	21	16.57	6.35	13.90	293.00	61.000	-3.610	.000
	EG	18	25.61	6.53	27.11	488.00			
Devising a Plan	CG	21	13.81	5.42	13.43	282.00	51.000	-3.897	.000
	EG	18	23.11	7.38	27.67	498.00			
Carrying Out the Plan	CG	21	12.28	4.98	14.31	300.50	69.500	-3.373	.001
	EG	18	20.05	6.67	26.64	479.50			
Looking Back	CG	21	9.71	4.82	16.02	336.50	105.500	-2.346	.018
	EG	18	14.05	6.01	24.64	443.50			
Problem-Solving Skills (General)	CG	21	52.38	19.64	14.00	294.00	63.000	-3.552	.000
	EG	18	82.83	24.89	27.00	486.00			

Table 12. Mann-Whitney U Post-Test Analysis Regarding Problem-Solving Skills of EG and CG

Table 12 presents the post-test analysis results for problem-solving skills of the EG and CG groups. Specifically, this table provides the Mann-Whitney U test results for each phase based on the groups' post-test scores obtained from the PSS. These results revealed a significant difference in each phase of problem-solving between the two groups: understanding the problem (U = 61.000, $p < .05$), devising a plan (U = 51.000, $p < .05$), carrying out the plan (U = 69.500, $p < .05$), and looking back (U = 105.500, $p < .05$). According to Table 12, the U-test results regarding the sum of the post-test scores for all phases of problem-solving skills for both groups revealed a significant difference between the groups (U = 63.000, $p < .05$). On the other hand, the mean ranks made it evident that the EG students scored higher than the CG students in each phase—understanding the problem, devising a plan, carrying out the plan, and looking back—as well as in problem-solving skills (general). This finding indicates that the PBL approach was effective in improving problem-solving skills in each phase and overall.

Findings and Interpretations of Wilcoxon Signed-Rank Test Analysis Results for PST Pre-Test and Post-Test Scores of EG and CG

EG PST Pre-test and Post-test Scores

Table 13 presents the Wilcoxon signed-rank test results, which indicates whether a statistically significant difference existed between the pre-test and post-test scores of the EG students' problem-solving skills.

Problem-Solving Phases	Post-test – Pre-test	*N*	Mean Rank	Rank Sum	*Z*	*p*
Understanding the Problem	Negative Rank	0	.00	.00		
	Positive Rank	18	9.50	171.00	3.728*	.000
	Equal	-	-			
Devising a Plan	Negative Rank	0	.00	.00		
	Positive Rank	18	9.50	171.00	3.724*	.000
	Equal	-	-	-		
Carrying Out the Plan	Negative Rank	1	1.00	1.00		
	Positive Rank	16	9.50	152.00	3.584*	.000
	Equal	1	-	-		
Looking Back	Negative Rank	0	.00	.00		
	Positive Rank	17	9.00	153.00	3.628*	.000
	Equal	1	-	-		
Problem-Solving Skills (General)	Negative Rank	0	.00	.00		
	Positive Rank	18	9.50	171.00	3.724*	.000
	Equal	0	-	-		

*Based on negative ranks

Table 13. Wilcoxon Signed-Rank Test Results for EG PST Pre-Test and Post-Test Scores

Table 13 revealed a statistically significant difference between the pre-test and post-test scores of the EG students in the phases of understanding the problem ($z = 3.728$, $p < .05$), devising a plan ($z = 3.724$, $p < .05$), carrying out the plan ($z = 3.584$, $p < .05$), and looking back ($z = 3.628$, $p < .05$). Additionally, considering the total scores of each phase before and after the intervention, a significant difference was found regarding their problem-solving skills (general) ($z = 3.724$, $p < .05$). This result indicates that the PBL approach improved the problem-solving skills of the EG students. The results in Table 13 show that the PBL approach is effective in developing each phase of problem-solving and general problem-solving skills of the EG students.

CG PST Pre-test and Post-test Scores

The Wilcoxon signed-rank test results, indicating whether a statistically significant difference occurred between the pre-test and post-test scores of the CG students' problem-solving skills, are presented in Table 14.

