On Adapting Theoretical Models from the Work of David Lewin

Michael Cherlin

The writings of David Lewin have that rare quality that forces us to re-evaluate that which had seemed familiar and well understood. The simplest and most ubiquitous ideas—interval, transposition, inversion—basic concepts that fill our musical imaginations at every turn, are recast. That which was familiar and easy takes on new meaning. Processes of reorientation are always difficult.

Yet here is an irony. Just because Lewin's thought cuts so deep, just because it addresses those aspects of our musical imaginations that are most fundamental, his ideas pertain to every level of our musical education. In the present paper, I attempt to express Lewin's thought in ways that might be accessible to an audience larger than those stalwart professionals in music theory who have come to appreciate Lewin's particular genius. Within the confines of this paper, I do little analysis except by way of explanation. I concentrate on the orientation that the theories provide us with, and I use examples only by way of explanation. The analytic cogency of Lewin's thought should be vivid nonetheless.

Useful Metaphors

Some colleagues and I first studied David Lewin's book, *Generalized Musical Intervals and Transformations*, during the summer of 1987 as a group project (no pun intended).[1] Early on, one member of the group puzzled about the title, one that sounds somehow as if it were translated from the German. It eventually became clear that the title expresses as succinctly as possible those two modes of thinking about music that are most fundamental to the text. The two basic models, formalized as Generalized Interval Systems and Transformation Graphs and Networks, provide the larger contexts within which musical objects and relationships are conceived. In fact, it does not seem to be misplaced to refer to the two modes as formalizing two basic, controlling metaphors through which we create and interpret music.

Without going into details as of yet, we can begin to clarify the nature of those metaphors. First, we can conceptualize musical objects (principally notes, time points and durations), and we can conceptualize a distance traversed, a move or measurement that carries the listener or thinker from object to object or that measures the distance between objects. Thus conceptualized, music is made analogous to extended space. We move between musical events much like we move between spatial objects in the physical world. Music thought of this way is likened to architecture or to geography, or even to cosmology for that matter. Within the larger metaphor, intervals thought of as measurements make an analogy to schematics or maps that represent relevant space as laid out on some grid. Intervals thought of as moves, or spaces traversed, are analogous to mental or physical travel, wherein all other locations are always relative to some present location.

In contrast, a transformation model is more analogous to organic growth and decay, wherein things exist in a state of flux. Hence, moves are not made from object to new object but rather from one state of being to a new state of being. A musical element or Gestalt becomes

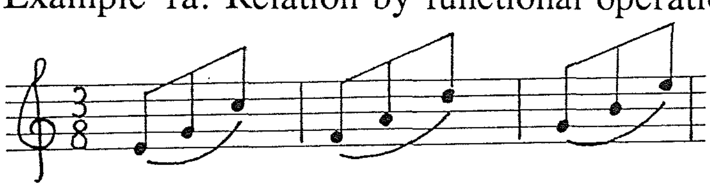
[1]David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987). The theoretical models discussed in this paper are formalized and applied in this book (henceforth GMIT).

another through modification, be it far ranging or subtle. And there is a shift in emphasis from relations among objects to processes of transformation.

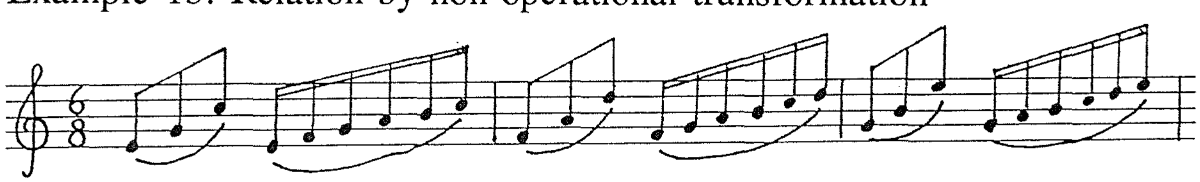
Before getting too carried away by our metaphors, however, we must admit and emphasize that Lewin's formalisms are not tied to the metaphors that we have placed onto them. More formally, the functions in both intervallic and transformational systems are defined as transformations. The formal transformations in an intervallic system, however, are restricted functions that move each element in an argument to a unique element in a value, with nothing left over on either side of the relationship. As a result of this property, each intervallic move has an inverse, a move that "undoes" it. In contrast, and in addition to other formal distinctions, a transformational system allows functions that do not move its arguments onto unique values, and it allows functions that do not have an inverse. The transformations in an intervallic system are named operations. The transformations in a transformational system may or may not be operations.

Example 1, which shows two types of familiar musical transformations, will clarify the distinction we have just made. In Example 1a the successive arpeggios, diatonic transpositions of one another, are related by a functional operation. In Example 1b the arpeggios are filled in with passing tones to form scalar gestures. Since there is not a l-to-1 relationship as we move from arpeggio to scale, the transformation in Example 1b is not an operation in the formal sense.

