A Mathematician's Take on Ethnomusicology

Modeling of Protest Music Transmission and Cultural Memory using Stochastic Partial Differential Equations and Non-Linear Dynamics

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Abstract

Protest music functions both as artistic expression and as a vehicle for collective identity and memory in social movements. Songs born out of resistance often spread across communities and generations, helping to sustain the movements' ideals and preserving historical memory of struggles. Understanding how protest songs propagate through space and time, and how they evolve within cultural contexts, calls for a rigorous framework that can capture dynamical and stochastic aspects of this diffusion. In this paper, I develop a theoretical framework using Stochastic Partial Differential Equations (SPDEs) and dynamical systems theory to model the transmission and evolution of protest music. This approach treats the spread of songs akin to a propagating wave or diffusive process on a cultural landscape, subject to nonlinear feedbacks (from social reinforcement) and random perturbations (due to unpredictable social events). I leverage concepts such as attractors**,** Lyapunov stability**,** bifurcation theory**,** stochastic resonance**,** and symbolic dynamics to analyze the model's behavior. My goal is to reveal structural insights into how protest music contributes to sociopolitical movements and cultural memory, providing quantitative measures for phenomena that ethnomusicologists have observed qualitatively – e.g. the way songs "have work to do" in coordinating and unifying communities and how they serve as repositories of cultural memory[1].

I begin by formulating a general SPDE model for the spatiotemporal transmission of

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[1] Owen, Andy. "Can Music Change the World?" *Culturico*, March 29th, 2025. Accessed March 28, 2025. https://culturico.com/2025/01/27/can-music-change-the-world/#:~:text=Collectively%2C%20songs%20are%20an%20important,neglected%20piece%20of%20cultural%20history.

protest songs. I then introduce relevant dynamical-systems definitions (attractors, stability, etc.) and analytical tools to study this model. Existing results from the literature on cultural and linguistic diffusion are incorporated to ground my approach: for example, prior works have used PDEs to model information or language spread in social systems[2]. I adapt such methods to the domain of music and resistance. Formal propositions are stated to characterize the stability of musical traditions and the conditions for their persistence or extinction. Next, I apply the framework to a detailed case study – the role of contemporary Tamil protest music in preserving the legacy of the Tamil Eelam struggle – demonstrating how the abstract model can illuminate real-world cultural dynamics. Finally, I discuss how this mathematical formalism can advance ethnomusicology by offering predictive insight into music as a dynamic carrier of resistance, identity, and memory.

Theoretical Framework for Modeling Protest Music Diffusion

2.1 Modeling Musical Transmission with SPDEs

State Variable and SPDE Formulation: I define a continuously varying field $u(x, t)$ to represent the “intensity” or prevalence of a particular repertoire of protest music at location x and time t. Here x can be a point in a geographic space or an abstract position in a social network of communities. High values of $u(x, t)$ indicate that protest songs are widely known,

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[2] Abrams, Daniel M., and Steven H. Strogatz. "Modelling the Dynamics of Language Death." *Nature* 424, no. 6951 (August 21, 2003): 900.

actively sung, or influential in community x at time t, whereas $u(x,t) \approx 0$ indicates little presence. I propose that $u(x,t)$ evolves according to a stochastic reaction–diffusion equation**:**

$$\partial_t u(x,t) \;=\; D\,\Delta u(x,t) \;+\; F(u(x,t);\,\theta) \;+\; \sigma\,\eta(x,t)\,,$$

where Δ is the Laplacian (modeling spatial diffusion with coefficient $D > 0$), $F(u;\theta)$ is a nonlinear reaction term with parameters θ capturing local transmission dynamics, and $\eta(x,t)$ is a noise term (often idealized as Gaussian white noise) scaled by $\sigma \geq 0$ to represent stochastic influences. This is a Stochastic Partial Differential Equation (SPDE) on the domain of interest. The choice of F encodes how protest music spreads or decays locally: for example, a logistic growth term $F(u) = \alpha\,u(1 - u/K) - \mu\,u$ could indicate that songs propagate via social reinforcement (growth rate α\alphaα up to a carrying capacity K) but can also be forgotten or suppressed at rate μ. More generally, $F(u;\theta)$ can include cultural transmission terms where the presence of music encourages further adoption (positive feedback) and saturation or loss terms. The stochastic term $\sigma\eta(x,t)$ accounts for random external shocks such as spontaneous protest events, media mentions, or the creative emergence of a new song, which perturb the cultural state unpredictably.

Boundary and Initial Conditions: To study transmission across space, one can consider a spatial domain Ω (e.g. regions or a continuous landscape) with appropriate boundary conditions. For instance, if Ω is a collection of community sites (a network), Δ may be replaced by a graph Laplacian. For simplicity, assume Ω is a continuous region with either periodic boundaries (if I imagine an idealized large repeating domain) or zero-flux (Neumann) boundaries (no cultural flow across the boundary of the considered region). An initial condition $u(x,0) = u_0(x)$ encodes the initial distribution of protest music – e.g. perhaps initially concentrated in a few locales (seed communities where the songs originate).

Interpretation: This SPDE treats protest music spread analogously to other diffusive processes (epidemics, information flow) but tailored to cultural context. The diffusion term $D\,\Delta u$ captures spatial transmission – protest songs spreading from one community to neighboring ones (through interpersonal contact, migration of activists, broadcast media, etc.). The reaction term $F(u)$ captures local social dynamics: for example, if few people know the song (small u), interpersonal sharing or the inspirational effect of the music can increase that number (growth); if many know it, it may saturate or face diminishing returns; and if interest is low or opposition is high, the prevalence may decay. The stochastic term reflects that cultural evolution is not deterministic – random events (a charismatic singer touring, a crackdown by authorities, etc.) can cause deviations. By formalizing these elements in an SPDE, I can apply analytical techniques from partial differential equations to understand pattern formation, propagation speed, and long-term behavior of protest song popularity.

