# SOCIAL RESPONSIBILITY IN SUPPLY NETWORKS

Han Zhang

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Doctoral Committee

Rodney P. Parker, PhD

Goker Aydin, PhD

Ruth Beer, PhD

Kyle D. Cattani, PhD

Daniela Puzzello, PhD

March 3, 2020

To my parents

Han Zhang

SOCIAL RESPONSIBILITY IN SUPPLY NETWORKS

I study how the structure of supply networks interacts with efforts to make supply chains more socially responsible. One example concerns mineral mining in the Democratic Republic of the Congo that funds armed conflicts. Nonprofits and legislative bodies pressure manufacturers to trace and disclose their mineral sources. My first essay studies the decisions of manufacturers and smelters in the mineral supply network. We show the equilibrium depends on the total demand of "compliance-prone" manufacturers, who would comply if the prices of certified and noncertified metals were equal. Our results imply once penalties make sufficiently many manufacturers compliance-prone, certified metal may become so expensive that some compliance-prone manufacturers will not comply.

Five companies established a common fund for auditing the mineral smelters. Since the list of smelters certified by the audits is public, companies have an incentive to free-ride. Despite this incentive the fund was a success and received subsequent contributions from dozens of other companies. My second essay studies why. We consider two factors: an early-stage alliance and status-seeking behavior. We model the funding initiative as a public goods game and test the results in laboratory experiments. Our experiments show that the invitation stage is key to high contribution and status-seeking behavior affects the forming of an alliance.

My third essay studies a buyer auditing suppliers within a network to identify noncompliance. If a supplier fails an audit, the buyer must rectify the supplier or drop it (along with dependent suppliers). The network topology evolves as the buyer drops suppliers. We show the buyer should first audit and drop some suppliers, then either rectify all remaining ones, or proceed directly to production. When focusing on an upper tier, the buyer should always audit the least valuable unaudited supplier, yielding greater balance in the network structure. We establish the condition under which the buyer may truncate auditing ("hear no evil, see no evil").

Rodney P. Parker, PhD

Goker Aydin, PhD

Ruth Beer, PhD

Kyle D. Cattani, PhD

Daniela Puzzello, PhD

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## 1 Introduction

Supply chain management has traditionally focused on maximizing the profits of the various parties within the supply chain. As supply chains extend vertically and horizontally into global supply networks new dimensions of problems arise, especially from suppliers operating in developing countries. Environmental sustainability, social and economic development, and business ethics are increasingly important for the firm. Incidents like the Dhaka garment factory fire in 2012, the Rana Plaza factory collapse in 2013, and the revelations of bonded labor in upstream suppliers in various countries repeatedly remind global businesses and supply chain researchers that a modern company can sustain its success only by incorporating into its objectives the triple bottom line of profit, people, and planet (Lee and Tang 2017).

Consider one case in point: Decades-long civil conflicts have entangled the Democratic Republic of the Congo, causing more than five million deaths from 1998 to 2007 alone. A primary source of funding for the war groups is minerals mined in the country, in particular, tin, tungsten, tantalum, and gold. These minerals are used by manufacturers around the globe, including many household names, such as Apple, Intel, and Microsoft. By using the minerals without full knowledge of the sources, the manufacturers may inadvertently funnel funds into the conflicts. Both NGOs and legislative bodies have pressured manufacturers to trace the sources of the minerals they use and avoid conflict sources (e.g., Section 1502 of the Dodd-Frank Act). A key challenge for compliance lies in a pinch point in the mineral supply network in the form of the smelters: numbering a few hundred globally, the smelters process the ore extracted from mines into metal before manufacturing use.

In Chapter 2 we study the equilibrium outcome arising from the mineral supply network, modeled in three tiers: mines, smelters, and manufacturers. The smelters and manufacturers make compliance and procurement decisions. A manufacturer faces a penalty when it chooses not to be compliant. We characterize the equilibria and discuss their implications for the manufacturers and regulators. We show that the penalties on non-compliant manufacturers, while useful in the beginning, are insufficient to eliminate the use of conflict sources. In practice, instead of indiscriminately targeting all manufacturers who may use the minerals, the NGOs and the legislation focus their efforts on particular subsets of manufacturers. We model the practice by allowing an NGO to choose to toggle on the penalty for selected non-compliant manufacturers. Our model shows that once a critical mass of manufacturers are compliant, it becomes more difficult for the NGO to convert additional manufacturers.

Recognizing the compliance challenges, Apple, GE, HP, Intel, and Microsoft contributed funds to launch the Initial Audit Fund under the Responsible Minerals Initiative, an industrial alliance that audits and certifies mineral smelters to facilitate responsible sourcing. Under the Responsible Minerals Initiative once a smelter passes an audit its certification is public: all companies have equal access to the certified smelters regardless of contribution. Because of the public certification, contributing to such a common fund has minimal direct benefit specific to the contributor. As such companies have an incentive to free-ride. Despite this incentive the funding initiative succeeded: dozens of other companies contributed subsequently to the Initial Audit Fund, which drove the Conflict Minerals Initiative to become the largest and most central industry-led effort on conflict minerals (Young 2015).

Inspired by the success of the Initial Audit Fund, in Chapter 3 we study how companies with a shared social responsibility problem can successfully form an alliance to solve the problem together. We propose two factors that explain the Initial Audit Fund's success despite the incentive to free-ride: the initial catalyst of an early-stage alliance and the status-seeking behavior of lower-status companies. To capture the companies' incentive to free-ride, we model the funding initiative as a public goods game and incorporate the two factors: (1) an invitation stage where a subset of companies can form an initiating alliance and contribute to the fund, and (2) the difference in the status of each company in the form of the company's brand value. A company may want to be associated with other companies with well-known brands beyond any direct economic benefit. We use the model to show how the combination of both factors leads to high contribution. We then conduct laboratory experiments to test the effect of the two factors and how they interact. Our experiments show that successfully forming an alliance significantly increases group contribution. As our model predicts, status-seeking behavior affects whether and how an alliance is formed. In particular a low-status company significantly prefers to invite a high-status company to jointly initiate an alliance than to invite a low-status company. Surprisingly, high-status companies not in the initiating alliance also contribute after the invitation stage. We attribute this unexpected act to the high-status companies' stronger sense of moral responsibility. Both our theoretical model and experimental results affirm that forming an initiating alliance is the key to the success of a shared social responsibility project.

Apart from conflict minerals, social responsibility violations may arise in diverse domains. When overlooked they may cause hefty economic and reputational damages to the buyer when uncovered by the media, NGO, or regulator (e.g., Phillips 2016, Gant 2019). As the conflict minerals example shows, many companies now take the initiative to audit their direct and indirect suppliers to assure compliance. For example Apple audits its suppliers regularly and publishes the results in its annual supplier responsibility progress report.

To understand such practice, Chapter 4 focuses on the dynamic decisions of auditing suppliers within a supply network for social responsibility. We consider a buyer auditing a three-tier supply network with general network configuration. While auditing all suppliers in the extended network may be overly expensive and impractical, a buyer may judiciously audit some suppliers to lower the expected penalty from violations. Beyond the results of audits, Apple also reports remedial actions taken against the violations the audits uncover. A remedial action may consist of requiring the supplier to undergo a rectification process (e.g., for debt-bonded labor: see Apple Inc. 2018b) or dropping the supplier from Apple's supply chains (e.g., for not meeting conflict-free sourcing standards: see Apple Inc. 2018a). We incorporate the two remedial actions in our model: if a supplier fails an audit, the buyer decides whether to rectify or to drop the supplier. If the buyer drops a supplier, it also drops other suppliers in the network dependent on this supplier. With fewer suppliers, the network becomes less competitive, raising the buyer's input cost. Throughout the process the buyer balances the cost of auditing, the cost of rectifying non-compliant suppliers, the profit lost due to dropping suppliers along with their dependent firms, and the potential penalty from violations at unaudited suppliers. The network topology evolves as the buyer conducts audits and drops suppliers.

We build a two-phase model: an auditing phase followed by a production phase. We prove the existence of a unique equilibrium arising from the competition in the production phase. The equilibrium determines the buyer's production profit and the value of each supplier to the buyer, aiding the buyer's auditing decisions. For the auditing phase we characterize the buyer's optimal auditing policy in two subphases: in the first subphase the buyer audits and drops some suppliers; in the second subphase the buyer either audits and rectifies all remaining suppliers, or passes up any further audits and proceeds directly to production. Within the first (audit-and-drop) subphase, when focusing on the upper tier, the buyer should always audit a *least valuable unaudited supplier*; this practice tends to yield greater balance in the structure of the network. We also establish the condition under which the buyer may truncate auditing altogether, carrying unaudited suppliers to production ("hear no evil, see no evil"). When the buyer audits more broadly, any supplier (not necessarily the least valuable) may be chosen. In particular the buyer may audit a supplier in a pivotal position to ascertain the viability of the network, informing subsequent decisions (a "litmus test").

## 2 Curbing the Usage of Conflict Minerals: A Supply Network Perspective

*Abstract.* An important source of funds for the conflict in the Democratic Republic of the Congo (DRC) is the revenue from minerals mined in the DRC. Non-profits and legislative bodies made efforts to require manufacturers that use "conflict minerals" to learn and disclose their sources. In the mineral supply chain, the critical link between mines and manufacturers is smelters. We study equilibrium sourcing decisions that arise in such a network consisting of manufacturers and smelters. We find the equilibrium depends on the total demand of "compliance-prone" manufacturers, who would choose to be compliant if the prices of certified and noncertified metals were equal. We identify the conditions for the existence of several types of equilibrium: an all-certified equilibrium in which all smelters become certified; an equilibrium in which both metal types co-exist with no shortage of certified metal; and an equilibrium in which both metal types co-exist with a shortage of certified metal. In the event that an all-certified equilibrium is out of reach, we identify how the usage of conflict minerals change as an NGO or a legislative body targets additional manufacturers. An implication of our equilibrium results is that imposing penalties on manufacturers goes only so far: If penalties induce enough manufacturers to become compliance-prone, certified metal may become so expensive that some compliance-prone manufacturers will not comply.

### 2.1 Introduction

The Democratic Republic of the Congo (DRC), located in the center of Africa, has suffered a string of civil wars since it gained its independence in 1960. The revenue from minerals mined in the DRC, in particular, tantalum, tin, tungsten, and gold (often referred to as 3TG minerals), has been an important source of funds for the parties involved in these armed conflicts, hence the term "conflict minerals." To address the humanitarian crisis in the DRC, several NGOs made efforts to draw public attention to the matter. Most notably, the Enough Project, which concerns itself with human rights abuses in Africa, has been effective in publicizing the issue of conflict minerals. As a consequence of continued pressure from NGOs such as the Enough Project, the U.S. Congress has developed regulations aimed at curtailing mineral purchases from questionable sources in the DRC. In particular, Section 1502 of the Dodd-Frank Act requires all public companies to disclose: (i) if they use any 3TG minerals in their production, (ii) if so, whether the countries of origin include the DRC, and (iii) if so, the chain of custody for these minerals from mine to manufacturer. Effectively, this legislation requires any public company that uses a 3TG mineral to establish its sources. A similar legislation was passed by the European Union (Lewis 2016).

The Securities and Exchange Commission (SEC), which was tasked with implementing Section 1502, issued a rule in 2012 for public companies to file conflict minerals reports annually. However, by mid-2017, the enforcement of Section 1502 faltered amid legal challenges in the Court of Appeals (Piwowar 2017). Thus, it is not clear what, if anything, a manufacturer must do to be in compliance with the regulation. Nevertheless, from 2014 through 2016, when the SEC was still enforcing the regulation, more than 1,000 companies filed disclosure forms with the SEC annually (Alali and Wang 2018). Prior, in response to the Enough Project's requests, many electronics manufacturers had disclosed their auditing and tracing efforts, which the Enough Project published in a 2012 ranking of manufacturers (Lezhnev and Hellmuth 2012).

As manufacturers made efforts to trace the sources of 3TG minerals, many observers have recognized a particular feature of the mineral supply chain: Even though the supply chain has several intermediaries as the mineral makes its way from the mine to the manufacturer, the most critical echelon along the way consists of smelters, who convert the ore supplied by mines to the metals used by manufacturers. This echelon is critical, because it is a "pinch point" of the supply chain: The number of smelters is relatively low — on the order of hundreds — compared to myriad manufacturers served by a multitude of mines (Schuh and Strohmer 2012). Hence, much of the tracing efforts have focused on the sourcing practices of smelters, thus creating a set of certified smelters. For example, the Responsible Minerals Initiative (formerly the Conflict-Free Sourcing Initiative) maintains a list of smelters who meet audit standards to document that they produce responsibly-sourced materials.

Given this context, our study is informed by the assumption that manufacturers who disregard the NGO pressure and legislative action face penalties implicitly imposed by reputational risk, if not explicitly imposed by legislation. Given the current ambiguity of what it means to be in compliance with the legislation, in this paper we adopt the following working definition of compliance for modeling and analysis purposes: We consider a manufacturer to be compliant if it transparently traces its sources, and buys as much as possible from certified sources. If the quantity of supply from certified sources is not sufficient to satisfy demand, a manufacturer can buy from uncertified sources, and we still consider it to be compliant as long as it acknowledges the portion of its metal coming from uncertified sources.

Our research goal is to study sourcing relationships that arise in a supply network with manufacturers and smelters, in the presence of penalties to the manufacturers who do not trace their sources. Alongside manufacturers and smelters who intend to ensure compliance, there may remain firms who choose not to do so. Therefore, one can claim that there exist two types of mineral — certified mineral, which is known to be conflict-free, and noncertified mineral, which is not known to be so — and the supply and demand for these two types of mineral are interdependent.

Our first research question is about the effect of penalties on the supply network. Specifically, if one were able to levy increasingly high penalties on noncompliant manufacturers, would one eventually reach a supply chain that uses 100% certified metal? On the surface, if more manufacturers want to buy certified metal, the price of certified metal should go up, thus enticing all smelters to become certified. However, with all smelters becoming certified, the audited and conflict-free mines might become over-utilized, thus driving up the cost for certified smelters. Hence, it is not necessarily clear that all smelters will find it in their best interest to become certified.

Our second research question adopts the perspective of an NGO or a policy-maker. If an NGO or policy-maker, whose goal is to achieve higher amounts of certified metal in the supply chain, were to impose the penalty selectively on a subset of manufacturers, which subset of manufacturers should it target? Both the manufacturer's volume of metal purchase and what the manufacturer stands to lose from bad publicity could play a role in this decision. The sourcing decisions of heavy users have a bigger effect on the quantities traded in the supply network, but the behavior of those vulnerable to bad publicity might be easier to change. We identify how an NGO or policy-maker should factor in such manufacturer characteristics.

## 2.2 Literature Review

A recent group of papers provide behavioral evidence that customers are willing to pay a premium for socially responsible products (e.g., Bartling et al. 2015, Pigors and Rockenbach 2016, Kraft et al. 2018). In keeping with this evidence, a stream of research in the supply chain literature studies interactions among manufacturers and their suppliers in the presence of a group of customers who are willing to pay more for socially or environmentally responsible products (e.g., Guo et al. 2016, Ha et al. 2018, Agrawal and Lee 2019, Kraft et al. 2019). A second stream of the supply chain literature, however, avoids explicit models of competition for consumer segments that value social responsibility, but assumes that there is a penalty for social responsibility violations. These papers dwell in more detail on compliance decisions on the part of suppliers and auditing decisions on the part of buyers (e.g., Aral et al. 2014, Plambeck and Taylor 2016, Chen and Lee 2016, Chen et al. 2018, Caro et al. 2018, Chen et al. 2019b). A third stream of research focuses more on decisions about whether or not to disclose noncompliance (e.g., Chen and Slotnick 2015, Kim 2015, Kalkanci and Plambeck 2019a,b).

Similar to the last two streams of research, we forego an explicit model of competition for socially conscious customer segments — our model assumes that there is a penalty for social responsibility violations instead — to cover compliance decisions on the part of suppliers and auditing decisions on the part of buyers. We do not use as granular a model of compliance and auditing decisions as in the second stream, or make a true distinction between auditing and disclosure decisions as in the third stream. For example, we do not model compliance and auditing efforts that can take a range of values. Instead, a supplier complies or not, and a buyer audits or not (and, if an audit is done, its result is disclosed). Simplifying these aspects of the model allows us to adopt a network-wide perspective of the supply chain, in which multiple buyers can source from multiple suppliers. This network perspective is what distinguishes our paper from the aforementioned literature.

Closer to our multi-buyer, multi-supplier setting, some recent work considers compliance and supplier selection in supply chains by allowing more than two tiers or multilateral auditing arrangements. Huang et al. (2017) study a three-tier supply chain, with one firm in each tier. Each firm may exert an effort to strengthen compliance at the most upstream tier. They illustrate the unique challenges of inducing compliance in multi-tier supply chains. Fang and Cho (2019) study a hub-and-spoke network with one supplier and multiple buyers, who can form coalitions to either jointly audit the supplier or audit independently yet share auditing results within the coalition. Chen et al. (2019a) study the coordination of auditing activities by two buyers, who share one supplier in addition to each having its exclusive supplier. Focusing on a single buyer's multi-tier supply network with fixed material flows, Feng et al. (2019) study whether the buyer should directly implement environmental and social responsibility standards at upstream suppliers or delegate the task to midtier suppliers. Zhang et al. (2019a) study a buyer's auditing decisions in a two-tier network that evolves as the buyer may drop suppliers that fail audits. In contrast to these papers, we study a setting where the suppliers' compliance decisions influence the sourcing decisions of multiple buyers procuring from multiple suppliers. In thematically related work, motivated by the success of the Responsible Minerals Initiative, Zhang et al. (2019b) develop a behavioral model of auditing alliances and conduct laboratory experiments to study manufacturers' incentives to participate.

Our work is methodologically related to research on models of supply networks. Corbett and Karmarkar (2001) study a supply network with a general number of tiers and general numbers of firms in each tier; firms in each tier play a Cournot quantity competition game in the downstream market, facing input cost derived from the upstream markets. Subsequent work in this area includes Adida and DeMiguel (2011) who study demand uncertainty, Belavina (2017) who examine the desirableness of relational sourcing, Ang et al. (2017) who study sourcing decisions in a network where a buyer's tier-1 suppliers have overlapping and risky tier-2 suppliers, Bimpikis et al. (2019) who study the optimal structures of supply networks facing disruptions, Bimpikis et al. (2018) who analyze multi-sourcing and miscoordination in a supply network, and Korpeoglu et al. (2018) who study the expansion and integration of supply networks.

Our equilibrium concept is a pure-strategy version of that presented in Schmeidler (1973), part of a literature on large games (e.g., Aumann 1964, Mas-Colell 1984, Housman 1988). A relatively recent stream of literature compares various large game equilibrium concepts and vindicates their usage as approximations to finite games (Al-Najjar 2008, Carmona and Podczeck 2009, Yang 2011). Recent operations-centric papers that apply large game models to competition settings include Yang and Xia (2013) and Yang et al. (2014).

### 2.3 Model

In our model of the supply network, mines supply ore to smelters, who in turn convert the ore into metal, used by manufacturers. In the upstream echelon, we treat mines as two aggregated pools, *verified mines* and *unverified mines*. Verified mines are those that have been confirmed to be conflict-free. We label the remainder "unverified mines." Some of these mines may very well be conflict-free, but they have not been verified to be so. In the middle echelon, each smelter decides whether to be certified: A *certified smelter* commits to purchasing only from the pool of verified mines. A *noncertified smelter*, on the other hand, is free to buy from both pools of mines. We refer to the metal produced by certified smelters as *certified metal*, which can be traced back to its verified conflict-free sources, and the metal produced by noncertified smelters as *noncertified metal*. In the downstream echelon, each manufacturer decides whether to be compliant: A *compliant manufacturer* commits to purchasing certified metal — produced by certified smelters — as long as there is sufficient supply of the certified metal. We model individual smelters and manufacturers as infinitesimal players, whose profit-maximizing decisions collectively give rise to an equilibrium from which no player has an incentive to deviate. We aim to explore the supply relationships that arise in this equilibrium.

#### 2.3.1 The Supply Network

In this subsection, we discuss entities in the supply network, and the material flows among these entities.

**Manufacturers.** Let $\mathcal{M} \subseteq \mathbb{R}$ be the set of infinitesimal manufacturers with manufacturer $i$ characterized by demand $D_i > 0$, penalty for noncompliance $\rho_i \geqslant 0$, and compliance cost $\gamma_i \geqslant 0$. We assume the joint distribution of manufacturer parameters $(D_i, \gamma_i, \rho_i)$ is continuous. Manufacturer $i$ incurs penalty $\rho_i$ when it chooses to be noncompliant, in which case the manufacturer is free to buy from certified and noncertified smelters. As discussed in the Introduction, this penalty may be imposed explicitly by the legislation or arise from reputational risk. On the other hand, manufacturer $i$ incurs compliance cost $\gamma_i$ when it chooses to be compliant. The compliance cost captures the manufacturer's administrative costs for documenting the chain of custody.

We represent manufacturer $i$'s compliance decision by $\zeta_i \in \{C, N\}$, with $C$ representing compliance and $N$ representing noncompliance. Let $r_i$ be the proportion of demand $D_i$ that manufacturer $i$ orders from certified smelters. Note that for a compliant manufacturer $i$ we must have $r_i = 1$. Given the manufacturers' compliance decisions, we use $M_{\text{CM}} \subseteq \mathcal{M}$ to denote the set of compliant manufacturers, and $M_{\text{NM}} \subseteq \mathcal{M}$ the set of noncompliant manufacturers.

Let $D_T = \int_{\mathcal{M}} D_i di$ be the total demand of all manufacturers in the supply network. The total demand of all compliant manufacturers is denoted by $D_{\text{CM}} = \int_{M_{\text{CM}}} D_i di$ and the total demand of all noncompliant manufacturers is denoted by $D_{\text{NM}} = \int_{M_{\text{NM}}} D_i di$. If manufacturer $i$ is noncompliant, it may split its demand between certified and noncertified smelters. Consequently, the total demand for certified metal includes all the demand from compliant manufacturers ($D_{\text{CM}}$) and some demand from noncompliant manufacturers (specifically, a fraction $r_i$ of noncompliant manufacturer $i$'s demand), and is given by $D_C = \int_{M_{\text{CM}}} D_i di + \int_{M_{\text{NM}}} r_i D_i di$. The total demand for noncertified metal is the remaining demand from noncompliant manufacturers, and is given by $D_N = \int_{M_{\text{NM}}} (1 - r_i) D_i di$.

**Smelters.** Let $\mathcal{S} \subseteq \mathbb{R}$ be the set of smelters, with smelter $j$ characterized by capacity $K_j > 0$ and unit processing cost $c_j \geqslant 0$. Assume the joint distribution of $c_j$ and $K_j$ is continuous. Denote the total capacity of all smelters by $K_T = \int_{\mathcal{S}} K_j dj$.