Example 1a. Relation by functional operation



Example 1b. Relation by non-operational transformation



Among the ways we might understand the pattern at Example 1a is through an interpretation that hears first inversion triads spanning a scalar ascent, E-F-G, in the bass. In our formal as well as in our metaphorical sense, that would be an intervallic hearing. In contrast, if we hear the second and third arpeggios as transformations of the first, in the sense that E-G-C becomes F-A-D which in turn becomes G-B-E, the three objects of our first hearing are reduced to one object that undergoes transformations. Although intervals still obtain in the formal sense, in our metaphoric sense our hearing is transformational. At Example 1b the association of arpeggio and scale is easily heard as one is becoming the next. We hear the example transformationally both formally and metaphorically. The alternating arpeggios and scales can each be related to one another by intervallic measures, but in doing so the cogent transformation of arpeggio into scale is ignored.

In music, as in other forms of experience, both modes of understanding, transformational and intervallic, constantly interact. For example, we typically experience and describe themes and motives that modify over time, thus thinking transformationally. In contrast, we typically describe the notes and rhythmic durations that comprise those motives and themes to be composed of discrete elements related to one another by temporal and spatial measurements, thus thinking intervallically. To a remarkable degree, we mix our metaphors, and slip without much forethought from one mode into the other. Although some musical relationships seem more simple to think of in terms of objects related by intervals, while others seem more intuitive as musical transformations, many relationships can be conceptualized either or both ways. Musical relationships are notoriously evanescent, and blurred distinctions are not necessarily the outcome of sloppy thought. Nonetheless, at the very least, an awareness of the alternatives should sensitize the analyst to those presuppositions that so profoundly affect the ways we hear and make sense of music.[2]

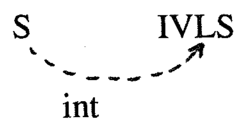
[2]Indeed, the argument can be made that analyses that restrict their reported hearings to a single mode of analysis falsify to the degree that compositions engage our imaginations in diverse and fluctuating ways.

Generalized Interval Systems and Transformation Networks

At this point we may move from general considerations to more specific descriptions, first of the Generalized Interval System and then of Transformation Graphs and Networks.[3] As shown at the top of Figure 1, the Generalized Interval System comprises three basic components.

Figure 1. Components of Generalized Interval System

GIS: (S, IVLS, int)



GIS is the acronym for Generalized Interval System. S, the first element of the ordered triple, denotes a family of musical objects. These are normally pitches, pitch-classes, time points, or durations, selected because they are suggestive in a given musical context. IVLS denotes the group of intervals within the system. Having said that, we should note that the description ''group of intervals'' can be misleading. The intervals of IVLS are abstract numerical relationships that in themselves are not assigned musical values. Essentially, IVLS may be thought of as an abstract grid upon which distances or moves between the objects of S will be measured. The group properties of IVLS assure certain relationships among intervals. Returning to our

---

[3] The following discussion assumes some background in mathematical semigroup and group properties. Lewin fully covers the theoretical grounding in the opening chapter of *Generalized Musical Intervals and Transformations*. In the context of classroom teaching, group properties can easily be introduced informally to musicians who are not comfortable with mathematical discourse. Thus, the cogent aspect of semigroup properties is that combinations of transformations in the semigroup reduce to simple transformations within the semigroup. To this, group property adds the constraint that every transformation must have an inverse, a way of ''undoing'' itself, and an identity, a transformation that leaves each object unmoved.

grid metaphor, we might say that certain kinds of relationships are insured as we move along the grid. These include that every move will have an inverse (a move that ''undoes'' it), that there will be an identity (an interval that leaves the musical object unmoved), and that there will be closure so that we will not be able to somehow move out of interval space into some relational never-never land. These properties correspond to the properties of operations that we have already discussed. The third component in the abstract GIS is denoted by the lower case int. While through S and IVLS we conceive our musical objects and the intervallic grid that will correlate those objects, int is the function that maps ordered pairs of musical objects into intervallic relationships. We might say that the function int places objects on the grid. The int function, mapping S into IVLS, is depicted by the curved, dotted arrow at the bottom of Figure 1. In order to qualify as a GIS structure, Lewin places two restricting conditions upon the int function. Condition one requires that if we measure or traverse an interval from a first object to a second and then measure or traverse an interval from the second object to a third the combined interval will be the same as that from the first object to the third. This property is shown on Figure 2.

Figure 2. Condition one of int

The interval from a to c having gone through b is equal to the interval from a to c.

a -----> b -----> c
a ---------------> c

The utility of this condition makes intuitive sense. Nonetheless, the quality of departing from a and arriving at c might be profoundly different from the quality of departing from a and going through b on the way to c. Perhaps it need not be said that the analyst needs to be cautious in measuring simple intervals when more complex contexts are involved.

Condition two for GIS structure requires that for every musical object *a* (members of S), and for every intervallic measure *i* (members of IVLS) there is a unique member of S that lies that interval i from a. In other words, if we can measure a distance on the grid from some musical object in the system then there must be another musical object (or the same object if the measure leaves the object unmoved) that will be uniquely placed in the relation that obtains. Thus, the objects of musical space and the intervals of intervallic space correlate completely.

