

antecedents and consequents are eternal sentences. The authors' analysis of conditionals is an extension of Stalnaker's:  $A > B$  is true at  $w, t$  iff  $B$  is true at  $f(A, w, t)$ , where  $f$  selects the world-time pair  $w', t'$  that is most similar to  $w, t$  among those at which  $A$  is true. They persuasively argue that a principle of past predominance obtains according to which similarity between worlds before times  $t$  and  $t'$  predominates over any similarity after these times. The final paper in the volume, *A temporal framework for conditionals and chance*, by van Fraassen, also develops a system for tensed conditionals (though one that is different in a number of respects from Thomason and Gupta's). Van Fraassen also formulates a connection between subjective probability and chance that is like Lewis's.

As indicated by this brief description of its contents, this volume is essential for anyone working on the logic of conditionals and the foundations of probability and decision theory. BARRY LOEWER

LEONARD GODDARD and RICHARD ROUTLEY. *The logic of significance and context*. Volume 1. Scottish Academic Press, Edinburgh and London 1973, and Halsted Press, New York 1974, xi + 641 pp.

The rejection of certain philosophical theses as nonsense has been a standard ploy in twentieth century philosophy—witness the rejection of metaphysics by logical positivism's identification of significance with verifiability. The ploy has been applied to nonphilosophical (but no less bothersome) sentences as well—witness Russell's resolution of the paradoxes in terms of the significance criteria of the ramified theory of types. What is needed in all these cases, according to the authors of this text, is "a general formal theory of significance in terms of which significance claims, and arguments by means of which they are made, can be assessed" (pp. 5–6).

A number of different significance logics are constructed by the authors, but all are based upon a common *context logic* CL in which the significance principles of each logic are understood to be restricted to a fixed context of use. A basic assumption of CL is that there are grammatically well-formed sentences (of the background natural language) which in some contexts are nonsignificant while in others (which might be nonstandard contexts) they are significant and either true or false. Strictly speaking, according to the authors, truth and falsity are properties only of the use of sentence tokens in a given context, and nonsignificance is a third value that such a use might have. The authors include a fourth semantic value here as well, namely incompleteness, for those uses of sentence tokens that are significant and yet neither true nor false (apparently because the tokens are about items which do not exist); but this fourth value is not involved in the significance logics developed within CL. With sentences identified with sentence tokens in a given context, sentences themselves are then said to be true, false, nonsignificant, or else significant but neither true nor false (i.e., incomplete), depending on the value their use has in that context. The contexts are restricted to those involving "intended statement making, or indicative, acts" (p. 24). A sentence will yield a statement in such a context if and only if it is both significant and complete, that is, both significant and either true or false.

The basic kind of assertion made in CL is that a quoted sentence  $qu(p)$  in a context  $c$  yields a statement that- $a$ . The sentential variable  $p$  has sentence tokens of the background natural language as its values and  $qu(p)$  gives the quotation expression of these tokens. A special that-operator,  $\S$ , is used to generate subject expressions that mention statements. Whether or not a sentence token is significant and complete (in a given context), the assertion that it yields a particular statement (in that context) is either true or false in CL. That is, CL is a two-valued metatheory of a four-valued object language. Of course, since three-valued significance logics are extensions of CL, the authors really reject any sharp distinction between CL as a metatheory and the various significance logics that are its formal object theories. This is achieved in part by what they call absorption principles, which allow them to avoid writing the quotation function symbol. For example, instead of writing  $sig\ qu(p)$ , where  $sig$  is the primitive significance predicate of CL, an absorption principle allows them to write  $Sp$ . Similarly, instead of writing  $comp\ qu(p)$ ,  $tru\ qu(p)$ , and  $fal\ qu(p)$ —where  $comp$ ,  $tru$ , and  $fal$  are the predicates definable in CL for completeness, truth, and falsity—the authors use absorption principles to write  $Cp$ ,  $Tp$ , and  $Fp$  instead. Thus, where two sentence tokens are extensionally equivalent in the strong sense if and only if they are both true, both false, both nonsignificant, or both incomplete, an absorption principle allows this relation to be expressed by the following thesis of CL:  $\{[p \simeq q] \equiv [(Tp \ \& \ Tq) \vee (Fp \ \& \ Fq) \vee (\sim Sp \ \& \ \sim Sq) \vee (\sim Cp \ \& \ \sim Cq)]\}(c)$ .