Problem-Solving Phases	Post-test – Pre-test	*N*	Mean Rank	Rank Sum	*Z*	*p*
Understanding the Problem	Negative Rank	7	6.57	46.00		
	Positive Rank	12	12.00	144.00	1.975*	.048
	Equal	2	-	-		
Devising a Plan	Negative Rank	7	6.71	47.00		
	Positive Rank	13	12.54	163.00	2.174*	.030
	Equal	1	-	-		
Carrying Out the Plan	Negative Rank	8	9.38	75.00		
	Positive Rank	13	12.00	156.00	1.413*	.158
	Equal	0	-	-		
Looking Back	Negative Rank	7	7.29	51.00		
	Positive Rank	11	10.91	120.00	1.506*	.132
	Equal	3	-	-		
Problem-Solving Skills (General)	Negative Rank	7	8.71	61.00		
	Positive Rank	14	12.14	170.00	1.896*	.058
	Equal	0	-	-		

*Based on positive ranks

Table 14. Wilcoxon Signed-Rank Test Results for CG Pre-Test and Post-Test Scores

Table 14 indicates a statistically significant difference between the pre-test and post-test scores of the CG students in the phases of understanding the problem (z = 1.975, p < .05) and devising a plan (z = 2.174, p < .05). However, according to Table 14, no statistically significant difference occurred between their pre-test and post-test scores in the phases of carrying out the plan (z = 1.413, p > .05), looking back (z = 1.506, p > .05), and problem-solving skills (general) (z = 1.896, p > .05). The results showed that the expository teaching method did not improve the students' skills in the phases of carrying out the plan, looking back, and problem-solving skills (general), but it contributed to the phases of understanding the problem and devising a plan.

Analysis of Data from SSIF

This research aimed to examine the impact of the PBL approach on students' problem-solving skills across each phase. In addition, example works performed by the EG students during the implementation phase are provided in Table 7 and Table 8. To better reflect the quantitative results of the study, the perspectives of the EG students on the implementation of the PBL approach were included. The students' views on the process indicated they absorbed the PBL approach, worked more effectively, and believed their knowledge was more permanent because they learned by experiencing and researching rather than memorizing. Although the results of this study—conducted with a limited sample—cannot be generalized, the students' opinions on the process suggest that the research findings are of significant importance. In this context, the EG students' views on the implementation can be summarized as follows.

A large majority of the EG students responded positively to the following questions:

- What do you think about teaching probability through scenarios?
- Did the teaching help you, and if so, in what ways?
- Did you face any difficulties during the study? If so, at which phases did you struggle and how did you overcome these difficulties? What did you do?
- How had previous mathematics lessons been conducted?
- Is there a difference, and if so, what is it? Can you explain?
- What did you learn from group work?
- Would you like future mathematics lessons to be conducted this way? Can you explain why? Do you think you have learned probability?

Students reported that the PBL approach helped them understand the lessons, improved their group work skills, made the lessons enjoyable, and resulted in more enduring knowledge. Similarly, they stated that the implementation allowed them to express themselves better, comprehend the topic more thoroughly through collaboration and discussions, and work in harmony. The students also mentioned that through group work and self-study, they grasped the relevance and importance of the topic in daily life, found that linking the topic with real life helped their understanding, and felt that group work boosted their confidence and made the lessons more fun. They also noted that they filled gaps in their knowledge and improved their friendships by helping each other. Excerpts from the students' responses are presented below:

- "I tried to express myself. I asked my friends about the questions I didn't understand and learned very easily; it became more permanent."
- "I didn't have trouble with group work. Because the teacher was there, I asked questions. When examples were given, I understood and solved them better. I also got help from my group mates."

- "Yes, we sometimes discussed the questions with our friends when we had different answers to the same question."
- "I hadn't liked group work before. But in this exercise, it became very enjoyable."
- "When we had different answers, I discussed mine with my group mates. These discussions made me reflect on my answers and realize my mistakes."
- "I believe that if a probability problem arises in daily life, I can solve it."