Two simple examples of GIS structure borrowed from Lewin are shown in Figure 3.[4]

Figure 3. Examples of GIS structure

> A) S is the twelve pitch-classes under equal temperament; IVLS comprises integers under addition, modulo 12; and int(s,t) is the number of hours clockwise from one pitch-class s to pitch-class t.
>
> B) S is a family of durations; IVLS is a multiplicative group of positive numbers; int(s,t) is the quotient of the t and s measurements, t/s. If s spans 4 time units and t spans 3 time units, then int(s,t) = 3/4. t is "3/4 the length of" s.

Figure 3 brings up another aspect of Lewin's work. Musical intervals are generalized to include various types of pitch and pitch-class intervals and also various types of temporal intervals. The abstract structure of a GIS allows it to be extremely flexible in analytic applications. The analyst is not given an intervallic system a priori, but instead chooses to address the types of intervals that are musically suggestive or appropriate in a specific musical context.

The three components of a GIS force us to ask three questions, and these have therapeutic as well as theoretic value. When we select our family of objects to fill S, we ask not only what is to be included, but what is to be excluded and why. When we choose our abstract grid, we reject others that might also work and we become self-conscious of

[4]GMIT, 17, 28.

that decision as well. And finally, when we apply an operation that places musical objects into relationships we become aware of how our ideas of musical relationships are mutually conditioned by the interaction of our first two choices. The musical objects and the abstract group of intervals are conceptually distinct, and the structure of GISs reminds us of that distinction.

We may now consider the structure of Transformation Graphs and Networks. A Transformation Graph is the more abstract system that lies at the core of every Transformation Network. In turn, even more basic is the two-component system of NODES and ARROW that provides the foundation for a Transformation Graph. Two simple NODES-ARROW systems are shown in Figure 4.

Figure 4. NODES-ARROW system

O ------> O;   O <------ O

Within the NODES-ARROW system, the nodes, depicted by empty circles in the figure, may be conceptualized as place holders, so far devoid of musical content. The nodes are filled with content at the most concrete stage of tranformational logic as depicted in a Transformation Network. The NODES-ARROW system depicts an arrangement in time, read left to right. Thus, the node to the left of the arrow occurs before the node to the right. The arrow indicates a transformation of the node at its tail into the node at its head. In the primitive NODES-ARROW system the nature of this transformation is not yet assigned. That assignment takes place in the second logical stage, that of the Transformation Graph. The arrow pointing from left to right may be conceptualized as asserting an active transformation of the left-hand node into the right. Thus a left to right arrow indicates a relation something like, left goes to right by way of some transformation. In contrast, the arrow pointing right to left is passive. Thus, a right to left arrow indicates a relation something like right comes from left by way of some transformation.

The Transformation Graph adds two new components to the

NODES-ARROW System. This is shown at the top of Figure 5.

Figure 5. Transformation Graph

(NODES, ARROW, SGP, TRANSIT)

NODES ARROW SGP

TRANSIT

The third component, SGP, denotes a mathematical semigroup. That is, a family of elements and an associative binary composition with closure.[5] Like IVLS in the GIS structure, SGP here provides the abstract means to measure relations. There are two important differences, however. First, if we take the transformational metaphor seriously, it implies a change from something to something else and not simply a relationship that may be measured on some sort of grid. Second, a semigroup does not necessarily have all of the properties that obtain in a group. Namely, transformations in a semigroup do not necessarily include an inverse for each transformation; they do not necessarily include an identity operation and they are not necessarily l-to-1. Thus, semigroup properties give rise to transformations that are not necessarily operations, an aspect of transformational systems we had noted earlier. We will explore some of the ramifications of all this in due course.

Just as in a GIS we needed a function to map musical objects into the group of intervals, in the Transformation Graph we need a function that will map the relational ARROW into the semigroup of transformations. That function is named TRANSIT and it is listed fourth in the ordered quadruple that comprises a Transformation Network. The mapping of ARROW into SGP is depicted by the curved

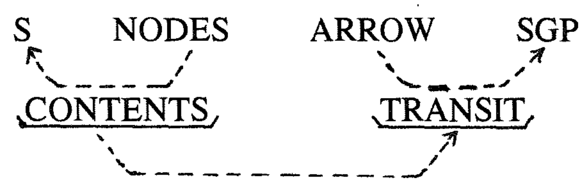
[5] A binary composition takes two inputs and produces one output. Closure assures that the output is a member of the semigroup. Associativity is the property a*(b*c) = (a*b)*c where * is a binary composition in the semigroup. Addition is a familiar binary composition that has the associative property.

arrow connecting them at the bottom of Figure 5. Since the contents of the nodes in a Transformation Graph are not assigned (they are still empty place holders), the same Graph may be used to correlate different realizations. This is suggestive compositionally as well as analytically.

A Transformation Network is shown in Figure 6.