In the predicate logic for CL the authors allow nonsignificant subject expressions (e.g. 'the purple number 7,' which might be significant in a nonstandard context) as well as contradictory descriptions (e.g. 'the round square') to be "values" of the subject variables. Quantification, in other words, is given a

substitutional interpretation. This is so, according to the authors, because significance has nothing to do with what sorts of things exist, but only with “the sorts of things which can be talked about” (p. 475). One benefit of such an interpretation is that it does not lead to significance essentialism. Wide quantifiers  $U$  and  $P$  (read “for every” and “for some out of all”) are used to cover all constant subject expressions, and a special existential quantifier  $\exists$  (read “there exists”) is used to cover those constant subject expressions that do refer to items that exist. Another “particular” quantifier  $q$  (read “for some”) is used in the three-valued significance logics as well. This quantifier is needed since  $(Px)fx$  will be nonsignificant if  $qu(fx)$  is nonsignificant for some value of  $x$ , which will generally be the case. That is, whereas  $v((Px)fx) = t$  iff  $v(fx) = t$  for some value of  $x$  and  $v(fx) = n$  for no value of  $x$ ,  $v(qx)fx = t$  iff  $v(fx) = t$  for some value of  $x$  simpliciter.

What a subject expression is about is the “item” it identifies; that is, what it is about depends on some identification being made in the context in question. In terms of the particular quantifier  $q$ , however, ‘Pegasus,’ ‘the purple number 7,’ and ‘the round square’ are all about “something.” Identity is not unique here, moreover, but is a determinable under which there are many specific or determinate identity relations, with extensional identity the weakest and a Leibnizian identity the strongest. Since “aboutness” depends on identity, there are therefore different “aboutness relations” as well. This leads to what the authors call *contractual semantics*, where a contraction is a function of three things: (1) the class of subject expressions on which it is made, (2) the identity relation used in forming equivalence sets, and (3) a special property. For example, where the special property is existence or subsistence (i.e., possible existence), respectively, *reference* is an aboutness relation (based on extensional identity) to items which exist, and *designation* is similarly an aboutness relation to items which subsist. Some items, nevertheless, neither exist nor subsist.

Included among the subject expressions, incidentally, are quotation expressions and that-clauses. In terms of the former, the primitive *yields* relation of CL is an aboutness relation based upon statement equality and the property of having a truth-value, with the class of subjects restricted to quoted sentence tokens. With ‘that’ as part of subject expressions, moreover, ‘Tom knows Mary’ and ‘Tom knows that there is no present King of France’ are taken as having the forms  $K(x, y)$  and  $K(x, \S a)$ , respectively, where  $K$  (for ‘knows’) is an extensional predicate. Indeed, for the authors there are no intensional predicates at all but only two intensional, or opaque, operators, namely,  $\S$  and  $qu$ , which resist the substitutivity of identicals. Thus, because of the intensionality of the that-operator,  $x = y \supset \S(x = x) = \S(x = y)$  is rejected for much the same reason that  $x = y \supset qu(x) = qu(y)$  is. With Leibnizian identity, however,  $x \approx y \supset \S(x = x) \approx \S(x = y)$  is valid, whereas  $x \approx y \supset qu(x) \approx qu(y)$  is invalid.

The significance logics constructed by the authors are developed in stages, beginning with a weak and unobjectionable three-valued logic which in successive stages is made more and more comprehensive. The stages are described first on the sentential level, both axiomatically and in terms of three-valued matrices. Several alternatives to the authors’ “classical” significance logic  $S_6$  are described and rejected on this level—for example, the three-valued matrices Kleene described for the context of partially recursive functions, and Halldén’s earlier significance logic in which  $n$  as well as  $t$  is taken as a designated value. According to the authors, the third value of Kleene’s logic is not properly interpreted as ‘non-truth-valued’ (and not, therefore, as ‘nonsignificant’), but only as “not truth-value determinable under the procedure” in question (p. 268).

Since modal connectives are not matrix-accessible over a finite set of values, the authors develop a model theory of situations and set-ups for the semantics of their multiply-modal significance logics. A substitutional interpretation of the quantifiers is then appended to this model theory. “Set-ups” are sets of sentences that, in effect, are generalizations of state descriptions. Situations are special set-ups that satisfy the negation rule:  $\sim A \in H$  iff  $A \notin H^*$ , where  $*$  is one-to-one and such that  $H^{**} = H$ . Statements are treated as members of situations. Set-ups need not, in general, be alternatives to the actual world situation, and in fact they may be inconsistent as well as incomplete. Intensional models are constructed on this basis in the usual way, and soundness and completeness theorems are proved.

A number of applications of significance logic are made and discussed by the authors, including, for example, a significance arithmetic and theory of partial recursive functions. The authors reject second and higher order logics (on grounds of absolute nonsignificance, strange to say) but instead develop many-sorted and what they call second-stage logics. The text closes with a discussion of the problem of formulating a theory of categories in the sense of logical kinds determined by characteristic predicates that are significant for the members of such a kind.