When comparing the current teaching with previous mathematics lessons, the students noted that the new approach was better. They asserted that the previous lessons lacked opportunities for group work and discussions, they could not ask the teacher questions freely, and they were not given the chance to speak whenever they wanted. On the other hand, the students emphasized that in the current teaching approach, they were able to discuss the topic; received opportunities to speak freely; shared their work results with their group mates and the rest of the class; and asked the teacher questions more comfortably, leading to a better learning experience. They also expressed frustration with the traditional methods, finding them tiring and difficult to understand. In response to the question "Do you think you have learned probability?" most of the students confirmed they had indeed learned. A few students claimed initial difficulties but reported a better understanding of the topic over time. Excerpts from the students' responses are as follows:

- "I had already learned the topic before the teacher explained it because we had done research, worked on our own, and done group work. These activities were very helpful; I learned very well."
- "Initially, I didn't understand, but later I learned the topic; it was very good."
- "In our previous mathematics lessons, the teacher would write questions on the board, we would solve them and write them in our notebooks. But I learned better with scenarios, and it was stuck in my mind."
- "In group work, I learned probability. When I didn't know certain questions, one of my friends helped me. I learned by working with friends within the group."
- "Absolutely, it became more permanent. The previous mathematics lessons had been very difficult, and I hadn't enjoyed them; I hadn't understood. But this time, the lessons went very well for the first time."
- "The topic of probability was well understood, and during the group work, everyone in our group spoke and we discussed all the questions. Group work made it more catchy."
- "I realized at home that I could easily solve probability problems, whereas I couldn't do tests on other topics."
- "I think using scenarios had a very positive effect on me. We used to just write down what was on the board in our notebooks, but this time we discussed the topic better together."
- "At the beginning, I was against this approach, but over time it helped me solve tests. I realized it had an incredible contribution. In previous mathematics lessons, I couldn't comfortably ask about topics or problems I didn't understand, but this group work was effective in learning the topic."
- "We didn't use to do group work with each other before. We couldn't discuss the topic during lessons. But during these lessons, I could ask both my friends and the teacher questions; group work was great."

Conclusion and Discussion

This research aimed to examine the impact of the PBL approach on middle school students' problem-solving skills in learning probability. In this section, the research findings first are summarized, and the limitations of the research are discussed. Then, the results are analyzed considering the existing literature. Based on these limitations, various recommendations are provided to serve as conclusions of this study and to be used in future research.

The study revealed a statistically significant difference between the problem-solving skills of the students in the EG, who were taught with the PBL approach, and the skills of the students in the CG, who were taught with the traditional expository teaching method. In the study conducted using Barrows' (1986) closed-loop PBL model and Lambros' (2004) PBL list, the significant difference was in favor of the experimental group. This study was conducted with a small sample size over a 4-week period. The results of the study should be evaluated in the context of these limitations. The most significant conclusion of this study is that effectively integrating the closed-loop PBL model with PBL lists plays a crucial role in enhancing students' problem-solving skills.

Additionally, using scenario-based PBL to bring probabilistic real-life situations for students into the classroom contributes to the learning of mathematical topics—such as probability—which require advanced reasoning. In modern societies, individuals must be able to make decisions in uncertain, probabilistic, and risky situations they may encounter in daily life. To do so, they need to analyze random variables, evaluate options, and predict developments. This necessity requires logical reasoning, which, in turn, depends on probabilistic reasoning. In this study, the problem scenarios derived from students' everyday experiences were presented using the PBL approach and applied to the EG for teaching probability. These problem situations were used as tools to enhance students' problem-solving skills. In this research, which compared the PBL approach with the traditional expository teaching method, statistical analyses revealed a significant difference in favor of the EG. This result aligns with findings from previous studies on the effectiveness of PBL (Er Nas, 2008; Süğümlü, 2009; Dorimana et al., 2022; Yahya & Zaman, 2008; Zamir et al., 2022).