Figure 6. Transformation Network

(S, NODES, ARROW, SGP, TRANSIT, CONTENTS)



The new components are: 1) S, which corresponds to the set of musical elements that might be presented in a GIS; and 2) CONTENTS, the function that maps nodes into members of set S and thus fills those nodes with musical content. The curved arrows at the bottom of Figure 6 show the functions CONTENTS and TRANSIT mapping NODES into S and ARROW into SGP. The functional relation between the two mappings is the underlying NODES-ARROW system.

In conjunction with Transformation Graphs and Networks, Lewin introduces a functional orthography that is contrasted with a more traditional labeling in Figure 7.

Figure 7. Two different labelings of a transformation

C Major Triad ----------> F Major Triad
TONIC

C Major Triad ----------> F Major Triad
DOM

The situation at the top of Figure 7 reflects the more traditional nomenclature. The tonicity of F major is asserted as the C Major Triad moves to its Tonic (arrow means: move to the tonic). The situation at the bottom of Figure 7 states the transformation differently. Here C major is given the force of a Dominant through the transformation that takes us to F major. The lower label thus names the functional transformation that the node on the left undergoes (arrow means: C undergoes a transformation that defines its function. C major becomes a dominant chord). In contrast to the transformational arrow DOM, the TONIC label at the top of the figure is more "thing" oriented. C major does not become a tonic but it goes to that which is tonic.

Figure 8 displays the alternative nomenclatures as they apply to arrows that point right to left. The example, adapted from Lewin, is meant to depict a situation such as that at a half cadence in C major.

Figure 8. Right-to-left labelings of a transformation

C Major Triad <---------- G Major Triad
TONIC

C Major Triad <---------- G major Triad
DOMINANT

The left-to-right temporal orientation of the nodes still holds in both cases of Figure 8. As we have said, the relational arrow, running right to left, can be characterized as a "passive" function. In a sense, the G major triad glances backwards to the previous tonic. The object orientation at the top thus names the arrow TONIC (i.e., G looks back in time to C, its tonic). The transformational orientation at the bottom names the arrow DOMINANT (i.e., G looks back in time to that which has transformed into a dominant by becoming G).

At this point, we can consider some of the formal distinctions between intervallic spans and other kinds of transformations. Transformation Graphs can depict essentially three different types of situations that cannot be expressed in simple intervallic terms: a) Two or more distinct transformations may span identical intervallic distances; b) Identical transformations may span distinct intervallic distances depending upon the musical Gestalt to which they are applied; and c) Transformations might elegantly express relations that cannot be measured in terms of simple intervals. In Figure 9 we consider an example of each case. Those functions labeled below each arrow are transformations; the functions labeled above each arrow are intervals.

Figure 9a. Distinct transformations - identical intervallic distances

5
C Major Triad ----------> A Minor Triad
REL

5
C Major Triad <---------- A Minor Triad
SUBMED

Figure 9b. Identical transformations - distinct intervallic distances

RI_0
A-B♭-E-D ----------> E-D-A♭-A
RICH

RI_2
C-B-A♭-G♭ ---------> A♭-G♭-E♭-D
RICH

Figure 9c. Transformation in which intervallic distance does not apply

{C♯,D,E♭,E,A} ----------> {C,F,F♯,G,G♯,B♭,B}
COMP

The intervallic labels in Example 9a are non-traditional and need to be explained. Our musical elements are the seven triads built upon the scale steps in the major mode. The group of intervals comprises integers under addition modulo 7. (That is, any sum larger than six is divided by seven with the remainder being our interval.) A unison spans interval 0, a move up one scale degree spans interval 1 and so forth. The intervallic function 5 measures the number of ascending steps from C to A. The two transformational functions express different functional relationships. REL, in this case, expresses the relationship of relative major to the triad at the right. (REL can also be applied to a minor triad moving to its relative major.) The functional transformation to A minor asserts the subordination of C major to it as its relative. (The network says that C becomes a relative major through the transformation that takes us to A minor.) In contrast, the SUBMED arrow points right to left—A minor transforms into submediant in its passive relation to C major. Thus, although the interval is constant in both cases of Example 9a, the transformational functions are very different.

Example 9b displays a context-sensitive transformation that Lewin has named RICH denoting a retrograde inverted chain. In RICH, the final two notes of any linear motive are preserved while the intervallic sequence is presented in reverse order. In other words, RICH results in the retrograde inversion that begins with the penultimate note of the shape being transformed. The intervallic analogue of RICH depends upon the intervallic structure of the motive. Thus, a retrograde inverted chain can display intervallic irregularities within transformational consistency.

The transformation in Example 9c displays an unordered pentachord moving into its complement, the heptachord that holds no notes in common with the antecedent pentachord. The transformation cannot be expressed as an intervallic move, yet transformations like it are common in much twentieth-century music.

The theoretic-therapeutic questions that are involved in selecting a GIS become more searching in our choice of a Transformation Network. Transformation Networks are highly interpretive by their very nature. The flexibility that semigroup transformations adds is matched by a more context-sensitive orientation toward functions, and this requires careful analytic choices with each move. The arrow/node foundation of the system, variably filled in with transit-functions and contents, potentially allows nodes to comprise anything from the simplest elements to large-scale "macro-structures."