Notably, this continuous approach complements existing discrete or network-based models of cultural diffusion. Recent studies of information spread on networks have similarly moved from purely empirical or ODE-based models to PDE frameworks that capture both temporal and spatial patterns. For example, Wang *et al.* (2013) describe reaction-diffusion equations on “cyber-space” to model online information diffusion, including influences of external media[3]. They highlight how PDE models can yield traveling wave solutions describing how information (or analogously, a protest song) propagates with a finite velocity across the network. Inspired by such approaches, my SPDE model aims to quantify how fast a protest song can spread geographically or socially, and under what conditions it saturates or dies out.

Example (Deterministic Limit): To ground the intuition, consider the deterministic special case $\sigma = 0$ with a simple reaction term. Suppose

$$F(u) = r\,u\left(1-\frac{u}{K}\right) - \delta\,u.$$

Here r is a base transmission rate (how quickly a known song inspires new singers), K is a notional saturation level (e.g. due to population limits or attention limits), and δ is a decay rate (loss of knowledge or suppression). The reaction term can be rewritten $F(u) = u(r - \delta - \frac{r}{K}u)$. The homogenous states $u = 0$ (no song presence) and $u = K\left(1 - \frac{\delta}{r}\right)$ (a positive equilibrium, if $r > \delta$) are fixed points. Basic linear analysis shows that $u = 0$ is stable if $r < \delta$ (music dies out as decay dominates) and becomes unstable if $r > \delta$, in which case a nonzero endemic state of the song emerges. This transcritical bifurcation at $r = \delta$ is analogous to a threshold in epidemiology or language survival – it suggests a critical transmission rate needed for the song to persist in the long run. I formalize this as:

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[3] Wang, Haiyan, Feng Wang, and Kuai Xu. "Modeling Information Diffusion in Online Social Networks with Partial Differential Equations." arXiv preprint arXiv:1310.0505v1 [cs.SI], October 1, 2013. http://arxiv.org/abs/1310.0505v1.

Lemma 2.1 (Local Transmission Threshold). In the deterministic reaction–diffusion model, the trivial solution $u(x,t) \equiv 0$ is Lyapunov stable if $R_0 := \frac{r}{\delta} < 1$ and becomes unstable if $R_0 > 1$. In the latter case, a positive equilibrium $u^*(x) \equiv K(1 - \frac{\delta}{r})$. exists and is locally asymptotically stable (attracts nearby states).

Proof: The linearization of $\partial_t u = D\Delta u + (r - \delta)u - \frac{r}{K}u^2$ around $u = 0$ gives $\partial_t u \approx D\Delta u + (r - \delta)u$. Its eigenvalues in a bounded domain have real parts $\lambda = -D\lambda_i + (r - \delta)$, where $\lambda_i \geq 0$ are spatial Laplacian eigenvalues. The principal mode (with $\lambda_0 = 0$) has growth rate $r - \delta$. If $r - \delta < 0$ ($R_0 < 1$), all perturbations decay; If $r - \delta > 0$ ($R_0 > 1$), the $u = 0$ state is unstable. A standard application of bifurcation theory (e.g. reaction-diffusion analog of a transcritical bifurcation) yields the stable uniform state $u^* > 0$ for $R_0 > 1$.

This simple result illustrates how mathematical modeling yields a structural insight: there is a threshold condition for a protest song's sustainability. If the song's "reproduction number" R_0 (the effective rate at which one singer creates new singers, relative to the rate of forgetting) exceeds 1, the musical tradition can sustain itself and even grow to a stable prevalence; if $R_0 < 1$, the song will eventually fade out in that community. While real cultural dynamics are more complex (with spatial variation, external inputs, etc.), this intuition underpins later sections when I consider how changes in social parameters cause songs to either flourish or vanish (bifurcations).

Spatial Propagation and Traveling Waves: In the above reaction–diffusion setting (with $r > \delta$ so that a positive state exists), one can further investigate traveling wave solutions connecting $u = 0$ (ahead of the wave) to $u = u^*$ (behind the wave). Such solutions, if they exist, represent a scenario where a protest song, once initiated in one region, expands its reach outward at some wave speed c. The mathematical criteria for existence of traveling waves can often be derived (e.g. via phase-plane analysis or monotone iteration for KPP-type equations). For Fisher–KPP type diffusion ($F(u) = ru(1 - u/K), \delta = 0$) it is well-known that minimal wave speed $c^* = 2\sqrt{Dr}$ arises; any disturbance spreads at least as fast as c^*, which sets a cultural diffusion velocity. In sociocultural terms, this speed would translate to how quickly a protest song can disseminate through contiguous communities under ideal conditions. Empirical analogs might be the speed at which a solidarity song spreads during a wave of protests. Prior PDE-based studies of information diffusion have indeed identified such wave fronts and propagation speeds[4]. In more complex or multi-stable cases (where $u = 0$ and u^* are both locally stable, as could happen if there is an adoption threshold), one gets front solutions only if the initial perturbation exceeds a critical size – reflecting that a critical mass of early adopters is needed for the protest song to successfully invade new territory (a phenomenon ethnomusicologists might observe as a song failing to catch on if too few people know it initially).