These studies also demonstrate that PBL has a significant and positive impact on EGs. A common feature of these studies is the strong connection established between the lesson content and real-life scenarios, which facilitates learning and helps students solve problems more easily. This similarity positively influenced our research, contributing to the improvement of problem-solving skills in EG. Additionally, previous research indicates that implementing the PBL approach enhances high school students' mathematical problem-solving skills (e.g., Dorimana et al., 2022; Zamir et al., 2022) and enables them to develop the MathS-Set instructional design model (e.g., Yahya & Zaman, 2008).

The findings indicated that the EG students were more successful in each phase of the problem-solving process and problem-solving skills (general) compared to the CG students. The study concluded that the PBL approach was more effective than the expository method in teaching probability to the eighth graders. This result can be attributed to the students' active engagement in the PBL process and the provision of learning settings that encouraged them to take responsibility for their learning. Throughout this process, the researcher guided the students, providing support and helping them stay within the context when needed. Thus, with PBL, the students had opportunities for both individual and group work (Delisle, 1997). The sharing of information, group discussions, and feedback sessions throughout the process prevented incomplete and incorrect learning. This finding supports Collins's (1994) view that presenting content in a narrative format enhances students' active participation in the learning process, making PBL effective. The students were able to articulate their ideas better, generate various solutions to problems, and decide on the best solution through group discussions (Schoenfeld, 1985).

Furthermore, the integration of Barrows' (1986) closed-loop PBL model with the lists recommended by Lambros (2004) contributed significantly to these results. The use of the closed-loop PBL model provided students with the opportunity to reflect on the problem-solving process through group work. This reflective process allowed them to review their actions throughout the study and correct any deficiencies or errors. Following the scenario activities, the students in the EG used activity worksheets to review the initial problem in the scenario, which allowed them to reinforce their learning and complete the cycle. The research findings indicated that the PBL approach significantly impacted the development of students' skills in each phase of problem-solving. These results were also supported by students' expressions during the implementation process. The students' feedback shows that they embraced the PBL approach, recognized the significant contributions of group work to their learning, and felt more comfortable and effective compared to previous lessons. The effectiveness of the PBL approach in enhancing learning by making students more active supports the view of Rybarczyk et al. (2007) that PBL is an effective method for increasing learning engagement.

In this study, it was observed that the expository teaching method used in the CG did not result in significant differences in students' problem-solving skills based on pre-test and post-test scores. On the other hand, it is important to note that the lessons for the EG were conducted by one of the researchers, while the lessons in the CG were taught by the students' current mathematics teacher. The CG students were accustomed to their teacher and had extensive experience with the teacher, making the significance of the research findings more apparent. In traditional methods, the teacher explains, and students take notes and attempt to internalize what they have learned independently. During this process, students may ask questions and occasionally express their ideas, but they may not have sufficient time to articulate their thoughts or engage in discussions to defend their arguments. This lack of time and opportunity may lead to learning losses or incomplete and incorrect learning. In contrast, in PBL, learning by doing is emphasized. Therefore, despite the CG students' familiarity with their teachers, the lack of emphasis on learning by doing in traditional approaches may have made problem solving more challenging for them. Kindley (2002) argues that learning in PBL can occur at the levels of analysis and synthesis (as cited in Sorin, 2013). Thus, PBL facilitates students taking responsibility for their learning, making efforts to acquire knowledge, constructing new knowledge by using background knowledge, and defending their arguments and solutions, thereby internalizing new knowledge. This theory supports the results obtained in our research.

The findings indicated that the PBL approach had a positive impact on each phase of the problem-solving process and problem-solving skills (general), as concluded by the comparison of post-test measurements with pre-test measurements in EG. In contrast, the CG, which employed an expository teaching method, showed a significant improvement only in the phases of understanding the problem and devising a plan. However, no significant differences were observed in the phases of carrying out the plan, looking back, or overall problem-solving skills. This result suggests that the expository teaching method was effective in the phases of understanding the problem and devising a plan but did not significantly affect the phases of carrying out the plan, looking back, or overall problem-solving skills. Furthermore, a noted inference is that the prior experience of CG students with problem-solving processes was beneficial during the stages of understanding the problem and devising a plan.

