Richard Montague and the Logical Analysis of Language

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Richard Montague was an exceptionally gifted logician who made important contributions in every field of inquiry upon which he wrote. His professional career was not only marked with brilliance and insight but it has become a classic example of the changing and developing philosophical views of logicians in general, especially during the 1960s and 70s, in regard to the form and content of natural language. We shall, in what follows, attempt to characterize the general pattern of that development, at least to the extent that it is exemplified in the articles Montague wrote during the period in question.

The articles to which we shall especially direct our attention are: 'Pragmatics' [1]; 'Pragmatics and Intensional Logic' [2]; 'On the Nature of Certain Philosophical Entities' [3]; 'English as a Formal Language' [4]; 'Universal Grammar' [5]; and 'The Proper Treatment of Quantification in Ordinary English' [7]. Needless to say, but many of the ideas and insights developed in these papers Montague shared with other philosophers and logicians, some of whom were his own students at the times in question. Montague himself was meticulous in crediting others where credit was due, but for convenience we shall avoid duplicating such references here.

1 Constructed Versus Natural Languages

In his earlier pre-1964 phase of philosophical development Montague thought a formalization of English or any of natural language was either impossible or extremely laborious and not rewarding ([10], p. 10). In his view there was an important difference between constructing a formal language for the purpose of analyzing concepts that were of interest to philosophers and investigating the behavior of the expressions in natural language that were ordinarily used to express those concepts. He held in low esteem the general methodology of so-called ordinary-language philosophers that was very popular during the 1950s and early 60s and thought that there was little or no philosophic interest in attempting to analyze ordinary language itself.

At the height of his later post-1964 phase of development, Montague had come to completely reject 'the contention that an important theoretical difference exists between formal and natural languages' ([4], p. 188), i.e., 'between natural language and the artificial languages of logicians' ([5], p. 222); and he set about constructing a formal language that 'may be reasonably regarded as a fragment of ordinary English' ([4], p. 188). He still rejected the methodology of ordinary-language philosophy but had by then come to believe that indeed there is philosophic interest in attempting to analyze ordinary language after all ([3], p. 186).

Nevertheless, despite this important change in view and attitude regarding natural language, Montague continued to maintain that ordinary language is 'an inadequate vehicle for philosophy' ([3], p. 186). This is partly because ordinary language is rich in ambiguities and needs to be disambiguated and partly because the semantical relation of logical consequence and logical truth cannot be characterized except with respect to a logically perspicuous language. It is also, and perhaps more especially, because of the dubious ontological nature and status of entities purportedly involved in the use of ordinary language, at least when compared to the entities to which they are reduced in the logically perspicuous language Montague constructs for the semantics of ordinary language. Indeed, here in the ontology of that logically perspicuous language we find another, and perhaps more fundamental, change or development in Montague's philosophy of language.

2 Set Theory Versus Intensional Logic

In his earlier phase, Montague had maintained that philosophy had 'as its proper theoretical framework set theory with individuals and the possible addition of empirical predicates' ([3], p. 154). Philosophical analyses, on this view, were to be carried out within definitional extensions of a set theory so supplemented; and, in consequence, nonextensional entities such as individual concepts, properties, relations-in-intension and propositions were not allowed to occur in such analyses. Intensional contexts and so-called propositional attitudes, such as knowledge and belief, were to be analyzed instead in terms of relations between persons and sentences or other similar expressions. Montague's own contributions to such analyses (and reprinted in *Formal Philosophy*) can be found in [8], [9], [7]. This last paper is especially important for its extension of Tarski's theorem to necessity, or necessary truth, as a property of sentences.

In his later phase, however, Montague developed a new theoretical framework that transcended set theory and in which individual concepts, properties, relations-in-in tension, and propositions are all allowed to occur in philosophical analyses and are found to be 'not much more mysterious than sets' ([3], p. 152). 'Philosophy', Montague wrote at the mid-point of this later phase, 'is always capable of enlarging itself; that is, by metamathematical or model-theoretic means—means available within set theory—one can "justify" a language or theory that transcends set theory, and then proceed to transact a new branch of philosophy within the new language' (*ibid.*, p. I55). This new theoretical framework was intensional logic.

There are two stages and two systems involved in Montague's development of his new theoretical framework for philosophy. The first is a second-order modal logic (which is readily extendible to third and higher-order modal logic), and the second is an extension of Alonzo Church's formulation of type theory in which primitive operators for the extension-intension, or sense-denotation, distinction have been introduced. The first, for the most part, allows only for intuitive translations into logical notation (as typified by the symbolizations found in elementary logic texts) and usually requires the use of circumlocution or paraphrase for the representation of certain intensional verbs such as 'seek', 'conceive', 'owe', etc. The second, on the other hand, allows for a precise translation of ordinary language and for a direct representation of intensional verbs. (See Gallin [11] for a description of both systems, together with other related systems and a number of proofs of results regarding all of these systems and their interrelationships.) Before turning to these systems, however, we shall first take up Montague's development of pragmatics, which involves only first-order languages and which in fact rather appropriately characterizes the initial phase in the development of Montague's new orientation to the form and content of language.

3 Pragmatics and Logical Semiotics

Prior to the 1960s most formal language philosophers deliberately avoided the question of how the different contexts in which language is used affects the meaning and reference of expressions. Logicians, with their proper concern for rigor and formal precision, tended to see the logical analysis of those features of ordinary language that were dependent on context as either impossible or hopelessly complex. They preferred instead to abstract from particular contexts of use and to provide analyses of sentences and arguments of ordinary language within a timeless and context-free setting.

Logical semiotics at that time, as opposed to descriptive semiotics, was in fact by definition restricted to logical (or pure) syntax (concatenation theory) and logical (or pure) semantics (model-theory as a branch of set-theory or as a form of higher-order logic), where the latter amounted in effect only to a theory of reference. Pragmatics, the third branch of the semiotic trinity, was not only excluded from logical or pure semiotics on this scheme, but was in fact considered to be the very basis of descriptive semiotics (Cf. Carnap [12]). Thus, for example, Rudolf Capnap considered pragmatics to be an empirical theory that was to be formulated in the metalanguage of a collection of languages, and in that regard was to be concerned with the relationship between the users and the signs of the object languages in question. The expressions of a pragmatic theory, on this view, including epistemic expressions for belief, doubt, etc., were considered to be *predicates* belonging to the metalanguage that were understood as designating psychological relations between language users and signs of the object languages. Indeed, Carnap clearly states even as late as 1963 that the relation of designation in pragmatics 'is a psychological concept' (cf. Carnap [13]).

Montague's development of pragmatics was not so much a rejection of this tradition as an alternative development within logical semiotics. What he proposed was an extension of the semantical notion of truth (in a model or interpretation) as defined for standard first-order languages but now supplemented with pragmatic operators, such as operators for the past and future tenses, as well as other indexical expressions for demonstratives and personal pronouns. Pragmatics, according to Montague, is to follow the lead of semantics and 'concern itself also with the notion of truth—but with respect not only to an interpretation but also a context of use' ([1], p. 96). Thus instead of dealing with descriptive pragmatic predicates of an empirical metalinguistic theory, Montague formulates a logical theory of first-order languages that include pragmatic operators and certain individual constants taken to represent the demonstratives and personal pronouns of natural language.

4 Pragmatic Languages

On Montague's approach then, a *pragmatic language* is a first-order language the basic symbols of which are drawn from the following categories: (1) logical constants, e.g., $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\forall$, $\exists$ and $=$ (with their usual readings); (2) auxiliary symbols such as parentheses, brackets and commas; (3) individual variables; (4) individual constants; (5) n-place predicate constants (for $n \in \omega$), including the one-place predicate constant '**E**' (for 'exists'); and (6) n-place operators, for $n \in \omega - \{0\}$. The terms and formulas of a pragmatic language L are built up in the usual way except that now whenever $\mathbf{M}$ is an n-place operator of L and $\varphi_1, ..., \varphi_n$ are formulas of L, then $\ulcorner \mathbf{M}\varphi_1...\varphi_n \urcorner$ is also a formula of L. Thus, e.g., where $\mathcal{P}$ and $\mathcal{F}$ are one-place operators (for the past and future tense) and φ is a formula, then $\ulcorner \mathcal{P}\varphi \urcorner$ and $\ulcorner \mathcal{F}\varphi \urcorner$ are also formulas and may be read as $\ulcorner$It was the case that $\varphi \urcorner$ and $\ulcorner$It will be the case that $\varphi \urcorner$, respectively.

In interpreting a pragmatic language, it is of course necessary that we take into account the possible contexts of use for that language. We need not consider these contexts in their full complexity, on the other hand, but need only consider those aspects of such a context that are relevant to the language in question. Montague calls such a complex of relevant aspects an *index* or, following Dana Scott, *a point of reference*. Thus for the tense operators each index will specify a moment of time, and for personal pronouns and demonstratives it will also specify speaker, a listener, objects indicated, etc.

Besides the indices or points of reference, an interpretation for a pragmatic language L will assign intensions (or meanings) to each of the operators and individual and predicate constants in L. The idea here is that an intension determines an extension of the relevant type in each context of use, i.e., at each index of a possible pragmatic interpretation. Intensions, or rather their set-theoretic representations, accordingly, can be identified with certain functions on the set of indices. Thus, e.g., the intension of an individual constant will be a function that assigns an individual to each index; and the intension of an n-place predicate constant will be a function that assigns a set of n-tuples of individuals, i.e., an n-ary relation (in extension) between individuals to each index. Montague called the first type of intension an *individual concept* and the second an n-ary *relation-in-intension* (between individuals). The intension of a one-place predicate constant he called a *property* (of individuals). Rudolf Carnap was the originator of this general type of approach to the set-theoretic representation of intensions. Carnap, however, as Montague noted, restricted his indices to models or state-descriptions.

Where Montague went beyond Carnap, aside from his disassociating indices from models or state-descriptions, was in his unified treatment of pragmatic operators—and in that regard in his development of a general pragmatic logic. In order to understand this treatment, let us note first that as an intension a proposition determines a truth-value in each context of use, and therefore at each index of a pragmatic interpretation. In this regard, the same proposition can be set-theoretically represented either by the set of indices (of the interpretation) at which it is true or by the function that assigns to each index the truth-value the proposition determines at that index. Relative to the former representation, the functions on indices that Montague associates with n-place operators are those that assign to each index an n-ary relation (in extension) between the propositions of the interpretation, i.e., those that assign to each index an n-ary relation on the set of all subsets of the set of indices of the interpretation. In other words, because the different subsets of indices are here taken as set-theoretic representatives of propositions, pragmatic operators were interpreted by Montague in effect as higher-order properties and relations-in-intension of propositions.

Formally stated, then, *a possible interpretation for a pragmatic language* L is an ordered triple $\langle I, U, F \rangle$ such that (1) I and U are the sets of indices and individuals, respectively, of the interpretation; (2) F is a function defined on the set of operators and individual and predicate constants in L; (3) for each individual constant $\mathbf{c}$ in L, $F(\mathbf{c}) \in U^I$; (4) for each n-plane predicate constant $\mathbf{P}$ in L, $F(\mathbf{P}) \in \{A : A \subseteq U^n\}^I$; and (5) for each n-place operator $\mathbf{M}$ in L, $F(\mathbf{M})$ is a function on I such that for each $i \in I$, $F(\mathbf{M})(i) \subseteq \{J : J \subseteq I\}^n$.

