Communications

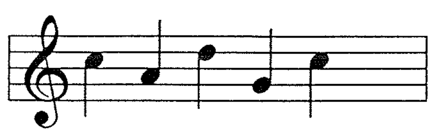
To the Editor:

In vol. 15, no. 2 of the *Indiana Theory Review*, Paul F. Zweifel gives a snippet of melody which he claims cannot be sung exactly in tune, i.e., a singer using just intonation on all intervals must necessarily go flat by a syntonic comma (81/80), the difference between a Pythagorean major third (81/64) and a just major third (5/4)—henceforth simply a comma.[1] This snippet is reproduced in Example 1; when all intervals are sung justly, Zweifel claims the resulting frequency of the final C will be

$$C_{final} = C_{init} \times 5/6 \times 4/3 \times 2/3 \times 4/3$$ (down a minor 3rd, up a perfect 4th, down a perfect 5th, up a perfect 4th)

$$= C_{init} \times 80/81$$ (a comma flat)

Example 1. Zweifel, Figure B-1, p. 110



[1]Paul F. Zweifel, "Just Tuning and the Unavoidable Discrepancies," *Indiana Theory Review* 15, no. 2 (Fall 1994): 110.

Zweifel uses this as ammunition to lash back at choral directors who get annoyed when choirs sing flat. Now, I do not want to dispute the deep emotional satisfaction that this analysis gives one; I must point out, however, that as it stands it is a red herring. Any good choir, having been harangued on the subject by directors for years, will think "high" when singing a descending interval to keep from going flat; i.e., they will shave off the comma when appropriate. In particular, in Zweifel's example a choir will not descend a full just minor third (5/6) but instead a smaller minor third (27/32)—this corrects the comma error, and the final C will be in tune.

Where does that smaller minor third come from? In a just tuning where there is only one A in the arbitrary octave C-C', at ratio 5/3, the A-C' is indeed 6/5; *in practice*, however, a good choir would use two As, differing in frequency by a comma, and switch between them as needed to keep in tune. The choir does not think of it that way, of course; they just know to keep in tune. This smaller minor third is not being pulled out of thin air—in the just tuning for C major, it exists between D and F, and is the cause for the mistuned triad D-F-A. 32/27 is in fact the Pythagorean minor third.[2]

There is a much longer history of examples of untunable chord progressions and investigations in how to deal with them than Zweifel mentions in his article. For example, Zarlino, in Book III of the *Istitutione harmoniche*, clearly distinguishes between two species of minor thirds,[3] directs a good deal of attention to the two sizes of whole tones and semitones,[4] notes that vocalists do in fact shade tunings,[5] and in fact demands that composers make use of the two sizes of whole

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[2]As calculated from C to E♭; this is slightly less (by 23.5 cents, the Pythagorean comma) than the interval C to D♯, 19683/16384.

[3]Gioseffo Zarlino, *Le Istitutione harmoniche III* (Venice, 1558), trans. Guy A. Marco as *The Art of Counterpoint* (New Haven: Yale University Press, 1967), chap. 16, pp. 32-33.

[4]Ibid., chaps. 18-19, pp. 36-38.

[5]"[Voices] can tune intervals higher or lower as desired and through this bring to perfection any composition." (Ibid., 35.)

tones in order to allow consecutive imperfect consonances.[6] (The distinction between the various flavors of intervals is still present as late as Rameau and Kirnberger.[7]) Curiously, Zarlino does not say anything specifically about using the two forms of minor thirds ("semiditones" in his terminology) to avoid comma errors, leaving the question as a matter of performance practice.

The real problem with Zweifel's example is his textural reduction to a solo line, since any single melodic line can be sung entirely justly by a judicious use of differently-shaded intervals that allow one to correct commas when needed. In a polyphonic texture, however, the additional constraints imposed by holding notes across sonority changes *can* produce inherently untunable sonorities. Consider Examples 2A and 2B: the As in the F and D sonorities differ by a comma as computed from their bass notes.[8] If the A is rearticulated (perhaps via a different text), then both chords can be sung justly by having all voices move by Pythagorean rather than just intervals. If it must be held across the change, though, the performers are trapped: either the D-A of the third chord will have to be a "wolf fifth" of 40/27 or else the final C sonority will be flat by a comma relative to the initial one.