Inclusion of Noise: The full SPDE with $\sigma > 0$ presents a random dynamical system. Rather than a single deterministic trajectory, I consider probability distributions over possible evolutions of $u(x, t)$ given random fluctuations. In practice, noise can model irregular events: e.g., a sudden political incident might cause a spike in interest in certain protest songs (positive noise impulse on u), or conversely, censorship might momentarily suppress performances (negative deviation).

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[4] Wang, Haiyan, Feng Wang, and Kuai Xu. "Modeling Information Diffusion in Online Social Networks with Partial Differential Equations." arXiv preprint arXiv:1310.0505v1 [cs.SI], October 1, 2013. http://arxiv.org/abs/1310.0505v1.

Mathematically, one may study the SPDE in the framework of random dynamical systems, asking whether it admits a random attractor or stationary distribution. A *random attractor* is a generalization of a global attractor to stochastic systems: it is a random set $A(\omega)$ (depending on the noise sample ω) that captures the long-term statistical behavior of the system. Existence of random attractors for various SPDEs under suitable conditions is known[5], ensuring that despite continual perturbations, the system's state distribution converges to a steady pattern in a statistical sense. In my context, a random attractor might correspond to a persistent but fluctuating presence of the protest music, where the exact level oscillates with contingencies but remains within a band (rather than drifting off to extinction or unbounded growth).

2.2 Dynamical Systems Concepts: Attractors, Stability, and Bifurcations

Having defined the SPDE model, I now introduce key concepts from dynamical systems theory to analyze the long-term behavior and robustness of protest music diffusion. These concepts provide formal definitions and theoretical tools:

- **Deterministic Attractors:** An attractor is a set in state space toward which trajectories evolve after transients. Formally, a compact set A is an attractor for a dynamical system $\dot{y} = G(y)$ if A is invariant (G carries points of A to A) and there is a neighborhood of A such that any trajectory starting in that neighborhood converges to A as $t \to \infty$. In my case, the state space is an infinite-dimensional function space (e.g. $H^1(\Omega)$) of profiles

[5] Gess, Benjamin, Wei Liu, and Michael Röckner. "Random Attractors for a Class of Stochastic Partial Differential Equations Driven by General Additive Noise." *arXiv preprint* arXiv:1010.4641v2 (July 20, 2011). https://arxiv.org/abs/1010.4641v2.

$u(x)$) over space. A simple example of an attractor is a stable equilibrium $u(x,t) \to u^*(x)$ as $t \to \infty$, which might represent either the disappearance of the song ($u^* = 0$) or a sustained pattern ($u^*(x)$ nonzero, possibly spatially varying if, say, some regions permanently adopt the song more than others). More interestingly, there could be pattern attractors – stable spatial distributions like periodic patterns or localized hotspots of musical activity – if the dynamics permit heterogeneity. Attractors can also be time-periodic or even chaotic (infinite-period) in more complex models, indicating persistent cycles or irregular revival/extinction cycles of musical popularity.

- **Lyapunov Stability and Stochastic Stability:** A state (or attractor) is Lyapunov stable if small perturbations to the state result in only small deviations in the trajectory. In deterministic terms, an equilibrium u^* is Lyapunov stable if for every $\varepsilon > 0$ there is a $\delta > 0$ such that $\| u(x,0) - u^*(x) \| < \delta$ implies $\| u(x,t) - u^*(x) \| < \varepsilon$ for all the $t > 0$. If moreover $\| u(x,t) - u^*(x) \| \to 0$, the equilibrium is *asymptotically stable*. For my cultural model, stability of the $u = 0$ state means a small presence of the song will die out (if below threshold), whereas instability means any small seed will grow (if above threshold, as in Lemma 2.1). In the stochastic context, one defines stability in probability or almost sure sense. For example, stochastic stability of $u = 0$ would mean that under random disturbances, the system still tends to return to $u \approx 0$ (the song does not take hold, even with noise). A useful quantity is the Lyapunov exponent associated with a perturbation mode; for random systems, one examines expected Lyapunov exponents. If the top Lyapunov exponent becomes positive as parameters change, an equilibrium loses stability in a stochastic bifurcation. In particular, if noise is present, I monitor when the *effective growth rate* of small perturbations crosses zero. This criterion is analogous to

$R_0 = 1$ threshold: indeed, in many SPDEs or stochastic ODEs, a change in sign of a Lyapunov exponent signifies a qualitative change in behavior (an equilibrium giving way to fluctuations of growing amplitude)[6].

- **Bifurcation Theory:** A bifurcation occurs when a small change in a system parameter causes a sudden qualitative change in its long-term behavior. Common bifurcations in my context might include: a transcritical bifurcation (as in the threshold example where the trivial and nontrivial equilibria exchange stability), a saddle-node (where a pair of stable/unstable states appear or annihilate, possibly modeling the sudden loss of a musical tradition when support falls below a critical level), or even a Hopf bifurcation (where a stable equilibrium gives rise to a stable limit cycle, which could correspond to oscillatory cycles of interest and dormancy in protest song popularity). Stochastic analogues of bifurcations are defined either via changes in the stationary distribution of a system (P-bifurcation) or changes in dynamical Lyapunov exponents (D-bifurcation)[7]. For example, in a stochastic Hopf bifurcation, increasing noise or other parameters might cause the system to start oscillating randomly around a mean state. In the protest music model, one could imagine a scenario whereas societal tension increases (a parameter in $F(u)$), the previously dormant state $(u = 0)$ becomes unstable and the system bifurcates to an active

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[6] Ashwin, Peter. "Minimal Attractors and Bifurcations of Random Dynamical Systems." *Proceedings: Mathematical, Physical and Engineering Sciences* 455, no. 1987 (1999): 2615–34. http://www.jstor.org/stable/53489.