Where $\mathcal{A}$ is a possible interpretation for a pragmatic language L, the fact that each nonlogical constant in L is assigned an intension in $\mathcal{A}$ allows us to follow the recursion clauses in the definition of the terms and formulas of L in such a way as to assign a unique intension of the appropriate type to each term and formula of L. This then leads in a natural and obvious way to a definition of *truth* for the sentences of L *at an index of* $\mathcal{A}$, and thereby to definitions of *logical truth* and *logical consequence* in the sense of general pragmatics. (We shall forgo the details of these definitions here, however.)

Now it should be noted that particular pragmatic languages, such as those for modal, deontic and tense logic, had of course been studied before, both by Montague and his students as well as by other logicians. But in each case, as Montague observed, 'truth and satisfaction had to be defined anew,' because no 'unified treatment of operators' had yet been realized ([2], p. 120). Indeed, it is in fact Montague's unified treatment of the semantics of operators that is his unique contribution to the development of pragmatics within logical semiotics. In particular, it is only with respect to such a unified treatment that the notions of logical truth and logical consequence in the sense of general pragmatics can be characterized at all.

Notwithstanding this development of pragmatics within logical semiotics, however, philosophers are typically not interested in general pragmatic logic but in certain specializations or disciplines, such as the tense, modal and deontic logics already mentioned. Such specializations or disciplines, relative to general pragmatic logic, amount to restricting the possible interpretations to certain classes of what might be considered the standard interpretations for the discipline in question. Thus where K is such a class of possible interpretations, we can restrict logical truth and logical consequence in the sense of general pragmatic logic to the interpretations in K, giving us instead, e.g., K-validity and K-consequence. Sometimes it will even be necessary to restrict the indices as well, designating some of them as the standard points of reference; and this will lead to further restrictions on validity and consequence, all serving the particular discipline in question (cf. [1], pp. 104f.).

5 Intensional Logic as Higher-Order Modal Logic

It is noteworthy that although the nonlogical constants of a pragmatic language are all assigned intensions in a possible interpretation for that language, the only entities quantificationally referred to under such an interpretation are the individuals of the interpretation. Pragmatic languages are after all first-order languages, and in that regard, what is indicated by the fact that pragmatic operators can be interpreted as higher-order properties and relations-in-intension of propositions is that pragmatic languages amount in effect to first-order reductions of (a part of) higher-order intensional languages, i.e., higher-order languages where the properties and relations-in-intension between propositions that operators in pragmatic languages represent are directly represented instead by higher-order predicate constants.

Alternatively, what this also indicates is that pragmatics, as a development within logical semiotics, might more appropriately be viewed as contained in intensional logic where quantificational reference to the intensional entities represented in pragmatic interpretations is a part of the object language itself. Indeed, according to Montague, the fact that the modalities that have usually been associated with operators coincide in a sense with properties and relations-in-intension of propositions 'is what lends interest to those modalities and provides intuitive sanction for using them to interpret operators' ([2], p. 137). And if that is the case, then we might just as well develop an intensional logic and proceed to do philosophy in that framework instead. This in fact is what Montague had gone on to do next in his developing new orientation to the philosophy of language.

Montague's first formulation of an intensional logic is really a second-order modal logic supplemented with third-order predicate constants. An intensional language on this formulation is one the basic symbols of which are (1) the logical constants of pragmatic languages; (2) parentheses and brackets; (3) individual variables and constants; (4) n-place predicate variables, for $n \in \omega$; (5) the one-place operator $\Box$ (read 'necessarily'); (6) the descriptive symbol $\mathbf{T}$ (read 'the unique ... such that'); and predicate constants of type s, for each finite sequence s of integers ≥ -1.

The type of a predicate constant, as described above, indicates the grammatical types of the argument-expressions it takes, with -1 being the type of an individual variable or constant and a nonnegative integer n being the type of an n-place predicate variable. We identify propositional variables and propositional constants here with 0-place predicate variables and predicate constants of type s, where s is the null sequence.

The *atomic formulas* of an intensional language L are expressions either of the form $\ulcorner \zeta = \eta \urcorner$, where ζ, η are individual variables or constants; $\ulcorner \mathbf{P} = \mathbf{Q} \urcorner$, where $\mathbf{P}$, $\mathbf{Q}$ are n-place predicate variables, for $n \in \omega$; $\ulcorner \mathbf{P}(\zeta_1, ..., \zeta_n) \urcorner$, where $\mathbf{P}$ is an n-place predicate variable and each ζ_i is either an individual variable or an individual constant; or $\ulcorner \mathbf{R}(\xi_1, ..., \xi_k) \urcorner$, where $\mathbf{R}$ is a predicate constant of type $\langle s_1, ..., s_k \rangle$ and either $s_1 = -1$ and ξ_i is an individual variable or constant or $s_i \geq 0$ and ξ_i is an s_i-place predicate variable.

Given the atomic formulas, the formulas of L are then built up in the obvious way. The descriptive symbol $\mathbf{T}$, however, is allowed to apply only to predicate variables, so that where $\mathbf{P}$ is an n-place predicate variable and φ is a formula, $\ulcorner\mathbf{TP}\varphi\urcorner$ is a complex n-place predicate expression. As used by Montague, these descriptive phrases are all eliminable and therefore, strictly speaking, the descriptive symbol $\mathbf{T}$ need not occur at all in the formulas of intensional logic. Its utility is really one of convenience for allowing certain more direct ways for representing complexly described intensions.

Because the set-theoretic counterparts or representatives of intensions are always functions on some set I of indices and with values that are either drawn or constructed from a set U of individuals, it is more accurate in some contexts if we speak of these counterparts as $\langle I, U\rangle$-intensions. Thus an $\langle I, U\rangle$-individual concept is a function on I with values in U; and an n-ary $\langle I, U\rangle$-relation-in-intension is a function on I with n-ary relations (in extension) on U, i.e., sets of n-tuples drawn from U, as values. An $\langle I, U\rangle$-property is a one-ary $\langle I, U\rangle$-relation-in-intension.

An $\langle I, U\rangle$-proposition, as already noted, can be identified either with a subset of I, namely, the set of indices at which the proposition is true, or with a function that assigns a truth-value to each index. If we use 1 to represent *the true* and 0 to represent *the false*, then an $\langle I, U\rangle$-proposition, as a function that assigns a truth-value to each index, amounts to the characteristic function of the set of indices at which the proposition is true, i.e., it amounts to that function on I with values in $\{0, 1\}$ such that 1 is assigned to all and only those indices in the set in question. This latter identification is more convenient in our present context since we identify 0 with both the null sequence and the empty set and take $1 = \{0\}$. Thus, because propositional variables are construed here as 0-place predicate variables, an $\langle I, U\rangle$-proposition which is the value of such a variable will be an $\langle I, U\rangle$-intension that has 0 or 1 as its value (extension) at each index. An $\langle I, U\rangle$-proposition, in other words, is a 0-ary $\langle I, U\rangle$-relation-in-intension.

A *possible interpretation for an intensional language* L, then, is a triple $\langle I, U, F\rangle$ such that (1) I and U are the sets of indices and individuals, respectively, of the interpretation; (2) F is a function defined on the nonlogical constants of L, i.e., the single operator $\Box$ and the individual and predicate constants in L; (3) for each individual constant $\mathbf{c}$ in L, $F(\mathbf{c}) \in U^I$, i.e., $F(\mathbf{c})$ is an $\langle I, U\rangle$-individual concept; (4) $F(\Box) = \{\langle i, \langle I\rangle\rangle : i \in I\}$, i.e. $F(\Box)$ is that property that propositions (as subsets of I) have when and only when they are true at every index[1]; and (5) for each predicate constant $\mathbf{P}$ of type

[1] Our interpretation here of $\Box$ as a one-place operator that is directly to be assigned an

$\langle s_1, ..., s_k\rangle$ in L, $F(\mathbf{P})$ is a function on I such that for $j \in I$, $F(\mathbf{P})(j)$ is a set of n-tuples $(V_1, ..., V_k)$ such that for all i, if $1 \leq i \leq \_k$, than either $s_i = -1$ and $V_i \in U$ or $s_i \geq 0$ and V_i is an s_i-ary $\langle I, U\rangle$-relation-in-intension.

As with pragmatic languages, we are able again to follow the recursion clauses for the construction of an arbitrary formula of an intensional language L and define, relative to a possible interpretation $\mathcal{A}$ for L, the intension in $\mathcal{A}$ of each formula of L; and, as before, this definition also leads in a natural way to a definition of *truth* (for the sentences of L) at an index of $\mathcal{A}$, and thereby to definitions of *logical truth* and *logical consequence* (in the sense of intensional logic).

With the syntax and semantics of intensional languages characterized as above, the exact sense in which pragmatics is contained in intensional logic can now be explained as follows. For each pragmatic language L, there is (easily constructible from L) an intensional language L' containing (1) all the individual constants of L; (2) a predicate constant $\mathbf{P}'$ of type $\langle s_1, ..., s_n\rangle$, where $s_i = -1$, for $i \leq n$, for each n-place predicate constant $\mathbf{P}$ in L; and (3) a predicate constant $\mathbf{M}'$ of type $\langle s_1, ..., s_m\rangle$, where $s_i = 0$, for $i \leq m$, for each m-place operator $\mathbf{M}$ in L. Similarly, for each pragmatic interpretation $\langle I, U, F\rangle$ for L there is an intensional interpretation $\langle I, U, F'\rangle$ for L' such that (1) $F'(\mathbf{c}) = F(\mathbf{c})$ for each individual constant $\mathbf{c}$ in L; (2) $F'(\mathbf{P}') = F(\mathbf{P})$ for each n-place predicate constant $\mathbf{P}$ in L; and (3) for each m-place operator $\mathbf{M}$ in L, $F'(\mathbf{M}')$ is that function on I such that for $i \in I$, $F'(\mathbf{M}')(i) = \{\langle cf(J_1), ..., cf(J_m)\rangle : \langle J_1, ..., J_m\rangle \in F(\mathbf{M})(i)\}$, where $cf(J_k)$ is the characteristic function of J_k, for $k \leq m$.

A translation function t that translates each formula of L into a formula of L' can be recursively defined as follows: $t(\ulcorner \varsigma = \eta \urcorner) = \ulcorner \varsigma = \eta \urcorner$; $t(\ulcorner \mathbf{P}(\varsigma_1, ..., \varsigma_n)\urcorner = \ulcorner \mathbf{P}'(\varsigma_1, ..., \varsigma_n)\urcorner$; $t(\ulcorner \neg\varphi \urcorner) = \neg t(\varphi)$; and similarly for $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$; $t(\ulcorner \forall v_i \varphi \urcorner) = \ulcorner \forall v_i t(\varphi)\urcorner$, and similarly for $\ulcorner \exists v_i \varphi \urcorner$; and finally $t(\ulcorner \mathbf{M}\psi_1 ... \psi_m^{\urcorner}) = \exists \mathbf{p}_1 ... \exists \mathbf{p}_m[\Box[\mathbf{p}_1 \leftrightarrow t(\psi_1)] \wedge ... \wedge \Box[\mathbf{p}_m \leftrightarrow t(\psi_m)] \wedge \mathbf{M}'(\mathbf{p}_1, ..., \mathbf{p}_m)]$. It is now easily proved that if φ is a sentence of L and $i \in I$, then φ is true at i in the pragmatic interpretation $\langle I, U, F\rangle$ if and only if $t(\varphi)$ is true at i in the intensional interpretation $(\langle I, U, F'\rangle$.

intension under an interpretation is an inessential modification of Montague's treatment.