Let $\sigma_j \in \{C, N\}$ denote smelter $j$'s decision to be certified or not, with $C$ and $N$ representing the smelter choosing to be certified or noncertified, respectively. If a smelter is noncertified, it is free to split its purchases between verified mines and unverified mines. Let $t_j$ be the proportion of ore smelter $j$ buys from the pool of verified mines. Note that $t_j = 1$ for certified smelter $j$. Given the certification decisions, let $S_C \subseteq \mathcal{S}$ denote the set of certified smelters, and $S_N \subseteq \mathcal{S}$ the set of noncertified smelters. The total capacity of certified smelters is $K_C = \int_{S_C} K_j dj$. The total capacity of noncertified smelters is $K_N = \int_{S_N} K_j dj$.

**Markets.** We assume that the metal will be traded in two distinct but related markets, one for certified metal, and another for noncertified metal. Even though the trade of 3TG minerals does not always take place in centralized markets — for example, the trade of tantalum is often through bilateral contracts — we use the term "market" to reflect the fact that the price of a metal type (certified or noncertified) will depend on the total demand and supply for it. Hereafter, for ease of reference, we use $C$ and $N$ as shorthands for certified and noncertified metal types, respectively.

Notice that in our model, there is a one-to-one correspondence between metal type $s$ and smelter type $s$, $s \in \{C, N\}$, in that certified metal is produced by certified smelters, and noncertified metal by noncertified smelters.

There can be a variety of different equilibrium outcomes, depending on how the total capacity of all smelters, $K_T$, compares to the total demand of all manufacturers, $D_T$. We focus on the case where the total capacity is equal to the total demand, that is, we assume a *balanced market*. The balanced market assumption is not only analytically convenient, but it is also a more reasonable approximation of markets where excess capacity, while it may exist, is not too large. As for the unit price functions of certified and noncertified metal, $p_s(D_s, K_s)$ for $s \in \{C, N\}$, for analytical convenience, we assume that the unit price functions are symmetric, and linear in capacity and demand. These assumptions are summarized below.

**Assumption 2.1.** *We focus on a balanced market with linear and symmetric unit price functions:*

*(a) The total smelting capacity is equal to the total demand for the metal, that is,* $K_T = D_T$.

*(b) The unit price of metal type s is linearly increasing in the demand for metal type s and linearly decreasing in the smelting capacity of metal type s, that is,* $p_s(D_s, K_s) = aD_s - bK_s + p_0$, $s \in \{C, N\}$, *where* $a, b, p_0 > 0$ *are constants, except in the boundary case* $K_s = D_s = 0$, *where we let* $p_s(0, 0) = 0$.

The unit prices for certified and noncertified metal are inter-dependent under Assumption 2.1. The larger the demand (or capacity) for one metal type is, the lower the demand (or capacity) for the other one, so the prices of the two metal types are linked. In the boundary case $K_s = D_s = 0$, that is, when there is no demand or supply for metal type $s$, we let the price $p_s(0, 0) = 0$. In this boundary, setting the price to zero captures the fact that any infinitesimal smelter choosing to produce metal type $s$ does not have any customer for it. Likewise, for any infinitesimal manufacturer who chooses to demand metal type $s$, the price is moot as there is no supply. In this boundary condition, if a subset of manufacturers demanded metal type $s$ to create non-zero demand for it, then we would have a shortage for metal type $s$, as we discuss next.

In our model, we allow the possibility that there will be a shortage for one of the two metal types, that is, we allow the demand for a metal type to exceed its capacity. If there is a shortage for metal type $s$, we assume that the excess demand will be met by the capacity of the other metal type, which we denote by $-s$. (Given our assumption of a *balanced market*, there cannot be simultaneous shortages of both metal types.) When there is a shortage for a metal type, we assume that the metal type in shortage will be more expensive than the metal type in surplus. Furthermore, we assume that each manufacturer receives an allocation proportional to its demand for the metal type in shortage. These assumptions are summarized below:

**Assumption 2.2.** *For $s \in \{C, N\}$, if $D_s > K_s$, then:*

*(a) $p_s(D_s, K_s) > p_{-s}(D_{-s}, K_{-s})$.*

*(b) Any manufacturer who has a non-zero demand for metal type $s$ receives a fraction $\frac{K_s}{D_s}$ of its demand for metal type $s$, and the rest of its demand for metal type $s$ must be met with metal type $-s$ instead.*

We remark on three consequences of Assumptions 2.1 and 2.2.

*Remark* 2.1. The coefficients $a$ and $b$ in the price functions must be equal (see Lemma 2.1 in Appendix 2.A). Therefore, the price of metal type $s$ changes linearly in the difference between its demand and capacity.

*Remark* 2.2. Given the market is balanced, both certified smelters and noncertified smelters will use up their entire capacity. Therefore, letting $Q_s$, $s \in \{C, N\}$, denote the quantity traded of each metal type, we must have $Q_s = K_s$ for $s \in \{C, N\}$.

*Remark* 2.3. If the demand for certified metal exceeds the capacity of it, then every compliant manufacturer will receive only a portion of its demand for certified metal, and the rest of its demand will have to be met using noncertified metal. Consequently, we allow a manufacturer to claim compliance even if it buys some noncertified metal due to shortages in the certified metal market. As discussed in the Introduction, this is the working definition of compliance we adopt in this paper, given the ambiguity surrounding the legislation. This conceptualization of compliance is in keeping with what the regulators and NGOs initially expected. For example, when publishing its company rankings in 2012, the Enough Project did not require a manufacturer to achieve 100% conflict-free sourcing for the manufacturer to earn its "green designation." Instead, the NGO awarded green designation for a sufficient degree of tracing and reporting (Lezhnev and Hellmuth 2012). Likewise, for a two-year period at the outset, the SEC rule allowed manufacturers to report "DRC Conflict Undeterminable" status, which amounts to the firm claiming that it has not been able to trace its sources back to the mines after exerting due diligence (U.S. Securities and Exchange Commission 2017)

An important driver of equilibrium results will be the total demand of *compliance-prone manufacturers*, which we define to be those manufacturers whose penalty $\rho_i$ exceeds their compliance cost $\gamma_i$. These manufacturers are compliance-prone in the sense that if the prices of certified and noncertified metals were the same, they would prefer to become compliant and buy certified metal to avoid the penalty cost. Of course, whether or not these manufacturers end up complying in equilibrium will depend on the relative prices of certified and noncertified metals in equilibrium. We let $\widehat{D}$ denote the total demand of compliance-prone manufacturers. We refer to this quantity as *compliance-prone demand*, and it is given by $\widehat{D} := \int_{\mathcal{M}} D_i \mathbb{1}_{\{\rho_i > \gamma_i\}} \,\mathrm{d}i$.

**Mines.** We model mines in two aggregated pools, verified and unverified. As we explain below, even though we do not explicitly model the mines as strategic players, our model takes into account how the sourcing decisions made by smelters and manufacturers might affect the available capacity of the mines and their extraction costs.

The demand for ore from verified mines (verified ore, for short) originates from two sources. First, all certified metal traded in the market, $Q_C$, must come from verified ore. Second, any noncertified smelter $j$ purchases a portion $t_j$ of its ore from verified mines. Therefore, noncertified smelter $j$ with capacity $K_j$ will direct a demand of $t_j K_j$ toward the verified mines. Consequently, the total demand for verified ore, denoted by $Q_V$ is as follows

$$Q_V = Q_C + \int_{S_N} t_j K_j dj. \tag{2.1}$$

Similarly, the demand for ore from unverified mines (unverified ore, for short) is denoted by $Q_U$ and given by

$$Q_U = \int_{S_N} (1 - t_j) K_j dj. \tag{2.2}$$

We do not impose an explicit capacity constraint on how much ore can be extracted from each pool of mines, so the demands for verified and unverified ores, $Q_V$ and $Q_U$, are met in full by the corresponding pools.

We next discuss the effects of smelters' and manufacturers' sourcing decisions on the mines in our model. One may speculate that the smelters' and manufacturers' sourcing decisions have two opposite effects on the price of ore paid by smelters to mines. First, what we will refer to as the *depletion effect*: The unit price of verified ore increases in the quantity extracted from the verified pool of mines (similarly for the pool of unverified mines). Second, what we will refer to as the *competition effect*: If the total demand of compliant manufacturers is larger (and, consequently, the demand of noncompliant manufacturers is smaller), then more mines would become verified. Hence, one would expect that the pool of verified mines would grow (and the pool of unverified mines would shrink), thus creating more competition among verified mines and driving down the price of verified ore (and less competition among unverified mines, driving up the price of unverified ore).

To capture the depletion effect, we assume that the unit price of ore coming from a particular pool will increase as one extracts more from that pool. To capture the competition effect, we assume that the higher the total demand of compliant manufacturers, the lower the price of verified ore will be, everything else being equal (and, similarly, the larger the total demand of noncompliant manufacturers, the lower the price of unverified ore). To summarize, using $o_V(Q_V, D_{\text{CM}})$ and $o_U(Q_U, D_{\text{NM}})$ to denote the unit prices of verified ore and unverified ore, respectively, we assume:

**Assumption 2.3.** *$o_V(Q_V, D_{\text{CM}})$ is continuous, strictly increasing in $Q_V$, and decreasing in $D_{\text{CM}}$; $o_U(Q_U, D_{\text{NM}})$ is continuous, strictly increasing in $Q_U$, and decreasing in $D_{\text{NM}}$.*

We expect that it is more plausible that the depletion effect is stronger than the competition effect in the case of a natural resource. Therefore, to capture the case where the depletion effect dominates the competition effect, we make the assumption below:

**Assumption 2.4.** *There exists $\overline{Q} \in [0, D_T]$ such that $o_V(x, x) < o_U(D_T - x, D_T - x)$ when $x < \overline{Q}$ and $o_V(x, x) > o_U(D_T - x, D_T - x)$ when $x > \overline{Q}$.*

To explain why the depletion effect dominates the competition effect under this assumption, suppose that we had a clear separation of markets so that all the metal used by compliant manufacturers came from the verified pool of mines (say, quantity $x$), and all the metal used by noncompliant manufacturers came from the unverified pool (which would amount to a quantity $D_T - x$) — such a clear separation does not have to arise in equilibrium, but the assumption is best understood through such a scenario. Then the unit cost of verified ore would be $o_V(x, x)$ and the unit cost of unverified ore would be $o_U(D_T - x, D_T - x)$. As $x$ increases, the verified ore's cost is driven further down by the competition effect, but driven further up by the depletion effect. According to the assumption, the verified ore is more expensive at large values of $x$. Therefore, as the verified pool of mines supply more ore, it must be that the depletion effect dominates the competition effect.

### 2.3.2 The Equilibrium Concept

In this subsection we discuss the problems solved by individual smelters and manufacturers, and how they come together in an equilibrium.

**The Smelter's Problem.** A smelter incurs a unit processing cost in addition to the price of ore it pays to the mines and accrues revenue from the metal it sells to manufacturers. Each smelter's goal is to maximize its profit. Let $\pi_{cs}(j)$ and $\pi_{ns}(j)$ be the profits of smelter $j$ if it decides to be certified or noncertified, respectively. If smelter $j$ chooses to be certified, then its profit is the smelter's margin multiplied by its capacity, where the margin is the price of certified metal net of the price of verified ore and the smelter's processing cost:

$$\pi_{cs}(j) = \left(p_C(D_C, K_C) - o_V(Q_V, D_{CM}) - c_j\right) K_j. \tag{2.3}$$

On the other hand, if smelter $j$ chooses to be noncertified, then it chooses $t_j \in [0,1]$, the fraction of ore it buys from the verified mines. Hence, the margin of a noncertified smelter is the price of noncertified metal net of its processing cost and a weighted average of the prices of verified and unverified ores:

$$\pi_{ns}(j) = \max_{t_j \in [0,1]} \left\{ \left[p_N(D_N, K_N) - \left(t_j o_V(Q_V, D_{CM}) + (1-t_j) o_U(Q_U, D_{NM})\right) - c_j\right] K_j \right\}. \tag{2.4}$$

Smelter $j$ must decide to be certified or not:

$$\max\{\pi_{cs}(j), \pi_{ns}(j)\}. \tag{2.5}$$

**The Manufacturer's Problem.** In addition to paying for the metal bought from the smelters, a manufacturer incurs a compliance cost if it is compliant, and a penalty cost if it is noncompliant.

Each manufacturer's goal is to minimize its total cost. Let $\nu_{cm}(i)$ and $\nu_{nm}(i)$ be the costs of manufacturer $i$ if it decides to be compliant or noncompliant, respectively. If manufacturer $i$ chooses to be compliant, then its cost is

$$\nu_{cm}(i) = p_C(D_C, K_C)\left(\frac{K_C}{D_C} \wedge 1\right) D_i + p_N(D_N, K_N)\left(1 - \frac{K_C}{D_C}\right)^+ D_i + \gamma_i, \quad \text{for } D_C > 0. \tag{2.6}$$

The first term is what the manufacturer pays for certified metal; in keeping with our assumptions, compliant manufacturer $i$ will meet as much of its demand $D_i$ as possible using certified metal. However, if there is a shortage of certified metal, that is, $D_C > K_C$, the supply of certified metal is rationed in proportion to the manufacturer's demand for certified metal. Consequently, in the event of shortage, compliant manufacturer $i$ uses noncertified metal to meet the shortfall, that is, $\left(1 - \frac{K_C}{D_C}\right)^+ D_i$, which gives the second term in the cost function above. The third term is the exogenously fixed compliance cost for manufacturer $i$.

When dealing with the manufacturer's cost, it will be convenient to define:

$$p_s^e(D_s, K_s) := p_s(D_s, K_s)\left(\frac{K_s}{D_s} \wedge 1\right) + p_{-s}(D_{-s}, K_{-s})\left(1 - \frac{K_s}{D_s}\right)^+ \quad \text{for } D_s > 0. \tag{2.7}$$

For each unit of metal type $s$ a manufacturer demands, what the manufacturer pays is either the unit price of that metal type (if there is no shortage of it), or a weighted average of the prices of both metal types (if there is a shortage of metal type $s$, thus requiring the manufacturer to meet the shortfall using the other metal type).[1] With this notation in place, the cost of compliant manufacturer $i$ is simply

$$\nu_{cm}(i) = p_C^e(D_C, K_C) D_i + \gamma_i. \tag{2.8}$$

Recall that noncompliant manufacturer $i$ must choose $r_i \in [0, 1]$, the proportion of its demand ordered from certified smelters. A noncompliant manufacturer's cost consists of what it pays for certified and noncertified metals and the penalty for noncompliance. Leveraging the definition in

[1] For the boundary case where there is no demand for metal type $s$, that is, $D_s = 0$, we define: (i) If there is no capacity for metal type $s$, that is, $K_s = 0$, then $p_s^e(D_s, K_s) := p_{-s}(D_T, K_T)$ (a manufacturer demanding metal type $s$ will pay the price of the other metal type, which is the only option available). (ii) If there is positive capacity for metal type $s$, that is, $K_s > 0$, then $p_s^e(D_s, K_s) := p_s(0, K_s)$ (this is the price a manufacturer has to pay for the next infinitesimal unit of metal type $s$ it demands).

(2.7), one can write noncompliant manufacturer $i$'s cost $\nu_{nm}(i)$ as

$$\nu_{nm}(i) = \min_{r_i \in [0,1]} \{p^e_C(D_C, K_C) r_i D_i + p^e_N(D_N, K_N)(1 - r_i) D_i + \rho_i\}\,. \tag{2.9}$$

It is worth remarking upon the boundary case in which there is no capacity for one type of metal, that is, $K_s = 0$ for $s = C$ or $s = N$. In that case, the effective prices defined in (2.7) are the same, that is, $p^e_C(D_C, K_C) = p^e_N(D_N, K_N)$, and a noncompliant manufacturer will be indifferent between the two metal types. As stated below, we make the natural assumption to break the tie: The noncompliant manufacturer will order the metal type in existence.

**Assumption 2.5.** *When $K_s = 0$ for $s = C$ or $s = N$, a noncompliant manufacturer orders only metal type $-s$.*

Manufacturer $i$ must decide to comply or not:

$$\min\{\nu_{cm}(i), \nu_{nm}(i)\}. \tag{2.10}$$

**Equilibrium.** Collectively, the manufacturers' and smelters' decisions determine the subset of manufacturers who are compliant, the subset of smelters who are certified, and for every manufacturer and smelter, how they allocate their purchases among sources in their respective upper echelons. Let $A = \{(C, 1)\} \cup (\{N\} \times [0, 1])$ be the set of (pure) strategies available to each manufacturer or smelter. If manufacturer $i$ plays strategy $(\zeta_i, r_i) \in A$, then it is compliant if $\zeta_i = C$, in which case it demands $D_i$ from certified smelters, and noncompliant if $\zeta_i = N$, in which case it demands a fraction $r_i$ of $D_i$ from certified smelters. Likewise, if smelter $j$ plays strategy $(\sigma_j, t_j) \in A$, then it is certified if $\sigma_j = C$, in which case it procures all of its ore from the verified pool, and noncertified if $\sigma_j = N$, in which case it procures fraction $t_j$ of its ore from the verified pool. A *strategy profile* is mapping $\phi : \mathcal{M} \sqcup \mathcal{S} \to A$.

As for the timing of events, we assume manufacturers and smelters make all their decisions simultaneously. Once all decisions are made, the market prices for certified metal and noncertified metal are observed, and the transactions occur and payoffs are collected.

An *equilibrium* is a strategy profile $\phi$ that simultaneously solves all the smelters' and manufacturers' problems.

### 2.4 Sourcing Decisions in Equilibrium

Noncompliant manufacturers are free to buy any mixture of certified and noncertified metal, but compliant manufacturers must first seek certified metal. In that sense, certified metal is a perfectly valid substitute for noncertified metal, but the opposite is not true. Hence, in practice, one would expect that if one type of metal is in shortage, it would be the certified metal. As we establish in this section, our model exhibits this intuitively appealing and practically valid property. Therefore, in our model, there can arise two types of equilibrium: with or without a shortage of the certified metal. Before we discuss the prices and sourcing relationships that would arise under these two types of equilibrium, we first dispense with two special cases.

It can be shown that there exists an equilibrium in which no smelter is certified and no manufacturer demands certified metal (i.e., $K_C = D_C = 0$) if and only if there is no compliance-prone manufacturer (i.e., $\widehat{D} = 0$). In the current landscape, where the manufacturers are concerned about implicit or explicit penalties for buying conflict minerals, we no longer expect to encounter such a zero-certification equilibrium, because it cannot be sustained even if just a small subset of manufacturers are compliance-prone — there would arise a subset of certified smelters to satisfy the demand of these manufacturers.

At the opposite end of the spectrum, one can also identify the conditions under which the entire market turns to certified metal. This type of equilibrium is described in the next proposition. The proof of the proposition, along with all the other proofs, is included in the Appendices.

**Proposition 2.1.** *If and only if the condition $p_0 \geqslant o_V(D_T, \widehat{D}) - o_U(0, D_T - \widehat{D})$ is satisfied, then there exists an "all-certified" equilibrium with the following properties:*

*(a) All smelters are certified (i.e., $K_C = K_T$);*

*(b) All compliance-prone manufacturers are compliant;*

*(c) All other manufacturers remain noncompliant (i.e., $D_{\mathrm{CM}} = \widehat{D}$); and*

*(d) All manufacturers, compliant and noncompliant, demand only certified metal ($D_C = D_T$).*

Note that, in the equilibrium described above, all smelters are certified, so all smelters buy from the pool of verified mines. Thus, the demand for verified ore, $Q_V$, is given by the total demand $D_T$, while there is no demand for unverified ore, that is, $Q_U = 0$. Such lopsided ore demands drive up the price of verified ore and drive down the price of unverified ore. The condition $p_0 \geqslant o_V(D_T, \widehat{D}) - o_U(0, D_T - \widehat{D})$ guarantees that, even with such ore prices, no smelter has any incentive to become noncertified. It seems that this type of equilibrium has already been achieved for tantalum. According to the Responsible Mineral Initiative, which maintains a list of performance indicators, 40 out of 41 eligible tantalum smelters are in conformant status at the time of writing (with the remaining supplier in 'active' status, meaning that it is scheduled for an assessment). This is not the case for tin, tungsten, or gold. Interestingly, in the case of cobalt, which was not one of the initially targeted 3TG minerals, but has since become the next front in conflict minerals, only 7 out of 31 eligible smelters are reported to be conformant or active (Responsible Minerals Initiative 2019b). Thus, we next turn to other equilibria, in which certified and noncertified metal coexist. The following theorem shows that, depending on the size of compliance-prone demand, there can be two types of equilibrium, distinguished by whether the certified metal is in shortage or not.

**Theorem 2.1.** *If the compliance-prone demand, $\widehat{D}$, is less than or equal to the threshold $\overline{Q}$, then there is no shortage of certified metal in equilibrium. Otherwise, there will be a shortage of certified metal in equilibrium (except when the entire market is certified as in Proposition 2.1).*

Recall that $\overline{Q}$ is the threshold at which the price of verified ore overtakes the price of unverified ore. Setting aside the special case of a fully certified market discussed above, the theorem indicates that if the compliance-prone demand is less than $\overline{Q}$, then there will be sufficient certified capacity to meet the eventual demand for certified metal. Otherwise, there will be a shortage of certified metal. In the remainder of the section, we characterize the properties of these two types of equilibrium.

**The Equilibrium Without Shortage.** We first consider the case when compliance-prone demand is less than the threshold, that is, $\widehat{D} \leqslant \overline{Q}$, in which case there is no shortage of certified metal. To explain the forces at play, imagine the firms in the supply network formed two completely detached channels, one for certified metal and the other for noncertified metal. In this hypothetical scenario, suppose that all compliance-prone manufacturers are compliant and buy from certified smelters, who then buy from the verified pool; all remaining manufacturers are noncompliant and buy only from noncertified smelters, who then buy from the unverified pool.

($i$) Consider the manufacturers' incentives in this scenario with detached markets. For each metal type, the capacity is equal to demand, which leads to the same price for both metal types (as a consequence of Remark 2.1). With the two metal prices being the same, compliance-prone manufacturers would indeed remain compliant and buy certified metal; all others would remain noncompliant.

($ii$) Consider now the smelters' incentives in this scenario. A total demand of $\widehat{D} \leqslant \overline{Q}$ is flowing through the certified channel to the verified pool. Given that the quantity demanded from the verified pool is less than $\overline{Q}$, the verified ore would be cheaper than the unverified ore. That being the case, the noncertified smelters would no longer be content buying only from the unverified pool. Instead, some noncertified smelters, who may buy from whichever pool is cheaper, would shift their purchases to the verified pool; they would do so until the prices of the two ores became equal.