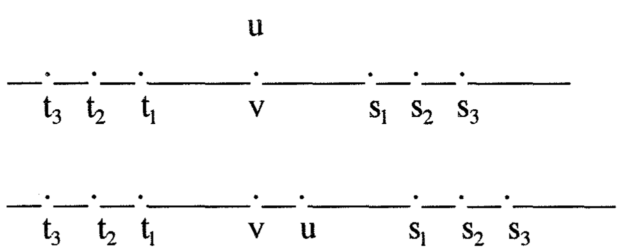
Generalized Inversion

Within the context of GMIT, inversion is introduced as an operation within GISs. Thus its formal definition is one bound up with the concept of interval. As we have noted, intervals are generalized to include not only spans or moves between pitches and pitch-classes but also spans or moves between time points, classes of time points, durations and classes of durations. Since inversion is defined as an operation on intervals it may be applied to each of the various types of intervals.

An abstract schematic for the inversional operation is given in Figure 10.

Figure 10. Inversional operation

$Iu/v; Iu/v(s_1) = t_1, Iu/v(s_2) = t_2, Iu/v(s_3) = t_3$



$int(t_3,v) = int(u, s_3); int(t_2,v) = int(u,s_2); int(t_1,v) = int(u,s_1)$

The figure traces intervals fron points t_3, t_2 and t_1 on the left to point v, and correlates those intervals with spans from point u to points s_1, s_2 and s_3. The inversion that maps u onto v will map s_1 onto t_1, s_2 onto t_2 and s_3 onto t_3. The same relations obtain if u and v coincide, or if u and v are distinct. Figure 10 can easily be read, for example, to denote a series of time points. The inversion that maps time point u onto time point v maps the time that is s_1 after u onto a time that is t_1 before v.

In Figure 11 the same model is applied to diatonic pitch inversion.

Figure 11. Inversion for diatonic pitches

Iu/v; $Iu/v(s_1) = t_1$, $Iu/v(s_2) = t_2$, $Iu/v(s_3) = t_3$

Where C = u = v:

C -	D -	E -	F		C -	B -	A -	G
u	s_1	s_2	s_3		v	t_1	t_2	t_3

Where C = u and A = v:

C -	D -	E -	F		A -	G -	F -	E
u	s_1	s_2	s_3		v	t_1	t_2	t_3

In Figure 11, the family of musical objects comprises the scale degrees of the C major diatonic collection, the group of intervals comprises integers under addition and the intervallic function counts the number of ascending scale steps between notes. The equations among the pitch intervals correspond to those temporal intervals we read in Figure 10. For example, where C = u = v, the space from B to C is one scale degree, as is the space from C to D. The inversion Iu/v maps B onto D (and D onto B). In contrast, where C = u and A = v, the span G to A correlates with C to D and the inversion Iu/v maps G onto D (and D onto G).

In Figure 12 "inversion" is applied to ratios among durations. This example departs remarkably from more restricted intuitions about musical inversion.

Here our family of objects comprises durations counted in eighth-notes. The intervallic function is s divided by u which maps by inversion onto v divided by t. The interval tells us how much a duration has shrunk or expanded. For example, s_1 last 2/3s as long as u and v lasts 2/3s as long as t_1.

So far, all of our examples of inversion can exist within GISs. We can also conceptualize inversional transformations that are excluded from interval systems. For example, inversional operations that are coordinated with the ordering of pitches or pitch-classes may be conceptualized as inversions mapping some ordinal position into another ordinal position. Figure 13 is derived from a network of transformations that Bach uses in his three-part invention in F minor. A cogent aspect of the transformation in each case is that the last pitch-class of each three note segment is held invariant in the succeeding segment. The TRANSIT function of our transformational arrows reflects this.

Figure 12. Inversional operation for durations

$Iu/v; Iu/v(s_1) = t_1, Iu/v(s_2) = t_2$

Duration in eighth-notes:

t_2	t_1	$v = u$	s_1	s_2
18	9	6	4	2

t_2	t_1	v	u	s_1	s_2
6	3	2	6	4	2

$int(u,s_1) = int(t_1,v) = 2/3$
$int(u,s_2) = int(t_2,v) = 1/3$

Figure 13. Non-operational inversional transformations

INVLAST
G - D♭ - C ----------> F - B - C

INVLAST
C - G♭ - F ----------> B♭ - E - F

INVLAST
B♭ - E - E♭ ----------> A♭ - D - E♭

The temporal inversion of larger Gestalts, such as first theme A followed by theme B and then theme B followed by theme A, a common procedure for example in Schoenberg's recapitulations, can also be notated by inversional transformations that are not operations.

In addition to the extended meaning given to inversion by the generalization of intervals, Lewin's model has the advantage of allowing us to name the key elements in an inversional operation, those elements we have been calling u and v, according to musical context. u and v might be at the center of inversional balance, or they might be chosen because they lie at registral extremes. u and v might be chosen because they are emphasized rhythmically or because they are emphasized in a serial context. Thus the notation for the inversional operation becomes analytically precise rather than theoretically universal.