6 Philosophical Applications of Intensional Logic

In turning to philosophical applications of his intensional logic, Montague took up such questions as whether we need 'tolerate such entities as pains, events, tasks and obligations' ([3], p. 148). Sentences in which we purportedly talk about these entities 'play a conspicuous role in philosophy, perceptual psychology, and everyday discourse' (*ibid.*); and 'it therefore appears desirable to investigate the nature of the entities in question, construct an exact and convenient language in which to speak of them, and analyze the pertinent notion of logical consequence' (*ibid.*). It is this last task in particular that, according to Montague, is 'a necessary preliminary to the rational treatment of certain philosophical paradoxes' (*ibid.*, p. 149).

Of course, if the entities in question can be reduced to the properties and relations-in-in tension of intensional logic, then this last task has in fact already been accomplished; and it is not surprising, therefore, to find Montague proposing just such a reduction. Thus, e.g., insofar as it is not unreasonable to regard an (instantaneous generic) event of the sun's rising as corresponding to the expression 'the sun rises at t', it is similarly not unreasonable, according to Montague, to identify the (generic) event in question with the property expressed by that expression. Instantaneous generic events, on this analysis, 'will form a certain class of properties of moments of time' (*ibid.*, p. 150); and the occurrence of such an event at a moment is to be explicated as the possession by that moment of the property that is identified with that event. Protracted generic events can of course be identified with properties of intervals, rather than of just moments, of time.

Tasks, according to Montague, 'should be regarded as certain two-place relations-in-intension between persons and moments' (*ibid.*, p. 151); and therefore the performance of a task R by a person x at a moment t is to be taken as x's bearing the relation-in-intension R to t. Pains, 'and more generally experiences, are of the same ontological sort as tasks: they form a certain class of relations-in-intension between persons and moments' (*ibid.*). Thus, for example, 'the experience of seeing a tree is the relation-in-intension born by x to t just in case x sees a tree at t' (*ibid.*); and therefore for x to *have* the experience R at t is for x to bear the relation-in-intension R to t. Finally, obligations, according to Montague, can also 'best be regarded as the same sort of thing as tasks and experiences, that is, as relations-in-intension between persons and moments' (*ibid.*); and therefore for a person x to *discharge* or *fulfill* an obligation R at a moment t is again for x to bear the relation-in-intension R to t.

One point to be noted in these reductions is that on Montague's view 'the basic notion of partaking of a property ... is expressed in ordinary English by several different verbs, depending on context; by 'perform' in the case of a task, 'have' in the case of an experience, and 'occur' in the case of an event' (*ibid.*); and of course by 'discharge' or 'fulfill' in the case of an obligation. It should also be noted, however, that in none of these reductions does Montague claim to have defined the notion of an experience, or of a task, etc. What he does claim is that the property of being an experience, of being a task, etc., is to be regarded as a property of relations-in-intension between persons and moments.

The reduction of these 'dubious ontological categories' to one, namely that of relations-in-intension, led Montague to place more emphasis on the ontological status of the indices and individuals involved in the set-theoretic construction of relations-in-intension. Instead of being merely possible contexts of use, indices are now spoken of as full-fledged possible worlds and the individuals are all the different *possibilia* that might exist in those different possible worlds. There is, however, some question here as to whether the notions of a possible world and of a possible individual have an absolute or only a relative status in the set-theoretic foundations of Montague's new theoretical framework; and we shall return to this question at a later point in this essay.

The important observation that should be made here in any case is that by reducing these 'dubious ontological categories' to that of relations-in-intension, Montague had in effect produced 'an exact language capable of naturally accommodating discourse about the dubious entities' as well as defined 'an intuitively satisfactory notion of logical consequence for the sentences of that language' (*ibid.*, p. 154); and in consequence we have at least the necessary preliminaries for a rational treatment of a variety of philosophical puzzles or paradoxes.

Toward the treatment of such puzzles in the present version of intensional logic, Montague utilizes the modal operator as a means for constructing names of specific relations-in-intension. Thus, adopting the following abbreviatory notation:

$$\hat{\mathbf{v}}_1 ... \hat{\mathbf{v}}_n \varphi =_{df} \mathbf{TP} \forall \mathbf{v}_1 ... \forall \mathbf{v}_n \Box [\mathbf{P}(\mathbf{v}_1, ..., \mathbf{v}_n) \leftrightarrow \varphi]$$

we can read $\ulcorner \hat{\mathbf{v}}_1 ... \hat{\mathbf{v}}_n \varphi \urcorner$ as $\ulcorner$the relation-in-intension between those (possible) individuals $\mathbf{v}_1, ..., \mathbf{v}_n$ such that $\varphi\urcorner$. If $n = 1$, then $\ulcorner \hat{\mathbf{v}}_1 \varphi \urcorner$ designates the property of those individuals that satisfy φ (as values of the individual variable '$\mathbf{v}_1$' and for $n = 0$, we use $\ulcorner\hat{\ }\varphi\urcorner$for $\ulcorner$the proposition expressed by $\varphi\urcorner$.

In general, at least when the formula in question is not too complex, Montague identifies $\ulcorner\hat{\mathbf{v}}_1\varphi\urcorner$ and $\ulcorner\hat{\mathbf{v}}_1\hat{\mathbf{v}}_2\varphi\urcorner$ with the use of infinitives and gerunds in English. E.g., where 'Bald' is a predicate constant of type $\langle -1\rangle$, $\ulcorner\hat{\mathbf{v}}_1\mathbf{Bald}(\mathbf{v}_1)\urcorner$ represents the different English expressions, 'to be bald', 'being bald' and 'the property of being bald'. The last expression is of course also represented by the predicate constant '**Bald**' itself so that

$$\mathbf{Bald} = \hat{\mathbf{v}}_1\mathbf{Bald}(\mathbf{v}_1)$$

is taken here to be logically true; and in fact it is an instance of the more general logical truth:

(1) $$\forall\mathbf{P}[\mathbf{P} = \hat{\mathbf{u}}\mathbf{P}(\mathbf{u})]$$

It is important to note here, however, that (1) cannot be instantiated to just any predicate expression and still result in a logical truth. E.g., suppose we restricted ourselves to the present moment and took experiences, tasks, etc., to be (relational) properties that a person may or may not now possess; and suppose that a certain person, Jones, is now having the experience of seeing a tree, although in some other possible world the same person, Jones, is having a completely different experience. Clearly, there is nothing inconsistent in this situation; and the assumption that (the property of) seeing a tree is an experience and that it is in fact the experience that Jones is now having can be plausibly represented by:

(2) $$\mathbf{Experience}(\hat{\mathbf{u}}[\mathbf{u\ sees\ a\ tree}]$$

(3) $$\mathbf{TP}[\mathbf{Experience}(\mathbf{P}) \wedge \mathbf{P}(\mathbf{Jones})] = \hat{\mathbf{u}}[\mathbf{u\ sees\ a\ tree}]$$

Moreover, the (modal) fact that Jones is not seeing a tree in some other possible world but is having some completely different experience in that world indicates that the property of having the experience that Jones is having is not identical with the property of seeing a tree, i.e., that

(4) $$\hat{\mathbf{u}}[\mathbf{TP}[\mathbf{Experience}(\mathbf{P}) \wedge \mathbf{P}(\mathbf{Jones})](\mathbf{u})] \neq \hat{\mathbf{u}}[\mathbf{u\ sees\ a\ tree}]$$

is also true in the situation described—because otherwise the two properties designated in (4) would have the same extension in every possible world, which is contrary to assumption.

As Montague points out, it is fallacious to attempt to derive a contradiction here by claiming that

$$\mathbf{TP}[\mathbf{Experience}(\mathbf{P}) \wedge \mathbf{P}(\mathbf{Jones})] = \hat{\mathbf{u}}[\mathbf{TP}[\mathbf{Experience}(\mathbf{P}) \wedge \mathbf{P}(\mathbf{Jones})](\mathbf{u})]$$

follows from (1) by universal instantiation, and that therefore, by (3) and Euclid's law that things equal to the same thing are equal to one another,

$$\hat{\mathbf{u}}[\mathbf{TP}[\mathbf{Experience}(\mathbf{P}) \wedge \mathbf{P}(\mathbf{Jones})](\mathbf{u})] = \hat{\mathbf{u}}[\mathbf{u\ sees\ a\ tree})$$

is true after all. For the fact is that the variable $\mathbf{P}$ occurs in (1) within the scope of the modal operator; and in consequence (1) can be instantiated only to those predicate expressions that (rigidly) designate the same property in every possible world, whereas of course $\ulcorner\mathbf{TP}[\mathbf{Experience}(\mathbf{P}) \wedge \mathbf{P}(\mathbf{Jones})\urcorner$, by assumption, does not designate the same property in every possible world. Thus, while the experience that Jones is having is indeed a property, it is nevertheless not the property of having the experience that Jones is having.

Another example to which Montague applies his intensional logic is in the analysis of:

Jones sees a unicorn having the same height as a table actually before him.

which according at least to some philosophers implies the existence of sense data. Here Montague takes 'sees' in its veridical sense and interprets its nonveridical sense as 'seems to see', where 'seems' is taken as a predicate constant of type $\langle -1, -1, 1\rangle$, as in 'x seems to y to see a unicorn', i.e., as in $\ulcorner\mathbf{Seems}(\mathbf{x}, \mathbf{y}, \hat{\mathbf{u}}[\exists\mathbf{z}(\mathbf{Unicorn}(\mathbf{z}) \wedge \mathbf{Sees}(\mathbf{u}, \mathbf{z}))])$. Montague's analysis of (6) then is as follows:

$$\begin{aligned}&\exists\mathbf{x}[\mathbf{Table}(\mathbf{x}) \wedge \mathbf{Before}(\mathbf{x}, \mathbf{Jones}) \wedge \mathbf{Seems}(\mathbf{Jones}, \mathbf{Jones}, \\ &\hat{\mathbf{u}}[\exists\mathbf{y}(\mathbf{Unicorn}(\mathbf{y}) \wedge \mathbf{Sees}(\mathbf{u}, \mathbf{y}) \wedge \mathbf{Has\text{-}the\text{-}same\text{-}height\text{-}as}(\mathbf{y}, \mathbf{x}))])]\end{aligned} \tag{7}$$

Note that no sense data are required on this analysis, so that an argument for the existence of sense data based on (6) must be deemed inconclusive (see [3], p. 171).