Therefore, starting with the hypothetical scenario of two detached markets, the network would settle in an equilibrium, in which the prices of the two metal types are the same, and the prices of the two ore types are the same, with some noncertified smelters buying verified ore. In this equilibrium, all compliance-prone manufacturers would be compliant and all other manufacturers would remain noncompliant and buy only noncertified metal. The ongoing discussion is formalized in the next theorem:

**Theorem 2.2.** *Suppose the total demand of compliance-prone manufacturers is less than the threshold, that is, $\widehat{D} \in (0, \overline{Q}]$. There exists an equilibrium where*

*(a) All compliance-prone manufacturers are compliant; all other manufacturers remain noncompliant, and buy only noncertified metal, that is, $D_{\text{CM}} = \widehat{D} = D_C$;*

*(b) The total certified demand and total certified capacity both equal $\widehat{D}$, that is, $D_C = K_C = \widehat{D}$;*

*(c) The prices of certified and noncertified metal are the same (i.e, $p_C(D_C, K_C) = p_N(D_N, K_N)$).*

Furthermore, the supply network would be able to sustain other versions of the above no-shortage equilibrium, in which the total demand for certified metal increases beyond $\widehat{D}$. The basic intuition behind this statement is that, since the prices of the two metal types are the same in the no-shortage equilibrium, a subset of noncompliant manufacturers would be willing to shift their purchases to certified metal. In fact, as the following corollary states, given any quantity $Q$ between $\widehat{D}$ and $\widetilde{Q}(\widehat{D})$ (which is a threshold below $\overline{Q}$), there exists an equilibrium in which the demand for the certified metal is $Q$.

**Corollary 2.1.** *Suppose the total demand of compliance-prone manufacturers is less than the threshold $\overline{Q}$, that is, $\widehat{D} \in (0, \overline{Q}]$. There exists another threshold $\widetilde{Q}(\widehat{D}) \in (\widehat{D}, \overline{Q}]$ such that, for any strictly positive quantity $Q \in [\widehat{D}, \widetilde{Q}(\widehat{D})]$, there exists an equilibrium where*

*(a) All compliance-prone manufacturers are compliant; all other manufacturers remain noncompliant, but they demand some certified metal, that is, $D_{\mathrm{CM}} = \widehat{D} \leqslant D_C$;*

*(b) The total certified demand and total certified capacity both equal $Q$, that is, $D_C = K_C = Q$;*

*(c) The prices of certified and noncertified metal are the same (i.e., $p_C(D_C, K_C) = p_N(D_N, K_N)$).*

An important implication is that one can obtain more-than-commensurate benefits from having a subset of compliance-prone manufacturers. Once a group of manufacturers become compliance-prone, they will create a market for certified metal, which will be utilized by noncompliant manufacturers as well. In equilibrium, the quantity of certified metal that exchanges hands is capped by a threshold ($\widetilde{Q}(\widehat{D})$), which is no less than the compliance-prone demand ($\widehat{D}$). Therefore, policymakers and NGOs, by focusing their efforts on a subset of manufacturers, may be able to get the ball rolling in the creation of a market for certified metal. In fact, this is in keeping with the current state of affairs. The efforts of NGOs such as the Enough Project led a group of well-known electronics manufacturers to take the initiative in sourcing "conflict-free" metal, which led to the emergence of certified smelters. Now that there are such certified smelters, they could very well be utilized even by manufacturers who have no incentive to be compliant (e.g., manufacturers that are not subject to the Dodd-Frank Act or that have not been on the radar of the Enough Project).

**The Equilibrium with Shortage.** Consider now the case where the compliance-prone demand, $\widehat{D}$, exceeds the threshold $\overline{Q}$. In this case, according to Theorem 2.1, there will be a shortage of certified metal. Once again, to explain the first-order effect that leads to a shortage, imagine the firms started in two completely detached channels, one for certified metal and the other for noncertified metal. In this hypothetical scenario, all compliance-prone manufacturers are compliant and buy from certified smelters, who then buy from the verified pool; all remaining manufacturers are noncompliant and buy only from noncertified smelters, who then buy from the unverified pool. Note that, in such a hypothetical scenario, there is no shortage of certified metal. However, in such a scenario, a total quantity of $\widehat{D}$, which now exceeds $\overline{Q}$, would be flowing through the certified channel to the verified pool. Thus, the verified ore would now be more expensive. Consequently, some certified smelters would prefer to give up their certified designation, so that they can shift their purchases to the unverified pool. This would create a shortage of certified metal, which would lead to a higher price, thus inducing some compliance-prone manufacturers to choose noncompliance (specifically, those for whom the noncompliance penalty is not too far above the compliance cost); others with sufficiently high penalty will continue to ensure compliance.

In conclusion, starting with the hypothetical scenario of two detached channels, the network would settle in an equilibrium, in which only a subset of compliance-prone manufacturers choose to comply, and even those who comply will not find sufficient certified metal to meet all their demand. This observation is formalized in the next theorem, which describes the properties of equilibria with a shortage of certified metal.

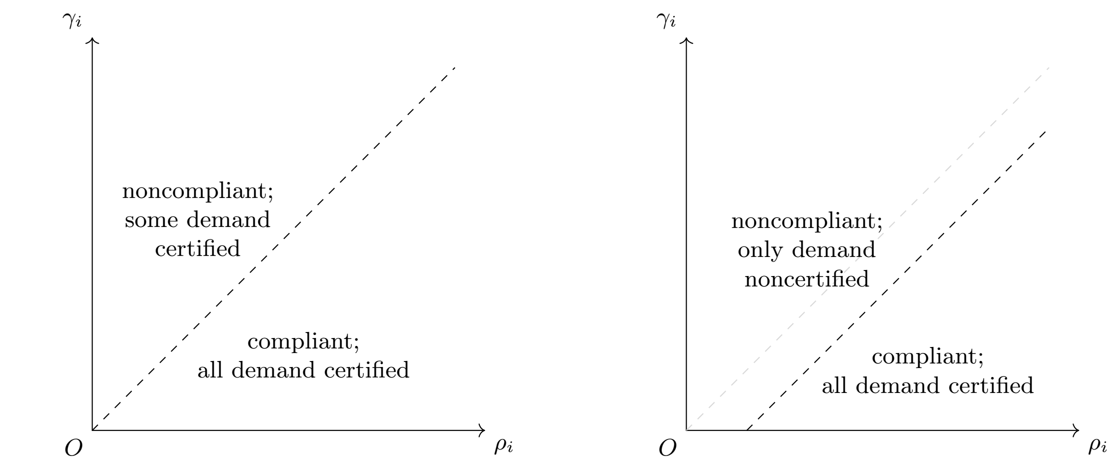
**Theorem 2.3.** *If the total demand of compliance-prone manufacturers, $\widehat{D}$, is higher than the threshold $\overline{Q}$, then there exists an equilibrium with a shortage of certified metal. In any equilibrium with shortage,*

*(a) The price of certified metal is higher than that of noncertified metal, that is, $p_C(D_C, K_C) > p_N(D_N, K_N)$;*

*(b) The total certified demand exceeds the threshold $\overline{Q}$, but does not exceed the compliance-prone demand, that is, $D_C \in (\overline{Q}, \widehat{D}]$;*

*(c) The total certified capacity exceeds the threshold $\overline{Q}$, but is less than the total certified demand, that is, $K_C \in (\overline{Q}, D_C)$; and*

*(d) Some compliance-prone manufacturers are compliant; all other manufacturers, possibly including some compliance-prone manufacturers, remain noncompliant, and they demand only noncertified metal, that is, $D_C = D_{\mathrm{CM}} \leqslant \widehat{D}$.*




(a) Manufacturer decisions when there is no shortage of certified metal (when $\widehat{D} \leqslant \overline{Q}$)

(b) Manufacturer decisions when there is a shortage of certified metal (when $\widehat{D} > \overline{Q}$)

Figure 2.1: Manufacturers' compliance and sourcing decisions

Figure 2.1 provides a summary comparison of compliance decisions in Theorems 2.2 and 2.3.[2] Recall that a manufacturer is compliance-prone if its compliance cost, $\gamma_i$, is less than its penalty cost, $\rho_i$. Panel $(a)$ is the case with no shortage of certified metal (Theorem 2.2 and Corollary 2.1); all compliance-prone manufacturers choose to be compliant and all other manufacturers choose to remain noncompliant, but they still buy some certified metal. In contrast, Panel $(b)$ is the case with a shortage of certified metal (Theorem 2.3); here some compliance-prone manufacturers choose to be noncompliant. As the figure shows, these are the manufacturers whose penalty cost, while larger than the compliance cost, is not steep enough to justify the purchase of certified metal, which is more expensive because of the shortage.

Figure 2.2 puts together Theorem 2.2, Corollary 2.1 and Theorem 2.3 to show the quantity of certified metal traded in equilibrium, $Q_C$, as a function of compliance-prone demand, $\widehat{D}$. In the region where the compliance-prone demand $\widehat{D}$ is below the threshold $\overline{Q}$ (Theorem 2.2), any quantity between $\widehat{D}$ and $\widetilde{Q}(\widehat{D})$ (shown as the shaded area) can be supported as the quantity of certified metal $Q_C$ in equilibrium. Therefore, in this region, the supply network uses certified metal in quantities that exceed the compliance-prone demand. In contrast, once the compliance-prone demand exceeds the threshold (Theorem 2.3), the certified metal is in shortage and the quantity of certified metal in

[2]For clarity of illustration, we assume for this figure that all manufacturers have the same demand, that is, $D_i$ is the same for all $i$. Otherwise, in addition to $\gamma_i$ and $\rho_i$, we would need to add a third axis for $D_i$ to characterize manufacturer $i$.

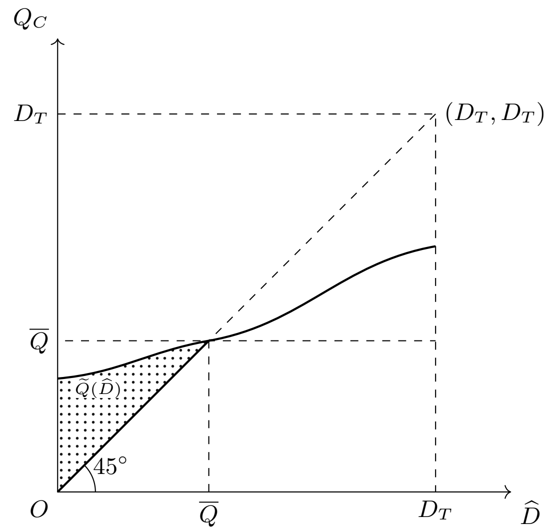



Figure 2.2: How the equilibrium $Q_C$ changes as a function of the compliance-prone demand, $\widehat{D}$

the network falls below the compliance-prone demand. We make the following observation from the figure: Suppose that the compliance-prone demand, $\widehat{D}$, is less than the threshold $\overline{Q}$, putting us in the region where there is no shortage of certified metal. If the compliance-prone demand increased by an amount $\delta$ so that it is now higher than the threshold $\overline{Q}$, putting us in the region where there is a shortage of certified metal, the increase in the quantity of certified metal traded will be strictly less than $\delta$.

The preceding observation has an important policy implication. Policy-makers or NGOs might have enough leverage to increase noncompliance penalties to manufacturers, and they could use this leverage to make more manufacturers compliance-prone. However, the above arguments imply that there may be a limit to the usefulness of such efforts. Starting with a status quo in which a small enough portion of the total demand is coming from compliance-prone manufacturers, if one levies additional penalties so that the compliance-prone demand becomes larger, there will be an incommensurate increase in the amount of certified metal traded as there will not be enough smelters to meet all the new demand for certified metal.

## 2.5 Whom to Target for Penalty?

NGOs such as the Enough Project, in their efforts to publicize the issue of conflict minerals, have targeted a particular subset of manufacturers using 3TG minerals. Specifically, the Enough Project focused its initial efforts on electronics manufacturers with wide brand recognition (e.g., Intel, Apple, Microsoft, Nintendo). These are certainly not the only, or even the largest, users of all 3TG minerals. For example, tungsten is used in large quantities by industrial tool manufacturers, who are not household names (Vital Metals 2016, International Tungsten Industry Association 2016). In short, by focusing their publicity efforts on a subset of manufacturers, NGOs and policy makers are ultimately choosing whom to target for penalties associated with bad publicity. When making such choices, one can speculate that there are two dimensions to consider. First, the better known a manufacturer's brand, the more vulnerable it is to bad publicity, so it might be easier to change the behavior of manufacturers with well-known brands. Second, the higher a manufacturer's demand for these minerals, the more impact one can have by changing the manufacturer's behavior. In this section, we adopt the perspective of an NGO who wishes to improve the amount of verified ore and certified metal in the supply chain, and we analyze which manufacturers the NGO should target.

In modeling this decision, we assume that each manufacturer has an exogenously fixed penalty $\rho_i$, but the penalty is active only if the NGO "toggles" it on by targeting that manufacturer. The assumption that the NGO can choose to turn on the penalty, but not the size of the penalty, is a reflection of the penalty being tied to the manufacturers' characteristics such as brand recognition (cell phone makers are more exposed than industrial tool makers), revenues (the manufacturers with high revenues have more to lose), and reputation (a manufacturer is more vulnerable if its brands appeal to consumers with higher awareness of issues).

Let $\tau_i$ be a binary variable that equals 0 when the penalty for manufacturer $i$ is off, and 1 when on. We then replace $\rho_i$ in the noncompliant manufacturer's cost, given by (2.9), with $\rho_i \tau_i$. As in Section 2.4, we refer to a manufacturer as compliance-prone if its exogenous penalty $\rho_i$ is greater than its compliance cost, $\gamma_i$. However, we now note that this penalty matters only when it is on, that is, when $\tau_i = 1$. We redefine $\widehat{D} := \int_{\mathcal{M}} D_i \mathbb{1}_{\{\rho_i \tau_i > \gamma_i\}} \, \mathrm{d}i$, so that it is the total demand of compliance-prone manufacturers whose penalties are on. Note that, unlike the original definition of $\widehat{D}$, the revised definition is no longer simply the total demand of all compliance-prone manufacturers, but the total demand of only those compliance-prone manufacturers that are "targeted."

Let $T$ and $T'$ be the sets of manufacturers whose penalties are on and off, respectively, and assume these sets are exogenously given in the status quo. Given the equilibrium in the status quo, let $V \subseteq T'$ be the set of manufacturers such that turning on the penalty of any manufacturer $i \in V$

will induce that manufacturer to become compliant. These manufacturers are "vulnerable" to being targeted by the NGO in the sense that they will be compliant if the NGO decides to turn on their penalties. The next theorem characterizes the set $V$:

**Theorem 2.4.** *Given the sets of manufacturers whose penalties are currently on ($T$) and off ($T'$), let $\widehat{D}$ be the total demand of compliance-prone manufacturers whose penalties are currently on.*

*(a) If $\widehat{D} \leqslant \overline{Q}$, then $V$ is the set of all compliance-prone manufacturers whose penalties are currently off, that is, $V = \{i : i \in T' \text{ and } \rho_i > \gamma_i\}$.*

*(b) If $\widehat{D} > \overline{Q}$, then $V$ is a subset of compliance-prone manufacturers whose penalties are currently off: $V = \left\{i : i \in T' \text{ and } \frac{\rho_i - \gamma_i}{D_i} > \xi\right\}$ where $\xi > 0$ depends on the equilibrium.*

According to Theorem 2.4a, when $\widehat{D}$ is sufficiently low, any compliance-prone manufacturer will become compliant once targeted by the NGO. In contrast, part b implies that, once $\widehat{D}$ becomes sufficiently large, only certain compliance-prone manufacturers could be induced to actually be compliant when targeted. Specifically, once $\widehat{D}$ is beyond the threshold $\overline{Q}$, a compliance-prone manufacturer is vulnerable to targeting only if its net penalty per unit (i.e., the portion of the penalty in excess of compliance cost, spread over the manufacturer's entire demand) is sufficiently large. Therefore, what makes a manufacturer vulnerable is not simply its penalty or its demand, but essentially the ratio of the two.

Parts a and b together have the following implication: Once a critical mass of manufacturers have become compliant (precisely, a group of manufacturers whose demands add up to $\overline{Q}$ or more), it becomes more difficult for the NGO to convert additional manufacturers: The set $V$, which is the set of manufacturers vulnerable to targeting, becomes smaller. Furthermore, there is another type of diminishing returns associated with targeting additional manufacturers beyond a critical mass: Once the total demand of compliant manufacturers exceeds the threshold $\overline{Q}$, we know from our earlier results that there will be a shortage of certified metal. Hence, if the NGO targets an additional manufacturer to become compliant after that stage, only some of the manufacturer's demand will be met by certified metal, with the rest of its demand still met by uncertified metal.

In the spirit of this result, when the Enough Project updated its conflict mineral rankings in 2017, it included jewelry retail companies for the first time, in addition to consumer electronics manufacturers that have been the sole subject of previous rankings (Callaway 2017). This may be seen as the NGO investing its efforts in a mineral for which it has not progressed too far up the curve of auditing and tracing: While significant progress has been made in tantalum, tin, and tungsten (with 98%, 79% and 79%, respectively, of eligible smelters listed as conformant by the Responsible Minerals Initiative), the progress has been much slower in gold (with 66% of eligible smelters listed as conformant) (Responsible Minerals Initiative 2019b).

### 2.6 Conclusion

The Democratic Republic of the Congo (DRC) is an important source of 3TG minerals (tantalum, tin, tungsten and gold), and the trade of these minerals helped to fund armed groups involved in ongoing hostilities in the DRC. In an effort to curb the flow of funds to such armed groups, there have been efforts by legislative bodies (e.g., Section 1502 of the Dodd-Frank Act in the U.S.) and NGOs (e.g., the Enough Project). These efforts put pressure on manufacturers so that they will trace their sources for 3TG minerals and avoid buying from questionable sources. In the supply chain for minerals, the critical link between a manufacturer (who uses the metal) and a mine (who supplies the ore) is the smelter, who converts the ore into the metal. In this paper, we study how legislative efforts and NGOs may influence sourcing relationships among multiple manufacturers and smelters in a supply network.

In our modeling framework, we aggregate mines into two pools, verified mines (known to be conflict-free) and unverified mines (not known to be so). Each smelter decides whether to be certified (committing to purchase only from the pool of verified mines), or noncertified (free to buy from both pools of mines). Each manufacturer decides whether to be compliant (committing to purchasing from certified smelters to the extent possible) or noncompliant (free to buy from both certified and noncertified smelters). Thus, we essentially model two parallel markets for metal, one for certified metal and the other for noncertified. The two markets interact because the supply and demand in each market are tied to the prices in the other market.

We find that an important predictor of outcomes is the "compliance-prone demand" in the network. The compliance-prone demand is the total demand of the manufacturers who would prefer to be compliant if the prices of certified and noncertified metals were the same. These are the manufacturers whose noncompliance penalty exceeds their compliance cost. If the compliance-prone demand is below a threshold, then there will be enough certified metal to go around. However, once the compliance-prone demand exceeds the threshold, there will arise a shortage of certified metal.

Even when the compliance-prone demand is low, it can still create a market for certified metal, which even noncompliant manufacturers take advantage of, by spreading their purchases between certified and noncertified metals until the prices of the two metal types meet. However, once the compliance-prone demand is high, there will be a shortage of certified metal, and even some compliant manufacturers will have to buy noncertified metal to make up for the shortage. In this equilibrium, certified metal will remain more expensive than noncertified metal.

Equipped with the equilibrium results above, we ask whether increased efforts by policy-makers (NGOs and legislative bodies) can induce a conflict-free supply network. Policy-makers can influence the "compliance-prone demand" in the network, because they are in a position to impose larger penalties for noncompliant manufacturers. A consequence of the results discussed above is that even if policy-makers imposed increasingly high noncompliance penalties on a broader group of manufacturers, such penalties might not be sufficient. Once the compliance-prone demand reaches a threshold, further increases in compliance-prone demand will be countered with less than commensurate increases in certified metal capacity, thus leading to shortages. In short, imposing higher penalties on more manufacturers might not be enough to achieve a conflict-free network.

We then ask which additional manufacturers an NGO should target in its efforts to curb the usage of conflict minerals. When the quantity of certified metal in the network is low, then the NGO can target any manufacturer vulnerable to bad publicity, and this will compel the manufacturer to become compliant. However, once the certified metal volume reaches a critical mass and the certified metal is in shortage, then the NGO would have to choose additional targets judiciously. Because of the shortage, certified metal will be more expensive, and it will be harder to convert the manufacturers to compliance. In this case, the NGO should target the manufacturers whose noncompliance penalty, spread over the volume of metal they need, is high enough to justify paying the premium for certified metal.

## Appendix 2.A Technical Assumptions and Preliminary Lemmas

We first state and prove preliminary lemmas that will be used throughout the appendix. Here we omit measurability and integrability assumptions and "almost everywhere" qualifiers in most of our statements.

**Lemma 2.1.** *In a balanced market, the linear unit price functions $p_s(D_s, K_s) = aD_s - bK_s + p_0$ for $S \in \{C, N\}$ must have $a = b$.*

*Proof.* Pick any $x \in (0, D_T)$ and any $\epsilon \in (0, x \wedge D_T - x)$. Let $D_s = x + \epsilon$ and $K_s = x$. In a balanced market, $D_{-s} = D_T - x - \epsilon$ and $K_{-s} = D_T - x$ (by Assumption 2.1a). Since metal type $s$ is in shortage, Assumption 2.2a yields $p_s(x + \epsilon, x) > p_{-s}(D_T - x - \epsilon, D_T - x)$. By Assumption 2.1b, this is equivalent to $a(x + \epsilon) - bx + p_0 > a(D_T - x - \epsilon) - b(D_T - x) + p_0$, which reduces to $b(D_T - 2x) > a(D_T - 2x - 2\epsilon)$.

Now, let $D_s = x - \epsilon$ and $K_s = x$. Therefore, $D_{-s} = D_T - x + \epsilon$ and $K_{-s} = D_T - x$. Since metal type $-s$ is in shortage, Assumption 2.2a now yields $p_s(x - \epsilon, x) < p_{-s}(D_T - x + \epsilon, D_T - x)$. By Assumption 2.1b, this is equivalent to $a(x - \epsilon) - bx + p_0 < a(D_T - x + \epsilon) - b(D_T - x) + p_0$, which reduces to $b(D_T - 2x) < a(D_T - 2x + 2\epsilon)$.