One reason why CG students were unsuccessful in the stages of carrying out the plan and looking back may have been their inability to grasp the topic of probability at both the procedural and conceptual levels. In traditional classroom settings, lessons taught with conventional methods that fail to convey the importance of learning often result in shallow learning and mere memorization (Duruhan, 2004; Schettino, 2016). In addition, further time was needed to enhance the problem-solving skills of the CG students. The potential lack of understanding of the topic at both the conceptual and procedural levels may have hindered the improvement of the students' problem-solving skills. Prior incorrect or incomplete knowledge (e.g., Figure 1) led to difficulties in the phases of devising a plan and carrying out the plan. Students who experienced the disadvantages of the expository teaching method struggled to completely grasp the topic, particularly given the higher level of qualitative understanding required for probability compared to other mathematical topics (Carter, 2005). Thus, learning a complex topic like probability becomes notably challenging in a traditional classroom setting where students predominantly listen to the teacher and often lack opportunities to express their thoughts. In this context, the lack of active engagement and the assumption of uniform student characteristics in the CG complicated their understanding of the topic, resulting in difficulties in the phases of carrying out the plan and looking back, despite success in understanding the problem.

The responses of some CG students to two questions in the PST (see Appendix B, Questions 3 and 9) after the intervention are significant in highlighting the difference in problem-solving skills between the EG and CG. One student in the CG answered incorrectly by disregarding the fact that "1 is not a prime number," while another student gave an incorrect response because they did not understand what the term *or* meant in the context of the problem. Since these concepts are fundamentally related to the ideas of sets and logic rules, the student's prior knowledge and readiness in probability likely influenced their ability to learn the topic. Fischbein and Schnarch (1997) state that one of the foundational concepts for learning probability is the concept of sets, which impacts how the topic of probability is learned.

This study examined the impact of the PBL approach on the problem-solving skills of the eighth graders using quantitative methods. This study was conducted over four weeks with a small sample size and focused exclusively on the teaching of probability. Another limitation of the study is the use of the closed-loop PBL model, one of the six different PBL models proposed by Barrows (1986), in combination with Lambros's (2004) PBL checklists. In this context, the effects of different PBL models on students' problem-solving skills could be investigated. Furthermore, the following recommendations can be made to mathematics teachers and researchers for future PBL studies:

- The PBL approach can be used in the problem-solving process in mathematics education. Qualitative research can deeply investigate the process.
- The PBL approach can be applied at different grade levels and on different topics in the problem-solving process, with results being compared.
- In the PBL process, to improve problem-solving skills, Barrows' (1986) closed-loop PBL model and Lambros's (2004) PBL lists can be combined and used to teach various mathematical topics.
- In the PBL process, where problem solving is targeted through instructional scenarios, learners can develop different solutions and models. In this context, mathematical problem-solving strategies and model-building processes of learners during PBL can be examined. As a continuation of this examination, the contribution of the stages of understanding the problem and generating strategies in the problem-solving process to mathematical literacy can be investigated.
- PBL provides learners with a free and constructive discussion environment. Building new and different knowledge in such environments suggests that, in addition to cognitive gains, mathematical beliefs can be strengthened, and the impact of PBL on mathematical communication can be studied. From this perspective, the development of mathematical language and terminology can also be facilitated.
- Considering that PBL structures learning in both sequential steps and a cyclical manner that forms a cohesive whole, it may help learners acquire 21st-century skills such as computational thinking. Research could explore the extent to which PBL supports computational thinking. Although this suggestion may not seem directly related to the study's focus, the similarities between problem solving and computational thinking make it valuable. By considering common points, various scenarios could be used to develop these skills.

Research in the literature indicates that traditional approaches are insufficient for effective mathematics instruction, particularly in developing strong problem-solving skills. One way to overcome this challenge is by using the PBL approach, in which complex real-life problems are used as a starting point in developing problem-solving skills. The result of this research reveals that the use of the PBL approach is effective in developing the problem-solving skills of eighth-grade students.