Circumlocutions, such as 'seems to see' for the nonveridical sense of 'sees', are rather common in philosophical analyses and are in general based on the assumption that opaque contexts in ordinary language, such as that of the direct-object position in the nonveridical sense of 'sees', are always to be analyzed by means of a paraphrase in which a sentence or formula occurs within the scope of a modal operator. It is the occurrence of a full sentence or formula within the scope of a modal operator, in other words, that explains on this assumption why, e.g., existential generalization or the substitutivity of identity fails for opaque contexts. Thus, e.g., while 'x sees a unicorn' can be represented by

$$\mathbf{Seems}(\mathbf{x}, \mathbf{x}, \hat{\mathbf{u}}[\exists \mathbf{y}(\mathbf{Unicom}(\mathbf{y}) \wedge \mathbf{Sees}(\mathbf{u}, \mathbf{y}))])$$

when 'sees' is taken in its nonveridical sense; and in the latter analysis the formula $\ulcorner \exists \mathbf{y}(\mathbf{Unicorn}(\mathbf{y}) \wedge \mathbf{Sees}(\mathbf{u}, \mathbf{y})^{\urcorner}$ occurs within the scope of a modal operator (which is implicit in the definition of $\ulcorner \hat{\mathbf{u}}[...]^{\urcorner}$), so that nothing follows about the existence of a unicorn that only seems to be seen.

As another example of the apparent need for circumlocution, consider the verb 'seek', which also generates an opaque context with respect to its direct-object position. When compared with the verb 'find', what we notice is that while

Jones finds a unicorn; therefore, there is a unicorn.

is valid, the apparently similar argument:

Jones seeks a unicorn; therefore, there is a unicorn.

is invalid;[2] and this amounts to a philosophical puzzle insofar as we need to explain how of two arguments of apparently the same logical form one can be valid and the other invalid.

Relative to the present version of intensional logic, Montague's own approach to this example is to regard 'tries to find' as a circumlocution for 'seeks', where the verb 'tries' is represented by a predicate constant of type $\langle -1, 1 \rangle$. The above argument is then analyzed by Montague as follows:

$$\mathbf{Tries}(\mathbf{Jones}, \hat{\mathbf{u}}[\exists \mathbf{y}(\mathbf{Unicorn}(\mathbf{y}) \wedge \mathbf{Finds}(\mathbf{u}, \mathbf{y}))]);\ \mathbf{therefore},\ \exists \mathbf{y} \mathbf{Unicorn}(\mathbf{y}).$$

[2]Strictly speaking, 'Jones seeks a unicorn' is ambiguous between a *de re* (referential) reading and a *de dicto* (nonreferential) reading. Such ambiguity is typical of intensional verbs. The *de re* reading is unambiguously given by 'Jones seeks a certain unicorn', from which, as premise, it does follow that there is a unicorn. We are of course understanding the sentence in the text to be given with its *de dicto* reading, from which it does not follow that there is a unicorn.

which of course is completely different in logical form from:

$$\exists \mathbf{y}(\mathbf{Unicorn}(\mathbf{y}) \wedge \mathbf{Finds}(\mathbf{Jones}, \mathbf{y})); \ \mathbf{therefore}, \ \exists \mathbf{y}\mathbf{Unicorn}(\mathbf{y}).$$

Thus, upon analysis through circumlocution and paraphrase, the two arguments are seen to have significantly different logical forms after all, so that the validity of the one need not, and in fact does not, carry over to the other.

Despite his use of circumlocution and paraphrase in a number of analyses, Montague nevertheless came to reject the general assumption that opacity is always to be accounted for by the occurrence in ordinary language of a sentence or formula within the scope of a modal operator and that therefore sentences containing intensional verbs, such as 'seek', 'conceive', 'worship', etc., that do not involve such an occurrence of a sentence or formula are to be paraphrased into sentences that do. This is because, for Montague, opacity is but an aspect of intensionality, which is present in the semantics of a number of transitive verbs. It is an aspect, however, that cannot be represented in the present formulation of intensional logic without resorting to circumlocution—though, as we shall see, it can be represented without circumlocution and paraphrase in Montague's second and later development of intensional logic.

7 Intensional Logic Based on the Sense-Denotation Distinction

It is noteworthy that the only entities explicitly acknowledged in Montague's first version of intensional logic, i.e., entities to which quantificational reference can be made in the object language itself, are possible individuals and, for each $n \in \omega$, n-ary relations-in-intension, where the latter include propositions (for $n = 0$) and properties (for $n = 1$). The apparently excluded ontological categories of individual concepts and (n-ary) relations-in-extension, it turns out, are directly reducible to relations-in-intension. E.g., as an $\langle I, U \rangle$-intension, we have identified an individual concept with a function f from I into U; and therefore the same individual concept can equally well be identified with a function f' from I into singleton subsets of U, where $f'(i) = \{f(i)\}$, for each $i \in I$. Thus, we can identify individual concepts with those properties satisfying the following formula (as values of the one-place predicate variable P) (cf. [2], p. 132):

$$\Box\exists \mathbf{u}\forall \mathbf{v}[\mathbf{P}(\mathbf{v}) \leftrightarrow \mathbf{v} = \mathbf{u}].$$

An n-ary relation-in-extension, on the other hand, can be identified with an n-ary relation-in-intension that has the same extension, namely the relation-in-extension in question, at every index. Thus, we can identify n-ary relations-in-extension with those n-ary relations-in-intension that satisfy the following formula (as values of the n-place predicate variable $\mathbf{P}$) (*ibid.*):

$$\forall \mathbf{v}_1, ...\forall \mathbf{v}_n[\Box\mathbf{P}(\mathbf{v}_1, ..., \mathbf{v}_n) \vee \Box\neg\mathbf{P}(\mathbf{v}_1, ..., \mathbf{v}_n)].$$

Having made these reductions, one might well maintain that the distinction between the intension and extension, or the sense and denotation, of an expression is neither fundamental to intensional logic—and therefore not a basic distinction for philosophical analysis—nor necessary, as apparently Gottlob Frege thought, for the analysis of ordinary language. One might well speak, accordingly, of the denotation *simpliciter* of individual and predicate expressions, even though the denotations of the latter would in this case be what others call their senses. Indeed, in [4], Montague explicitly endorsed and carried through just such a view in his semantics for a fragment of ordinary English. 'It is wrong to maintain', Montague argued there, 'that an analysis of ordinary English (or German) requires a notion of sense as well as one of denotation' (p. 217).

In [5], on the other hand, Montague withdrew his emphasis 'on the possibility of doing without a distinction between sense and denotation'; and he claimed that 'while such a distinction can be avoided in special cases, it remains necessary for the general theory, and probably provides the clearest approach even to the special cases in question' (p. 222). And having changed his view on this matter, Montague proceeded to formulate his second version of intensional logic in which the distinction between sense and denotation was then taken as fundamental.

The ontological framework of this new intensional logic is based on a type-theoretical distinction regarding the various entities that are to be explicitly acknowledged in the logic, i.e., the entities to which quantificational reference is to be meaningful; and, in effect, a theory of types that is essentially an extension of Alonzo Church's theory of simple types. To set-theoretically characterize the types, let e, t, s be three objects, none of which is an ordered pair; and let *Type*, the set of types, be inductively defined as the smallest set such that (1) $e, t \in Type$; (2) $\langle a, b\rangle \in Type$ whenever $a, b \in Type$; and $\langle s, a\rangle \in Type$ whenever $a \in Type$.

Entities of type e are understood to be the possible individuals of intensional logic, while entities of type t are the two truth-values. An entity of type $\langle a, b\rangle$ is a function from entities of type a to entities of type b; and therefore entities of type $\langle a, t\rangle$ are the characteristic functions of sets of entities of type a, all of which we will hereafter speak of as being those sets themselves; i.e., we will not distinguish here between a set and its characteristic function. Also, we will not distinguish here between an n-ary relation-in-extension between entities of type $a_1, ..., a_m$, respectively, and the function of type $\langle a_1, \langle a_2 ..., \langle a_n, t\rangle\rangle ...\rangle\rangle$. E.g., a set of individuals and a binary relation-in-extension between individuals are construed here as entities of types $\langle e, t\rangle$ and $\langle e, \langle e, t\rangle\rangle$, respectively.

Finally, where $a \in Type$, the sense, or intension, or an entity of type a is an entity of type $\langle s, a\rangle$; and thus senses, or intensions, as can be seen from the definition of $Type$, are always senses of an entity of some type, i.e., there are no entities of type s *simpliciter*. Senses, or intensions, of sets of entities of type a, i.e., senses of type $\langle s, \langle a, t\rangle\rangle$ are understood in this framework to be the *properties* of entities of type a; and senses of n-ary relations-in-extension between entities of type $a_1, ..., a_n$, are understood to be the n-ary *relations-in-intension* between entities of these types. Propositions, moreover, are understood to be the senses of type $\langle s, t\rangle$.

In regard to the syntax of intensional logic, the only primitive syncategorematic signs, besides parentheses and brackets, that Montague adopts are $=$, the identity sign; λ, Church's function-forming λ-abstraction operator; and $\hat{}$ and $^{\vee}$, the intension-forming and extension-forming operators. (Here we see already how fundamental the intension-extension, or sense-denotation, distinction is on this version of intensional logic.) We also assume, for each $a \in Type$, the availability of denumerably many variables of type a as well as of denumerably many constants of type a. The set of meaningful expressions of type a, ME_a, is recursively defined (on the set of types) as follows:

1. every variable and constant of type a is in ME_a;
2. if $\boldsymbol{\alpha} \in ME_{\langle a,b\rangle}$ and $\boldsymbol{\beta} \in ME_a$, then $\ulcorner \boldsymbol{\alpha}(\boldsymbol{\beta}) \urcorner \in ME_b$;
3. if $\alpha \in ME_a$ and $\mathbf{u}$ is a variable of type b, then $\ulcorner \boldsymbol{\lambda} \mathbf{u} \boldsymbol{\alpha} \urcorner \in ME_{\langle b,a\rangle}$;
4. if $\boldsymbol{\alpha}, \boldsymbol{\beta} \in ME_a$, then $\ulcorner \mathbf{a} = \boldsymbol{\beta} \urcorner \in ME_t$;

5. if $\boldsymbol{\alpha} \in ME_a$, then $\ulcorner {}^{\wedge}\boldsymbol{\alpha} \urcorner \in ME_{\langle s,a \rangle}$;

6. if $\boldsymbol{\alpha} \in ME_{\langle s,a \rangle}$, then$\ulcorner {}^{\vee}\boldsymbol{\alpha} \in ME_a$.