Thus, we obtain $a(D_T - 2x - 2\epsilon) < b(D_T - 2x) < a(D_T - 2x + 2\epsilon)$, which must hold for any arbitrarily small $\epsilon > 0$. Therefore, we must have $a(D_T - 2x) = b(D_T - 2x)$ and, hence, $a = b$. □

**Lemma 2.2.** *Let $h : \mathbb{R} \to \mathbb{R}$ be defined as $h(y) = \int_{\mathcal{M}} D_i \mathbb{1}_{\{\mu_i > y\}} \,\mathrm{d}i$, where $\mu_i = \frac{\rho_i - \gamma_i}{D_i}$. Then, $h(y)$ is continuous.*

*Proof.* Since the joint distribution of $(D_i, \gamma_i, \rho_i)$ is continuous, the distribution of $\mu_i$ is continuous. For any $y \in \mathbb{R}$, $|\mathbb{1}_{\{\mu_i > y_n\}} - \mathbb{1}_{\{\mu_i > y\}}| \to 0$ a.e. for any $y_n \to y$. Then $|h(y_n) - h(y)| = \left|\int_{\mathcal{M}} D_i \mathbb{1}_{\{\mu_i > y_n\}} \,\mathrm{d}i - \int_{\mathcal{M}} D_i \mathbb{1}_{\{\mu_i > y\}} \,\mathrm{d}i\right| \leqslant \int_{\mathcal{M}} D_i |\mathbb{1}_{\{\mu_i > y_n\}} - \mathbb{1}_{\{\mu_i > y\}}| \,\mathrm{d}i \to 0$ by the integrability of $D_i$ and invoking the Lebesgue dominated convergence theorem. □

**Definition 2.1.**

(a) Define function $R : [\overline{Q}, D_T] \times [\overline{Q}, K_T] \to \mathbb{R}$ by

$$R(x, y) := 2a(x - y) - (o_V(y, x) - o_U(D_T - y, D_T - x)). \tag{2.11}$$

(b) Define $\kappa : [\overline{Q}, D_T] \to \mathbb{R}$ to be the implicit function $y = \kappa(x)$ that solves $R(x, y) = 0$, such that $\kappa(x) \in [\overline{Q}, x], \forall x \in [\overline{Q}, D_T]$.

**Lemma 2.3.** *The implicit function $\kappa$ is well defined and has the following properties: $(i)$ $\kappa(x)$ is monotonically increasing; $(ii)$ $\kappa(x)$ is continuous; $(iii)$ $\kappa(x) \in (\overline{Q}, x)$ for $x \in (\overline{Q}, D_T]$, and $\kappa(\overline{Q}) = \overline{Q}$.*

*Proof.* We first show that $\kappa$ is well-defined. Given $x \in [\overline{Q}, D_T]$, first observe that $R(x, \overline{Q}) = 2a(x - \overline{Q}) - (o_V(\overline{Q}, x) - o_U(D_T - \overline{Q}, D_T - x)) \geqslant 2a(x - \overline{Q}) - (o_V(\overline{Q}, \overline{Q}) - o_U(D_T - \overline{Q}, D_T - \overline{Q}))$, where the inequality follows from $x \geqslant \overline{Q}$ and Assumption 2.3. Therefore, it now follows from the definition of $\overline{Q}$ in Assumption 2.4 that $R(x, \overline{Q}) \geqslant 2a(x - \overline{Q}) \geqslant 0$. Second, observe that $R(x, x) = 0 - (o_V(x, x) - o_U(D_T - x, D_T - x)) \leqslant 0$ because $o_V(x, x) \geqslant o_U(D_T - x, D_T - x)$ for $x \in [\overline{Q}, D_T]$ due to Assumption 2.4. By virtue of these two observations, the continuity of $R$, and the intermediate value theorem, there exists $y \in [\overline{Q}, x]$ s.t. $R(x, y) = 0$. Moreover, by the strict monotonicity of $o_V$ and $o_U$, $R(x, y)$ is strictly decreasing in $y$; so there exists a unique $y \in [\overline{Q}, x]$ s.t. $R(x, y) = 0$.

Next, we show that $\kappa(x)$ satisfies properties $(i)$ through $(iii)$:

$(i)$ Because $R(x, y)$ is increasing in $x$ and decreasing in $y$, it follows that $\kappa(x)$ is increasing in $x$.

$(ii)$ Extend $R$ to $\overline{R} : \mathbb{R}^2 \to \mathbb{R}$ by extending $o_V$ and $o_U$ monotonically outside of $[0, D_T]^2$ in such a way that they remain continuous throughout, and are strictly increasing in their respective first argument, and strictly decreasing in their respective second argument, and given any $x$, $\lim_{y \to \pm\infty} \overline{R}(x, y) = \mp\infty$. Now, $\overline{R}$ is continuous. Given any $x$, $\overline{R}(x, y)$ is strictly decreasing in $y$. Consequently, (a) $\overline{R}(x, \cdot)$ is one-to-one, hence, locally one-to-one; (b) by the intermediate value theorem, there exists a unique $y$ s.t. $\overline{R}(x, y) = 0$. We write this unique $y$ as $y = Hx$. By Theorem 1.1 in Kumagai (1980), $H$ is continuous. Because $\kappa$ coincides with $H$ in the relevant domain, we conclude that $\kappa$ is also continuous.

$(iii)$ We first note that $R(\overline{Q}, \overline{Q}) = 0$, so $\kappa(\overline{Q}) = \overline{Q}$. Now, consider $x > \overline{Q}$. In that case, we have $R(x, \overline{Q}) > 0$ and $R(x, x) < 0$ by (2.11) and Assumptions 2.3 and 2.4. Because $R(x, y)$ is continuous and strictly decreasing in $y$, it follows that $R(x, y) = 0$ for some $y \in (\overline{Q}, x)$. In other words, $\kappa(x) \in (\overline{Q}, x)$. □

## Appendix 2.B Proof of Preliminary Equilibrium Results

We first state the following result, which implies only certified metal could be in shortage in equilibrium.

**Lemma 2.4.** *In equilibrium, noncertified metal is never in shortage, that is, $D_N \leqslant K_N$.*

*Proof.* By way of contradiction, suppose noncertified metal is in shortage, that is, $D_N > K_N$, in equilibrium. Then $D_N > 0$. By (2.7), we have

$$p_N^e(D_N, K_N) = p_N(D_N, K_N)\frac{K_N}{D_N} + p_C(D_C, K_C)\left(1 - \frac{K_N}{D_N}\right). \tag{2.12}$$

Note that since noncertified metal is in shortage, certified metal cannot be in shortage, that is, we have $K_C > D_C$. Therefore, given certified metal has sufficient capacity, its effective price $p_C^e(D_C, K_C)$, is simply $p_C(D_C, K_C)$. In the remainder, we will show a contradiction arises by showing that $D_N = 0$.

By Assumption 2.2a, $p_N(D_N, K_N) > p_C(D_C, K_C)$. If $K_N > 0$, it follows from (2.12) that $p_N^e(D_N, K_N)$ is greater than $p_C(D_C, K_C)$, which is equal to $p_C^e(D_C, K_C)$. Therefore, no manufacturer would demand noncertified metal, and we would have $D_N = 0$, thus yielding a contradiction. If $K_N = 0$, by Assumption 2.5, all manufacturers demand only certified metal. Thus, once again, we would have $D_N = 0$, yielding a contradiction. Hence in equilibrium, $D_N \leqslant K_N$. □

We next prove a result that describes all possible cases that may arise in equilibrium. Subsequent equilibrium results will be derived by studying these cases in detail.

**Theorem 2.5.** *Any equilibrium falls into one of the following two cases:*

*(a) Certified metal is in shortage: In this case, certified metal is more expensive than noncertified metal and only compliant manufacturers demand certified metal, that is, $p_C(D_C, K_C) > p_N(D_N, K_N)$, $D_{\mathrm{CM}} = D_C > K_C$, and $D_{\mathrm{NM}} = D_N < K_N$.*

*(b) Certified metal is not in shortage: In this case, certified metal has the same effective price as noncertified metal and noncompliant manufacturers also demand some certified metal, that is, $p_C^e(D_C, K_C) = p_N^e(D_N, K_N)$, $D_{\mathrm{CM}} \leqslant D_C = K_C$, $D_{\mathrm{NM}} \geqslant D_N = K_N$.*

*Proof.* We divide all possible equilibria into three mutually exclusive and collectively exhaustive cases, and we show that one case cannot arise in equilibrium and the two remaining cases correspond to cases a and b.

- *Case A: Equilibria in which $D_C > K_C$.* In this case, given that the certified metal is in shortage, it follows from Assumption 2.2a that $p_C(D_C, K_C) > p_N(D_N, K_N)$. If $K_C = 0$, then there is no capacity for certified metal and, by Assumption 2.5, all noncompliant manufacturers would demand only noncertified metal, that is, $D_N = D_{\text{NM}}$. This yields a special case of the equilibrium in part a with $K_C = 0$. In the remainder, we consider the case with $K_C > 0$. By (2.7),

$$p_C^e(D_C, K_C) = p_C(D_C, K_C)\frac{K_C}{D_C} + p_N(D_N, K_N)\left(1 - \frac{K_C}{D_C}\right) \tag{2.13}$$

Given $K_C > 0$, it follows from (2.13) that $p_C^e(D_C, K_C) > p_N(D_N, K_N)$. Furthermore, because $K_N > D_N$, $p_N^e(D_N, K_N) = p_N(D_N, K_N)$. Therefore, we have $p_C^e(D_C, K_C) > p_N^e(D_N, K_N)$. Consequently, all noncompliant manufacturers will demand only noncertified metal, that is, $D_N = D_{\text{NM}}$. This also implies that the demand for certified metal is only the demand of compliant manufacturers, that is, $D_C = D_{\text{CM}}$. These equilibria correspond to case a.

- *Case B: Equilibria in which $D_C < K_C$:* This case cannot occur in equilibrium. Suppose that $D_C < K_C$. Under Assumption 2.1a, $D_C < K_C$ implies $D_N > K_N$, contradicting Lemma 2.4.

- *Case C: Equilibria in which $D_C = K_C$:* In this case, we also have $D_N = K_N$ by the balanced market assumption (Assumption 2.1a). We consider three cases: $0 < K_C < K_T$, $K_C = 0$, $K_C = K_T$. In the first case, with $0 < K_C < K_T$, we observe from the price function $p_s(D_s, K_s) = aD_s - bK_s + p_0$ for $s \in \{C, N\}$, that $p_C(D_C, K_C) = p_N(D_N, K_N)$ (because we have $a = b$ by Lemma 2.1 and $D_s = K_s$ for $s \in \{C, N\}$). Hence, it is also true that $p_C^e(D_C, K_C) = p_N^e(D_N, K_N)$. In the second case, with $K_C = 0$, we note that we must have $K_N = K_T$. In addition, we have $p_C^e(0, 0) = p_N(D_T, K_T)$ (see footnote 1 — with zero demand and capacity for the certified metal, the effective price of the certified metal is the price of the noncertified metal) and $p_N^e(D_T, K_T) = p_N(D_T, K_T)$. Thus, $p_C^e(D_C, K_C) = p_N^e(D_N, K_N)$ holds in the case with $K_C = 0$. In the third case, with $K_C = K_T$, we note that we must have $K_N = 0$. Similar to the previous case, $p_N^e(0, 0) = p_C(D_T, K_T)$ (with zero demand and capacity for the noncertified metal, the effective price of the noncertified metal is the price of the certified metal) and $p^e_C(D_T, K_T) = p_C(D_T, K_T)$. Thus, $p^e_C(D_C, K_C) = p^e_N(D_N, K_N)$ holds in the case as well. Finally, because any compliant manufacturer always demands certified metal only and a noncompliant manufacturer may also demand certified metal, we have $D_C \geqslant D_{\text{CM}}$ and $D_N \leqslant D_{\text{NM}}$. Such equilibria correspond to case b. □

## Appendix 2.C Proofs of Results in Section 2.4

In this section, we state and prove Proposition 2.2. Subsequently, we show that Proposition 2.1, Theorem 2.1, Theorem 2.2, Corollary 2.1, and Theorem 2.3 in Section 2.4 all follow from Proposition 2.2. For any given $D \in [0, D_T]$, define $\widetilde{Q}(D)$ as the value of $Q$ at which $o_V(Q, D) = o_U(D_T - Q, D_T - D)$.[3]

**Lemma 2.5.** *If $\widehat{D} > \overline{Q}$, then there exists $x \in (\overline{Q}, D_T]$ such that*

$$x = \int_{\mathcal{M}} D_i \mathbb{1}_{\left\{\mu_i > \frac{2a\kappa(x)}{x}(x - \kappa(x))\right\}} \, \mathrm{d}i. \tag{2.14}$$

*Proof.* Define $F : [\overline{Q}, D_T] \to \mathbb{R}$ by $F(x) := x - \int_{\mathcal{M}} D_i \mathbb{1}_{\left\{\mu_i > \frac{2a\kappa(x)}{x}(x-\kappa(x))\right\}} \, \mathrm{d}i$. It suffices to show that $F$ has a zero in $(\overline{Q}, \widehat{D}]$. Since both $\kappa$ (Definition 2.1) and $h$ (defined in Lemma 2.2) are continuous, $F$ is continuous. Observe $F(\overline{Q}) = \overline{Q} - \int_{\mathcal{M}} D_i \mathbb{1}_{\{\mu_i > 0\}} \, \mathrm{d}i = \overline{Q} - \widehat{D} < 0$ (where the first equality holds because, by Lemma 2.3, $\kappa(\overline{Q}) = \overline{Q}$; the second equality follows from the definition of $\widehat{D}$; the last inequality holds because $\widehat{D} > \overline{Q}$ by the premise of the lemma). In addition, observe that $F(\widehat{D}) = \widehat{D} - \int_{\mathcal{M}} D_i \mathbb{1}_{\left\{\mu_i > \frac{2a\kappa(\widehat{D})}{\widehat{D}}(\widehat{D} - \kappa(\widehat{D}))\right\}} \, \mathrm{d}i \geqslant \widehat{D} - \widehat{D} = 0$ (where the inequality holds because, by Lemma 2.3, $0 \leqslant \overline{Q} < \kappa(x) < x$ for $x \in (\overline{Q}, D_T]$). Hence, by the intermediate value theorem, there exists $x \in (\overline{Q}, \widehat{D}]$ s.t. $F(x) = 0$. □

**Proposition 2.2.** *The following lists all possible equilibria and the respective conditions (on model parameters) for the existence of each of them:*

*(1) There exists an equilibrium of the type in Theorem 2.5b, in which $K_C = D_C = 0$, if and only if $\widehat{D} = 0$.*

*(2) There exists an equilibrium of the type in Theorem 2.5b, in which $K_C = K_T$, $D_C = D_T$, and $D_{\text{CM}} = \widehat{D}$, if and only if $p_0 - (o_V(D_T, \widehat{D}) - o_U(0, D_T - \widehat{D})) \geqslant 0$.*

[3] For any given $D \in [0, D_T]$, the existence and uniqueness of $\widetilde{Q}(D)$ follows from Assumptions 2.3 and 2.4.

*(3) There exists an equilibrium of the type in Theorem 2.5b, in which* $p_C(D_C, K_C) = p_N(D_N, K_N)$, $0 < D_C = K_C = Q < D_T$, *and the compliant manufacturers are precisely those manufacturers who are compliance-prone, that is,* $D_{\text{CM}} = \widehat{D}$, *if and only if* $Q \in [\widehat{D}, \widetilde{Q}(\widehat{D})]$ *and* $Q > 0$.

*(4) There exists an equilibrium of the type in Theorem 2.5a, in which* $p_C(D_C, K_C) > p_N(D_N, K_N)$, $\overline{Q} < K_C < D_C = D_{\text{CM}} \leqslant \widehat{D}$, *and only some compliance-prone manufacturers are compliant, if and only if* $\widehat{D} > \overline{Q}$.

*Proof.* To exhaust all possible types of equilibrium that might exist, we analyze both types of equilibrium listed in Theorem 2.5 to show if and when such an equilibrium exists. In the remainder of the proof, we will utilize the condition for smelter $j$ to be certified. Smelter $j$ is certified if and only if $\pi_{\text{cs}}(j) \geqslant \pi_{\text{ns}}(j)$.[4] This condition is equivalent to

$$(p_C(D_C, K_C) - o_V(Q_V, D_{\text{CM}}) - c_j)K_j \geqslant [p_N(D_N, K_N) - (o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}})) - c_j]K_j. \tag{2.15}$$

Under Assumption 2.1, recall that $p_s(D_s, K_s) = aD_s - bK_s + p_0$ (except when $D_s = K_s = 0$, in which case $p_s(D_s, K_s) = 0$) and furthermore, by Lemma 2.1, $a = b$. Therefore, except when $D_s = K_s = 0$ for $s = C$ or $s = N$, the above expression simplifies as follows:

$$a(D_C - K_C) + p_0 - o_V(Q_V, D_{\text{CM}}) - c_j \geqslant a(D_N - K_N) + p_0 - (o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}})) - c_j. \tag{2.16}$$

In what follows, we take up each possible type of equilibrium listed in Theorem 2.5. For each type of equilibrium, (a) we derive additional properties of this type of equilibrium, (b) supposing such an equilibrium exists, we identify the conditions (on model parameters) that must be satisfied, and (c) we show that if these conditions are satisfied, then this equilibrium exists.

- *Case b of Theorem 2.5.* Since the effective prices of certified and noncertified metals are equal in this type of equilibrium, the manufacturers' problems will be solved when all manufacturers with $\rho_i > \gamma_i$ are compliant and all manufacturers with $\rho_i < \gamma_i$ are noncompliant, that is, manufacturer $i$ is compliant if and only if it is compliance-prone. Hence, in this type of equilibrium, the total

[4]More precisely, smelter $j$ gets certified if $\pi_{\text{cs}}(j) > \pi_{\text{ns}}(j)$; smelter $j$ does not get certified if $\pi_{\text{cs}}(j) < \pi_{\text{ns}}(j)$; smelter $j$ is indifferent between certification or otherwise if $\pi_{\text{cs}}(j) = \pi_{\text{ns}}(j)$.

demand of compliant manufacturers will be the same as the total demand of compliance-prone manufacturers, that is, $D_{\text{CM}} = \widehat{D}$. We continue our analysis by examining three mutually exclusive and collectively exhaustive subcases, depending on where $K_C$ lies in the range $[0, K_T]$:

– *Subcase a:* $K_C = 0$. In this subcase, all smelters are noncertified. First, in addition to $D_{\text{CM}} = \widehat{D}$ as argued above, we observe the following properties of the type of equilibrium in this subcase. (i) We must have $D_C = 0$ (because $D_C \leqslant K_C$ according to Theorem 2.5b). (ii) Given $D_C = 0$, we must also have $\widehat{D} = D_{\text{CM}} = 0$ (because $D_{\text{CM}} \leqslant D_C$). (iii) Given $D_C = K_C = 0$, we must have $K_N = D_N = D_T$.

From property (ii), we get the condition: $\widehat{D} = 0$. In addition, the condition $\pi_{\text{cs}}(j) < \pi_{\text{ns}}(j)$ must hold for all smelters (i.e., all smelters prefer to be noncertified), which can be written as

$$(p_C(D_C, K_C) - o_V(Q_V, D_{\text{CM}}) - c_j)K_j < [p_N(D_N, K_N) - (o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}})) - c_j]K_j.$$

When $D_C = K_C = 0$, we have $p_C(D_C, K_C) = 0$ and $p_N(D_N, K_N) = p_0$ (because $K_N = D_N = D_T$), so the above condition simplifies to $(-o_V(Q_V, D_{\text{CM}}) - c_j)K_j < [p_0 - (o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}})) - c_j]K_j$ for all $j \in S$. Using property (ii), the condition can be further simplified as $-o_V(Q_V, 0) < p_0 - (o_V(Q_V, 0) \wedge o_U(Q_U, D_T))$. Observe from the smelter's problem in (2.4) that, in any equilibrium where $K_N = D_T$ (i.e., all smelters are noncertified), we must have $o_V(Q_V, 0) = o_U(Q_U, D_T)$.[5] Now, given that $o_V(Q_V, 0) = o_U(Q_U, D_T)$, the previous condition reduces to $p_0 > 0$, which holds by assumption.

Next, we prove the converse, that is, if $\widehat{D} = 0$, then there exists an equilibrium of the type in Theorem 2.5b with $K_C = D_C = 0$. To that end, consider the following strategy profile: *All manufacturers are noncompliant and demand only noncertified metal. All smelters are noncertified and buy a total quantity of $\widetilde{Q}(0)$ from the verified pool, and buy the rest from the unverified pool.* Note that, by construction, $K_C = D_C = 0$ in this strategy profile. We will show that this strategy

[5] Suppose for a contradiction that $o_V(Q_V, 0) < o_U(Q_U, D_T)$. Then all smelters would set $t_j = 1$ and buy only verified ore, that is, $Q_V = D_T$ and $Q_U = 0$. However, with $Q_V = D_T$ and $Q_U = 0$, Assumptions 2.3 and 2.4 together imply that $o_V(Q_V, 0) > o_U(Q_U, D_T)$, yielding a contradiction. For another contradiction, suppose that $o_V(Q_V, 0) > o_U(Q_U, D_T)$. Then all smelters would set $t_j = 0$ and buy only unverified ore, that is, $Q_V = 0$ and $Q_U = D_T$. In this case, with $Q_V = 0$ and $Q_U = D_T$, Assumptions 2.4 implies that $o_V(Q_V, 0) \leqslant o_U(Q_U, D_T)$, yielding a contradiction once again. Thus, it must be that $o_V(Q_V, 0) = o_U(Q_U, D_T)$.

profile is an equilibrium (i.e., solves the manufacturers' and smelters' problems simultaneously). Here are some implications, expressed in our notation, of the strategy profile specified above: (i) $p^e_C(D_C, K_C) = p^e_N(D_N, K_N)$ (since $K_C = D_C = 0$, we have $p^e_C(0,0) = p_N(D_T, K_T)$ (see footnote 1) and since $K_N = D_N = D_T$, we have $p^e_N(D_T, K_T) = p_N(D_T, K_T)$), and (ii) $Q_V = \widetilde{Q}(0), Q_U = D_T - \widetilde{Q}(0)$. Now, given the condition $\widehat{D} = 0$ — equivalently, $\rho_i \leqslant \gamma_i, \forall i$ — and property (i), one can compare (2.8) and (2.9) to verify that the manufacturer's problem (2.10) is indeed solved when the manufacturer's strategy is to choose noncompliance and to demand only noncertified metal. Given property (ii) and recalling that $o_V(\widetilde{Q}(0), 0) = o_U(D_T - \widetilde{Q}(0), D_T)$ by the definition of $\widetilde{Q}(\cdot)$, one can verify from (2.4) that $\pi_{\text{ns}}(j) = [p_N(D_T, K_T) - o_V(\widetilde{Q}(0), 0) - c_j]K_j$, which further simplifies to $\pi_{\text{ns}}(j) = [p_0 - o_V(\widetilde{Q}(0), 0) - c_j]K_j$ (by noting that $p_N(D_T, K_T) = p_0$). Similarly, one can verify from (2.3) that $\pi_{\text{cs}}(j) = [p_C(0,0) - o_V(\widetilde{Q}(0), 0) - c_j]K_j$, which further simplifies to $\pi_{\text{cs}}(j) = [-o_V(\widetilde{Q}(0), 0) - c_j]K_j$ (by noting that $p_C(0,0) = 0$). Therefore, $\pi_{\text{ns}}(j) > \pi_{\text{cs}}(j)$ holds for all $j \in S$, and the smelter's problem (2.5) is indeed solved when all smelters choose to be noncertified and buy a total quantity of $\widetilde{Q}(0)$ from the verified pool. In summary, under the condition $\widehat{D} = 0$, the strategy profile we described is an equilibrium, whose outcome is as described in Theorem 2.5b with $K_C = D_C = 0$.