[7] Baxendale, Peter H., and Levon Goukassian. "Lyapunov Exponents for Small Random Perturbations of Hamiltonian Systems." *Annals of Probability* 30, no. 1 (2002): 101–134.

state of persistent music (a *resonant* cultural state). I will later see how the Tamil protest music case might be viewed through a bifurcation lens – e.g. major political events can act as bifurcation parameters toggling the community from a quiescent state to one where protest songs are widespread.

- **Stochastic Resonance in Cultural Context:** Stochastic resonance (SR) is a phenomenon usually studied in physics and biology, where adding a certain level of noise to a nonlinear system can actually amplify a weak periodic signal. In my context, think of an annual commemoration or a recurring protest anniversary as a weak periodic input to the cultural system – on its own, it might not sustain the memory of a protest song. However, the presence of moderate noise (random smaller protests or discussions throughout the year) could cooperate with the annual event to resonate, yielding a strong collective response at each anniversary (i.e. everyone suddenly sings the old songs at the commemorative protest) even if without noise the signal would be too weak to elicit a response. Mathematically, SR requires a bistable setup and a sub-threshold periodic input[8]. In a cultural interpretation, "bistable" could mean there are two semi-stable states: one where the song is mostly forgotten, and one where it is in active circulation. The periodic forcing (annual memorial events) alone might not push the community from the forgetful state to the active state, but random cultural fluctuations can occasionally tip the system. At an optimal noise intensity, the timing of those random song revivals lines up with the periodic forcing, causing almost regular oscillations (song is remembered and

[8] DYKMAN, M.I., and P.V.E. McCLINTOCK. "Stochastic Resonance." *Science Progress (1933-)* 82, no. 2 (1999): 113–34. http://www.jstor.org/stable/43424141.

sung at each anniversary) – thus effectively preserving the song exactly at those moments of need. I highlight this concept because it offers a novel insight: random cultural "noise" need not always be detrimental; it can actually help maintain a rhythm of musical remembrance. This might help explain why diasporic communities with a lot of "chatter" and varied cultural activity manage to put together large memorial events every year with protest music, whereas a more orderly but silent community might let the tradition die out between anniversaries.

- **Symbolic Dynamics and Cultural Pattern Formation:** Symbolic dynamics provides a way to encode the trajectories of a dynamical system as sequences of symbols, which can be useful for analyzing complex or chaotic behavior. To apply this to protest music, I need to identify a discrete alphabet of states or events. For example, one could coarse-grain the level of musical activity in a community as "H" (high) or "L" (low), or record the presence (1) or absence (0) of a certain song in each generation. By doing so for successive time intervals or generations, each community yields a bi-infinite sequence of symbols like ...L, L, H, H, L, ... The space of allowed sequences (for instance, perhaps two consecutive generations of "L, L" is forbidden if one generation will inevitably learn from their parents, etc.) forms a subshift in the sense of symbolic dynamics. I can then leverage concepts like topological entropy – which measures the complexity (exponential growth rate of distinct allowed sequences) – as a proxy for the cultural richness or unpredictability. A higher entropy would indicate more variability in how protest music might evolve over time (many possible sequences of revival and decline), hinting at an intrinsically chaotic dynamic in the social spread of music[9]. Symbolic dynamics has been widely used to study chaotic nonlinear systems and even been applied in economic and social contexts to detect chaos in time series. In my framework, if the SPDE or its Poincaré map (stroboscopic map, say year to year) exhibits chaotic behavior, one could construct a symbolic coding of its trajectories. For example, partition the state space by whether $u(x, t)$ is above or below some threshold in each region, and encode this as symbols. An intricate patchwork of musical activity across regions over time could then be studied with symbolic sequences, where standard results (e.g. existence of a semi-conjugacy to a Bernoulli shift) would confirm positive entropy and thus chaos. While this level of mathematical abstraction might be beyond immediate practical necessity, it underscores a point for ethnomusicology: the diffusion of music in society can generate extremely complex, even chaotic temporal patterns under some conditions, defying simple periodic models of rise-and-fall. Symbolic analysis might help identify repeating motifs or the "grammar" of protest song evolution (for instance, a song might nearly disappear (L) but is almost always followed by a revival (H) in the next generation – a kind of cultural rule that could be deduced from the sequence data).

- **Numerical Simulation and Scheme Considerations:** Many SPDEs cannot be solved in closed form, so numerical methods are crucial to explore the model's predictions. For deterministic PDEs of this type, one might use finite difference or spectral methods in

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[9] Matilla-García, Mariano, and Manuel Ruiz Marín. "A New Test for Chaos and Determinism Based on Symbolic Dynamics." *Journal of Economic Behavior & Organization* 76 (2010): 600–614. https://doi.org/10.1016/j.jebo.2010.09.017.

space combined with time-stepping (e.g. Euler or Runge–Kutta). In the stochastic case, one can implement an Euler–Maruyama scheme (for time discretization of the stochastic term) along with the spatial discretization. Alternatively, a spectral Galerkin method could be employed where the Laplacian is diagonalized (using eigenfunctions of the domain) and then one integrates the resulting set of stochastic ODEs. Care must be taken to resolve sufficient spatial detail because cultural diffusion might form sharp fronts. Agent-based simulations can also complement the continuum model: I could simulate individuals spreading songs on a network, then fit or compare to the SPDE results, as has been done in other cultural diffusion studies[10]. Verification of numerical schemes (ensuring convergence in probability, etc.) is a mathematically non-trivial but important aspect if I were to quantitatively calibrate the model with data.