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Appendix A

Equivalence Test (Questions)

1) What is the result of $(15 – 6) + 3^2 + 5$ =?
A) 21 B) 22 C) 23 D) 24

2) What is the result of $75: 5 + 2^2. 3 - 5^2$?
A) 155 B) 72 C) 12 D) 2

3) If the difference between a number and 5 times itself is 40, what is 3 times this number?
A) 24 B)30 C) 36 D) 42

4) Since the sum of four consecutive numbers that are multiples of 2 is 108, what is the smallest of these numbers?
A) 30 B)28 C) 26 D) 24

5) Which of the following is the smallest?
A) $|-9|$ B) -1 C) +2 D) -8

6) Ahmet receives 10 liras from his father every day and spends 4 liras at the school canteen for lunch. How many liras will Ahmet have at the end of the fourth day?
A) 32 B) 28 C) 24 D) 20

7) Which of the following is a prime number?
A) 24 B) 14 C) 11 D) 6

8) What is the value of 5a + 2 for a = 7?
A) 35 B) 36 C) 37 D) 38

9) If Gamze buys 200 files of 0.05 lira each, how much will she pay?
A) 10 B) 15 C) 20 D) 30

10) What is the result of 1/3+ 2/5 =?
A) 3/8 B) 3/15 C) 11/15 D) 7/15

11)I spent 3/4 of 24 liras. How many liras do I have left?
A) 6 B) 12 C) 14 D) 16

12) Which of the following rational numbers is not between 3/5 and 5/7?
A) 22/35 B) 23/35 C) 24/35 D) 26/35

13) What is the result of $\frac{0,7}{0,07} + \frac{0,018}{0,006} - \frac{9}{0,9}$ =?
A) - 6 B) 0 C) 3 D) 5

14) What is the result of
$3.100 + 7.10 + 5.\frac{1}{10} + 8.\frac{1}{100}$ =?
A) 370,58 B) 37,58
C) 37,058 D) 375,8

15) If a vehicle travels 75 km with 10 liters of gasoline, how many kilometers will it travel with 40 liters of gasoline?
A) 100 B) 200 C) 300 D) 400

16) What is the perimeter of a square whose area is 100 cm^2?
A) 10 B) 100 C) 40 D) 400

17)

AB ∩ CD = {O} olduğuna göre, **s($\widehat{COB}$) kaç derecedir?**

A) 80 B) 100 C) 120 D) 160

18) What is the sum of the possible values of "a" for the number "427a" to be divisible by 2 without remainder?
A) 10 B) 12 C) 18 D) 20

19) What is the result of $\frac{1+\frac{2}{3}}{2-\frac{1}{3}}$ =?
A) 1 B) 2 C) 3 D) 4

20) What is the explementary angle of the angle whose measure is 35°?
A) 55° B) 65° C) 145° D) 155°

Appendix B

Questions of the Problem-Solving Test

Q.No/Question

1 Each letter of the word PROBABILITY is written on a piece of paper. The papers are put in a bag, and one is drawn. How many possible outcomes are there?
What is the probability of drawing the letter O?
What is the probability of drawing the letter L?

2 In a committee consisting of 3 men and 2 women, the chairman is chosen randomly from the committee members. If the event is "choosing a lady," how many outcomes does this event have?

3 In the experiment of rolling a dice, find the probability that the number that comes up is a prime number.

4 Could there be a 5/3 probability of an event happening? Why?

5 Two marbles are drawn one after the other from a bag containing 5 red and 4 blue marbles of the same size, and they are not thrown back. Find the probability that both drawn marbles are blue.

6 There are 4 balls marked 1, 3 balls marked 2, and 6 balls marked 3 in a bag. What is the probability that the ball drawn is less than 2? What are the outputs of this question?

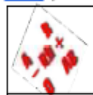
7 A rabbit at the crossroads heads for one of these roads at random. Each of these four roads divides into two narrow paths. When the rabbit comes to a fork in the road, it chooses one of the two narrow roads randomly and continues on its way. What is the probability that the rabbit will meet the turtle waiting on one of the narrow roads?

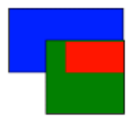
8 There are 36 coloured beads of the same size in a bowl. Some of these beads are blue, some are green, and the rest are yellow. If a bead is drawn from the bowl without regarding its colour, the probability of being blue is 4/9. How many blue beads are there in the bowl?