Generalized Transposition Compared to Interval-Preserving Operations

Lewin makes a distinction between generalized transposition and interval-preserving operations. It is safe to say that this distinction, at least at first attempts toward understanding, will provide one of the most difficult problems for the majority of his readers. The reason for this is largely because in most musical contexts, transposition is precisely the operation that preserves intervals. For example, if I transpose Beethoven's Third Symphony to E major, all of the internal relations are preserved (that is, all temporal and pitch intervals remain the same as they interrelate internal to the piece), while everything is globally moved in relation to an external standard, say A = 440 Hertz. In a similar way, if I begin my performance a half hour later than anticipated, the internal temporal relations are not affected (assuming that other factors are not involved; for example, I might hurry through the piece because I am late!). My performance has been transposed one half hour later, internal intervals are preserved while all is globally transposed in relation to an external standard, say Eastern Standard Daylight Time.

Lewin's definition for transposition is easily followed:[6]

> 3.4.1 Definition: *transposition by i*, denoted by T_i; $T_i(s)$ is that unique member of S which lies interval i from s; int(s, $T_i(s)$) = i

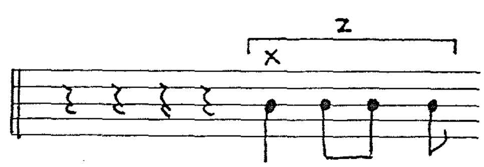
The definition for interval preservation also makes intuitive sense:[7]

> 3.4.6 Definition: given a GIS (S, IVLS, int), a transformation X on S will be called *interval-preserving* if for each s and each t:
> int(X(s),X(t))=int(s,t)

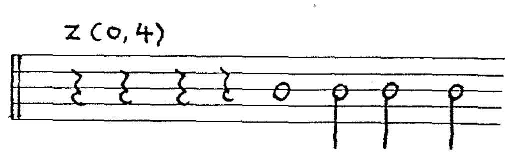
The distinction between transposition and interval preservation becomes particularly clear and particularly useful when the transformations involve intervallic placement (e.g., move some musical object to a position earlier or later in the piece) plus span (e.g., multiply the intervals internal to our object—duration or pitch interval, etc.—by some factor). Example 2 serves as an example.

Example 2. Transformations involving intervallic placement and time span

a.



b.

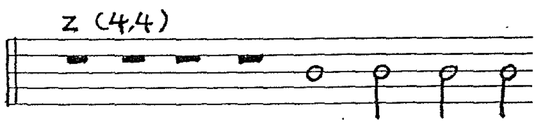


[6]Adapted from GMIT, 46.

[7]Adpated from GMIT, 48.

Example 2 (continued).

c.



Example 2a will serve as a source idea that will undergo transformation. The label x denotes a duration of quarter-note length, and z denotes the rhythmic figure quarter-note followed by three eighth-notes. In Example 2a the figure z begins four x-units after the double bar. The transformation is an ordered duple involving the placement and the duration of z. The ordered pair (0,4) applied to z denotes: 1) move z (by the transformation) zero (0) units, and then 2) multiply the durations in z by a factor of four. In transposition, z is transposed zero x-units and then augmented by a factor of four. Example 2b is the result. In contrast, interval preservation (Example 2c) requires that in transforming the Gestalt of Example 2a all of the relative intervallic spans internal to the passage remain intact. If z is to be augmented by a factor of four, then the x-unit, which places z in reference to the double bar, must also be augmented by that factor, before z is moved zero units.

Put another way, transposition maintains the intervallic unit (here x-spans) but does not maintain the ratios between the transformed object (here z) and the larger Gestalt (Example 2a). Confusingly, interval preservation does not preserve the interval size but rather preserves relationships among the intervals internal to some Gestalt. Interval preservation better describes transformations such as those in prolation canons, or such as those often used, for example, by Elliott Carter.

Interval Function

Our final topic is Lewin's interval function. Given a set of objects a and another set of objects b, the interval function counts the number of times some interval i is spanned between the sets. Once again, the generalization of musical intervals has powerful ramifications. The interval function may tell us of pitch or pitch-class intervals that are spanned, or of time spans between objects or of ratios between durations. The type of interval function we choose depends upon the type of GIS we operate within, and as we have said, that is an analytic choice. Just as important, the interpretation of data—what to make of the intervals that are spanned—is a complex analytic procedure. Generally speaking, in any musical situation, those intervals that are unique and those intervals that are most prevalent tend to be of interest. The prevalent intervals saturate a musical event and provide contextual normalcy. Unique intervals, like the tritone in tonal pitch-class intervals, or like the notes of long duration in a Bach chorale, generally have special status because of their distinction. But the generalization is rough at best.