The logical constants of the earlier version of intensional logic can now of course be taken as defined rather than as primitive constants. E.g., where $a \in Type$, $\boldsymbol{\alpha}$ is the first variable of type a, $\mathbf{p}$ is a variable of type t, $\boldsymbol{\beta}$ is a variable of type $\langle t,t \rangle$, and $\boldsymbol{\varphi}, \boldsymbol{\psi}$ are formulas, i.e., $\boldsymbol{\varphi}, \boldsymbol{\psi} \in ME_t$:

$$\forall \boldsymbol{\alpha}\boldsymbol{\varphi} =_{df} \boldsymbol{\lambda}\boldsymbol{\alpha}\boldsymbol{\varphi} = \boldsymbol{\lambda}\boldsymbol{\alpha}[\mathbf{a} = \mathbf{a}]$$

$$\neg\boldsymbol{\varphi} =_{df} \boldsymbol{\varphi} = \forall \mathbf{p}\mathbf{p}$$

$$\boldsymbol{\varphi} \wedge \boldsymbol{\psi} =_{df} \forall \boldsymbol{\beta}(\boldsymbol{\psi} = [\boldsymbol{\beta}(\boldsymbol{\varphi}) = \boldsymbol{\beta}(\boldsymbol{\psi})])$$

The necessity operator and our earlier formal counterpart for infinitive constructions are definable here as follows, where $\boldsymbol{\varphi}$ is a formula, $\boldsymbol{\alpha}$ is a variable of arbitrary type a and $\mathbf{u}$ is a variable of type e:

$$\Box\boldsymbol{\varphi} =_{df} [\,\hat{}\,\boldsymbol{\varphi} = \hat{}\,\forall \mathbf{u}(\mathbf{u} = \mathbf{u})]$$

$$\hat{\boldsymbol{\alpha}}\boldsymbol{\varphi} =_{df} [\,\hat{}\,\boldsymbol{\lambda}\boldsymbol{\alpha}\boldsymbol{\varphi}]$$

It should also be noted here that if $a \in type$, $\boldsymbol{\alpha} \in ME_a$ and $\boldsymbol{\delta} \in ME_{\langle a,t \rangle}$, i.e., if $\boldsymbol{\delta}$ is of the type of a set of entities of type a, then $\ulcorner \boldsymbol{\delta}(\boldsymbol{\alpha}) \urcorner$ expresses the 'membership' of the entity denoted by $\boldsymbol{\alpha}$ in the set denoted by $\boldsymbol{\delta}$. On the other hand, if $\boldsymbol{\delta} \in ME_{\langle s,\langle a,t \rangle \rangle}$ instead, i.e., if $\boldsymbol{\delta}$ is of the type of a property of entities of type a, then we use:

$$\boldsymbol{\delta}\{\mathbf{a}\} =_{df} [{}^{\vee}\boldsymbol{\delta}](\boldsymbol{\alpha}).$$

to express the 'possession' by the entity denoted by $\boldsymbol{\alpha}$ of the property denoted by $\boldsymbol{\delta}$.

In constructing a set-theoretic semantics for the present version of intensional logic, we again need to regard certain sets I and U as being the set of possible worlds and the set of possible individuals, respectively. Where $a \in Type$, the set of *possible denotations* (or *values of variables*) *of type a as based on U* and I, in symbols$D_{a,U,I}$, is defined recursively as follows:

1. $D_{e,U,I} = U$;

2. $D_{t,U,I} = \{0, 1\}$;

3. if $a, b \in Type$, $D_{\langle a,b\rangle,U,I} = (D_{b,U,I})^{D_{a,U,I}}$;

4. if $a \in Type$, $D_{\langle s,a\rangle,U,I} = (D_{a,U,I})^{I}$.

In addition to the set of possible worlds I, Montague wants us to also consider a set J of contexts of use in the possible worlds in I. The indices or points of reference now are neither possible worlds alone nor contexts of use alone but are rather ordered pairs of each, i.e., a reference point is a pair $\langle i, j\rangle$, where $i \in I$ and $j \in J$. It should be noted, however, that Montague now takes moments, when they are needed in the interpretation of tenses, to be components of members of I rather than components of members of J (cf. [5], p. 228).

Relative to the sets U, I, J, and a type a, Montague takes the set of *possible meanings of type* a, in symbols $M_{a,U,I,J}$, to be functions from the points of reference into possible denotations of type a, i.e., $M_{a,U,I,J} = (D_{a,U,I})^{I \times J}$. Meanings, on this interpretation, are functions of two arguments: a possible world and a context of use. Senses, or intensions, on the other hand, are functions of only one argument, namely, a possible world (which may include specification of a moment of time). It should be noted, however, that only senses, and not meanings, are explicitly acknowledged in the ontological framework of the theory, i.e., it is senses, and not meanings, that can be quantificationally referred to within the logic itself. Meanings, rather than senses, are needed to serve as the interpretations of expressions, on the other hand, because the interpretation of a compound is always to be a function of the interpretation of its components, including of course indexical components such as pronouns and demonstratives; and functions on possible worlds alone will not always suffice to satisfy this condition.

An *interpretation* for the present version of intensional logic, accordingly, can be taken to be a 4-tuple $\langle I, J, U, F\rangle$, where (1) I, J, U are nonempty sets representing respectively, the possible worlds, the contexts of use, and the possible individuals of the interpretation; and (2) F is a function defined on the set of constants such that whenever $a \in Type$ and $\boldsymbol{\alpha}$ is a constant of type a, then $F(\boldsymbol{\alpha}) \in M_{a,U,I,J}$. Naturally, relative to such an interpretation we can again follow (but shall avoid doing so here) the recursion clauses in the definition of a meaningful expression of a given type so as to characterize the meaning each such expression is assigned under the interpretation; and of course, where $a \in Type$, the meaning of an expression of type a will be a member of $M_{a,U,I,J}$.

Where φ is a sentence, i.e., a member of ME_t in which no variable has a free occurrence, and $\langle i, j \rangle$ is a point of reference of an interpretation as defined above, then φ is *true at* $\langle i, j \rangle$ (under the interpretation) if the meaning φ has under that interpretation has the value 1 at the reference point $\langle i, j \rangle$. Definitions of logical truth and logical consequence do not immediately follow, however, because it is possible that under some interpretations some of the logical constants might not be assigned their standard or usual meanings. We can of course restrict our considerations to the class of those interpretations under which all the logical constants do receive their usual meanings and define logical truth and logical consequence with respect to this class. We can also extend this class, however, by including interpretations under which the logical constants receive their usual or standard extensions of all *designated* or standard points of reference but not also at certain other 'unactualizable' points of reference (cf. [5], p. 231). Sentences that are synonymous, i.e., that have the same meanings, under all the interpretations in this latter class will then also be logically equivalent; but then there will also be sentences that are logically equivalent, i.e., that have the same truth-value at all of the designated points of reference of all the interpretations in the class, that are not synonymous under every interpretation in the class.

This divergence between synonymy and logical equivalence, Montague notes, allows for 'a natural treatment of belief contexts that lacks the controversial property of always permitting interchange on the basis of logical equivalence' (*ibid.*). It should also be noted in this regard, however, that Montague's treatment of belief contexts in his first version of intensional logic had in fact possessed this controversial property (cf. [2], pp. 137ff.); and, indeed, even here in his second version of intensional logic it is not so much that Montague rejects his earlier analysis as that he wishes only to point out that that approach 'has genuine alternatives and is not forced upon us' ([15], p. 231).

8 On the Intensional Reality of Possible Worlds

We have already noted that the only entities to which quantificational reference can be made in Montague's first version of intensional logic are possible individuals and, for each $n \in \omega$, n-ary relations-in-intension. And the situation is really not that much different in the new version of intensional logic, although now quantificational reference to individual concepts (as senses of type $\langle s, e\rangle$) and relations-in-extension can be effected directly rather than indirectly through relations-in-intension. The question naturally arises then as to the status of possible worlds in the ontological framework of intensional logic. That is, in what sense, if any, does the adoption of intensional logic as a new theoretical framework for philosophy presuppose the existence or 'being' of possible worlds? In particular, are possible worlds among the entities to which quantificational reference can be made within this new theoretical framework?

Now it may seem natural to maintain that possible worlds have been assumed to exist all along in Montague's set-theoretic semantics for intensional logic; and, indeed, it is true that Montague even considered adding to the language of set theory 'two definite singular terms, one to designate the set of possible worlds, and the other the set of possible individuals' ([3], p. 154). But he also went on in the same context to reject that suggestion; and in his definition of an interpretation $\langle I, U, F\rangle$ for an intensional language, or of an interpretation $\langle I, J, U, F\rangle$ for intensional logic, we are not told that the sets I and U are the set of possible worlds and the set of possible individuals *simpliciter* but only that they are nonempty sets that are *to be regarded* as the set of possible worlds and possible individuals, respectively, of the interpretation in question. And in that sense, the notions of a possible world and of a possible individual have only a relative, and not an absolute, status from the ontological point of view.

What would render this notion absolute rather than relative in Montague's set-theoretic semantics, or at least what would be a necessary condition for such, would be an assumption to the effect that if $\langle I, U, F\rangle$ and $\langle I', U', F'\rangle$ are two interpretations for an intensional language L, then $I = I'$ and $U = U'$. And if Montague had added to the language of set theory the two special singular terms mentioned above, then he might well indeed have made just this assumption. But in fact Montague rejected adding such singular terms and there is no indication at all that he intended to make any such assumption. Indeed, to the contrary, Montague explicitly said that intensional logic *transcends* set theory, and that therefore it is not reducible to set theory even though it can be 'justified' by metamathematical or model-theoretic means available within set theory ([3], p. 155).

The question of Montague's commitment to possible worlds, accordingly, applies not to his set-theoretic semantics but to his adoption of intensional logic as a new theoretical framework for philosophy. For possible individuals, the commitment is quite explicit: these are the entities indicated by bound individual variables, i.e., variables of type e; and it is clear, moreover, that whatever else a possible world is within the framework of intensional logic, it is not an individual, since in that case relations-in-intension would reduce to relations-in-extension.[3]

Perhaps the most plausible suggestion is the identification of possible worlds with certain 'world propositions', where the notion of a world as a property of propositions is definable as follows, where $\mathbf{p}, \mathbf{q}$ are variables of type $\langle s, t\rangle$:

$$\mathbf{World}_{\langle s,\langle\langle s,t\rangle,t\rangle\rangle} =_{df} \hat{\mathbf{p}}[\Diamond {}^{\vee}\mathbf{p} \wedge \forall \mathbf{q}(\Box[{}^{\vee}\mathbf{p} \rightarrow {}^{\vee}\mathbf{q}] \vee \Box[{}^{\vee}\mathbf{p} \rightarrow {}^{\vee}\neg\mathbf{q}])$$

A world-proposition is true in Montague's set-theoretic semantics in one and only one possible world (of any given interpretation) and may therefore be taken as the intensional counterpart, i.e., the counterpart within the framework of intensional logic, of that possible world.[4] Understood in this sense, Montague's commitment to possible worlds is now quite clear; for the sentence:

$$\Box\exists\mathbf{p}(\mathbf{World}\{\mathbf{p}\} \wedge {}^{\vee}\mathbf{p})$$

is true at ever reference point of every interpretation.

9 On Quantifying over Possible Individuals

Quantificational reference to individuals that do not exist (at a given index) is sometimes objected to by philosophers who otherwise have no qualms about quantifying over propositions and relations-in-intension. Quantification with respect to individual variables, these philosophers maintain, should be restricted in content so as to apply only to the individuals that exist at the index in question. This view, at least with respect to Montague's overall semantic approach, is quite unwarranted and can in fact be refuted if the

[3] It has been suggested that possible worlds be added as a new type of entity altogether. E.g., let s be a basic type along with e and t, i.e., let $s \in Type$, and set $D_{s,U,I} = I$. (Cf. Gallin [11], §8.)