In summary, my theoretical framework combines an SPDE model of musical spread with dynamical systems analysis tools. The formal definitions (attractors, stability, etc.) allow me to describe scenarios like "song extinction vs. persistence" in rigorous terms, and to identify parameters (e.g. transmission rate, social connectivity, repression level) that serve as tipping points (bifurcations) for the system's behavior. I have also emphasized the role of stochasticity – acknowledging that real social movements are noisy – and even highlighted potentially counter-intuitive effects like stochastic resonance where noise assists cultural memory. In the next

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[10] An, Zhecheng, Qiuhui Pan, Guangying Yu, and Zhen Wang. "The Spatial Distribution of Clusters and the Formation of Mixed Languages in Bilingual Competition." *Physica A* 391 (2012): 4943–4952. https://doi.org/10.1016/j.physa.2012.05.023.

section, I put this framework to the test by examining a concrete ethnomusicological case study, using it to articulate how protest music can preserve the legacy of a political struggle.

Case Study: Tamil Protest Music and the Legacy of the Tamil Eelam Revolution

3.1 Background: Tamil Eelam Struggle and Its Music

The Tamil Eelam Revolution refers to the decades-long struggle of the Sri Lankan Tamil population (in the north and east of Sri Lanka) for autonomy or independence, led by movements such as the LTTE (Liberation Tigers of Tamil Eelam). Throughout the armed conflict (which lasted from the 1980s until 2009) and its aftermath, music has played a pivotal role in Tamil resistance culture. A rich genre of Tamil protest songs – often called *uprising songs* or *struggle songs* – emerged, portraying the emotions and challenges of Tamil Eelam's fight. These songs were used to inspire fighters and civilians alike, fostering unity and common purpose. Many were composed or popularized by the resistance organizations themselves or sympathizers in the Tamil diaspora. For instance, poets like Puthuvai Rathinathurai and musicians from Tamil Nadu and Sri Lanka collaborated to create songs that became anthems of the movement. The *Tamil Eelam Music Band* formed in the 1990s is one example of an organized effort to produce and disseminate such music[11].

[11] Telibrary. "Tamil Eelam Music Band." TELibrary. Accessed March 29, 2025. https://telibrary.com/en/tamil-eelam-music-

After the civil war's brutal end in 2009, the context changed: the Tamil population faced severe repression at home and a growing diaspora abroad. In this post-war era, contemporary Tamil protest singers and artists have sought to preserve the legacy of the Eelam struggle through music, even as active conflict ceased. This includes:

- *Diasporic hip-hop and rap:* Young Tamil artists in the diaspora (e.g. in Canada, the UK, Australia) mixing rap with Tamil lyrics that reference the war, loss, and continuing resistance. Tamil rap has become "a vital voice for a community displaced by war and oppression," giving expression to the diasporic Tamil youth's discontent and identity. By rapping in Tamil and fusing it with global genres, these artists keep the language and the memory of struggle alive in new forms. The syncretic blend of Tamil traditional musical elements with hip-hop has been noted to *"serve to keep traditional Tamil sound cultures alive, whilst raising the voices of Tamils"* in the global music scene[12].
- *Folk and community singers:* In Tamil Nadu (India) and within Sri Lankan Tamil refugee communities, folk musicians (like the Parai drummers or street theater singers) perform songs about the Eelam martyrs and injustices, often at memorial events or protests against ongoing discrimination. These performances are culturally resonant, drawing on Tamil

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band/#:~:text=The%20uprising%20songs%20arose%20during,Vaithiyanathan%20composed%20many%20songs.

[12] International Orange. "Eelam Style: The Rise of Tamil Rap." International Orange. Accessed March 29, 2025. https://internationalorange.io/eelam-style-the-rise-of-tamil-rap/#:~:text=sounding%20drill%20beat,propelling%20them%20into%20the%20limelight.

oral poetic traditions and thereby connecting contemporary political grievances with historic cultural memory.

- *Commemorative events:* Each year, the Tamil diaspora commemorates Mullivaikkal Remembrance Day (in May, marking the war's end and mass atrocities of 2009). Songs that were once battle anthems are sung in ceremonies from Toronto to London. New songs and poems are also composed to narrate the Tamil experience and trauma. This annual cycle is a clear example of a periodic forcing in cultural memory (analogous to the periodic signal in stochastic resonance terms).

The case study thus provides a rich scenario: a dispersed community with strong cultural diffusion links (via migration networks), possessing a repertoire of protest songs that need active effort to be transmitted to the next generation, under the influence of both supportive factors (community gatherings, activism) and suppressive factors (assimilation, trauma, lack of public space in Sri Lanka due to government restrictions). I will interpret and analyze this scenario using my SPDE/dynamical systems framework.

3.2 Applying the Model to Tamil Protest Song Dynamics

To tailor my general model to this case, I identify the "state" $u(x, t)$ with the prevalence of Tamil protest music and its associated memory in community x at time t. The domain of x could be taken as the Tamil transnational community – essentially a set of locales such as Jaffna (Sri Lanka), Chennai (India), Toronto, London, Sydney, etc., where Tamil populations reside. For simplicity, imagine x parameterizes a one-dimensional chain of these communities ordered by geographical or social proximity. Diffusion $D\Delta u$ then represents exchange of musical culture between neighboring communities (e.g. Tamil Nadu and Sri Lanka share cultural ties, diaspora communities interact via online networks and travel).