This subcase proves Proposition 2.2(*1*).

– *Subcase b:* $K_C = K_T$. In this subcase, all smelters must be certified. Once again, in addition to $D_{\text{CM}} = \widehat{D}$, we observe the following properties of the type of equilibrium in this subcase. (i) We must have $K_N = 0$, in which case $D_N = 0$ as well (by Lemma 2.4); hence, $D_C = D_T$. (ii) Given all smelters are certified, we must have $Q_V = D_T$.

Next, supposing that this type of equilibrium exists, we identify the conditions that must hold. The condition $\pi_{\text{cs}}(j) \geqslant \pi_{\text{ns}}(j)$ must hold for all smelters, (i.e., all smelters prefer to be certified). This condition can be written as

$$(p_C(D_C, K_C) - o_V(Q_V, D_{\text{CM}}) - c_j)K_j \geqslant [p_N(D_N, K_N) - (o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}})) - c_j]K_j.$$

When $D_N = K_N = 0$, we have $p_N(D_N, K_N) = 0$ and $p_C(D_C, K_C) = p_0$ (because $K_C = D_C = D_T$), so the above condition simplifies to $(p_0 - o_V(Q_V, D_{\text{CM}}) - c_j)K_j \geqslant [-(o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}})) - c_j]K_j$ for all $j \in S$. Using property (ii) and recalling $D_{\text{CM}} = \widehat{D}$, the condition can be further simplified to $p_0 - o_V(D_T, \widehat{D}) \geqslant -(o_V(D_T, \widehat{D}) \wedge o_U(0, D_T - \widehat{D}))$. From Assumptions 2.3 and 2.4, we can verify that $o_V(D_T, \widehat{D}) \geqslant o_U(0, D_T - \widehat{D})$. Hence, the condition reduces to $p_0 \geqslant o_V(D_T, \widehat{D}) - o_U(0, D_T - \widehat{D})$.

Next, we prove the converse, that is, if $p_0 \geqslant o_V(D_T, \widehat{D}) - o_U(0, D_T - \widehat{D})$, then there exists an equilibrium of the type in Theorem 2.5b with $K_C = K_T$ and $D_C = D_T$. To that end, consider the following strategy profile: *A manufacturer $i$ is compliant if and only if $\rho_i > \gamma_i$; all manufacturers demand only certified metal; all smelters are certified, and buy only verified ore.* Note that, by construction, $K_C = D_C = D_T$ and $D_{\text{CM}} = \widehat{D}$ in this strategy profile. We will show that this strategy profile is an equilibrium (i.e., solves the manufacturers' and smelters' problems simultaneously). Some implications of this strategy profile, expressed in our notation, are as follows: (i) $p^{\text{e}}_C(D_C, K_C) = p^{\text{e}}_N(D_N, K_N)$ (since $K_N = D_N = 0$, we have $p^{\text{e}}_N(0, 0) = p_C(D_T, K_T)$ (see footnote 1) and since $K_C = D_C = D_T$, we have $p^{\text{e}}_C(D_T, K_T) = p_C(D_T, K_T)$); (ii) $Q_V = D_T, Q_U = 0$. Given (i), a noncompliant manufacturer's cost is the same no matter how it allocates its demand between certified and noncertified metal. Also by (i), $\nu_{\text{cm}}(i) < \nu_{\text{nm}}(i)$ if and only if $\rho_i > \gamma_i$. Hence, manufacturer $i$'s problem (2.10) is indeed solved when its strategy is to choose compliance if and only if $\rho_i > \gamma_i$, and demand only certified metal regardless of compliance decision. Given property (ii) and recalling that $D_{\text{CM}} = \widehat{D}$, one can verify from (2.4) that $\pi_{\text{ns}}(j) = [p_N(0, 0) - (o_V(D_T, \widehat{D}) \wedge o_U(0, D_T - \widehat{D})) - c_j]K_j$, which further simplifies to $\pi_{\text{ns}}(j) = [-o_U(0, D_T - \widehat{D}) - c_j]K_j$ (by noting that $p_N(0, 0) = 0$ and $o_V(D_T, \widehat{D}) \geqslant o_U(0, D_T - \widehat{D})$ by Assumptions 2.3 and 2.4). Similarly, one can verify from (2.3) that $\pi_{\text{cs}}(j) = [p_C(D_T, K_T) - o_V(D_T, \widehat{D}) - c_j]K_j$, which further simplifies to $\pi_{\text{cs}}(j) = [p_0 - o_V(D_T, \widehat{D}) - c_j]K_j$ (by noting that $p_C(D_T, K_T) = p_0$). Therefore, if $p_0 \geqslant o_V(D_T, \widehat{D}) - o_U(0, D_T - \widehat{D})$, then $\pi_{\text{cs}}(j) \geqslant \pi_{\text{ns}}(j)$ holds for all $j \in S$, and the smelter's problem (2.5) is indeed solved when all smelters choose to be certified and (as required by the model) buy only certified metal. In summary, under the condition $p_0 \geqslant o_V(D_T, \widehat{D}) - o_U(0, D_T - \widehat{D})$, the strategy profile we described is an equilibrium, which leads to the outcome described in Theorem 2.5b with $K_C = K_T$, $D_C = D_T$, and $D_{\text{CM}} = \widehat{D}$.

This subcase proves Proposition 2.2(*2*).

– *Subcase c:* $0 < K_C < K_T$. In this subcase, there are both certified and noncertified smelters. In addition to $D_{\text{CM}} = \widehat{D}$, we observe the following properties of the type of equilibrium in this subcase: (i) Because $0 < K_C = D_C = Q < K_T$, it is also true that $0 < K_N = D_N < K_T$. (ii) We must have $K_C \leqslant Q_V$ (because certified smelters can only buy verified ore). (iii) Putting together the observations made throughout this subcase, we can also write $\widehat{D} \leqslant D_C = K_C = Q \leqslant Q_V$.

Next, supposing that this type of equilibrium exists, we show that $Q \in [\widehat{D}, \widetilde{Q}(\widehat{D})]$. Noting that $K_C = D_C$ and $K_N = D_N$ in this subcase, observe that condition (2.16), which must hold for smelter $j$ to be certified, reduces to $o_V(Q_V, \widehat{D}) \leqslant o_V(Q_V, \widehat{D}) \wedge o_U(Q_U, D_T - \widehat{D})$, which is independent of $j$. Note that $o_V(Q_V, \widehat{D})$ can never be strictly less than $o_V(Q_V, \widehat{D}) \wedge o_U(Q_U, D_T - \widehat{D})$. On the other hand, since there exist some certified smelters ($K_C > 0$), it cannot be the case that $o_V(Q_V, \widehat{D}) > o_V(Q_V, \widehat{D}) \wedge o_U(Q_U, D_T - \widehat{D})$. Hence it must be that condition (2.16) holds as equality. Therefore, we must have $o_V(Q_V, \widehat{D}) \leqslant o_U(Q_U, D_T - \widehat{D})$, which would then imply that $o_V(Q_V, \widehat{D}) = o_U(Q_U, D_T - \widehat{D})$.[6] Hence, by definition of $\widetilde{Q}(\cdot)$, we have $Q_V = \widetilde{Q}(\widehat{D})$. Given property (iii) in the paragraph above, we conclude $Q \in [\widehat{D}, \widetilde{Q}(\widehat{D})]$.

Next, we prove the converse, that is, if $Q \in [\widehat{D}, \widetilde{Q}(\widehat{D})]$ and $Q > 0$, then there exists an equilibrium of the type in Theorem 2.5b with $K_C = D_C = Q$ and $D_{\text{CM}} = \widehat{D}$. To that end, consider the following strategy profile: *Manufacturer $i$ is compliant if and only if it is compliance-prone (i.e., $\rho_i > \gamma_i$); all compliant manufacturers demand only certified metal, and a total of $Q - \widehat{D}$ of the noncompliant manufacturers' demand goes to the certified metal, while the rest goes to the noncertified metal. A set of smelters whose total capacity amounts to $Q$ are certified, and the rest noncertified; the certified smelters buy only verified ore; $\widetilde{Q}(\widehat{D}) - Q$ of the noncertified smelters' demand goes to verified ore, and the remaining goes to unverified ore.* We will prove that if $Q \in [\widehat{D}, \widetilde{Q}(\widehat{D})]$ and $Q > 0$, then this strategy profile is an equilibrium (i.e., solves the manufacturers' and smelters' problems) and it yields the outcome described in Theorem 2.5b with $K_C = D_C = Q$ and $D_{\text{CM}} = \widehat{D}$. Here are some implications of such a strategy profile: (i) $p_C(D_C, K_C) = p_N(D_N, K_N) = p_0$ (because $D_C = K_C = Q$, $D_N = K_N = D_T - Q$); (ii) hence $p^{\text{e}}_C(D_C, K_C) = p^{\text{e}}_N(D_N, K_N)$; (iii) $Q_V = \widetilde{Q}(\widehat{D})$; (iv) since $0 < Q \leqslant \widetilde{Q}(\widehat{D}) < D_T$, $D_C = K_C = Q > 0$ and $D_N = K_N = D_T - Q > 0$. Now, given (ii), one may compare (2.8) and (2.9) to verify that the manufacturer's problem (2.10) is indeed solved when, for all $i \in \mathcal{M}$, manufacturer $i$ chooses to be compliant if and only if $\rho_i > \gamma_i$.

[6] Suppose for a contradiction that $o_V(Q_V, \widehat{D}) < o_U(Q_U, D_T - \widehat{D})$. Then all smelters would set $t_j = 1$ and buy only verified ore, that is, $Q_V = D_T$ and $Q_U = 0$. In this case, we would have $o_V(Q_V, \widehat{D}) > o_U(Q_U, D_T - \widehat{D})$ (according to Assumptions 2.3 and 2.4), yielding a contradiction.

Given $o_V(\widetilde{Q}(\widehat{D}), \widehat{D}) = o_U(D_T - \widetilde{Q}(\widehat{D}), D_T - \widehat{D})$ (by the definition of $\widetilde{Q}(\cdot)$) along with properties (i), and (iv), one can compare (2.3) and (2.4) to verify that the smelter's problem (2.5) is indeed solved when the smelters collectively follow the strategy profile described above (i.e., a set of smelters whose total capacity amounts to $Q$ are certified, and the rest noncertified; the certified smelters buy only verified ore; $\widetilde{Q}(\widehat{D}) - Q$ of the noncertified smelters' demand goes to verified ore, and the remaining goes to unverified ore). In summary, under the conditions $Q \in [\widehat{D}, \widetilde{Q}(\widehat{D})]$ and $Q > 0$, the strategy profile we described is an equilibrium, which leads to the outcome described in Theorem 2.5b with $K_C = D_C = Q$ and $D_{\text{CM}} = \widehat{D}$.

This subcase proves Proposition 2.2(*3*).

- *Case a of Theorem 2.5.* First, we show that an equilibrium of the type in Theorem 2.5a must satisfy a number of properties in addition to those already listed in Theorem 2.5a. Second, we show that if such an equilibrium exists, then we must have $\widehat{D} > \overline{Q}$. Third, we show that if $\widehat{D} > \overline{Q}$, then there exists an equilibrium of the type in Theorem 2.5a. These complete the proof of Proposition 2.2(*4*).

First, we observe the following properties of the type of equilibrium in this case. (i) The type of equilibrium in Theorem 2.5a, by definition, has $D_C = D_{\text{CM}} > K_C$. (ii) We must have $D_C - K_C = -(D_N - K_N)$ (because $D_C + D_N = K_C + K_N$ by Assumption 2.1).

Given property (ii) above, we substitute $D_C - K_C = -(D_N - K_N)$ in Condition (2.16) to reduce it to $2a(D_C - K_C) \geqslant o_V(Q_V, D_{\text{CM}}) - (o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}}))$. Note that this condition is independent of $j$, so if any relationship ("$>$", "$=$", or "$<$") between $2a(D_C - K_C)$ and $o_V(Q_V, D_{\text{CM}}) - (o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}}))$ holds for a smelter $j$, then it must hold for all smelters. Next we show that, in this type of equilibrium, Condition (2.16) must hold as equality. For a contradiction, suppose that $2a(D_C - K_C) > o_V(Q_V, D_{\text{CM}}) - (o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}}))$ for all smelters. Then, all smelters will choose to be certified, that is, $K_C = K_T$, but this contradicts the equilibrium's property that $D_C > K_C$. For another contradiction, suppose that $2a(D_C - K_C) < o_V(Q_V, D_{\text{CM}}) - (o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}}))$ for all smelters. Then, all smelters will choose to be noncertified, that is, $K_N = K_T$ and $K_C = 0$, in which case we must have $o_V(Q_V, D_{\text{CM}}) = o_U(Q_U, D_{\text{NM}})$ (the reasoning is the same as in Footnote 5), which implies that we must have $2aD_C < 0$, which yields another contradiction. Therefore, we must have

$$2a(D_C - K_C) = o_V(Q_V, D_{\text{CM}}) - (o_V(Q_V, D_{\text{CM}}) \wedge o_U(Q_U, D_{\text{NM}})). \tag{2.17}$$

Since $D_C > K_C$, it follows from (2.17) that $o_V(Q_V, D_{\text{CM}}) > o_U(Q_U, D_{\text{NM}})$. Therefore, noncertified smelters would buy only from the unverified pool, so $Q_V = K_C$ and $Q_U = K_N = D_T - K_C$. Thus, we can write the last inequality as $o_V(K_C, D_{\text{CM}}) > o_U(D_T - K_C, D_{\text{NM}})$. Furthermore, because $D_C = D_{\text{CM}} > K_C$ (property (i) above), the last inequality combined with Assumption 2.3 allows us to conclude $o_V(K_C, K_C) > o_U(D_T - K_C, D_T - K_C)$. Now, Assumption 2.4 allows us to observe property (iii): $K_C > \overline{Q}$.

Using the definitions of $p^e_C(D_C, K_C)$ and $p^e_N(D_N, K_N)$ from (2.7), we obtain $p^e_C(D_C, K_C) = p_C(D_C, K_C)\frac{K_C}{D_C} + p_N(D_N, K_N)\frac{D_C - K_C}{D_C}$, and $p^e_N(D_N, K_N) = p_N(D_N, K_N)$. Since $p_C(D_C, K_C) > p_N(D_N, K_N)$ and $K_C > 0$ in this equilibrium, $p^e_C(D_C, K_C) > p^e_N(D_N, K_N)$. By observing (2.10) and using the definitions of $\nu_{\text{cm}}(i)$ and $\nu_{\text{nm}}(i)$ from (2.8) and (2.9) respectively, we note that manufacturer $i$ chooses to be compliant if and only if

$$p^e_C(D_C, K_C)D_i + \gamma_i < p^e_N(D_N, K_N)D_i + \rho_i. \tag{2.18}$$

Since $p^e_C(D_C, K_C) > p^e_N(D_N, K_N)$, if (2.18) holds for manufacturer $i \in \mathcal{M}$, then manufacturer $i$ must have $\gamma_i < \rho_i$, that is, manufacturer $i$ must be compliance-prone. Therefore, in the type of equilibrium in Theorem 2.5a, any compliant manufacturer must be compliance-prone, so we have property (iv): $D_{\text{CM}} \leqslant \widehat{D}$.

Second, supposing that the type of equilibrium in Theorem 2.5a exists, we conclude that $\widehat{D} > \overline{Q}$ must hold (by the equilibrium property that $D_{\text{CM}} > K_C$ along with $K_C > \overline{Q}$ (property (iii)) and $D_{\text{CM}} \leqslant \widehat{D}$ (property (iv)).

Third, we prove that if $\widehat{D} > \overline{Q}$, then there exists an equilibrium of the type in Theorem 2.5a. To do so, assuming $\widehat{D} > \overline{Q}$, we will specify a particular strategy profile, which we will show is an equilibrium (i.e., solves the manufacturers' and smelters' problems) and leads to the outcome described in Theorem 2.5a. Let $x \in (\overline{Q}, D_T]$ be a number that satisfies (2.14) (the existence of $x$ is by Lemma 2.5). Now, consider the following strategy profile: *Manufacturer $i$ complies if and only* *if*

$$\mu_i > \frac{2a\kappa(x)}{x}(x - \kappa(x)) \tag{2.19}$$

*and only compliant manufacturers demand certified metal; the set of certified smelters is such that the total certified capacity is equal to $\kappa(x)$, and only certified smelters purchase verified ore.* Here are some implications of such a strategy profile:

(*i*) $D_{\text{CM}} = \int_{\mathcal{M}} D_i \mathbb{1}_{\left\{\mu_i > \frac{2a\kappa(x)}{x}(x-\kappa(x))\right\}} \,\mathrm{d}i = x$ (by the definition of $x$).

(*ii*) $D_C = D_{\text{CM}} = x$ (since only compliant manufacturers demand certified metal).

(*iii*) $\overline{Q} < K_C < D_C$ (because $K_C = \kappa(x)$ and $\kappa(x) \in (\overline{Q}, x)$ by Lemma 2.3).

(*iv*) $p_C(D_C, K_C) > p_N(D_N, K_N)$ (by (*iii*) and Assumption 2.2).

(*v*) $p^e_C(D_C, K_C) = p_C(D_C, K_C)\frac{K_C}{D_C} + p_N(D_N, K_N)\frac{D_C - K_C}{D_C}$ and $p^e_N(D_N, K_N) = p_N(D_N, K_N)$ (by (*iii*) and (2.7)).

(*vi*) $p^e_C(D_C, K_C) > p^e_N(D_N, K_N)$ (by (*iv*) and (*v*)).

(*vii*) $Q_V = K_C$ (since only certified smelters purchase verified ore).

(*viii*) $o_V(Q_V, D_{\text{CM}}) - o_U(Q_U, D_{\text{NM}}) = 2a(D_C - K_C) > 0$: Given properties (*ii*), (*iii*), and (*vii*), we note that $o_V(Q_V, D_{\text{CM}}) = o_V(\kappa(x), x)$ and $o_U(Q_U, D_{\text{NM}}) = o_U(D_T - \kappa(x), D_T - x)$. Therefore, $o_V(Q_V, D_{\text{CM}}) - o_U(Q_U, D_{\text{NM}}) = o_V(\kappa(x), x) - o_U(D_T - \kappa(x), D_T - x) = 2a(x - \kappa(x))$ where the last equality follows from the definition of $\kappa(x)$ (Definition 2.1). The property follows because $x = D_C$ and $\kappa(x) = K_C$ by (*ii*) and (*iii*), respectively.

We first show that the strategy profile we specified above solves the manufacturers' problems. Because of (*vi*), a noncompliant manufacturer facing problem (2.9) would demand only noncertified metal. Hence, if manufacturer $i$ chooses to be noncompliant, then its profit is $\nu_{\text{nm}}(i) = p^e_N(D_N, K_N) + \rho_i$. Therefore, by examining the manufacturer's problem (2.10), we observe that manufacturer $i$ chooses to comply if and only if $p^e_C(D_C, K_C) + \gamma_i < p^e_N(D_N, K_N) + \rho_i$. This last inequality is equivalent to $\mu_i > \frac{2aK_C}{D_C}(D_C - K_C)$ (which can be shown by recalling that $\mu_i := \frac{\rho_i - \gamma_i}{D_i}$, and substituting the expressions for $p_C(D_C, K_C)$ and $p_N(D_N, K_N)$ from Assumption 2.1 in $p^e_C(D_C, K_C)$ and $p^e_N(D_N, K_N)$ given in (*v*)). Noting from properties (ii) and (iii) above that $D_C = x$ and

$K_C = \kappa(x)$, the last inequality is equivalent to (2.19). Thus, manufacturer $i$ chooses to comply if and only if (2.19) holds, which is what is what manufacturers do in the strategy profile we specified.

We next show that the strategy profile we specified above solves the smelters' problems. Because of (*viii*), a noncertified smelter facing problem (2.4) would demand only unverified ore. Hence, if smelter $j$ chooses to be noncertified, then its profit is $\pi_{\text{ns}}(j) = (p_N(D_N, K_N) - o_U(Q_U, D_{\text{NM}}) - c_j)K_j$. Therefore, by examining the smelter's problem (2.5), we observe that smelter $j$ chooses to be certified if and only if $(p_C(D_C, K_C) - o_V(Q_V, D_{\text{CM}}) - c_j)K_j \geqslant (p_N(D_N, K_N) - o_U(Q_U, D_{\text{NM}}) - c_j)K_j$. This last inequality is equivalent to $2a(D_C - K_C) \geqslant o_V(Q_V, D_{\text{CM}}) - o_U(Q_U, D_{\text{NM}})$ (which can be verified using Assumption 2.1). We note from property (*viii*) that the last inequality holds as an equality, which implies that any smelter $j$ is indifferent between being certified or noncertified. Therefore, all smelters' problems are indeed solved when the strategy profile is as we specify.

□

Proposition 2.1, Theorem 2.1, Theorem 2.2, Corollary 2.1, and Theorem 2.3 all follow from Proposition 2.2 as we argue next. By reviewing parts (*1*) through (*4*) of Proposition 2.2, observe that there exists an equilibrium with $K_C = D_C = D_T$ and $D_{\text{CM}} = \widehat{D}$ if and only if $p_0 \geqslant o_V(D_T, \widehat{D}) - o_U(0, D_T - \widehat{D})$. This observation yields Proposition 2.1. Once again, reviewing parts (*1*) through (*4*) of Proposition 2.2, we notice that if $\widehat{D} < \overline{Q}$, there is no equilibrium with shortage of certified metal. In contrast, if $\widehat{D} > \overline{Q}$, the equilibria described in parts (*1*) and (*3*) of Proposition 2.2 are not possible. Therefore, if $\widehat{D} > \overline{Q}$, either there is a shortage and the equilibrium is the type in part (*4*), or the entire market is certified as in the equilibrium of part (*2*). This observation yields Theorem 2.1. Theorem 2.2 and Corollary 2.1 are direct consequences of part (*3*) of Proposition 2.2. Theorem 2.3 is a direct consequence of part (*4*) of Proposition 2.2.

## Appendix 2.D Proof of Results in Section 2.5

*Proof of Theorem 2.4.* In this proof, we assume that the current equilibrium is not the "all-certified" equilibrium (i.e., $D_C = K_C = D_T$ as in part (*2*) of Proposition 2.2); in that case, there is no need to target any additional manufacturers. When $\widehat{D} \leqslant \overline{Q}$, the equilibrium is part (*1*) or (*3*) in Proposition 2.2. In all of these equilibria, the effective prices of certified and noncertified metals are equal (in part (*1*), the effective prices of both metal types are $p_0$, and part (*3*) has the property that $p_C(D_C, K_C) = p_N(D_N, K_N)$). Therefore, a manufacturer is compliant if and only if $\rho_i \tau_i > \gamma_i$; that is, turning on the penalty of any manufacturer in $\{i : i \in T' \text{ and } \rho_i > \gamma_i\}$ induces it to become compliant.