[4] From the algebraic point of view of Montague's set-theoretic semantics, a world-proposition might also be called an *atomic* proposition. (Cf. Gallin [11], § 11.) It should be noted here that we are assuming that all of the logical constants involved in the above definition have been defined in terms of the primitives of intensional logic and that, as so defined, they cannot be assigned 'nonstandard' meanings.

claim is that direct quantification over possible individuals cannot be justified using only the restricted quantifiers referring to existing individuals in modal or intensional contexts.

Consider, for example, the context of tense logic where the restricted individual quantifiers are supposed to refer only to the individuals existing at the time of utterance; and, for convenience, let us use $\forall^e$ and $\exists^e$ for these restricted quantifiers. Then the view in question, as applied to this context, maintains that the only significant way we can quantificationally refer to past or future individuals is *indirectly*, i.e., where these restricted quantifiers occur within the scope of the past or future tense operators; and, *a fortiori*, on this view the only significant way we can quantificational refer to possible individuals in the context of modal or intensional logic is also indirectly, i.e., where the restricted quantifiers occur within the scope of the modal or intension-forming operator.

Our counter-thesis is that by taking advantage of the pragmatic operators available within Montague's general framework we can fully justify *direct* quantification over past and future individuals in the context of tense logic and *direct* quantification over merely possible individuals in the context of modal or intensional logic. To show this, consider the problem of how we might formalize the English sentence:

(8) There did exist someone who is an ancestor of everyone now existing.

using only indirect quantification over past individuals. In this case it is clear that the symbolizations (where, for convenience, the variables are understood to range only over persons):

$$\mathcal{P}\exists^e \mathbf{u}\forall^e \mathbf{v}\mathbf{Ancestor}(\mathbf{u},\mathbf{v})$$

will not do, because this formula represents the different English sentence:

There did exist someone who was an ancestor of everyone then existing.

Clearly, what we need here is a way, whether primitive or defined, of directly quantifying over past as well as over presently existing individuals. E.g., using $\forall^p$ and $\exists^p$ for such quantifiers, we can formalize (8) above as:

$$\exists^p \mathbf{u}\forall^e \mathbf{v}\mathbf{Ancestor}(\mathbf{u},\mathbf{v})$$

And of course a completely analogous argument shows the need for a way, whether primitive or defined, of directly quantifying over future individuals.

Now the point of our counter-thesis is not that there are examples that clearly show the need for directly quantifying over past and future individuals in the context of tense logic, or for directly quantifying over possible individuals in the context of modal or intensional logic; but rather that by fully utilizing the pragmatic orientation of Montague's semantics we can in fact actually define the quantifiers needed for such direct quantification in terms of the restricted quantifiers occurring within the scope of the tense, modal or intension-forming operators.

As it turns out, what is needed to realize this type of definition is the pragmatic operator $\mathcal{N}$ for 'it is now the case that', as developed by Hans Kamp, one of Montague's former students (Kamp [14]). The introduction of this operator, Kamp has noted, requires a procedure of double (rather than single) indexing and a redefinition of *truth at an index* to *truth at an index when part of an utterance in the present context of use*; and of course it also requires specifying that index in each interpretation that is to be regarded as the present context of use in that interpretation. This supplementation of the relevant parameters involved in the analysis of truth is of course completely within the spirit of the pragmatic orientation of Montague's semantics.

Using the pragmatic now-operator in the analysis of (8), we see now how to avoid assuming the prior introduction of the quantifiers $\forall^p$ and $\exists^p$. For in this case, the formula

$$\mathcal{P}\exists^e \mathbf{u}\mathcal{N}\forall^e \mathbf{v}\mathbf{Ancestor}(\mathbf{u},\mathbf{v})$$

fully captures all that we want to say with (8) using only indirect quantification over past individuals. And of course what this analysis indicates is that (at least with respect to the present context of use) we can contextually define direct quantification over past individuals in general:

$$\forall^p \mathbf{u}\boldsymbol{\varphi} =_{df} \neg\mathcal{P}\neg\forall^e \mathbf{u}\mathcal{N}\boldsymbol{\varphi}$$

$$\exists^p \boldsymbol{\varphi} =_{df} \mathcal{P}\exists^e \mathbf{u}\mathcal{N}\boldsymbol{\varphi}$$

Naturally, essentially the same pattern of definition suffices to define direct quantification over future individuals; and assuming that the modal operator is interpreted either as in Montague's first version of intensional logic or is defined as above in terms of the intension-forming operator, we can then similarly define direct quantification over possible individuals as follows:

$$\forall \mathbf{u}\boldsymbol{\varphi} =_{df} \Box\forall^e \mathcal{N}\boldsymbol{\varphi}$$

$$\exists \mathbf{u}\boldsymbol{\varphi} =_{df} \Diamond \exists^e \mathbf{u} \mathcal{N} \boldsymbol{\varphi}$$

Thus the claim that only indirect quantification over possible individuals should be allowed in intensional logic is, as we have said, quite unwarranted. Whether, on the other hand, there are reasons to prefer defining $\forall$ and $\exists$ as above in terms of $\forall^e$and $\exists^e$ or for defining $\forall^e$ and $\exists^e$, as Montague (in effect) does, in terms of $\forall$ and $\exists$ and a predicate for 'exists' is another question altogether; but one which we shall not go into here. (This question is discussed in Cocchiarella [15].)

10 Universal Grammar

While the development of intensional logic as a new theoretical framework for philosophy marked one important shift in Montague's philosophy of language, the development of his universal grammar as a universal syntax and semantics applicable to natural as well as to artificial languages has come to mark an even more important shift not only for Montague but for many logicians and linguists as well. Natural languages, according to Montague, are no longer to be distinguished from the artificial languages of logicians on the grounds that only the latter have a mathematically precise syntax and semantics; and, indeed, in this regard Montague's universal grammar is intended 'to comprehend the syntax and semantics of both kinds of languages within a single natural mathematically precise theory' ([5], p. 222).

It is important to note here that the mathematically precise theory Montague has in mind for universal grammar is in effect a definitional extension of set theory (with proper classes). This is particularly noteworthy in that it may seem in the present context that Montague has thrown over his new theoretical framework for philosophy in favor of the old. To the contrary, however, intensional logic serves for Montague not only as an example within universal grammar of an important artificial language but also, and more importantly, as a semantical framework in its own right and into which extensive portions of natural language are to be translated by means of a rigorously described theory of translation that Montague also formulates as part of universal grammar. Such a rigorously characterized translation into intensional logic induces, according to Montague, a more perspicuous interpretation for the expressions of a natural language then would otherwise be given them in purely set-theoretic terms. The sense, moreover, in which intensional logic transcends set theory even while being metamathematically 'justified' by set-theoretic constructions remains no less in force in universal grammar than when these constructions were given independently of such a single comprehensive theory.

Despite this transcendence, Montague does formulate his universal grammar in purely set-theoretical terms and distinguishes in that grammar both a universal syntax and universal semantics. (The term 'universal' is meant here only in the logico-mathematical sense of utmost generality). As applied to any given language, the basic aim of the syntax is to characterize the various syntactical categories, including especially the category of declarative sentences, of that language, while the basic aim of the semantics is to characterize the notions of a true sentence (under an interpretation) and of entailment between the sentences of that language ([5] , p. 223). The two components are intimately related by Montague in that where L is a language and $\mathcal{A}$ is an interpretation for L, there exists in $\mathcal{A}$ a semantical operation corresponding to each structural operation in the syntax of L; and, furthermore, in that part of his semantics that Montague called the theory of reference, there will in addition be a unique semantic category corresponding to each syntactic category of L (though the correspondence will in general only be many-one and not one-to-one).

Montague's theory of reference, we should note, is essentially a model theory based upon the set of types already described for intensional logic and involves assigning the model-theoretic counterparts of senses or intensions as well as the more usual extensions to expressions. Senses differ in this regard from meanings in Montague's semantics in that they, but not meanings, are 'intensional entities that are sometimes *denoted* by expressions' ([5], p. 228). The types in general, whether extensional or intensional, are in fact the semantic categories that are to be associated with the syntactic categories of a language L; and, moreover, it is precisely because a type is assigned to each syntactic category of L that the more perspicuous interpretation of L that is induced by its translation into intensional logic can be effected. Thus, in a way the real point of the set-theoretical framework is to serve first as a syntactical metalanguage within which the syntax of any language whatsoever might be described, including the language of intensional logic, and, secondly, to serve in that regard as the syntactical basis of a theory of translation within or by means of which each such language might be translated into intensional logic and thereby be given its most perspicuous semantical representation.

Montague's set-theoretical description of a language L amounts in effect to identifying L with a system $\langle L^*, R\rangle$, where L^* is a (syntactically) disambiguated language (and therefore an artificial construct) and R is a binary relation (of syntactical analysis) with the set of proper expressions of L^* as its domain and the set of proper expressions of L as its range. R will in general by many-one, relating several disambiguated proper expressions of L^* (as alternative analyses) with each syntactically ambiguous proper expression of L. The disambiguated construct L^* is defined by Montague to be a system $\langle A, F_\gamma, X_\delta, S, \delta_0\rangle_{\gamma\in\Gamma,\delta\in\Delta}$, where $\langle A, F_\gamma\rangle_{\gamma\in\Gamma}$ is a free algebra generated by $U_{\delta\in\Delta}X_\delta$, with A as the set of proper expressions of L^* and each F_γ a structural operation on strings of members of A (resulting in new members of A) and with the sets X_δ being the sets of basic phrases of each of the categories of L^*, Δ being the set of indices of these syntactic categories and δ_0 being the index of the category of the declarative sentences of L^*; and where S, the syntactical rules of L^*, is a set of sequences of the form $\langle F_\gamma, \langle\delta_\xi\rangle_{\xi<\beta}, \eta\rangle$, where $\gamma \in \Gamma$, β is the number of places of the operation F_γ, $\delta_\xi \in \Delta$ for all $\xi < \beta$, and $\eta \in \Delta$. (A syntactical rule as described in effect stipulates that the structural operation F_γ operates on a β-place string of proper expressions, each of category δ_ξ, respectively, for all $\xi < \beta$, and results in a proper expression of category η). The family of phrases of the syntactic categories of L^* that are generated in this way in L^* is inductively characterized as the smallest indexed family $\langle P_\delta\rangle_{\delta\in\Delta}$ of subsets of A such that (1) $X_\delta \subseteq P_\delta$, for all $\delta \in \Delta$, and (2) whenever $\langle F_\gamma, \langle\delta_\xi\rangle_{\xi<\beta}, \eta\rangle \in S$ and $a_\xi \in P_{\delta_\xi}$, for all $\xi < \beta$, then $F_\gamma(\langle a_\xi\rangle_{\xi<\beta}) \in P_\eta$.