The reaction term $F(u;\theta)$ should incorporate key processes:

- **Intergenerational transmission:** Parents or elders teaching the songs (or their context) to youth. This might be modeled as positive feedback that increases u proportional to current u (if more people know the songs, more can teach others).
- **Forgetting/Assimilation:** Younger generations may forget or not learn the songs due to integration into other cultures or simply passage of time. This is a decay term $-\mu u$.
- **New creation and adaptation:** Contemporary artists (like Tamil rappers) create new songs or adapt old ones, effectively adding to the stock of protest music memory. This can be a source term independent of u or dependent on related cultural activity (e.g. general level of political activism).
- **Repression effects:** In Sri Lanka, open performance of these songs is dangerous; this could be modeled by a higher decay rate or even an explicit "censorship" term that actively drives u toward 0 when u is above a low threshold (a piecewise or nonlinear effect).
- **Threshold effects:** It might require a critical mass of individuals in a community to actively sustain singing traditions (if too few, they might not bother, or individuals feel shy to perform alone). This suggests an Allee-effect type nonlinearity: $F(u)$ could be negative for very small u (indicating difficulty of maintaining at very low prevalence) but positive once u passes a threshold.

To illustrate, I propose a stylized form:

$$\partial_t u = D\,\Delta u + \underbrace{\rho\, u\,(u-\kappa)\,(1-u)}_{\text{bistable reaction term}} + I(x,t) \;+\; \sigma\eta(x,t).$$

Here u is scaled to a $[0,1]$ range (fraction of community engaged with the protest music). The cubic reaction term has zeros at $u = 0$, $u = \kappa$, $u = 1$. I choose $0 < \kappa < 1$ so that:

- For $0 < u < \kappa$, the growth term $\rho\, u(u-\kappa)(1-u)$ is negative, meaning very low song prevalence tends to drop towards 0 (difficult to sustain if only a few know it – they might stop singing unless they recruit more).
- For $u < \kappa < 1$, the term is positive (if enough people know the songs, reinforcement dominates and it tends towards the upper state $u = 1$, i.e. widespread retention in that community).

This is a bistable model for each well-mixed community: $u = 0$ (cultural extinction of the songs) and $u = 1$ (full retention) are stable equilibria, with an unstable intermediate κ representing the minimum sustainable fraction. The term $I(x,t)$ represents external input or forcing – in this case, I can use it to model periodic commemoration or spikes due to major events. For example, $I(x,t)$ could be nonzero for diaspora communities during May (anniversary of war's end) to simulate the annual memorial events, or a shock at the time of a political controversy. The noise $\sigma\eta(x,t)$ captures random fluctuations like spontaneous interest or local events.

This model is admittedly qualitative, but it captures a plausible mechanism: If a diaspora community falls below a critical involvement κ, memory of the songs might rapidly fade (everyone assimilates, no one sings protest songs); if it stays above, it moves toward high preservation (a vibrant community that all know the songs, perhaps via formal teaching, cultural schools, archives, etc.). The connectivity D means even if one community falls into forgetfulness, it might be “reseeded” by neighbors where the music is still strong (for instance, activists from London might travel to a smaller community and reintroduce the songs). This coupling could effectively enlarge the basin of attraction of the $u \approx 1$ (preservation) state in the joint system.

3.3 Analysis: Persistence, Bifurcation, and Memory in the Tamil Context

Using the above model, I can reason about the sustainability of Tamil protest music:

Attractors and Stability: Each community individually is bistable, with either long-term disappearance or long-term preservation of the musical tradition. The presence of diffusion coupling means the entire spatial system’s attractors can be more complex. In one extreme, if most communities are initially in a high state (diaspora actively singing, etc.), they may collectively drag a few lower ones up (a pull-up effect), leading to a spatially homogeneous attractor $u(x) \approx 1$ (everyone preserves the songs). Conversely, if many communities have lost the music, they might pull others down. A nontrivial possibility is a patterned attractor: some communities end up preserving, others not, in a stable patchwork. However, given strong connectivity in the diaspora (via the internet, global Tamil gatherings), it’s reasonable that the model tends toward a consensus (all or none). The critical parameter is the overall supportiveness of the environment – which in my model could be reflected in how large $I(x,t)$ and σ can keep pushing small uuu values upward, helping them escape the $u < \kappa$ trap. In the deterministic skeleton, increasing the amplitude of periodic forcing $I(x,t)$ could cause the unstable κ\kappaκ state to vanish in a resonance-like bifurcation, leaving only a single stable high state – meaning any small remnant of music eventually grows (the community becomes excitable). From an ethnomusicological view, this suggests that consistent commemoration and external support can *guarantee* preservation: the songs become an attractor of the cultural dynamics.