When $\widehat{D} > \overline{Q}$, the equilibrium is part (*4*) in Proposition 2.2 (given that we exclude the possibility of all-certified equilibrium in part (*2*) of Proposition 2.2). Manufacturer $i$ is compliant if and only if (2.18) holds, where $p_C^e(D_C, K_C) > p_N^e(D_N, K_N)$ (implication (*vi*) on p. 42). Let $\xi :=$ $p_C^e(D_C, K_C) - p_N^e(D_N, K_N)$. It then follows from (2.18) that turning on the penalty of manufacturer $i$ induces it to become compliant in equilibrium if and only if $i \in T'$ and $\rho_i > \xi D_i + \gamma_i$. □

## 3 Building Alliances for Corporate Social Responsibility

*Abstract.* Following the Dodd-Frank Act's mandate that public companies disclose the use of conflict minerals in their products and processes, five well-known companies established an alliance and initiated a fund to audit mineral suppliers. Since contributing to a common fund has minimal direct benefits—audits are public and companies have equal access to certified suppliers regardless of contributions—companies have an incentive to free ride. Nevertheless, the initiative was successful: the alliance was established and dozens of other companies also contributed. We propose two factors that explain this success: the initial catalyst of an alliance and the *status-seeking behavior* of lower-status companies that subsequently contributed. To capture companies' incentives to free-ride, we model the funding initiative as a public goods game and incorporate the two factors: (1) an invitation stage where some of the companies can form an alliance and make an initial contribution to the public good, and (2) status-seeking behavior, by which low-status firms want to be associated to high-status firms. We use the model to show how the combination of both factors leads to high contributions. We then conduct a laboratory experiment to test the effect of the two factors and how they interact. Our experiment shows that the invitation stage is key to high contributions. In particular, the formation of an alliance significantly increases group contribution. As our model predicts, we find evidence of status-seeking behavior which influences whether and how an alliance forms. Surprisingly, we find that high-status companies not in an alliance also contribute after the invitation stage. We attribute this unexpected result to *higher moral responsibility* for the high-status companies. Our findings demonstrate important insights for companies that seek to address industry-wide social responsibility problems.

## 3.1 Introduction

In response to pressure from legislation, non-governmental organizations (NGOs), and investors, manufacturers have been working to avoid "conflict minerals" from the Democratic Republic of the Congo (the Dodd-Frank Act, see U.S. Securities and Exchange Commission 2013; Development International 2017, Walt and Meyer 2018).[1] Tracing the sources of minerals used to finance armed conflicts is challenging for two reasons. First, it requires working closely with smelters in remote regions less subject to pressure from regulation and NGOs. Second, only a few hundred smelters worldwide process the minerals (King 2014), while countless companies worldwide—including thousands of publicly traded companies in the U.S.—use them (Bayer and de Buhr 2011). As a result, each smelter may supply many manufacturers, sharply limiting the influence of an individual manufacturer over the smelter's practices.

A group of well-known manufacturers recognized the need for an industrial alliance to strengthen their influence as they work with the smelters to trace the sources of minerals. In addition such an alliance avoids individual manufacturer audits which may lead to "auditing fatigue" for a smelter facing numerous downstream manufacturers. As a result, the manufacturers established the Responsible Minerals Initiative (RMI) to audit and certify smelters who shun conflict minerals (Zhang et al. 2017).[2] Using a consistent protocol, the RMI independently audits smelters (Responsible Minerals Initiative 2018). A key component of the RMI is the Initial Audit Fund (IAF) that reimburses smelters for their first annual audit and relies entirely on corporate donations (D'Esposito 2012, Responsible Minerals Initiative 2019a).[3] Initially, executives from Intel successfully persuaded their counterparts at Apple, GE, HP, and Microsoft to help launch the IAF (Weaver 2012, Responsible Minerals Initiative 2019a). Subsequently, more than twenty other companies contributed (Responsible Minerals Initiative 2019a).

Similar industrial alliances exist for other social responsibility causes. For example, in the pharmaceutical industry, a group of companies formed the Pharmaceutical Supply Chain Initiative to jointly audit suppliers for ethics, fair labor, health and safety, environmental protection, and management systems. By avoiding duplication of auditing efforts, these audits achieve cost and resource

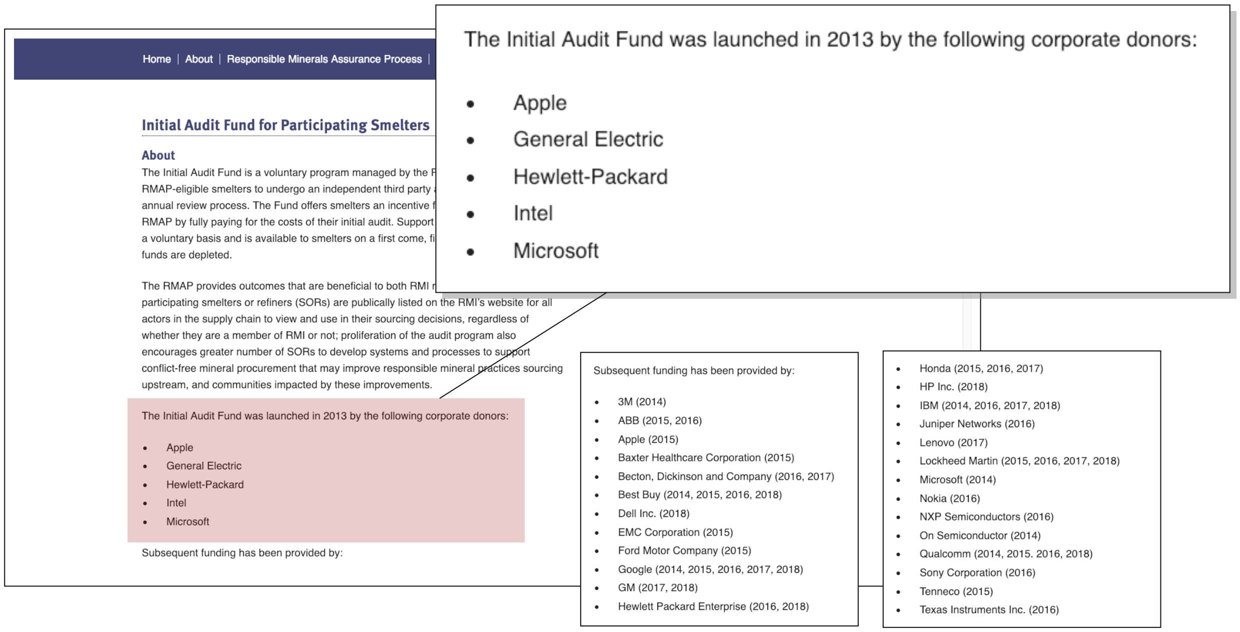
[1]The minerals are commonly known as 3TGs for their initials (tin, tungsten, tantalum, and gold ore) and are used in the manufacturing of various devices such as consumer electronics.
[2]The RMI was formerly the Conflict-Free Sourcing Initiative.
[3]The IAF was formerly the Early-Adopters Fund.

savings (Pharmaceutical Supply Chain Initiative 2018b,c,a, Chen et al. 2019a). Similarly, after the tragic Rana Plaza collapse that killed 1,100 people in 2013, more than 150 brands set up the Accord on Fire and Building Safety to improve safety in the Bangladeshi garment industry. The signatories annually contribute to the Accord Foundation to maintain the programs (Thomasson 2014, Accord on Fire and Building Safety in Bangladesh 2017, Chen et al. 2019a). Finally, in business education, a coalition of corporate and academic organizations founded The PhD Project to support underrepresented minorities in business doctoral programs and establish a pipeline of minority business leaders (The PhD Project 2018a,b). The common feature of these alliances is that many companies share the same social responsibility issue. In some cases an alliance of organizations with combined resources and market power have greater clout in addressing the problem; in other cases, coordinating the compliance effort helps spread the cost, avoid duplication, and possibly achieve economies of scale.

Among the alliances, the RMI is especially successful (Young 2015) and motivated our research. The success of the RMI's Fund is particularly interesting for two reasons. First, its benefits are equally shared among all manufacturers—those who contribute and those who do not. In fact, the RMI publicly displays the list of certified smelters for all manufacturers to view and use. In the RMI's own words, "Participating smelters or refiners . . . are publicly listed on the RMI's website for all actors in the supply chain to view and use in their sourcing decisions, regardless of whether they are a member of RMI or not" (Responsible Minerals Initiative 2019a). As a result, contributing to the Fund does not give a manufacturer a procurement advantage over other manufacturers. Second, manufacturers derive minimal direct benefits from contributing to the Fund. A manufacturer cannot appease an NGO or a regulator merely by donating to a fund; it appeases them by demonstrating that its minerals come from legitimate sources. Furthermore, manufacturers who contribute do not actively advertise their contribution to appeal to customers.

These characteristics make the IAF resemble a public good, for which "use of a unit of the good by one agent does not preclude its use by other agents" (Mas-Colell et al. 1995, p. 359). Interestingly, while public goods typically suffer from the free-rider problem and hence underprovision, the IAF has received continued corporate donation, making it the most successful industry-led effort on conflict minerals (Young 2015). In this paper, we seek to understand what drove the success of the IAF and why companies contribute to the IAF in spite of the incentive to free ride.

Figure 3.1: Screenshot of the IAF's Website




A screenshot of a part of the web page of the Initial Audit Fund (in three segments to save space) (source: Responsible Minerals Initiative 2019a).

A screenshot of the IAF's website (Figure 3.1) exhibits two salient features. First, the website includes a complete listing of all companies that have contributed to the IAF, highlighting the five initial contributors above the subsequent contributors. Second, the initial contributors generally have more valuable brands than the subsequent contributors. In *Forbes World's Most Valuable Brands* ranking, all five initial contributors were highly ranked, while the majority of the subsequent contributors fell out of the top 100 (Forbes 2013; see electronic companion 3.A for the list).[4] Based on these two focal features of the IAF, we conjecture that two factors could help explain the IAF's success and, more broadly, should be considered by companies seeking to address a common social responsibility problem: (1) the existence of an early *invitation stage*, in which a company can invite others to form an alliance and commit to contribute to the initiative before all other companies decide whether to contribute; and (2) the heterogeneity in the *status* of the companies—in terms of their brand values—leading to *status-seeking behavior* of low-status companies. Status-seeking behavior has been documented extensively in the literature, both theoretically and experimentally (see, for example, Ball et al. 2001 and Kumru and Vesterlund 2010 for experimental evidence on individuals, Ball et al. 2001 for a discussion on earlier theoretical work, and Heffetz and Frank 2011

[4]2013 is the year the IAF was launched.

for a survey on empirical and experimental evidence on social status). In our setting, status-seeking behavior would imply that low-status companies—those with a less known brand—may want to be associated with high-status companies, resulting in an additional incentive to contribute to the Fund when high-status companies are among the initial contributors.

This paper addresses the following research questions: (1) Are the invitation stage and the heterogeneity in companies' statuses relevant in establishing a successful alliance with high contributions? (2) If so, do the two factors interact?

We study, both from a theoretical and experimental perspective, success drivers for social responsibility initiatives that address a problem shared among several companies. Our contribution is two-fold. Our theoretical model and experimental design allow us to uncover the behavioral drivers leading to high contributions, and our experimental results derive important managerial insights for companies seeking to start such initiatives.

To study the problem, we analyze a two-stage public goods game with four players each representing a company, two with high status and two with low status.[5] An *invitation stage* in which one company may invite another to form an alliance and commit to contribute is followed by a *contribution stage* in which any company can make a contribution. We compare four variants of the game: a *baseline* case with neither heterogeneity in status nor an invitation stage; a case with *heterogeneity in status* but no invitation stage; a case with no heterogeneity in status but with an *invitation stage*; and a *full* case with both heterogeneity in status and an invitation stage. We solve for the subgame perfect equilibrium in each case. We show that both the invitation stage and the heterogeneity in status are necessary for contributions to arise, and that the total contribution depends on the difference between the levels of status and the extent to which companies value it.

We test the predictions of the theoretical model with a laboratory experiment. The experimental results show that the invitation stage is key to generating high contributions. Interestingly, the successful formation of an alliance significantly increases group contribution, with or without heterogeneity in status. Nevertheless, status does matter. When there is heterogeneity in companies' statuses, the decisions in the invitation stage are consistent with "status-seeking" behavior. In particular, a low-status company significantly prefers to invite a high-status company to jointly

[5] The game is modified version of the voluntary contribution mechanism (VCM) public goods game (Plott et al. 1985, Isaac and Walker 1988a,b).

establish an alliance than to invite another low-status company. However, we find that among non-alliance members facing the same type of alliance, a high-status company is even more likely to contribute than a low-status company. Our results suggest that while status-seeking preferences explain low-status companies' behavior well, a sense of higher "moral responsibility" seems to be the behavioral driver behind the high-status companies' contributions.

Our results provide useful managerial insights for companies seeking to address a shared social responsibility problem. Both our theoretical model and experimental results indicate that establishing an alliance is key to reaching an overall high contribution. While in the formation of the IAF all initial contributors were companies with well-known brands, our results suggest that a company with a relatively lower brand value can also initiate an effective alliance—to do so, they should invite a company with high brand value, who will be more likely to join. In addition, while the IAF was established by a company with high brand value inviting other companies with high brand value to join, our results suggest that a company with high brand value can also start a successful alliance by inviting a company with low brand value. Once the alliance is established, other companies with high brand value may also contribute out of a sense of moral responsibility to be part of the initiative.

### 3.2 Literature Review

Our work is related to various literature streams. Kim and Davis (2016) review conflict minerals reports that firms submit to the Securities and Exchange Commission and identify the complexity in the structure of the supply chain as a difficulty for monitoring. Noting the emergence of NGOs and industrial efforts to overcome the obstacle, Berman et al. (2017) show empirically that transparency and traceability initiatives have been able to reduce the risk of conflict, and Zhang et al. (2017) show with a supply network model that an industrial alliance to pool the auditing efforts is effective in reducing the use of noncertified minerals. Focusing on the various programs on conflict minerals, Young (2015) identifies the RMI—which motivates our research—as the largest and most central industry-led effort. We contribute to this literature by exploring the incentives at work in the formation of an industrial alliance that coordinates the compliance efforts.

More broadly, our work complements the literature on socially responsible supply chain management. The majority of the work in this area focuses on how an individual buyer induces a supplier to comply (Plambeck and Taylor 2016, Chen and Lee 2016, Cho et al. 2018, Kraft et al. 2019). Also focusing on a single buying firm, Kalkanci and Plambeck (2019a) study how managers can optimize the investors' valuation of the buying firm through auditing, remediation, and disclosure decisions. Instead of focusing on an isolated buyer, our work is closest to existing research on multiple buyers auditing shared suppliers. Caro et al. (2018) and Chen et al. (2019a) study different supply chain models with two buyers and show the myriad benefits that arise when an alliance of buyers audits a shared supplier. In particular, Chen et al. (2019a) demonstrate the tendency of buyers to avoid auditing the shared supplier when acting independently without an alliance. Fang and Cho (2019) study a supply chain in which multiple buyers audit a single supplier and show that buyers are able to sustain coalitions for joint auditing. We complement the existing literature by exploring the conditions that favor the formation of an alliance by a group of buyers for a shared social responsibility concern.

Given the main features of the IAF, we model the companies' decision to join the Fund as a decision to contribute to a *public good* (see Marwell and Ames (1981), Andreoni (1988), Isaac and Walker (1988b), Andreoni (1995) for seminal work; see Ledyard (1995) and Vesterlund (2016) for an extensive survey). In particular, we incorporate two main characteristics of the IAF: an invitation stage where an alliance of initial contributors can be formed, and heterogeneity in companies' status. The existence of an invitation stage relates our work to the literature on sequential public goods games. Existing research shows that whether sequential moves increase contribution highly depends on the context. In games with two players Andreoni et al. (2002) and Nosenzo and Sefton (2011) show that when the benefits from the public good are heterogeneous the effect of sequential moves on group contribution is mixed, while Potters et al. (2005) find that when there is imperfect information, letting the informed player contribute first increases group contribution. In finitely repeated games with four players (with fixed matching), Halloran et al. (2006) find that allowing players to commit a portion of their endowment to the public good does not affect the group contribution. In contrast, Güth et al. (2007) find that allowing only one player to contribute early significantly increases group contribution, and Rivas and Sutter (2011) find that a voluntary first mover is more effective in inducing high group contribution than an exogenously designated first mover. Arbak and Villeval (2013) study a one-shot game with multiple players and focus on identifying what motivates players to lead when one volunteer can make an initial contribution. Our work is different from the previous literature in that it focuses on the establishment of an *alliance* and its effects on group contribution.

We also study the role of *status* on the formation of alliances and contribution—this leads to a second stream of public goods literature related to our work. Kumru and Vesterlund (2010) find that, in a simultaneous move game with two players, a low-status player mimics a high-status player, inducing the high-status player to contribute more, together leading to greater group contribution when the high-status player contributes first. Eckel et al. (2010) designate one player as the central player in a group of four and find that the peripheral players mimic the central player more when the central player has high rather than low status. We contribute to the stream by studying how status influences the formation of an alliance of early contributors and subsequent contribution.

At a higher level, our research falls within the field of behavioral supply chain management, surveyed by Chen and Wu (2019). In the domain of social preferences in particular, the extant research mostly focuses on the interaction between one buyer and one supplier. Factors such as fairness (Cui et al. 2007, Loch and Wu 2008, Katok and Pavlov 2013), trust and trustworthiness (Özer et al. 2011, 2014, Spiliotopoulou et al. 2016, Özer and Zheng 2017, Beer et al. 2017), and long-term relational concerns (Davis and Hyndman 2017) have shown to be important to understand buyer-supplier relations. In the domain of social responsibility, Kraft et al. (2018) find that consumers value greater visibility regarding a company's social responsibility practices in the upstream supply chain. We contribute to this literature by studying the interactions among same-tier companies (buyers) in the context of raising funds for a shared social responsibility initiative.

## 3.3 Theoretical Model

We consider a game with four companies where each company chooses to contribute or not to a social responsibility project that benefits every company in the group. Motivated by the characteristics of the IAF, we assume that—as in standard public goods games—the marginal benefit of contributing does not cover the cost of contributing. Individual companies have no incentive to contribute, but every company is better off if all companies contribute than if no company contributes. To isolate the effects of an invitation stage (where an alliance can be formed) and of status, we analyze separately four different variants of the game. First, we present a baseline model with neither an invitation stage nor a difference in status.[6] Second, we incorporate heterogeneity in status in the model: companies have either high or low status and the utility of a company depends on the status of others she interacts with. In particular, we incorporate in the model the status-seeking behavior of low-status companies. Third, we put aside the model with status momentarily and add to the baseline model an invitation stage (before the contribution stage) where a company may invite another to form an alliance. Fourth, we study a full model with both heterogeinity in status and an invitation stage. Note that four is the minimum number of companies that allows us to study high- (and low-) status companies in the roles of both initiators and invitees in the full model. We study the equilibrium in pure strategies of the static games without the invitation stage, and the subgame perfect equilibrium in pure strategies of the extensive games with the invitation stage.

### 3.3.1 Baseline Model

We consider a single-stage (i.e. *contribution stage*) baseline model with four companies. Let $N = \{1, 2, 3, 4\}$ be the group of companies (players) in the game. Each company has an endowment $w > 0$ and, motivated by Kumru and Vesterlund (2010), we make the assumption that each company either contributes her entire endowment or does not contribute at all . That is, company $i$'s contribution is $c_i \in \{0, w\}$ for $i \in N$ (and we denote $c_{-i} := (c_k)_{k \in N - i}$ the contributions of other companies in the group). A binary decision captures the fact that when companies contribute to the IAF, they are automatically included on the list of contributors—independent of the amount contributed. In fact, the website does not publish each companies' contribution amounts nor does any source easily accessible to the public.[7] In the contribution stage, companies make their decisions simultaneously. The total amount the group contributes to the social responsibility project generates a benefit of $b \geqslant 0$ per unit to each company, regardless of the company's contribution. Thus a company's profit is given by her initial endowment, minus her contribution, plus the return of the project. More

[6] Note that this setting resembles a canonical voluntary contribution mechanism public goods game with binary decisions.

[7] In addition, binary decisions allow us to model status (in section 3.3.2) in a way that is consistent with the previous literature. In our status-seeking model, a low-status company derives utility from "being associated" to a high-status company. A binary contribution allows this association to either exist or not (and not depend on the contribution *level* each company chooses), consistent with the literature (Ball et al. 2001, Kumru and Vesterlund 2010).

specifically, company $i$'s profit is:

$$\pi_i(c_i, c_{-i}) = w - c_i + b \sum_{k \in N} c_k. \tag{3.1}$$

To ensure the game captures the main features of the IAF, we make the following canonical assumptions for public goods games (Fehr and Gächter 2000, Kumru and Vesterlund 2010, Reuben and Riedl 2013):

**Assumption 3.1.** *(a) The marginal benefit of contribution to the contributor herself does not cover the marginal cost of contribution, that is $b < 1$;*

*(b) Every company is better off if all companies contribute than if no company contributes: $|N|b > 1$ (i.e., $b > \frac{1}{|N|} = \frac{1}{4}$).*

Assumption 3.1a implies that independent of the other companies' decisions, a company seeking to maximize her own profit should never contribute. If this assumption did not hold, all the companies would have incentive to contribute unilaterally and the IAF would not have been needed. Assumption 3.1b implies that if every company contributes, the benefit the social responsibility project brings to each company, $b|N|w$, exceeds the cost $w$ the company incurs to contribute to the project. In this case, every company is better off than if they had all held on to their own endowment, rendering the project efficient.

There is a unique pure-strategy equilibrium in the baseline model:

**Proposition 3.1.** *In equilibrium, no company contributes to the social responsibility project in the baseline model.*

The proofs of all results are in the electronic companion.

### 3.3.2 Heterogeneity in Status

We now incorporate heterogeneity in companies' statuses into the baseline model and assume two companies have a higher status than the other two companies. We represent company $i$'s status by a *status score* $s_i \in \{s_L, s_H\}$ corresponding to *low-status* and *high-status* companies respectively with $s_L < s_H$.

To capture the *status-seeking* behavior of low-status companies, we assume that company $i$'s utility from contributing to the social responsibility project alongside another company $j$ has an additional status term given by:

$$f_{ij}(c_i, c_j) = \gamma \cdot (s_j - s_i)^+ \cdot \mathbb{1}_{\{c_i c_j > 0\}} \tag{3.2}$$

where the behavioral parameter $\gamma > 0$ represents a company's *concern for status*.[8] Note that $f_{ij}(c_i, c_j) = \gamma(s_H - s_L)$ if $s_i = s_L, s_j = s_H$ and $c_i > 0, c_j > 0$, and $f_{ij}(c_i, c_j) = 0$ otherwise. Combining the monetary profit and the utility from status, company $i$'s *utility* is

$$u_i(c_i, c_{-i}) = \pi_i(c_i, c_{-i}) + \sum_{k \in N-i} f_{ik}(c_i, c_k) \tag{3.3}$$

where $\pi_i(c_i, c_{-i})$ is as defined in (3.1).[9]

We show that in equilibrium it is never optimal to contribute: heterogeneity in status by itself—without an invitation stage—results in no contributions in equilibrium. Intuitively, a high-status company derives no utility from status (her utility is as in Proposition 3.1) and does not contribute. Since high-status companies do not contribute, low-status companies do not derive utility from status and do not contribute either.

