mathematics. Addressing these issues are here seen as one aspect of addressing *the needs* of the student as a holistic being. Embedding Hackenberg's concept of mathematical caring into the larger framework of an ethic of care will allow the keeping together of caring and learning mathematics. However, this connection is qualified by a more comprehensive consideration of the students' needs, with priority given to a larger purpose of teaching and schooling:

> If the school has one main goal that guides the establishment and priority of all others, it should be to promote the growth of students as healthy, competent, moral people. (Noddings, 1992, p. 10)

It seems to me that an integration of Hackenberg's notion of mathematical caring into a larger framework of an ethic of care as suggested here does share some of the benefits Hackenberg lists for her model of mathematical learning and caring relations in the conclusion of her article. For her model, she suggests that

> holding learning and caring together disrupts the traditional and harmful separation […] of intellectual activity from emotional, embodied states. (p. 49)

If mathematical caring is seen within a care-ethical framework, which has a holistic view of the human need structure, the intellectual and emotional aspects of being human will also find their joint consideration in the teacher's caring.

Hackenberg writes that

> [her] model explicates how student-teacher interaction can affect engagement with mathematical activity that is essential for acts of learning to occur. (p. 49)

In an ethic of care, social interaction is principal to being human, since humans are conceptualized as relational beings (Noddings, 2002, chapter 5). An ethic of care explicates the effect of human interaction on human functioning in general and students' and teachers' engagement in mathematical activities in particular. Placing mathematical caring within a care-ethical framework gives social interaction a central role in the teaching and learning of mathematics.

Finally, Hackenberg argues that her

> model points toward the necessary involvement of ethical issues in considering the role of social interaction in learning. (p. 50)

Viewing mathematical caring within an ethic of care will also have this benefit, since ethical issues are at the very centre of an ethic of care.

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Response to Falkenberg

AMY HACKENBERG

I agree with Falkenberg that engendering mathematical caring relations (MCRs) should be considered part of an overall project to engender general caring relations with students. However, I describe how I formulated mathematical caring as distinct from general caring in order to address aspects of mathematical interaction with students for whom general caring seemed insufficient. That is, my formulation and use of MCRs is an attempt to build a model of caring in the context of mathematics teaching and learning. Such models are crucial for making the type of inferences about students' mathematical needs that Falkenberg and I both deem important. In fact, in my research, I have found that remaining at the level of general care theory does not provide a powerful enough tool for me to make such inferences in the process of establishing caring relations with my students in mathematical interaction.

My experience with Bridget, one of the four sixth grade (11-12 years old) students I taught in a year-long constructivist teaching experiment [1], can provide an example of the specificity I came to understand was necessary to establish and maintain a mathematical caring relation with a student.

At the start of the experiment in October, both Bridget and her partner Deborah could solve problems like this one,

> *The Money Problem*: Tanya has $16, which is 4/5 of what David has. How much does David have?

Toward the end of the teaching experiment in May, we worked on problems like this one,

> *The Sub Problem*: A mini-sub sandwich is 5 inches long, and that's 3/4 of the length of a regular sub sandwich. How long is the regular sandwich?

The *Sub Problem* is considerably more complex than the *Money Problem* because the *Sub Problem* involves determining how to divide a five-unit quantity into three equal parts. During the experiment, we had worked on problems that might engender the construction of schemes for doing so (see Hackenberg, 2005a). However, in early May, Bridget experienced great difficulty in solving problems that required further partitioning a sub-divided quantity into some number of parts in order to determine a fractional amount of it.

Although I understood very soon that Bridget was experiencing considerable depletion, in the moment I did not know how to adapt my mathematical interaction with her so that she could act more independently, as well as feel more autonomous and in control. I resorted to coaching her through making drawings so that she might have some visual material on which to operate. This coaching was laborious for both of us. Because my suggestions were based on my perception of what breaking down the problems might involve (*e.g.*, for someone with operations like mine), they were not very effective for her and indicated that I did not decenter enough cognitively. Thus my suggestions did not alleviate her depletion, and the longer she remained in a depleted state the more depletion I felt!