Role of Noise – Stochastic Resonance: I specifically consider the annual Mullivaikkal memorial as a periodic input. By itself, if a community is below κ, one day of commemoration might not be enough to raise u significantly (they might participate ritually and then forget again). But random events spread throughout the year – say a famous singer releases a protest song video that goes viral, or a local youth group organizes a small event on Martyrs' Day in November – serve as noise that agitates the system. At low noise, these might be too few to matter; at very high noise, the cultural signal becomes incoherent (if completely random influences flood the community, they might pick up other music, diluting focus on protest songs). SR predicts an intermediate optimal noise intensity: roughly, enough random boosts that occasionally $u(t)$ is pushed above κ around the times of commemoration, but not so high that the cultural focus is lost. When this happens, the community will respond strongly at each annual event – singing the songs with renewed interest – effectively syncing to the period. Indeed, Tamil diaspora communities often report that during May each year, there is a surge of cultural memory activity (exhibitions, music sessions)[13], which then recedes. My model attributes this to a kind of resonance condition: the timescale of one year and the timescale of natural forgetting (or noise-driven fluctuations) are matched such that once a year the community "refreshes" its memory just in time. If the period were much longer, memory might decay completely in between; if

[13] Seioghe, Rachel. "Memory and Resistance in the London Tamil Diaspora: Reflections from the 'Tamils of Lanka: a Timeless Heritage' Exhibition." *The Tamil Academic Journal* 21 (n.d.): 21–50.

much shorter, the events might saturate interest and cause fatigue. This qualitative insight – that there is an optimal frequency of collective remembrance aided by incidental reminders – is an example of how mathematical analogies can deepen understanding of cultural practice.

Bifurcations – Sociopolitical Changes: Consider a parameter like ρ (which could correspond to how zealously people propagate the songs, or how relevant they feel the songs are to current struggles). In peacetime, ρ might decrease (people feel the songs of war are less relevant). If ρ drops below a bifurcation value, the high- u state might lose stability, meaning even a community that had preserved the songs could gradually let them slip away. This would manifest as a cultural bifurcation: a once-dominant tradition suddenly eroding. Something like this is observed in second-generation diaspora who often have weaker connection to the old war songs unless special effort is taken. On the other hand, if repression in Sri Lanka softens (imagine a hypothetical scenario of transitional justice allowing public remembrance), that would effectively reduce μ or the negative pressures. The threshold κ might then shrink, making it easier for communities in Eelam regions to sustain the songs again – potentially a bifurcation restoring a positive state. The framework can thus simulate what-if scenarios: e.g., if technology allows every Tamil song to be shared globally (huge increase in D connectivity), does that automatically preserve everything or could it also lead to homogenization and loss of diversity? (High D usually synchronizes regions, for better or worse – it could prevent local extinction, but also if a powerful region abandons the music, it can drag others with it.)

Symbolic Dynamics Perspective: If I take a step back and look at the sequence of states for, say, the Jaffna Tamil community over decades, I might encode each year as H (songs actively remembered) or L (largely dormant). A possible sequence might be LLLLHLHHLH ... over 15 years. By analyzing such symbolic sequences across multiple communities (or multiple trials of the model), patterns might emerge. I might discover a rule like “whenever there are two L in a row, the next symbol is almost surely L” (indicating that if the community forgets for two consecutive years, the tradition is essentially lost, echoing the threshold). Or I might find long-term periodicity (every N years an H occurs due to generational turnover). These are speculative, but it shows how the qualitative dynamics captured by the model can generate hypotheses for ethnomusicological research: for instance, researchers might look for evidence of these patterns in oral history or event archives – are there cases where a tradition nearly died (a gap) and then was revived by a single key event (an H after many L’s, which could correspond to, say, a commemorative monument unveiling accompanied by music that reignited interest)?

3.4 Discussion: Insights and Limitations for Ethnomusicology

By applying my SPDE and dynamical systems framework to the Tamil protest music case, several insights surface:

- **Protest Music as a Dynamic Equilibrium:** The persistence of songs in collective memory is not a static given, but a dynamic equilibrium that can tip one way or another. Mathematical models make explicit the factors needed to maintain that equilibrium. In Tamil diaspora communities, a combination of network connectivity (diffusion D), cultural investment (parameters like ρ or external input I), and periodic reinforcement (commemorations) all work in tandem to keep the protest music alive. If any of these factors weaken beyond a critical point (a bifurcation parameter), the equilibrium could shift to loss of musical knowledge. This formalizes the ethnomusicological argument that cultural memory requires active reinforcement – here I quantify "active reinforcement" with parameters and stability criteria.

- **Quantifying Cultural Diffusion:** My use of a diffusion term and traveling wave analysis provides a way to estimate how fast a protest song can spread. If one wanted, one could fit the model to data (for instance, how many villages learned a certain famous Eelam song over time) and estimate a diffusion constant. This is analogous to measuring how fast a viral song spreads on the internet, but for protest music in a movement. Prior PDE models in social contexts have successfully fit data (e.g., PDE models of online news spreading were validated on Twitter cascades[14]). Likewise, one could envision calibrating a model for the spread of the song *"Tholvi Nilayena Ninaithal"* (a well-known Eelam revolutionary song) across Tamil communities worldwide, given sufficient historical data. This would represent a novel quantitative historical approach in ethnomusicology.
- **Resonance and Ritual:** The notion of stochastic resonance gives a new lens to view the importance of ritual and random cultural "noise". It suggests that the *timing* and *frequency* of protest music performances might be as crucial as their content. A community that hears these songs regularly at just the right frequency might remember better than one that hears them either too rarely or too frequently (where they might become background noise). This potentially explains why annual events are powerful: yearly might be an optimal period to prevent forgetting while avoiding fatigue.

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[14] Wang, Haiyan, Feng Wang, and Kuai Xu. "Modeling Information Diffusion in Online Social Networks with Partial Differential Equations." arXiv preprint arXiv:1310.0505v1 [cs.SI], October 1, 2013. http://arxiv.org/abs/1310.0505v1.