**Proposition 3.2.** *In the one-stage game with two low-status companies and two high-status companies with utilities given by* (3.3)*, no company contributes in equilibrium.*

### 3.3.3 Invitation Stage

In order to reproduce the sequence of events that allowed for the IAF to arise, we now modify the baseline model and incorporate an initial stage where an alliance can be formed. Specifically, we

[8] We model status-seeking behavior in a way analogous to Kumru and Vesterlund (2010) and Ball et al. (2001). Note that, as in Kumru and Vesterlund (2010), we assume that a company's aversion to being associated with a lower status company is negligible.

[9] We also consider an alternative model where a low-status company gains the same amount of utility from association with high-status companies regardless of whether one or more high-status companies contribute. Formally, the utility of company $i$ is

$$u_i(c_i, c_{-i}) = \pi_i(c_i, c_{-i}) + \max_{k \in N-i} f_{ik}(c_i, c_k).$$

The equilibrium is similar to that of the model we present in the paper except that more stringent assumptions on the parameters are needed for an alliance and contributions to arise.

consider a two-stage game with an *invitation stage* followed by a *contribution stage*. In this section we consider the case without heterogeneity in and preference for status; we reintroduce status in the full model presented in the next subsection.

In the *invitation stage*, an *initiator* is exogenously and randomly selected from among the four companies. The initiator decides whether (or not) to invite one other company to form an alliance to commit to contribute early. If the initiator decides to invite a company, she chooses which company to invite; then the *invitee* decides whether to accept the invitation or to reject it.[10] If the initiator sends an invitation and the invitee accepts it, the two companies form an alliance and the invitation stage ends with the two companies in the alliance *committed* to contribute. The commitment is binding: the committed companies will contribute to the social responsibility project. If either the initiator decides not to send an invitation or the invitee declines the invitation, no alliance is formed and the invitation stage ends without any company committed to contribute. When an alliance is not formed, each of the four companies can decide whether to contribute in the subsequent contribution stage, which reduces the game to the baseline case (in which no company contributes by Proposition 3.1). If no alliance is formed the companies other than the initiator and the invitee do not directly observe the invitation process or who the initiator and the invitee are. On the other hand if an alliance is formed, the two members of the alliance will be announced publicly (without differentiating the initiator and the invitee) at the end of the invitation stage. In the *contribution stage*, every company who has committed in the invitation stage automatically contributes her endowment $w$ into the social responsibility project. Each uncommitted company (the two companies who are not part of the alliance if there is an alliance, or all four companies if there is no alliance) can choose whether to contribute or not.

We assume that the initiator is exogenously and randomly selected for simplicity.[11] This allows us to focus on how the status of the initiator influences the equilibrium and to have opportunities where companies from either status are the initiators. In addition, we focus on the interesting case where, when only two companies contribute, the benefit from the project to each company is not large enough to justify contribution. Otherwise any invitation would be accepted in equilibrium, as accepting always makes the invitee better off. In the case of the IAF, this is reflected by the fact

[10] Note that in this section the other companies are homogeneous, so the initiator is indifferent between which company to invite.

[11] In the experiment, the initiator is randomly selected by the computer program.

that the initial contributors actively seek to extend the initiative to the other members of the RMI. To this end, we make the following assumption:

**Assumption 3.2.** *A company is better off holding onto her own endowment than if only she and one other company contribute while the others do not:* $2bw < w$ *or* $b < \frac{1}{2}$.

**Proposition 3.3.** *In the case with an invitation stage (and without utility from status), in equilibrium the initiator is indifferent between sending an invitation or not, and between which company to invite (if an invitation is sent). If an invitation is sent, the invitee will decline the invitation. No company contributes to the social responsibility project in equilibrium.*

With the invitation stage (but without utility from status), no alliance is formed and no contributions are made in equilibrium.

### 3.3.4 Full Model (Heterogeneity in Status and Invitation Stage)

Lastly, we analyze the full model with an invitation stage and heterogeneity in and concern for status. We solve by backward induction starting with the final contribution stage and rolling back in sequence to the previous decisions in the game.

#### Contribution Stage.

We first analyze the uncommitted company's optimal strategy in the contribution stage. Note that a committed company does not have a decision to make in the contribution stage as her endowment will be automatically contributed to the social responsibility project.

The concern for status $\gamma$ is critical to the low-status uncommitted company's decision. We analyze three cases corresponding to three levels of $\gamma$:[12]

**Case LCS** (low concern for status). $\gamma < \frac{1}{2}\frac{(1-b)w}{(s_H-s_L)}$.

**Case MCS** (moderate concern for status). $\frac{1}{2}\frac{(1-b)w}{(s_H-s_L)} < \gamma < \frac{(1-b)w}{s_H-s_L}$.

**Case HCS** (high concern for status). $\gamma > \frac{(1-b)w}{s_H-s_L}$.

[12]Throughout the paper, we omit the relatively uninteresting cases of equality in the condition on the parameters (e.g., $\gamma = \frac{1}{2}\frac{(1-b)w}{s_H-s_L}$ precisely), where the company will be indifferent between two decisions from the two adjacent cases of inequality in the condition.

**Proposition 3.4.** *In the contribution stage,*

*(a) When the concern for status is low (case LCS), a low-status uncommitted company will not contribute. When the concern for status is moderate (case MCS), a low-status uncommitted company will contribute if and only if both high-status companies contribute. When the concern for status is high (case HCS), a low-status uncommitted company will contribute if and only if at least one high-status company contributes.*

*(b) A high-status uncommitted company will not contribute.*

Because a high-status company does not gain any utility from status by being associated with another company in the group, in the contribution stage a high-status uncommitted company is concerned only with her monetary profit and does not contribute. A low-status company gains utility from being associated with high-status companies; therefore, her contribution decision critically depends on her concern for status, $\gamma$. When the concern for status is low, the additional utility from status is insufficient to justify contribution. When the concern for status is moderate, a low-status uncommitted company contributes if both high-status companies contribute and, in doing so, derives a utility from status equal to $2\gamma(s_H - s_L)$. When the concern for status is high, a low-status uncommitted company contributes even if only one high-status company contributes, in which case she derives $\gamma(s_H - s_L)$ in utility from status.

We note that when the concern for status is low (case LCS), no company contributes in the contribution stage and therefore, no company has incentive to form an alliance in the invitation stage. Thus, no company contributes at any point in the game:

**Proposition 3.5.** *Under case LCS, an alliance will not be formed in the invitation stage and there will be no contribution in the contribution stage.*

In the remaining analysis, we focus on the more interesting cases MCS and HCS where the utility a low-status company can derive from being associated with a high-status company is sufficently high that alliance formation and contributions are possible.

**Invitee's Decision to Accept.**

We next analyze the invitee's decision to accept an invitation, assuming that every company follows its optimal strategy in the subsequent contribution stage. There are four types of invitations: from a high-status company to a high-status company (H→H), from a high-status company to a low-status company (H→L), from a low-status company to a high-status company (L→H), and from a low-status company to a low-status company (L→L). If the invitation is accepted, we represent the alliance type as HH, HL, LH, or LL, where the first letter represents the status of the initiator and the second that of the invitee.

In preparation for the analysis, we distinguish between the following two cases:

**Case LMB** (low marginal benefit)**.** The benefit to a contributor when three companies contribute is less than the cost to the contributor: $3bw < w$ or equivalently, $b < \frac{1}{3}$.

**Case HMB** (high marginal benefit)**.** The benefit to a contributor when three companies contribute is greater than the cost to the contributor: $3bw > w$ or equivalently, $b > \frac{1}{3}$.

This distinction is relevant because whether a high-status invitee accepts an invitation from a low-status initiator depends on whether the benefit to the invitee (when three companies contribute) exceeds the invitee's cost of contribution.

We note that, whether an invitee profits from accepting an invitation depends on (1) whether the invitee gains utility from status by contributing along with the initiator and (2) how many uncommitted companies will contribute along with the alliance in the contribution stage.

We must analyze separately the invitee's decision for each of the four possible invitation types. A low-status invitee will decline an invitation from a low-status initiator because there is neither utility gain from status nor anticipated subsequent contribution. Since a high-status company does not derive utility from status, a high-status invitee will accept an invitation from a low-status initiator only when the total number of contributors is sufficient to make a profit. This happens if and only if the low-status uncommitted company contributes subsequently (case HCS) and three contributors are sufficient to make a profit (case HMB). A low-status invitee will accept an invitation from a high-status initiator when her concern for status is high (case HCS), or when it is intermediate (case MCS) and $\gamma \geqslant \frac{(1-2b)w}{s_H - s_L}$. In these cases, the utility from contributing alongside a high-status initiator is sufficient to justify the low-status invitee's acceptance (even if no other company contributes in the following stage). If her concern for status is not sufficiently high (case MCS and $\gamma < \frac{(1-2b)w}{s_H - s_L}$), even if an HL alliance is formed, having a single high-status contributor in the alliance is not sufficient to induce the remaining low-status uncommitted company to contribute, thus the low-status invitee has no incentive to accept in the first place. Finally, a high-status invitee will accept an invitation from a high-status initiator both in HCS and MCS since the invitee anticipates the resulting HH alliance will induce contribution from both low-status companies, ensuring that the invitee derives a profit under Assumption 3.1b.

**Proposition 3.6.** *An L→L invitation will be declined; an L→H invitation will be accepted if and only if both cases HCS and HMB hold; an H→L invitation will be accepted if and only if case HCS, or case MCS with* $\gamma \geqslant \frac{(1-2b)w}{s_H - s_L}$ *holds; an H→H invitation will be accepted both in HCS and MCS.*

**Equilibrium.**

Finally, we characterize the equilibrium strategy of the initiator and the subgame-perfect equilibrium.

An initiator chooses to invite another company or not depending on whether the invitee will accept and whether the initiator can profit from the group contribution or gain utility from status. By Proposition 3.6, an L→L invitation will always be declined, so for a low-status initiator, inviting another low-status company is equivalent to not inviting any company. Hence, a low-status initiator can only meaningfully invite a high-status company. By Proposition 3.6, an L→H invitation is accepted if and only if both cases HCS and HMB hold. These conditions also ensure that the low-status initiator is compensated for the cost of contributing, resulting in the only equilibrium in which an alliance is formed with a low-status initiator.

**Theorem 3.1** (equilibrium with low-status initiator)**.** *If the initiator has low status,*

*(a) Under case MCS or LMB, in equilibrium, the low-status initiator is indifferent between inviting either company and not inviting; any invitee will decline the invitation; and no company contributes at any stage of the game;*

*(b) When cases HCS and HMB hold, in equilibrium the low-status initiator invites a high-status company, who accepts the invitation; the remaining low-status uncommitted company also contributes in the contribution stage; the remaining high-status uncommitted company does not contribute.*

Under either case HCS or MCS, a high-status initiator maximizes her utility by inviting the other high-status company who accepts the invitation (Proposition 3.6), inducing both low-status uncommitted companies to contribute in the following stage. In equilibrium, all four companies contribute. Note that in case HCS, a high-status initiator invites the other high-status company in equilibrium—if the initiator chose to invite a low-status company (off equilibrium), the invitee would also accept the invitation, resulting in an alliance of type HL. Nevertheless, a high status initiator prefers inviting the other high-status company over a low-status company, as this results in four companies contributing in total rather than three.

**Theorem 3.2** (equilibrium with high-status initiator)**.** *If the initiator has high status, in the unique equilibrium under either case HCS or case MCS, the initiator invites the other high-status company, who accepts the invitation; the two low-status uncommitted companies contribute in the following stage; and every company contributes to the social responsibility project.*

## 3.4 Experimental Design

We design a laboratory experiment to test the predictions from our theoretical model. We program the experiment using z-Tree (Fischbacher 2007). At the beginning of each session the instructions are displayed on each subject's screen and simultaneously read out loud by the experimenter. Each session consists of (1) ten rounds of a *corporate social responsibility game* (*CSR game*) that exactly conforms to our theoretical model, (2) a pro-social orientation task (3) a risk aversion task, (4) a debriefing questionnaire and (5) a demographics questionnaire. At the end of the experiment, each subject is paid based on her performance in the CSR game (one of the ten rounds of the CSR game played is randomly selected for payment), the pro-social orientation task, and the risk aversion task.

### 3.4.1 Corporate Social Responsibility Game

Subjects take the role of one of four companies who may contribute to a common fund to address a social responsibility problem the companies share. The process takes the form of a voluntary contribution game that follows the model in section 3.3. Subjects play ten rounds of the CSR game, and are randomly and anonymously matched into groups of four companies in each round. At the beginning of each round, each company is endowed with $w = 20$ points. Through subsequent decisions she may choose to either keep all 20 points in her private fund or to contribute all 20 points into a public fund for a *shared social responsibility project*. At the end of each round, the total amount in the public fund is multiplied by a factor of 1.6 and then distributed equally among the four companies in the group; that is, the marginal benefit $b = \frac{1.6}{4} = 0.4$. The return from the public fund is added to each company's private fund.

We conduct four different treatments varying in two dimensions—following a $2 \times 2$ design. The baseline game consists of homogeneous companies (in terms of status) and a single "contribution stage." All four companies in a group simultaneously decide whether to contribute their full endowment or not to contribute. Three variants on the baseline game arise from (1) the addition of an invitation stage at the beginning of the game, (2) inducement of status differences among the companies, or (3) both.

**Invitation Stage.** With an invitation stage, two companies may commit to contribute (form an alliance) prior to a contribution stage where all uncommitted companies simultaneously decide whether to contribute or not. The invitation stage begins with the computer randomly selecting one company in each group as the initiator. The initiator may choose to invite one other company to form an alliance or not to invite anyone. If the invitee accepts the invitation, the initiator and the invitee form an alliance and are committed to contribute their entire endowment to the fund. If the initiator does not invite any other company or if the invitee declines the invitation, they remain uncommitted and may each choose to contribute or not in the contribution stage. The companies who are neither the initiator nor the invitee are uncommitted. Capturing a main feature of the IAF, at the end of each round the computer displays to all the subjects in a group the list of all companies who have contributed. Also analogous to the IAF's website, in the treatments where an invitation stage is available and an alliance was formed, the computer lists those companies who are part of the alliance (initial contributors) separately and above the uncommitted companies who contributed later in the contribution stage.[13] The computer calculates each company's profit and displays it to all the companies in the same group, and then random re-matches subjects for the following round.

[13] A contributor is identified as "Company 1, high brand value" or "Company 3, low brand value," etc. The subjects always remain anonymous.

**Inducement of Status.** When heterogeneity in status is induced, the instructions include an explanation of the concept of "brand value" in business.[14] Then each company is randomly assigned a status, either "high brand value" or "low brand value", and the instructions indicate that a high-brand-value company has a "much higher" brand value than a low-brand-value company. In the treatments with induced status, each group consists of exactly two high-status companies and two low-status companies, and each company sees the status of the other companies in the same group. To sustain the difference in status throughout the session, high-status companies will see a golden star on their screen when the status is assigned and in all decision stages in the CSR game, while the low status subjects will not see this star. In addition, a subject's assignment to a high- or low-status company remains fixed throughout the experiment.

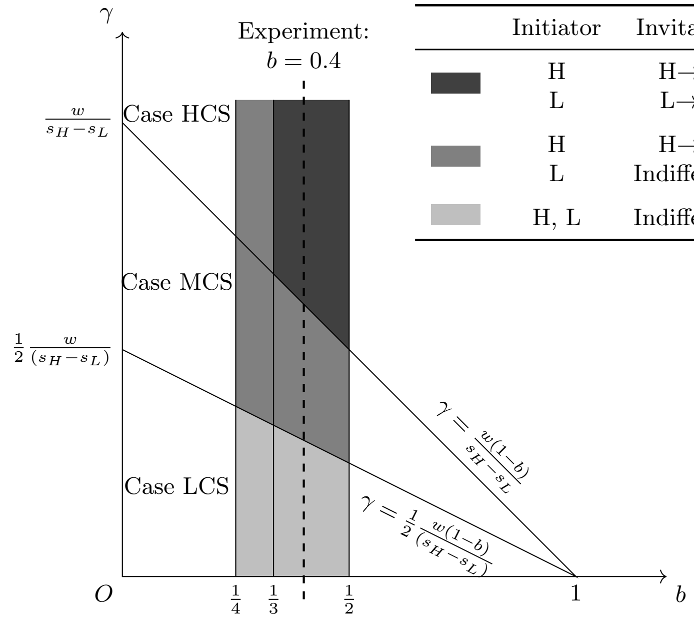
The four treatments of the CSR game are labeled: treatment B (for "baseline") which has neither induced status nor an invitation stage, treatment S ("status") with status inducement but no invitation stage, treatment I ("invitation") with no status inducement but with an invitation stage, and treatment F ("full") with both status inducement and an invitation stage. We follow a between subject design, i.e., each subject participates in only one treatment.

### 3.4.2 Additional Tasks

**Pro-social Orientation Task.** We conduct a dictator game (Forsythe et al. 1994) to assess subjects' pro-social orientation (Kraft et al. 2018). In this game, player 1 is initially endowed with 10 points and player 2 is endowed with 0 points. Player 1 decides how much of her endowment (between 0 and 10) to give player 2. Player 1's payoff is her initial endowment minus the amount given to player 2, and player 2's payoff is the amount received from player 1. Following the strategy method, all subjects make decisions as player 1 and then the computer randomly determines which subjects are assigned the role of player 1 and which subjects are player 2. Each player 1 is randomly and anonymously matched with a player 2 and player 1's decision is implemented for payment.

[14] The explanation is the following: "*Brand value* is the net present value of the estimated future cash flows attributable to the brand. According to *Forbes*, an American business magazine, a firm creates brand value if, for example, it invests in innovative ideas and research; it positively impacts the everyday lives of its customers; it maintains high standards of quality in its products; and it understands and addresses unique needs."

Figure 3.2: Full Model Equilibrium




| | Initiator | Invitation | Accepted? | Alliance | Contributors |
|---|---|---|---|---|---|
| | H | H→H | Yes | HH | HHLL |
| | L | L→H | Yes | LH | HLL |
| | H | H→H | Yes | HH | HHLL |
| | L | Indifferent | No | × | × |
| | H, L | Indifferent | No | × | × |

Equilibrium behavior in the full model as a function of the marginal benefit $b$ and concern for status $\gamma$. The legend on the right lists all possible equilibria in each shaded region. "x" indicates no alliance or no contribution.

**Risk Aversion Task.** This task aims to assess subjects' risk preferences (Holt and Laury 2002). Subjects choose between a fixed payoff of a specific number of points, ranging from 0.5 point to 7.5 points in 0.5 point increments, and a 50-50 chance lottery between a payoff of 0 or 8 points. Subjects' decisions are elicited following the strategy method (subjects make a decision for each possible situation) and then one of the decisions is randomly selected for payment.

After finishing these tasks, subjects complete a *debriefing questionnaire*, which includes questions about subjects' reasoning behind their decisions in the CSR game, and a *demographics questionnaire*.

## 3.5 Hypotheses

By Propositions 3.1, 3.2, and 3.3 we expect that no alliance is formed in treatment I and no contributions are made in treatments B, S, and I. In contrast, Theorems 3.1 and 3.2 demonstrate that in the full model, an alliance and positive contributions can arise.

**Hypothesis 3.1** (Comparison Between Treatments)**.** Alliances and contributions arise only in treatment F.

For the rest of the section we focus on treatment F. Figure 3.2 illustrates the equilibrium behavior in the full model given marginal benefit $b$ and concern for status $\gamma$. By Assumptions 3.1 and 3.2 we focus on the region $\frac{1}{4} < b < \frac{1}{2}$. Consider the shaded regions above the line $\gamma = \frac{1}{2}\frac{w(1-b)}{(s_H - s_L)}$ (dark gray ▬ and intermediate gray ▬ regions). These regions correspond to cases MCS (below the line $\gamma = \frac{w(1-b)}{s_H - s_L}$) and HCS (above the line $\gamma = \frac{w(1-b)}{s_H - s_L}$). If the initiator has high status, by Theorem 3.2, in these regions the initiator will invite the other high-status company and form an alliance of the type HH, and both low-status companies will contribute in the contribution stage. We list the contributors as HHLL (two high status and two low status). If the initiator has low status, in the dark gray (▬) region (corresponding to the intersection of cases HCS and HMB) by Theorem 3.1 the initiator invites a high-status company and forms an alliance of the type LH; the remaining low-status company contributes in the contribution stage. We list the contributors as HLL (one high status and two low status). In the intermediate gray (▬) region (corresponding to case MCS or case LMB) by Theorem 3.1 the low-status initiator is indifferent between inviting another company or not as no alliance will be formed in equilibrium and no company will contribute. Finally in the light gray (▬) region (corresponding to case LCS) by Proposition 3.5 no alliance is formed and no contribution is made in equilibrium. We detail the equilibrium corresponding to each scenario in the legend in Figure 3.2.

In the experiment, we choose marginal benefit $b = 0.4$ (shown with a dashed line in Figure 3.2) to allow for the maximum variety of equilibria to arise. In particular, we are most interested in the regions above the line $\gamma = \frac{1}{2}\frac{w(1-b)}{(s_H - s_L)}$ where our theory predicts that some companies will successfully form an alliance and, when they do, some other companies will contribute to the project along with the members of the alliance. We state our next hypothesis about the type of alliance that arises according to the equilibrium predictions for this region:

**Hypothesis 3.2** (Treatment F, $\frac{1}{4} < b < \frac{1}{2}$: Alliance Types). (a) A high-status initiator invites another high-status company, who accepts the invitation, and they form an alliance of the type HH.

(b) If the concern for status is high (case HCS), a low-status initiator invites a high-status company, who accepts the invitation, and they form an alliance of the type LH. If the concern for status is moderate (case MCS), a low-status initiator may or may not invite another company; if she does, the invitee declines the invitation and an alliance is not formed.

## 3.6 Experimental Results

We collected the data between September of 2017 and April of 2018 at a large public university in the Midwest of the United States.[15] Each session had 12 to 20 subjects, and lasted for about an hour. Subjects received \$0.25 per point plus a \$5 show-up fee, and earned on average \$14 (including the show-up fee). In total, 264 subjects participated in the experiment (60, 48, 52, and 104 subjects in treatments B, S, I, and F, respectively).