This text contains a number of novel systems and ideas, not all of which will find ready acceptance from logicians and linguists, but most of which deserve careful scrutiny and discussion. Space precludes any critical commentary here, other than to say that the text is in dire need of an index.

NINO B. COCCHIARELLA

CRISPIN WRIGHT. *Wittgenstein on the foundations of mathematics*. Duckworth, London, and Harvard University Press, Cambridge, Mass., 1980, xix + 481 pp.

This is a book on Wittgenstein's later philosophy of mathematics, that is, the philosophy of mathematics continuous with the philosophical ideas Wittgenstein developed *after* the publication of the *Tractatus* (2812). (The *Tractatus* itself is hardly mentioned.) One of the major aims of the book is to show the organic unity between Wittgenstein's philosophy of mathematics and his philosophy of language—that philosophy of language set forth, for instance, in the *Philosophical investigations*. For example, Wright successfully shows that much of the criticism of Wittgenstein's posthumously published *Remarks on the foundations of mathematics* misses the fact that much of what Wittgenstein says in that book is of a piece with, or follows from, Wittgenstein's celebrated treatment of "following a rule" in the *Investigations*. Indeed, some of the very philosophers who dismiss the *Remarks* take the *Investigations* seriously as a great work of philosophy!

Unfortunately, Wright's book is very hard to read. The reader is hard-pressed to follow the twists and turns of the arguments. Themes are introduced suddenly, interrupting the flow of argument. Wright reverses himself again and again; as he puts it, "the whole is a document of ideas in progress and should be read accordingly" (p. viii). Essentially, what Wright has done is to publish lecture notes from his classes in Oxford in 1974, 1975, and 1977 (sure enough, there are three sections to the book). I find it hard to imagine a reader agreeing with Wright when he says "Originally . . . it was my intention wholly to jettison the format of those lectures and attempt systematically to rewrite my notes from a unified point of view; but it soon became clear that this would be a massive task, quite out of proportion to any likely gain" (p. viii). To the contrary, the book should not have been published in its present form. The book's length could have been cut in half, with great gain in clarity. With book prices as they are, why inflict upon the reader all of one's false starts, bad moves, etc., in the order they occurred to one? On the other hand, the analytical table of contents and the index are quite good, and, given the complexity of the book in its present form, quite necessary.

As I read it, the book encompasses two main themes. Connected with each theme is a puzzle. The first theme is Wittgenstein's hostility to Platonism, or to mathematical realism, hostility based on what Wright annoyingly calls, throughout the book, the "rule-following considerations." The puzzle is, why did Wittgenstein not call for the revision of mathematical *practice*, since classical mathematics is based, it seems, on the very kind of mathematical realism that Wittgenstein opposed so strongly? Why did he not go even beyond the intuitionists in opposing classical mathematics? And yet Wittgenstein explicitly states that he will not "interfere with the mathematicians" (*Wittgenstein's lectures on the foundations of mathematics*, edited by Cora Diamond, Cornell University Press, 1976, p. 13). The second theme is Wittgenstein's *conventionalism*—his special views regarding mathematics and logic, views that do not flow from general considerations of language alone—namely, that an element of *decision* enters into accepting a mathematical proposition or, in general, any *necessary* proposition. The associated puzzle is, how do we characterize the propositions that we find important to render true by convention? To put it another way, *why* do we bother to make such conventions? I think a great virtue of the book is its attempt to distinguish clearly these themes.

Of course, there are many other themes in the book. Wright wrestles with Dummett's views on realism almost as much as he does with Wittgenstein's ideas. He discusses the destructive consequences of Wittgenstein's ideas for Davidson's "theory of meaning" project based on Tarski-type truth theories. (He regards it as "scandalous" (p. 279) that he is the first one to have done so, thirty years after the appearance of the *Investigations*. On the other hand, Wright himself ignores the Quine of *Truth by convention* (I 42) in his discussion of truth and convention, and *Word and object* is mentioned—in the bibliography—but not used very much in the text. So Wright's discussion of Quine on analyticity and logical truth ignores virtually everything but the cryptic and programmatic formulations of *Two dogmas of empiricism* (XLVII 230). Also, attention to Kripke's masterpiece, *Naming and necessity* (*Semantics of natural language*, D. Reidel and Humanities Press, 1972, pp. 253–355 and 763–769, also Harvard University Press, 1980), which contains powerful attacks on some of Wittgenstein's main doctrines, would have helped Wright on