Ethnomusicologists can look at different communities – those with more frequent gatherings vs. annual – to see which maintains songs longer, thus empirically testing a prediction of the model.

- **Evolution of Music – Beyond Persistence:** My framework can also accommodate evolution of the songs themselves. Symbolic dynamics could be used to model how lyrical themes or melodies change over time (each "state" includes which version of a song is prevalent). A chaotic dynamic might indicate active innovation in music, whereas convergence to an attractor could mean canonization of a fixed repertoire. In the Tamil case, I see innovation: traditional tunes have given rise to rap and fusion, i.e., the system is not static. One could model multiple state variables $u_i(x,t)$ for different musical styles (traditional, rap, etc.) interacting in a competitive or synergistic way. Bifurcation might then signify a cultural shift where one style overtakes another in conveying the protest message. This is akin to language competition models in dynamics (where two languages vie for speakers; here two musical forms vie for audience). Indeed, Walters (2014) introduced a reaction–diffusion model for two competing languages, showing how spatial areas can end up dominated by one language each[15]. Similarly, Tamil resistance music could split into domains – perhaps older folk style in rural Sri Lanka and rap in diaspora – each dominating their niche. The mathematics of multi-component SPDEs and pattern

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[15] Gaspard, Mallory, Peter Craig, and Erik Bergland. "An Integro-Differential Model of Language Competition." Preprint, Department of Mathematical Sciences, Rensselaer Polytechnic Institute, April 2, 2019.

formation (e.g. Turing instabilities) could predict under what conditions a patchwork versus a single unified musical evolution occurs.

- **Validation and Data:** I must note that while the theory is rich, empirical validation in this context is challenging. However, the framework at least suggests what data to collect: e.g., longitudinal surveys of protest song knowledge in various communities, records of events where songs were performed, and social network connectivity metrics for the communities. With such data, one could attempt to estimate parameters (like half-life of memory μ^{-1}, or influence range related to D). If the model predictions align with observed trends (e.g., communities with certain connectivity and event frequency indeed maintain songs while others lose them), that strengthens the case for using such quantitative models in cultural studies.

Conclusion

I have developed a mathematically rigorous framework to model the spread, sustainability, and evolution of protest music, leveraging tools from SPDEs and dynamical systems theory. By treating the diffusion of protest songs as a stochastic dynamical process, I can formally describe how these songs transmit across space and time and how they become embedded (or lost) in cultural memory. Key theoretical constructs – from attractors (stable states of cultural memory) to bifurcations (tipping points in musical continuity) – have been used to gain insight into phenomena documented by ethnomusicologists, such as the crucial role of songs in unifying movements and carrying historical narratives.

My case study on contemporary Tamil protest music demonstrates the practical applicability of the framework. It highlights how the legacy of a political revolution (the Tamil

Eelam struggle) can be sustained through music, if certain dynamical conditions are met: sufficient transmission rates, periodic reinforcement (rituals), and network connectivity to counteract decay. I saw that the model can incorporate real-world complexities like diaspora networks and state repression, and in doing so, it yields hypotheses (e.g. on optimal commemoration frequency, or on the threshold size of a community needed to keep songs alive) that bridge quantitative theory and qualitative cultural scholarship.

In broader perspective, this type of mathematical formalism can significantly advance the field of ethnomusicology by providing a systematic language for dynamics. Ethnomusicology often deals with evolution of musical traditions, diffusion of styles, and the interplay between music and social context – all of which are dynamic processes. By applying models from epidemiology, ecology, and network science (here unified in an SPDE approach), researchers can move from metaphorical descriptions to testable models. For instance, one could explore “cultural attractors” in music: are there attractor states corresponding to syncretic musical forms that movements worldwide gravitate toward? Does protest music exhibit critical mass behavior analogous to percolation, where a movement suddenly gains an anthemic song that everyone adopts? Using dynamical systems theory, such questions can be framed precisely.

Moreover, incorporating stochasticity acknowledges the unpredictability inherent in cultural change yet allows one to quantify the probability of various outcomes (persistence vs. extinction of a song, etc.). This merges with the concept of cultural resilience – the probability that a cultural trait survives perturbations – which could be estimated via my model (e.g. relating to Lyapunov exponents or stationary distributions). There is also room for synergy with data science: given large-scale data (like YouTube view statistics for protest songs over time across regions), one could fit SPDE-inspired models to detect underlying parameters of cultural diffusion. Such interdisciplinary work would enrich both the mathematical theory (by presenting new complex systems to analyze) and ethnomusicology (by validating theories of music and resistance with quantifiable metrics).

In conclusion, the spread of protest music is a multi-faceted process that benefits from the marriage of qualitative insight and quantitative rigor. My study illustrates how SPDEs and dynamical systems can serve as a powerful framework to model and understand this process. By formalizing concepts like transmission, diffusion, and memory in mathematical terms, I gain the ability to uncover hidden structures – be it a threshold phenomenon ensuring a song's survival, a resonance effect of collective rituals, or a chaotic variation in musical narratives. These insights not only deepen theoretical understanding but can also inform practical efforts in cultural preservation: for example, identifying when a movement's musical heritage is at risk and what interventions (increasing exchanges between communities, organizing more frequent music events) might avert its loss. Thus, mathematical modeling emerges not just as an abstract exercise, but as a tool of analysis with meaningful implications for preserving the sounds of resistance that are so integral to human struggles for justice and identity.

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