9 There are 24 students in a class. 10 of these students are female students. 5 of the female students and 6 of the male student's wear glasses. What is the probability that a student selected from this class is male or with glasses?

Q.No/Question

10 The blood types of the students in a class are determined and given in the table below. What is the probability that the blood group of a randomly selected student from this class is 'A Rh(+)' or '0 Rh(-)'?

	Kan Grubu							
	A Rh (+)	A Rh (-)	B Rh (+)	B Rh (-)	AB Rh (+)	AB Rh (-)	0 Rh (+)	0 Rh (-)
Kişi Sayısı	10	6	5	0	3	3	2	1

11 
The side lengths of the blue garden in the figure are 6m and 10m. The side lengths of the red garden are 2m and 4m. The side lengths of the green garden are 7m and 8m. When a ball is thrown, what is the probability of landing in the green or red garden?

12 Below are the numerical lotto coupons played by some people. Which of these coupons do you think is more likely to gain?

a) 1,2,3,4,5,6	e) 44,45,46,47,8,49
b) 5,10,15,20,25,30	f) 1,10,20,30,40,49
c) 2,14,18,30,36,44 d) 2,11,18,23,37,48	g) All are equal

13 A and B are two boxes. There are red and blue marbles in these boxes. You want blue marbles. You have to choose marbles without looking inside the boxes. Which box would you choose?

box a ☐ box b ☐ all the same ☐

A
7 BORDO
3 MAVİ

B
70 BORDO
30 MAVİ

14 A father has 4 daughters and a son. What are the chances of his newborn child being a male student?

Appendix C

Progressive Scoring Scale

Problem		
A		*Understanding the Problem*
	3	Understanding the problem entirely
	2	Understanding some points of the problem
	1	Being not able to understand the problem
	0	Not showing any efforts to understand the problem
B		*Devising a Plan (Choosing a Strategy)*
	3	Choosing a strategy that can lead to an appropriate solution
	2	Choosing only one part of strategy that can help solution
	1	Choosing an inappropriate strategy
	0	Not choosing any strategies
C		*Carrying Out the Plan*
	3	Finding suitable and correct solution
	2	Finding a solution which is partly correct
	1	Finding a solution which is inappropriate and incorrect
	0	Not being able to find any solutions
D		*Evaluation*
	3	Solving the problem and newly created problem based on it
	2	Verifying the results reasonably
	1	Verifying the results partially
	0	Not knowing how to verify the results

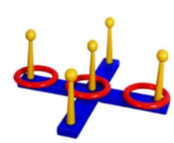
Appendix D

Scenarios and Activities

SCENARIO 1. WEEKEND AT THE AMUSEMENT PARK

(URL-1, 2022)

Arda, Sinan, Melis, and Umut go to the amusement park at the weekend. After getting into the bumper cars, they want to play the ring toss game. Sinan, who has a broken arm, watches his friends. Umut makes 36 tosses and manages to put 9 rings into the target. Melis makes 95 tosses and puts 19 rings. Arda makes 64 tosses and puts 16 rings. Here are a few questions regarding this scenario. Sinan and his friends are curious about your opinions on this matter; please help them.



(URL-2, 2022)

1) Have you ever been to an amusement park? If you have been, share with your friends what you went through, the entertainment vehicle that interested you the most, and your experiences.
2) Have you played the ring toss game before? If you have played, write about your experiences. How many tosses did you make, and how many rings did you put in the target? Could you share it with your classmates?
3) Sinan, who is watching his friends, is confused. He wants to prepare a Table using his non-broken arm with the information at hand. How can you help Sinan?
4) Umut, Melis, and Arda, three of the four friends who wanted to spend the weekend at the amusement park, made tosses. Who do you think hit the target the most and hit the least? Explain the reason for your answer.
5) Whom do you think Sinan should support in this game? Explain why.
6) If an equal number of tosses are made, who would hit the target the most?
7) What conclusion have you drawn from what you have done so far? Express your result mathematically.
8) Can you also write about a problem for the situation given in the scenario above? Solve the problem you wrote about.