11 The Translation of English into Intensional Logic

We shall forego the set-theoretical details of Montague's general theory of translation and briefly illustrate instead the particular translation Montague gave for the fragment of English formulated in [6]. The fragment is philosophically interesting in its own right, moreover, just for the novelties and insights it exhibits, such as, e.g., Montague's unified treatment of quantifier phrases and terms of English. Indeed, it is the particular type of formulation Montague gave for this fragment of English as well as the form of its translation into intensional logic that has been developed in recent years into an active, and apparently fruitful, field of research at the interface of linguistics and logic.[5]

Montague defines the set of syntactic categories for the fragment of English in question to be the smallest set K such that (1) the two basic types t and e of intensional logic are in K, and (2) whenever A and B are in K, then so are A/B and $A//B$. (K. Ajdukiewicz, the originator of categorial grammar, allowed only one compound category, A/B, to be generated from the more basic categories A and B. Montague's introduction and use of two such compounds is in itself already a novelty of his grammar, and he notes that 'in connection with other languages it is quite conceivable that a larger number would be required' ([6], p. 249).) Some of the more important traditional categories that are in K are:

$IV = t/e$	(the category of intransitive verb phrases)
$T = t/IV$	(the category of terms)
$TV = IV/T$	(the category of transitive verbs)
$IAV = IV/IV$	(the category of IV-modifying adverbs)
$CN = t//e$	(the category of common noun phrases)

Associated with each of the above compound categories, and several others, are the sets of words or phrases that are taken by Montague to be the *basic* phrases of English belonging to these categories and from which, by means of the syntactical rules, the more complex phrases of these categories are to be constructed. Thus, e.g., words like 'run', 'walk', 'talk' and 'rise' are stipulated as being basic expressions of category IV; and adverbs like 'rapidly' and 'slowly' belong to category IAV, while adverbs like 'necessarily' belong to category t/t. Phrases for propositional attitudes, e.g., 'believe that', 'assert that', etc., appropriately belong to category IV/t, while phrases like 'try to', 'wish to', etc., belong to category $IV//IV$, which is syntactically quite different from the category of adverbs $IAV = IV/IV$.

One of the general forms of the syntactical rules by means of which the more complex phrases of English belonging to these categories are to be generated can be described as follows: if $\boldsymbol{\alpha}$ is a phrase of the compound category A/B and $\boldsymbol{\beta}$ is a phrase of the category B, then $F_i(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is a phrase of category

[5] See especially the collection of papers in Partee (ed.) [16]. Partee's essay 'Montague Grammar and Transformational Grammar' [17] is an excellent introduction to a variety of the issues involved in combining Montague grammar with transformational grammar.

A, where F_i is the structural operation associated with the syntactical rule in question. (As a set-theoretical construction of universal grammar the syntactical rule in question is identified with the three-tuple $\langle F_i, \langle A/B, B\rangle, A\rangle$. A similar rule $\langle F_i, \langle A//B, B\rangle, A\rangle$ is given for the compound category CN, as defined above, and for $IV//IV$.[6]) Rules of this form are called rules of functional application; and taken together with the other syntactical rules Montague described they yield a simultaneous inductive definition of the sets P_A of phrases of English of each syntactic category A.

In regard now to Montague's translation of this fragment of English into intensional logic, let us note that the translation is to be based first upon a certain mapping of the syntactic categories of English into the semantical types of intensional logic and then upon the construction of a rule of translation corresponding to each of the syntactical rules involved in the simultaneous inductive definition of the sets P_A. The translation relation constructed in this way will not be a function, of course, given the phenomena of semantic ambiguity in English; but by means of an analysis relation, which Montague defined in terms of analysis trees (cf. [4], pp. 204-207), we can speak instead of the translation of a meaningful expression of English corresponding to any given analysis tree for that expression. For convenience, however, we shall ignore further mention of analysis trees here.

The form of the mapping of syntactic categories into semantic types cannot be ignored here, however; for aside from the obvious association of the simple syntactic categories e and t with the simple semantic types e and t, the mapping Montague defined is based on two fundamental assumptions regarding the semantics of English constructions. These are (1) that despite important differences in their syntactical roles in English, the compound categories A/B and $A//B$ are to be assigned the same semantical type; and (2) that in those cases where English constructions are generated by syntactical or grammatical relations associated with compound categories, the construction is to be interpreted as a function whose arguments are always to be senses or intensions. In particular, the general form of the semantical rules of translation that correspond to the rules of functional application as described above is paradigmatic for this type of interpretation: if $\boldsymbol{\alpha} \in P_{A/B}$ or $\boldsymbol{\alpha} \in P_{A//B}$, $\boldsymbol{\beta} \in P_B$, and $\boldsymbol{\alpha}$ translates into $\boldsymbol{\alpha}'$ and $\boldsymbol{\beta}$ translates into $\boldsymbol{\beta}'$, then

[6]Consider the two converse structural operations $F_6(\boldsymbol{\alpha}, \boldsymbol{\beta}) = \ulcorner\ \boldsymbol{\alpha}\boldsymbol{\beta}\urcorner$ and $F_7(\boldsymbol{\alpha}, \boldsymbol{\beta}) = \ulcorner\ \boldsymbol{\beta}\boldsymbol{\alpha}\urcorner$. The syntactical rules $S8$ and $S10$ of [6] apply as described above when $\boldsymbol{\alpha} \in P_{IV//IV}$ and $\boldsymbol{\beta} \in P_{IV}$ or when $\boldsymbol{\alpha} \in P_{IV/IV}$ and $\boldsymbol{\beta} \in P_{IV}$. Thus $S8$ generates, e.g., 'try to run' as an IV-phrase, while $S10$ generates 'run rapidly' as an IV-phrase.

$F_i(\boldsymbol{\alpha}, \boldsymbol{\beta})$ and $F_j(\boldsymbol{\alpha}, \boldsymbol{\beta})$ translate into $\ulcorner \boldsymbol{\alpha}'(\hat{}\,\mathbf{b}') \urcorner$.

The mapping that is based upon these two fundamental assumptions is defined as follows, for arbitrary categories A and B:

$$f(e) = e$$

$$f(t) = t$$

$$F(A/B) = f(A//B) = \langle\langle s, f(B)\rangle, f(A)\rangle.$$

12 Quantifier Phrases as English Terms

Now although there are expressions of type e in intensional logic denoting individuals, there are no such related expressions in Montague's fragment of English. There are of course English terms, i.e., phrases of category T, such as 'John', 'Mary', 'ninety', and even term variables 'he$_1$', 'he$_2$', etc. The category T, however, as defined above, is not a basic but a compound category t/IV; that is, a term phrase when combined with an intransitive verb phrase under a certain structural operation F_i results in a (declarative) sentence-phrase, i.e., a phrase of category t (which is either a sentence of English or like a sentence of English except for the occurrence of a variable 'he$_n$'). Consequently, since $f(T) = f(t/IV) = \langle\langle s, f(IV)\rangle, t\rangle$, and since $f(IV) = f(t/e) = \langle\langle s, e\rangle, t\rangle$ = the type of sets of individual concepts, then, instead of denoting an individual, an English term denotes a set of properties of individual concepts. Prima facie, this may seem somewhat anomalous, but it really falls rather nicely into place once we notice that the terms of English are in effect what linguists call noun phrases and that noun phrases include quantifier phrases as well as proper nouns and definite descriptions. The anomaly then, if any, is not that a term denotes a set of properties, but that it denotes a set of properties of individual concepts rather than a set of properties of individuals. (But more of this anon.)

Proper nouns and term variables, accordingly, are not the only term phrases in Montague's fragment of English. In particular, there are three syntactical rules $\langle F_i, CN, T\rangle$, for $i = 0, 1, 2$, such that if ζ is a common noun phrase, where 'man', 'woman', 'fish', 'unicorn', etc., are examples of *basic* CN-phrases, then $F_0(\zeta) = \ulcorner$every $\zeta\urcorner$', $F_1(\zeta) = \ulcorner$the $\zeta\urcorner$, and $F_2(\zeta) = \ulcorner$a(n) $\zeta\urcorner$ are all phrases of category T.

This is of course a significant departure from standard logical analysis. But then it is also very much in line with the traditional (pre-Fregean) subject-predicate analysis of English sentences; and, not surprisingly, it allows in this last regard for a more direct connection between the syntax and semantics of English. It also allows, it turns out, for a more direct representation of intensional verbs that avoids the type of circumlocution described earlier, such as paraphrasing 'seeks' as 'tries to find'; and it provides in that regard for a more realistic alternative to the rather common assumption that opaque constructions in English are always to be paraphrased into constructions involving the occurrence of a sentence or formula within the scope of a modal operator (cf. Partee [18]).

Consider, for example, certain individual constants 'j', 'm' and 'n' of intensional logic understood to denote the individuals John, Mary and the number ninety. The English terms 'John', 'Mary' and 'ninety', as noted above, are not translated by Montague, into 'j', 'm' and 'n', respectively, but into $\ulcorner \boldsymbol{\lambda} \mathbf{PP}\{\hat{}\,\mathbf{j}\}\urcorner$,$\ulcorner \boldsymbol{\lambda} \mathbf{PP}\{\hat{}\,\mathbf{m}\}\urcorner$' and $\ulcorner \boldsymbol{\lambda} \mathbf{PP}\{\hat{}\,\mathbf{n}\}\urcorner$ where $\mathbf{P}$ is a variable of type $\langle s, \langle\langle s, e\rangle, t\rangle\rangle$, i.e., of the type of properties of individual concepts. Similarly, where $\boldsymbol{\zeta}$ is a CN-phrase of English and $\boldsymbol{\zeta}'$ is its translation into intensional logic, then $\ulcorner$every $\boldsymbol{\zeta}\urcorner$, $\ulcorner$the $\boldsymbol{\zeta}\urcorner$, and $\ulcorner$a(n) $\boldsymbol{\zeta}\urcorner$ translate into:

$$\boldsymbol{\lambda}\mathbf{P}\forall\mathbf{x}(\boldsymbol{\zeta}'(\mathbf{x}) \rightarrow \mathbf{P}\{\mathbf{x}\}),$$

$$\boldsymbol{\lambda}\mathbf{P}\exists\mathbf{x}(\boldsymbol{\zeta}'(\mathbf{x}) \leftrightarrow \mathbf{y} = \mathbf{x}) \wedge \mathbf{P}\{\mathbf{x}\}),$$

$$\boldsymbol{\lambda}\mathbf{P}\exists\mathbf{x}(\boldsymbol{\zeta}'(\mathbf{x}) \wedge \mathbf{P}\{\mathbf{x}\}),$$

respectively, where '$\mathbf{x}$' and '$\mathbf{y}$' are variables of type $\langle s, e\rangle$, i.e., of the type of individual concepts. Here, of course, proper nouns, definite descriptions and quantifier phrases are all translated into expressions of intensional logic that denote sets of properties of individual concepts.

Simple predication of a term and an IV-phase on this interpretation, it should be noted, takes what the term denotes as the function and the intension of what the IV-phrase denotes as the argument. Thus, e.g., the English sentence 'John runs' is translated by Montague into intensional logic as follows:

$$\boldsymbol{\lambda}\mathbf{PP}\{\hat{}\,\mathbf{j}\}(\hat{}\,\mathbf{runs}') \tag{10}$$

where '$\mathbf{runs}'$' is the translation of the English IV-phrase 'runs', and is therefore of type $f(IV) = \langle\langle s, e\rangle, t\rangle$, i.e., of the type of sets of individual concepts; and 'runs'' therefore is of type $\langle s, \langle\langle s, e\rangle, t\rangle\rangle$, i.e., of the type of properties of individual concepts. However, by obvious principles of intensional logic (10) reduces to:

$$\textbf{runs}'(\hat{}\,\mathbf{j}) \tag{11}$$

which is now more nearly in line with standard logical analysis.