One might say that in this situation I tried to use general caring with Bridget, in decentering to 'receive' her emotional experience of darkness and frustration, and her general cognitive experience of not being able to operate. However, I did not sufficiently consider her specifically *mathematical* cognitive experience of being bothered by finding 1/3 of a 5-part quantity, let alone conceiving of a 5-part quantity as 3/4 of another quantity. In the moment of interaction with a student, decentering from my own ways of operating mathematically is quite challenging, in part because I may not have created explanatory constructs that I can formulate outside of the interaction. In this case, I had yet to formulate that solving the *Sub Problem* seems to require coordinating two different three-levels-of-units views of a quantity – *i.e.*, viewing the mini-sub's length as composed of a unit containing five units, each of which can be further partitioned into three (small) units, so that the length can be conceived of as a unit of three units each containing five (small) units (*cf.* Hackenberg, 2005a).

Yet even prior to this type of analysis, relying solely on general caring seems insufficient when trying to facilitate mathematical learning. That is, general caring does not seem to help a teacher address a student's specifically mathematical cognitive experience so that her mathematical activity might continue, albeit in ways that may be different from what the teacher originally had planned or envisioned.

My interactions with Bridget in early May set off a rather fervent search for better ways to communicate with her mathematically. Out of that search came a gradual reestablishment of our mathematical caring relation over the final episodes of the experiment. I began to take more seriously that Bridget could not yet view a quantity like 5 inches as partitioned into both 5 equal parts and 3 equal parts (without erasing the marks that made the 5 equal parts). Hence, I carefully planned a sequence of tasks for her that involved only unit quantities (1 meter or 1/2 meter) and fractional relationships. These plans could not be considered successful mathematical care for her unless she experienced some alleviation of her depletion, or received them with some openness. In fact, she responded quite positively to the sequence, both cognitively and affectively (Hackenberg, 2005a).

I contend that my increased attention to Bridget's mathematical ways of operating was an example of the engrossment that, as Falkenberg notes, is featured in Noddings' care theory (1984, 2002) and that I include in my notion of harmonizing with students' mathematical ways of operating. My motivation was also harnessed in service of posing problems in which Bridget could operate autonomously without excessive coaching, and yet in which she might have opportunities to make new coordinations that she herself may not have imagined. In this sense, I believe I practiced both Noddings' motivational displacement and expansion of a student's world, other key features of her theory of general caring. In turn, Bridget's positive and productive responses to our interaction over the May episodes indicate that she received my care for her. Thus, I fully support Falkenberg's assertion that "[t]he issues captured in Hackenberg's notion of mathematical caring are integrated as issues of (general) caring in the teaching and learning of mathematics" (p. 29).

However, I propose that my receptivity to, and formulation of, Bridget's ways of operating as different from my own, and as not addressed by my initial coaching attempts, required more than engrossment and motivational displacement as described by Noddings and Falkenberg. That is, I needed to formulate inferences (or at least hunches) about Bridget's ways of operating that would allow me to pose 'effective' problems for her (*i.e.*, to decide what mathematical activity could be productive for her at that point in her mathematical education) in order to address both her cognitive and emotional states. To do so, I call on constructs regarding mathematical learning (Hackenberg, 2005b). This level of specificity regarding students' mathematical ways of operating is, in my current thinking, an essential tool in caring mathematically for another and can allow a teacher to develop, as Falkenberg says, *"empathetic understanding of how individual students see and do mathematics"* (p. 28).

Making these specific formulations about students' mathematics is one way I can operationalize caring in my work with students. Doing so also demonstrates the crucial activity of building models of caring in the context of doing research, rather than attempting to apply a general theory of care to research situations (Steffe and Wiegel, 1996).

Notes

[1] The purpose of the teaching experiment was to understand how sixth graders (11-12 year old students) construct algebraic reasoning based on their evolving quantitative reasoning in interaction with a teacher-researcher (myself) who endeavored to establish and maintain MCRs with them (Hackenberg, 2005a). A central focus was how students constructed quantitative schemes underlying their construction and solution of linear equations of the form $ax = b$.

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On logical thinking in mathematics classrooms

KEITH WEBER

A comment on 'Talking about logic', Reid and Inglis, **25***(2)*: Earlier this year, my colleague and I published an article in FLM **25**(1) defining an implication as warranted if there is a socially accepted general mathematical principle for deducing its conclusion from its antecedent. We then argued for the importance of considering warranted implications when reading a mathematical proof (Weber and Alcock,