Figure 14. Interpretation of the opening of Schoenberg's *Fourth String Quartet* using Lewin's interval function

<C B G A♭> ---> <E♭ D♭ D B♭> ---> <G♭ F E A>
--- x --- --- y --- --- z ---

a. Pitch intervals spanned from x to y

int(C,E♭) = int(B,D) = int(G,B♭) = 3;
IFUNC(x,y)(3) = 3

int(C,D) = int(B,D♭) = int(A♭,B♭) = 2;
IFUNC(x,y)(2) = 3

int(G,D♭) = int(A♭,D) = 6;
IFUNC(x,y)(6) = 2

int(G,D) = int(A♭,E♭) = 7;
IFUNC(x,y)(7) = 2

Figure 14. (continued)

int(C,D♭) = 1 int(C,B♭) = -2
int(B,E♭) = 4 int(B,B♭) = -1
int(G,E♭) = 8 int(A♭,D♭) = 5

where i is 1 or 4 or 5 or 8 or -1 or -2
IFUNC(x,y)(i)=1

for all other values of i, IFUNC(x,y)(i) = 0

b. Pitch intervals spanned from y to z

int(E♭,G♭) = int(D♭,E) = int(D,F) = 3
IFUNC (y,z)(3) = 3

int(E♭,F) = int(D,E) = 2;
IFUNC(y,z)(2) = 2

int(D♭,F) = int(D,G♭) = 4;
IFUNC(y,z)(4) = 2

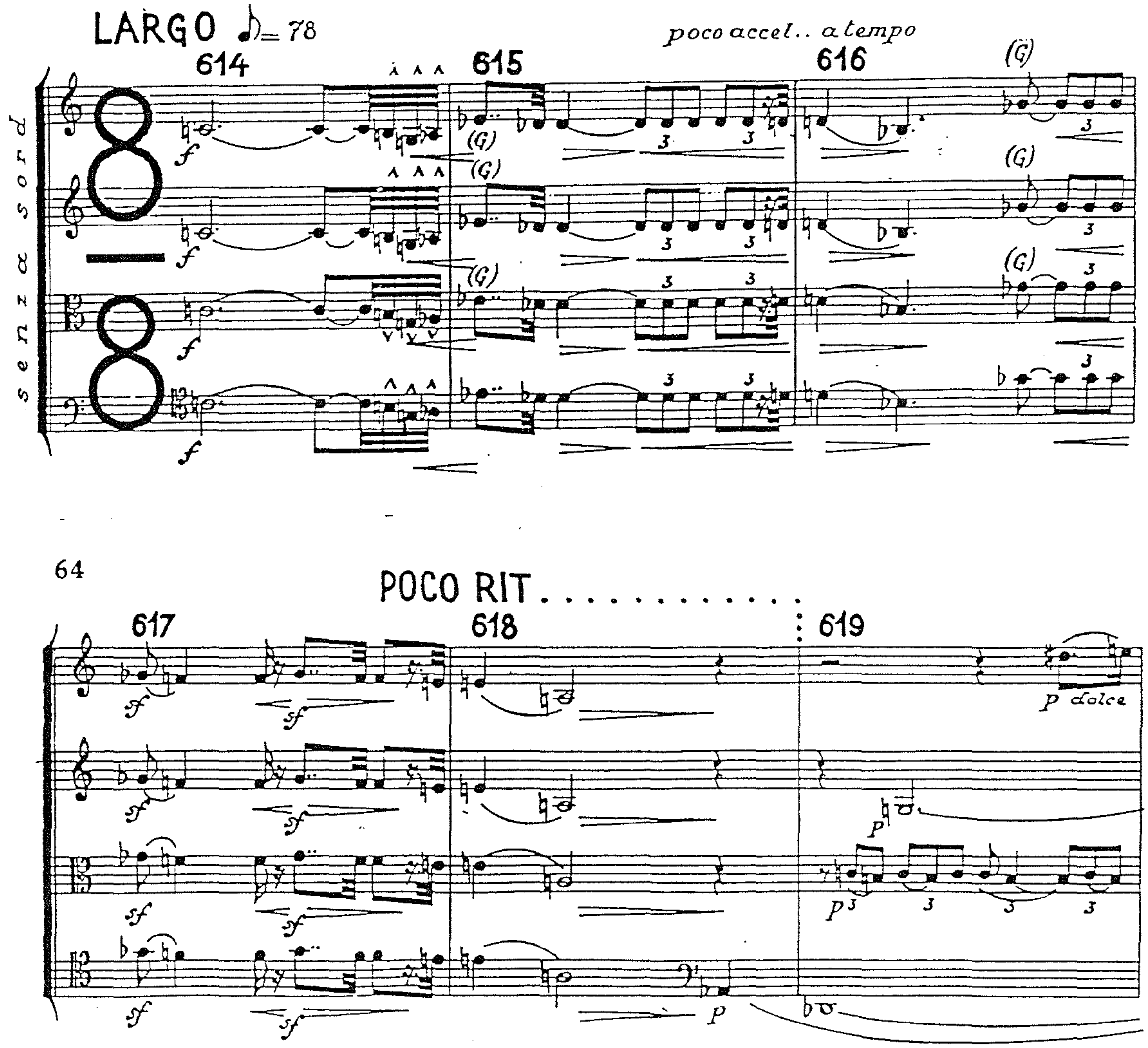
int(E♭,E) = 1 int(E♭,A) = -6
int(D♭,G♭) = 5 int(D♭,A) = -4
int(D,A) = -5 int(B♭,G♭) = 8
int(B♭,F) = 7 int(B♭,E)= 6 int(B♭,A) = -1