Of course, for an extensional IV-phrase such as 'runs', the (reduced) translation (11) is still unduly intensional. Montague rectifies this by first defining:

$$\boldsymbol{\delta}_* =_{df} \boldsymbol{\lambda}\mathbf{u}\boldsymbol{\delta}(\hat{}\,\mathbf{u})$$

where $\boldsymbol{\delta} \in ME_{f(IV)}$ and '$\mathbf{u}$' is an individual variable of type e, and by then giving a meaning postulate of the form:

$$\Box[\boldsymbol{\delta}(\mathbf{x}) \leftrightarrow \boldsymbol{\delta}_*(^{\vee}\mathbf{x})]$$

for any $\boldsymbol{\delta}$ that translates an extensional IV or CN-phrase. (Note that because $\boldsymbol{\delta}$ is of type $\langle\langle s, e\rangle, t\rangle$, i.e., of the type of sets of individual concepts, then $\boldsymbol{\delta}_*$ is of type $\langle e, t\rangle$, i.e., of the type of sets of individuals.) Thus, given that 'runs' is an extensional IV-phrase, (11) reduces to

$$\textbf{runs}'_*(\mathbf{j}) \tag{12}$$

which is exactly what the standard logical analysis of 'John runs' comes to.

Needless to say, but an entirely similar analysis applies to the English sentence 'Every man runs', which is directly translated as:

$$\boldsymbol{\lambda}\mathbf{P}\forall\mathbf{x}[\textbf{man}'(\mathbf{x}) \rightarrow \mathbf{P}\{\mathbf{x}\}](\hat{}\,\textbf{runs}')$$

and which first reduces to:

$$\forall\mathbf{x}[\textbf{man}'(\mathbf{x}) \rightarrow \textbf{runs}'(\mathbf{x})]$$

but which can be further reduced to:

$$\forall\mathbf{u}[\textbf{man}'_*(\mathbf{u}) \rightarrow \textbf{runs}'_*(\mathbf{u})]$$

assuming that 'man' is an extensional CN-phrase.

For extensional transitive verbs, i.e., English phrases of category TV, the situation is analogous, though somewhat more involved since $TV = IV/T$, and therefore $f(TV) = \langle\langle s, f(T)\rangle, f(IV)\rangle$. That is, what a transitive verb denotes in intensional logic is a function whose arguments are the intensions of terms; and for sentences with extensional transitive verbs, this means that not only must we reduce the intensionality of the subject term of the resulting IV-phrase but also that of the term that is the direct object of the transitive verb. Thus, e.g., where '**love′**' translates the English TV-phrase 'loves', we translate 'Mary loves John' and 'Mary loves every man' first into:

$$\boldsymbol{\lambda}\mathbf{PP}\{\,\hat{}\,\mathbf{m}\}(\,\hat{}\,\mathbf{love}'(\hat{\mathbf{P}}\mathbf{P}\{\,\hat{}\,\mathbf{j}\}))$$

and

$$\boldsymbol{\lambda}\mathbf{PP}\{\,\hat{}\,\mathbf{m}\}(\,\hat{}\,\mathbf{love}'(\hat{\mathbf{P}}\forall\mathbf{x}[\mathbf{man}'(\mathbf{x}) \rightarrow \mathbf{P}\{\mathbf{x}\})),$$

which, again by obvious principles of intensional logic, reduce to[7]:

$$\mathbf{love}'(\,\hat{}\,\mathbf{m}, \hat{\mathbf{P}}\mathbf{P}\{\,\hat{}\,\mathbf{j}\})$$

and

$$\mathbf{love}'(\,\hat{}\,\mathbf{m}, \hat{\mathbf{P}}\forall\mathbf{x}[\mathbf{man}'(\mathbf{x}) \rightarrow \mathbf{P}\{\mathbf{x}\}])$$

which in turn, by the meaning postulate regarding the extensionality of 'loves'[8], reduce to:

$$\mathbf{love}'_*(\mathbf{m}, \mathbf{j})$$

and

$$\forall\mathbf{x}[\mathbf{man}'(\mathbf{x}) \rightarrow \mathbf{love}'_*(\mathbf{m}, \mathbf{u})],$$

the second of which reduces finally to:

$$\forall\mathbf{u}[\mathbf{man}'_*(\mathbf{u}) \rightarrow \mathbf{love}'_*(\mathbf{m}, \mathbf{u})].$$

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[7] Regarding the use of relational notation in intensional logic, we set

$$\boldsymbol{\delta}(\boldsymbol{\alpha}, \boldsymbol{\beta}) =_{df} \boldsymbol{\delta}(\boldsymbol{\beta})(\boldsymbol{\alpha})$$

$\boldsymbol{\delta} \in ME_{\langle b,\langle a,t\rangle\rangle}$, $\boldsymbol{\alpha} \in ME_a$ and $\boldsymbol{\beta} \in ME_b$.

[8] Given the definition:

$$\boldsymbol{\delta}_* =_{df} \boldsymbol{\lambda}\mathbf{v}\boldsymbol{\lambda}\mathbf{u}\boldsymbol{\delta}(\,\hat{}\,\mathbf{u}, \boldsymbol{\lambda}\mathbf{PP}\{\,\hat{}\,\mathbf{v}\})$$

the meaning postulate for when δ translates an extensional transitive verb is:

$$\Box[\boldsymbol{\delta}(\mathbf{x},\mathcal{P}) \leftrightarrow \mathcal{P}\{\hat{\mathbf{y}}\boldsymbol{\delta}_*(\,^\vee\mathbf{x}, \,^\vee\mathbf{y})\}]$$

where $\mathcal{P}$ ranges over properties of properties of individual concepts and '**x**', '**y**' range over individual concepts.

The fact that transitive verbs are interpreted as functions whose arguments are the intensions of term phrases is of course but one example of Montague's general assumption, cited earlier, regarding the interpretation of English constructions based upon compound categories. It is this fact in particular that, together with his treatment of quantifier phrases as term phrases, enabled Montague to explain and represent the opacity of transitive verbs, such as 'seek' and 'conceive', without resorting to circumlocution or paraphrase. Indeed, the fact is that this opacity is but a feature of the intensionality that is assigned to all transitive verbs, and it is precisely this opacity that needs to be eliminated by meaning postulates in those contexts in which the verb is to be given an extensional interpretation.

Consider, for example, the different treatment that 'seek' receives in the translations of 'John seeks a unicorn' when this is given its *de dicto* reading:

$$\boldsymbol{\lambda}\mathbf{P}\mathbf{P}\{\hat{}\,\mathbf{j}\}(\hat{}\,\mathbf{seek}'(\hat{\mathbf{P}}\exists\mathbf{x}[\mathbf{unicorn}'(\mathbf{x}) \wedge \mathbf{P}\{\mathbf{x}\}])),$$

which reduces to:

$$\mathbf{seek}'(\hat{}\,\mathbf{j}, \hat{\mathbf{P}}\exists\mathbf{x}[\mathbf{unicorn}'(\mathbf{x}) \wedge \mathbf{P}\{\mathbf{x}\}])$$

from when it is given its *de re* reading:

$$\exists\mathbf{x}[\mathbf{unicorn}'(\mathbf{x}) \wedge \mathbf{seek}'(\hat{}\,\mathbf{j}, \hat{\mathbf{P}}\mathbf{P}\{\mathbf{x}\})],$$

which, given the extensionality of 'unicorn', can be reduced to:

$$\exists\mathbf{u}[\mathbf{unicorn}'_*(\mathbf{u}) \wedge \mathbf{seek}'(\hat{}\,\mathbf{j}, \hat{\mathbf{P}}\mathbf{P}\{\hat{}\,\mathbf{u}\})],$$

but which can be reduced even further in this case to:

$$\exists\mathbf{u}[\mathbf{unicorn}'_*(\mathbf{u}) \wedge \mathbf{seek}'_*(\mathbf{j}, \mathbf{u})].$$

Here, it is because intensionality is the rule that the opacity of 'seek' can be directly represented without resorting to circumlocution and paraphrase; and in those cases in which the rule is not to apply, a meaning postulate must be invoked so as to eliminate the intensionality and consequent opacity of the verb, and context, in question.

It is not only transitive verbs that are assigned this intensionality, according to Montague. We already saw above that meaning postulates are needed in the case of extensional *IV* and *CN*-phrases as well. But the fact is, however, that not all CN and IV-phrases are extensional, and a uniform interpretation that would render them all as such would therefore be quite inappropriate. Consider in this regard the following clearly invalid argument:

The temperature is ninety;
the temperature rises;
therefore, ninety rises.

Montague's analysis, or reduced translations, of the sentences in this argument are:

$$\exists \mathbf{x}[\forall \mathbf{y}(\mathbf{temperature}'(\mathbf{y}) \leftrightarrow \mathbf{y} = \mathbf{x}) \wedge [{}^{\vee}\mathbf{y}] = \mathbf{n}],$$

$$\exists \mathbf{x}[\forall \mathbf{y}(\mathbf{temperature}'(\mathbf{y}) \leftrightarrow \mathbf{y} = \mathbf{x}) \wedge \mathbf{rise}'(\mathbf{x})]),$$

and

$$\mathbf{rise}'({}^{\wedge}\mathbf{n})$$

where '**x**' and '**y**' range over individual concepts. Here we should especially note that while the identity in question in the first premise is not of two individual concepts but only of their extensions at the present moment in the real world, the truth of the second premise, according to Montague, depends on the fact that 'the temperature', loosely speaking, 'denotes' an individual concept and not an individual, and that 'rise', unlike most verbs 'depends for its applicability on the full behavior of individual concepts, not just on their extensions with respect to the actual world and ... [present] moment of time' ([6], p. 268). Thus, 'temperature' and 'rise' are particular examples of an intensional CN and IV-phrase, respectively; and it is precisely in order to explain the invalidity of the above type of argument that Montague interprets IV and CN-phrases in general as denoting sets of individual concepts rather than sets of individuals. It is for this reason of course that English terms, including quantifier phrases, are interpreted by Montague as denoting sets of properties of individual concepts rather than sets of properties of individuals.

13 Concluding Remarks

There are many other important features of Montague's grammar for English and of his translation of English by means of that grammar into intensional logic that we cannot go into here. The highly intensional nature of his semantics, for example, provides not only a more direct analysis of the opacity of intensional verbs but also a more direct analysis of the opacity of infinitive phrases as well. And then there is his treatment of relative clauses and of attributive adjectives, which we have not touched upon at all.

In closing then, it will no doubt have crossed the reader's mind that there may be some irony in the fact that Montague began his philosophical career as an extensionalist who took set theory as the proper theoretical framework for philosophy and as a formal-language philosopher who viewed the formalization of ordinary language as either impossible or extremely laborious, and in any case as certainly not philosophically rewarding. For the fact is that Montague has made important and philosophically innovative contributions toward a fully formalized syntax and semantics for natural language and that the semantics in question is most perspicuously described in terms of an intensional logic that transcends set theory and that in effect constitutes a new theoretical framework for philosophy. If this is not a revolution, it is at least a form of progress in the logical analysis of language.

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