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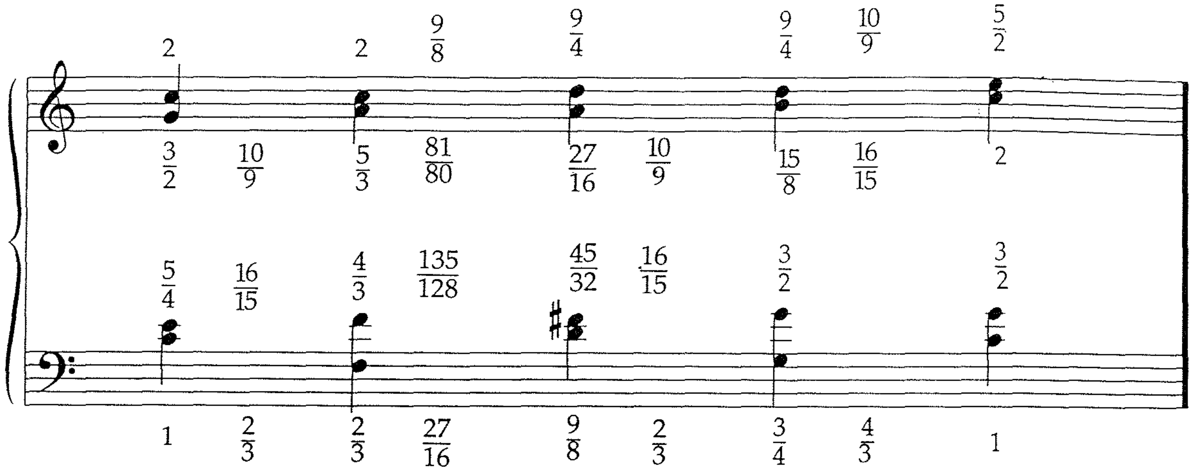
[6]Ibid., chap. 20, p. 62ff.

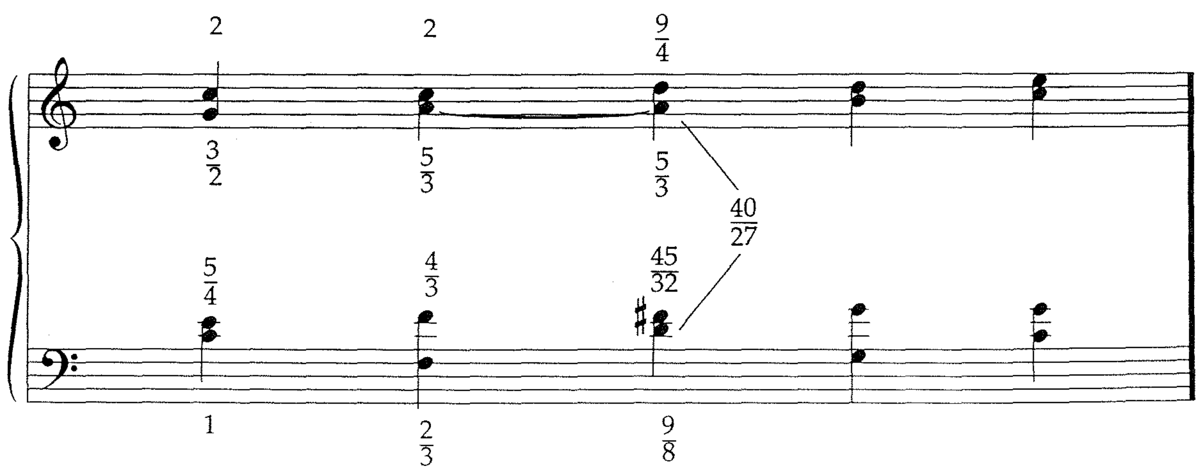
[7]Jean-Philippe Rameau, *Traité de l'harmonie* (Paris, 1722), trans. Philip Gossett as *Treatise on Harmony* (New York: Dover Publications, 1971), 27-40; in particular, see the table on pp. 30-31. Johann Philipp Kirnberger, *Die Kunst des reinen Satzes in der Musik*, 2 vols. (Berlin, 1771-79), trans. David Beach and Jürgen Thym as *The Art of Strict Musical Composition* (New Haven: Yale University Press, 1982), chap. 1, especially pp. 20-24.

[8]The ratios are the frequencies of the various notes in relation to the initial low C, which is arbitrarily assigned a value of 1. The ratios between chords are the interval by which the previous frequencies (notes) must be multiplied to reach the next ratio. Integer ratios are far more useful for such analyses than cents.

Example 2A.



Example 2B.



Had Zweifel quoted in full the examples he mentions from Hall[9] or Pierce[10] (which do use four-voice textures) or made up his own examples as I have done here, his position would have been unassailable.[11]

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[9]Donald E. Hall, ''Quantitative Evaluation of Musical Scale Tunings,'' *American Journal of Physics* 42 (1974): 543-52.

[10]John R. Pierce, *The Science of Musical Sound* (New York: Scientific American Library, 1983), 67 (p. 91 in the revised 1992 edition).

[11]Some other historical examples are given in Mark Lindley and Ronald Turner-Smith, *Mathematical Models of Musical Scales* (Bonn: Verlag für systematische Musikwissenschaft, 1993), 16, n.10.