where i is 1 or 5 or 6 or 7 or 8 or -1 or -4 or -5 or -6:
IFUNC(y,z)(i)=1

for all other values of i, IFUNC(y,z)(i) = 0

Figure 14 uses interval functions to study successive pitch relationships among the first three tetrachords in Schoenberg's *Fourth String Quartet* (Example 3). Of course, much more than the relationships among the tetrachords informs the passage. A more comprehensive study would consider, at the very least, the intervallic spans within each tetrachord as well as the intervallic spans within and between the hexachords that overlap and thus contradict the tetrachordal articulations.

Example 3. Schoenberg's *Fourth String Quartet,* I, mm. 1-6



Our musical objects are pitches, the group of intervals is integers under addition and the intervals spanned are measured by the number of ascending semitones from pitch to pitch. The interval function for each interval tabulates the multiplicity of that interval as we move from x to y or from y to z. Two of the interval functions are printed in boldface; they are singled out by analytic decision. Not only does interval 3 appear maximal in both the move from x to y and from y to z, but the three semitone intervals in both cases are spanned from the first three elements in each argument. That is, the first three pitches in x mapped by a three-semitone interval onto pitches in y, and the first three pitches in y mapped by a three-semitone interval onto pitches in z.

However, it is the ordinal permutations that correlate with those three-semitone spans that make those spans particularly interesting within the passage. Figure 15 studies the same relations we have highlighted in Figure 14 from a different point of view by selecting intervals derived from a different GIS. Now each note is read as an ordinal position in its respective tetrachord, the group of intervals remains integers under addition and the interval from a member of x to y or y to z is computed by counting from ordinal number to ordinal number.

For example, C is the first member of x, E♭ is the first member of y and G♭ is the first member of z. The interval from C to E♭ is 0, as is the interval from E♭ to G♭. In a similar way, the span from the second note in each tetrachord to the second note in the following tetrachord would also be computed as an interval 0. From first to second is +1, from second to first is -1, and so forth.

Figure 15. Transformations through ordinal positions

Selected from x to y,
int(C,E♭) = 0
int(B,D) = int(G,B♭) = 1

Selected from y to z,
int(E♭,G♭) = 0
int(D♭,E) = 1
int(D,F) = -1

The selected ordinal intervals from x to y display the single digit "right shift" of intervals (B,D) and (G,B♭) that moves the descending linear four-semitone space from the center of x to the right side of y. The selected intervals from y to z display the neighbor note shift among ordinal members that correlates with the linear transformation of E♭, D♭, D within y to G♭, F, E within z.

Returning to Figure 14, we will close our discussion of the Schoenberg by noting that among the interval functions moving x to y and then y to z, there are only two functions that are recursive. That is, there are only two intervals spanned from x to y, then z that maintain the same interval function mapping a member of x to y and then to z. These are shown in Figure 16.

Figure 16. Examples of recursive functions

x to y y to z
int(B,B♭) = int(B♭,A); IFUNC(x,y,z)(-1) = 1

x to y y to z
int(A♭,D♭) = int(D♭,G♭) IFUNC(x,y,z)(5) = 1

The first of these recursive functions, moving B to B♭ to A, proves to be musically cogent. To interpret it, however, we need to take into account the intervallic structure that is internal to x. This is a structure we have not addressed in our examples. The first interval in x is the descending semitone. The descending contour and chromatic space is interrupted immediately by the leap to G followed by the ascent to A♭. The remainder of the space that was opened by the leap from B to G is filled in across the tetrachords by the recursive descending semitone function. Moreover, the pitches that fill that space, B♭ and then A, each expresses closure within its respective tetrachord by articulating the largest intervallic span between adjacencies internal to those tetrachords.

Even given the small amount of analysis that we have done here, it is clear that the most difficult problem involved in applying interval functions toward musical interpretation is in separating the chaff from the grain. Since every moment in every piece can be connected, artificially or not, to every other moment in that and every other piece, the model can be easily abused to create rubbish heaps of data.

Yet, any and every hearing is necessarily a winnowing process. Fixed, unalterable hearings are certainly not desirable and thankfully impossible. As in all of the functions we have discussed, Lewin's work is not designed to alleviate the necessity of making choices (there is never a given-that-choose-this-situation), but rather to heighten an awareness of choices that can be made.

Our conceptual models inform that part of our musical imaginations that forms our musical intuition, yet imagination and intuition are never quite formalized and—I suspect (and hope)—never quite *can* be formalized. The ideas this paper has touched on feed our intuitions, but it is always back to intuition that we must slide. It is the final arbiter. If Lewin's or any other formalisms are to inform our imaginations and help form our intuitions they must become transparent. Otherwise the model becomes the center of focus and not the thing it models. In a sense, the most successful formalism is the one that disappears. This is just as it should be.