### 3.6.1 Comparison Between Treatments

We first compare the total group contribution across the four treatments. Recall that Hypothesis 3.1 predicts that only treatment F results in positive contributions. Interestingly, Table 3.1 shows that the group contributions are significantly different across the four treatments (Kruskal-Wallis test $p < 0.001$), but treatment I is not significantly different than treatment F.[16] Specifically, pairwise comparisons show that group contributions are not different across treatments B and S (11.1 vs. 12.0 points; Wilcoxon rank-sum test $p = 0.738$) nor across treatments I and F (26.6 vs. 25.1 points; $p = 0.625$). In addition, Figure 3.5 shows that the higher contribution in treatments I and F relative to B and S remains quite consistent as rounds in a session progress.[17] This result indicates that the two treatments where there is an invitation stage lead to significantly higher contributions than the two treatments that do not have an invitation stage—note that, treatments S and F have significantly different group contributions (12.0 vs. 25.2 points; $p < 0.001$) as do treatments B and I (11.1 vs. 26.6 points; $p < 0.001$). The results are confirmed using a linear regression of group contribution on treatment indicator variables with subject random effects controlling for round, as shown in Table 3.11. Taken together, these results demonstrate the importance of having an invitation stage (where an alliance can be formed) on group contributions.[18]

[15] The average age of the subjects was 21 years. 94% were undergraduate students and the remainder graduate students. 53% of the subjects were female. 62% identified themselves as white, 22% as Asian or Pacific Islander, and the remainder as Hispanic, black/African American, multiracial, or other. 50% declared business administration or economics as one of their majors.

[16] The Kruskal-Wallis equality-of-populations rank test tests the hypothesis that multiple samples are from the same population. It is a generalization of the two-sample Wilcoxon rank-sum test to multiple samples.

[17] Figure 3.5 also shows that the contribution levels decline as rounds pass by in a session in all treatments, a common result in public goods experiments (see for example Isaac and Walker 1988b, Kumru and Vesterlund 2010, Arbak and Villeval 2013).

[18] Also note that in both treatments I and F, the frequency of companies forming an alliance is quite high (43%) and companies' average profit is significantly higher when an alliance is established than when it

Table 3.1: Group Contribution by Treatment

| | Average group contribution (points) |
|---|---|
| Treatment B | 11.1 |
| Treatment S | 12.0 |
| Treatment I | 26.6 |
| Treatment F | 25.2 |
| | *p*-value |
| Test 1: B=S=I=F | <0.001 |
| Test 2: B=S | 0.738 |
| Test 3: I=F | 0.625 |

Average group contribution by treatment. Test 1: Kruskal-Wallis test on the hypothesis that the group contributions from the four treatments are from the same population. Tests 2 & 3: Wilcoxon rank-sum test on the hypothesis that the group contributions from the two treatments tested are from the same population.

**Lab Result 1.** *The existence of an invitation stage, where an alliance can be formed, significantly increases group contribution, regardless of whether companies are heterogeneous in status.*

Our finding that an alliance can be successfully formed and high contributions may arise in treatment I differs from Hypothesis 3.1, but is consistent with extant research on *sequential* public goods experiments with multiple players. For example, Güth et al. (2007) find in a repeated game setting that letting one subject in a group of four contribute early increases contribution. This result is attributed to *conditional cooperation* by which a subject becomes more cooperative the more cooperative the other subjects are. Arbak and Villeval (2013) find in a one-shot game with three players that some subjects persistently volunteer to contribute early. The study identifies three motives for this behavior: strategically distilling personal gains (anticipating others to reciprocate), altruism, and a positive social image associated with leadership. We believe that some of these explanations may also explain the high contributions in treatment I in our experiment. The result suggests that if companies were homogeneous in their brand values, implementing an invitation mechanism (allowing for the formation of an initial alliance) would be sufficient to ensure high contributions. However, as we observed in the case of the IAF, in most real examples companies are heterogenous in their brand values. In the next section we investigate whether and how the heterogeneity in status plays a role in the alliance formation and in the subsequent contribution by uncommitted companies. To do so we focus on treatment F, which has both heterogeneity in status and an invitation stage.

is not, confirming the importance that alliances have on contribution (see Table 3.12 for details).

Figure 3.3: Treatment F—Frequency of Each Type of Alliance

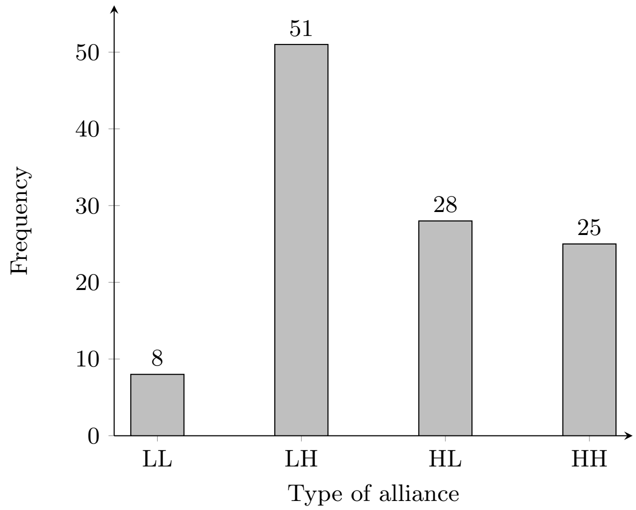



Table 3.2: Treatment F—Frequency of Each Type of Alliance

| Type of alliance | Frequency | Percent | Test 1: Freq. = 25% ($p$-value) |
|---|---|---|---|
| HH | 25 | 22.3 | 0.586 |
| HL | 28 | 25.0 | >0.999 |
| LH | 51 | 45.5 | <0.001 |
| LL | 8 | 7.1 | <0.001 |
| Test 2: HH = HL | | | $p = 0.784$ |
| Test 3: LH = LL | | | $p < 0.001$ |

Frequency of each alliance type. The percentage is relative to the total number of rounds in which an alliance is formed (43% of all rounds). Test 1 is a two-sided binomial probability test on the hypothesis that the type of alliance occurs with probability 25% among all rounds in which an alliance is formed. Test 2 is a two-sided binomial probability test on the hypothesis that types HH and HL occur with equal frequency. Test 3 is a two-sided binomial probability test on the hypothesis that the types LH and LL occur with equal frequency.

### 3.6.2 Treatment F: Interaction Between Alliance and Status

**Status Matters: The Type of Alliance That Arises.**

Hypothesis 3.2 predicts that only HH and LH types of alliance will arise, and only if the concern for status is sufficiently high. Table 3.2 and Figure 3.3 show the frequency of each alliance type across all rounds in the experiment.

Figure 3.3 shows that the four types of alliance are not equally likely to arise (frequencies: 25 HH, 28 HL, 51 LH, and 8 LL; Pearson's $\chi^2$-test $p < 0.001$). If subjects were oblivious to status, we would expect the four types of alliance to arise with the same probabiliy, each at 0.25. However, Test 1 in Table 3.2 shows that while the frequencies of the types HH and HL are not significantly different from 0.25 (0.22, $p = 0.587$ for HH; 0.25, $p > 0.999$ for HL), the frequency of the LH type is significantly higher than 0.25 (0.46, $p < 0.001$) and that of LL is significantly lower than 0.25 (0.07, $p < 0.001$). The fact that all alliance types are not equally likely to arise implies that subjects perceive a difference in status and incorporate it in their decisions leading to the formation of an alliance.

Pairwise comparisons of the frequency of alliance types with the same initiator status show that an alliance LH arises significantly more frequently than an alliance LL (51 vs. 8; binomial probability test $p < 0.001$, Test 3 in Table 3.2). This result is consistent with the theoretical prediction resulting from low-status companies' status-seeking behavior. While an LH alliance arises in equilibrium if the concern for status is sufficiently high (case HCS), an LL alliance does not arise in equilibrium in any case (HCS, MCS, or LCS). When we compare alliance types HH and HL, we find that their frequencies are not significantly different (25 vs. 28; $p = 0.784$, Test 2 in Table 3.2) while, by Hypothesis 3.2, we expect more HH types than HL types. Note that, in theory, both low- and high- status invitees would accept and invitation from a high-status initiator (case HCS); however, in theory a high-status initiator prefers to invite a high-status company over a low-status company as it results in a total of four contributors in equilibrium rather than three.

Overall, the results generally are consistent with Hypotheses 3.2: both HH and LH alliance types arise with a relatively high frequency (25 and 51 out of 112 rounds in which an alliance is formed). In addition, alliance type LL only arises 8 out of 112 rounds, consistent with Hypothesis 3.2. A result not predicted by our theory is that the frequency of alliance type HL is relatively high, 28 out of 112 rounds in which an alliance is formed. To understand this result, in the following sections we take a closer look at the initiator's, the invitee's, and the uncommitted company's decision making.

**Lab Result 2.** *The four types of alliance do not arise with equal probability. Alliance type LH arises significantly more often than LL; the frequencies of alliance types HH and HL are not significantly different.*

**Initiator's Decision.**

Theorems 3.1 and 3.2 predict that, with the marginal benefit $b$ we use in the experiment, if the concern for status is high (case HCS), then both the high- and low-status initiators will invite a high-status company; if the concern for status is moderate (case MCS), the high-status initiator will

Table 3.3: Treatment F—Initiator's Decision

| | High-status initiator | Low-status initiator |
|---|---|---|
| | (1) | (2) |
| Invite | 89 | 97 |
| Not invite | 39 | 35 |
| Test 1: Invite = Not invite ($p$-value) | <0.001 | <0.001 |
| Test 2: Both initiators equally likely to invite ($p$-value) | 0.481 | |
| Invite a high-status company | 43 | 79 |
| Invite a low-status company | 46 | 18 |
| Test 3: Either status equally likely ($p$-value) | 0.832 | <0.001 |
| Test 4: Each company equally likely ($p$-value) | 0.003 | 0.002 |

Test 1: Binomial probability test on the hypothesis that an initiator is equally likely to invite and not to invite. Test 2: Wilcoxon rank-sum test on the hypothesis that the high- and low-status initiators are equally likely to invite some company. Test 3: Binomial probability test on the hypothesis that the event of an initiator inviting a high-status company and the event of the initiator inviting a low-status company are equally likely. Test 4: Binomial probability test on the hypothesis that an initiator invites each of the other three companies equally likely, regardless of status.

invite another high-status company, and the low-status initiator will be indifferent between inviting anyone and not inviting, since any invitation from the low-status initiator will be declined.

**Invite or Not?** The upper half of Table 3.3 shows that both a high-status initiator and a low-status initiator are more likely to invite another company than not (frequency: 89 vs. 39 for high status, binomial probability test $p < 0.001$; 97 vs. 35 for low status, $p < 0.001$). In addition, the likelihood of making an invitation is no different across the two types of initiator (the high-status initiator invites 69.5% of the time and the low-status initiator 73.5%; Wilcoxon rank-sum test $p = 0.481$). Thus, the result is consistent with theoretical prediction for the case with high (and moderate) concern for status.

**Lab Result 3.** *Both the high- and low-status initiators are more likely to invite some company than not to invite any company.*

**Whom to Invite?** We first consider the invitation decision of a low-status initiator. We test the hypothesis that a low-status initiator invites a company of either status equally likely, i.e., given that the initiator invites some company, she chooses a high-status company with probability 0.5. Test 3 in column (2) of Table 3.3 shows that a low-status initiator invites high-status companies significantly more often than low-status companies (79 vs. 18 counts; binomial probability test $p < 0.001$). Since a group consists of two companies of each status (high and low), a low-status initiator chooses

Table 3.4: Treatment F—Likelihood of Being Invited by Status

| | Company is invited (Initiator has high status) | Company is invited (Initiator has low status) |
|---|---|---|
| | (1) | (2) |
| Company has high status | 0.492*** | 0.583*** |
| | (0.173) | (0.163) |
| Round | -0.039 | -0.051** |
| | (0.026) | (0.023) |
| Constant | -0.723*** | -0.859*** |
| | (0.172) | (0.171) |
| Observations | 384 | 396 |
| N. of subjects | 99 | 98 |

Probit regression with subject random effects. Robust standard errors in parentheses. Significance is denoted * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. *Company is Invited* is an indicator variable that takes the value 1 if the company is invited. *Company has high status* is an indicator variable that takes value 1 if the company's status is high and zero otherwise. The data includes companies who are not the initiator in the current round.

from a pool of two high-status companies and one low-status company. If the initiator disregards status, she will invite a high-status company twice as often as a low-status company. In contrast, Test 4 in column (2) of Table 3.3 shows that the low-status initiator's preference for inviting a high-status company is significantly high, even after taking into account group composition. A low-status initiator invites high-status companies significantly more than twice as often as she invites low-status companies (79 vs. 18 counts; binomial probability test $p = 0.002$). These results largely confirm that low-status initiators prefer to invite a high-status company over the other low-status company. Finally, Column (2) in Table 3.4 confirms that, among those companies who are not the initiator, high-status companies are more likely than low-status companies to receive an invitation from a low-status initiator (probit regression with subject random effects, $\beta = 0.583$, $p < 0.001$). Overall, these results are consistent with status-seeking behavior and the theoretical predictions in section 3.3.

We next turn to the invitation decision of a high-status initiator. Again, we begin by testing whether an initiator is equally likely to invite a company with either status. Test 3 in column (1) of Table 3.3 shows that the frequencies with which a high-status initiator invites a high- and low-status company are not significantly different (43 vs. 46 for high and low respectively, binomial probability test $p = 0.832$). However, this does not imply that the high-status initiator is oblivious to status. Since the group consists of two companies of each status, a high-status initiator chooses from a pool of two low-status companies and one high-status company. If the initiator was oblivious to status, rather than observing her inviting high- and low-status companies equally likely, we should observe low-status companies being invited twice as often as high-status companies. On the contrary, Test 4 in column (1) of Table 3.3 shows that when the initiator has high status the frequency of inviting a low-status company is significantly lower than twice the frequency of inviting a high-status company (binomial probability test $p = 0.003$). In addition, the regression in column (1) of Table 3.4 confirms that a high-status company is more likely than a low-status company to receive an invitation from a high-status initiator ($\beta = 0.531$, $p = 0.003$). Taken together, these results suggest that a high-status initiator distinctly recognizes the difference in status and is more likely to invite the high-status company when choosing among two low-status and one high-status companies.

**Lab Result 4.** *A low-status initiator is more likely to invite a high-status company than a low-status company. A high-status initiator, choosing between two low-status and one high-status companies, is more likely to choose the high-status company out of the three.*

Overall, the experimental results show that both high- and low- status initiators clearly distinguish between the high- and low-status companies. In the following sections, we explore the decisions of the invitee and the uncommitted companies.

**Invitee's Decision.**

We take two perspectives in the analysis. We first take the invitee's perspective, and look into whether an invitee is equally likely to accept an invitation from an initiator of either status. We then take the initiator's perspective, and investigate whether both types of invitees are equally likely to accept her invitation.

**Invitee's Perspective.** For a high-status invitee, accepting an invitation from either a high- or a low-status initiator is in line with the predictions of our theory. For a low-status invitee, accepting an invitation from a high-status initiator is in line with our theory, while accepting an invitation from a low-status initiator is not expected.

Table 3.5 presents the frequency with which an invitation is accepted for either status of the initiator and of the invitee separately. We observe that, while high-status invitees accept an invitation from an initiator with either status with relatively similar frequency, low-status invitees are (directionally) more likely to accept an invitation from a high-status company than an invitation from a

Table 3.5: Treatment F—Frequency of Acceptance

| | | Invitee status | | |
|---|---|---|---|---|
| | | High | Low | |
| | | (1) | (2) | Test 2 ($p$-value) |
| Initiator status | High | 58.1 % (43) | 60.9 % (46) | 0.794 |
| | Low | 64.6 % (79) | 44.4 % (18) | 0.117 |
| | Test 1 ($p$-value) | 0.487 | 0.237 | |

Frequency of acceptance. Number of observations reported in parentheses. Test 1: Wilcoxon rank-sum test on the hypothesis that an invitee accepts an invitation from an initiator of either status equally likely. Test 2: Wilcoxon rank-sum test on the hypothesis that an invitee of either status is equally likely to accept an invitation.

low-status company. These results are consistent with the theoretical predictions, particularly for the case where concern for status is high. Test 1 shows that the difference between accepting an invitation from a high- or low-status initiator is not statistically significant for an invitee of either status (58.1% H→H vs. 64.6% L→H, Wilcoxon rank-sum test $p = 0.487$ for a high-status invitee; 60.9% H→L vs. 44.4% L→L, $p = 0.237$ for a low-status invitee). The results are confirmed when we regress an indicator variable which takes value 1 if an invitee accepts an invitation on the initiator's status (probit regression with subject random effects $\beta = 0.129$, $p = 0.533$ for high-status invitees; $\beta = 0.735$, $p = 0.584$ for low-status invitees; see Table 3.13).

**Lab Result 5.** *A low-status invitee is (directionally) more likely to accept an invitation from a high-status initiator than from a low-status initiator. A high-status invitee is equally likely to accept an invitation from an initiator of either status.*

**Initiator's Perspective.** A high-status initiator's invitation being accepted by an invitee of either status is in line with our theory. On the other hand, according to theory, a low-status initiator's invitation is only accepted by a high-status invitee. Consistent with the theoretical prediction, Table 3.5 shows that a high-status initiator's invitation is equally likely to be accepted by a company of either status (58.1% vs. 60.9%), while a low-status initiator's invitation is (directionally) more likely to be accepted by a high-status invitee (64.6% vs. 44.4%). Test 2 in Table 3.5 shows no significant difference for an initiator of either status (Wilcoxon rank-sum test $p = 0.794$ when the initiator is of high status; $p = 0.117$ when the initiator is of low status). Table 3.13 shows similar results with probit regressions with subject random effects.

**Lab Result 6.** *A high-status initiator's invitation is equally likely to be accepted by an invitee of either status. A low-status initiator's invitation is directionally more likely to be accepted by a high-status invitee than by a low-status invitee.*

**Summary of the Invitation Stage.**

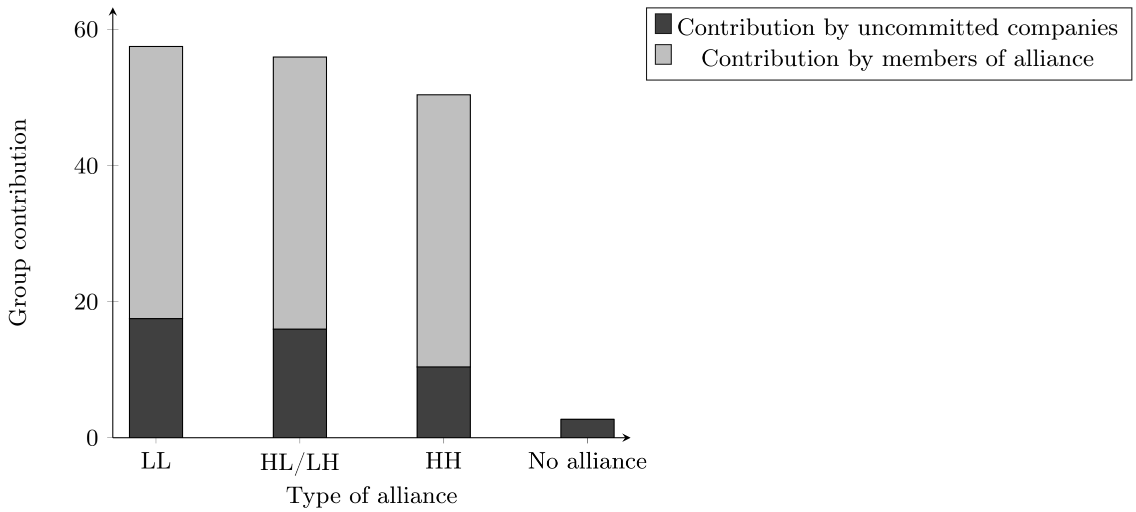
By analyzing the invitation stage, we find (1) evidence of a relatively high concern for status, and (2) results that are consistent with status-seeking behavior of low-status companies. High-status initiators are equally probable to invite a high- or low-status company, although since the high-status initiator is choosing from a pool of two low-status companies and just one high-status company, we observe a statistically significant preference for inviting the high-status company. Nonetheless, the result that a high-status initiator invites companies of either status with equal probability explains our surprising observation that the frequencies of the HL and HH types of alliance are not statistically different: a high-status initiator makes H→L and H→H invitations equally probable, which the respective invitee is equally likely to accept. Then, a follow-up question is, why does a high-status initiator not show a stronger preference for inviting a high-status company? In theory, the high-status initiator prefers to invite another high-status company because the resulting HH alliance will induce both remaining uncommitted companies of low status to contribute. In comparison, an HL alliance will only induce the one remaining low-status uncommitted company to contribute (Proposition 3.4). To answer this question, in the next section we explore whether uncommitted companies behave according to theory in the contribution stage.

**Contribution Stage.**

We now look at the contributions of uncommitted companies in the contribution stage. Note that we consider three alliance types HH, HL/LH, and LL. This is because the uncommitted companies who are not part of an alliance cannot distinguish which of the two members of the alliance is the initiator and which is the invitee. Therefore, alliances HL and LH are indistinguishable for the uncommitted companies.

**Effect of Alliance on Total Contribution.** Proposition 3.4 predicts that if the concern for status is high (case HCS), an HH alliance will induce both low-status uncommitted companies to

Figure 3.4: Treatment F—Group Contribution by Alliance Type




Average group contribution when each type of alliance is formed and when no alliance is formed. The bottom part of the bar corresponds to the average contribution by uncommitted companies–two companies when there is an alliance and four companies when there is no alliance (dark gray); the top part of the bar corresponds to the contribution by the two alliance members, when formed (light gray).

contribute in the contribution stage; an HL/LH alliance will induce the low-status but not the high-status uncommitted company to contribute; and an LL alliance will not induce any high-status uncommitted company to contribute. If the concern for status is moderate (case MCS), an HH alliance will induce both low-status uncommitted companies to contribute; no other alliance type will be able to induce any uncommitted company to contribute. Finally, Proposition 3.4 predicts no contributions when an alliance was not formed.

Figure 3.4 shows the total group contribution under each alliance type. The total contribution by the uncommitted companies is indicated in the bottom part of the bars (dark gray), while the total contribution by the alliance members is indicated in the top part of the bars (light gray). We note a few things from the figure. First, as discussed in section 3.6.1, the group contributes significantly more when there is an alliance than when there is not. Second, the average contribution by uncommitted companies is significantly higher when an alliance is formed than when no alliance is in place. The average contribution by an uncommitted company is 7.41 points when there is an alliance, and it is 0.676 points when an alliance was not formed (Wilcoxon rank-sum test $p < 0.001$). Table 3.6 confirms this result with a regression of a dummy variable that takes the value one if the uncommitted company contributes and zero otherwise, on a dummy variable that takes the value one when an alliance was formed and zero otherwise. We observe that both high-

Table 3.6: Treatment F—Contribution by Uncommitted Companies when Alliance Is Formed

| | Contribution by uncommitted company | |
|---|---|---|
| | High-status | Low-status |
| | (1) | (2) |
| Alliance formed | 2.380*** | 1.461*** |
| | (0.433) | (0.312) |
| Round | -0.118** | -0.111*** |
| | (0.053) | (0.041) |
| Constant | -1.879*** | -1.693*** |
| | (0.321) | (0.328) |
| Observations | 391 | 425 |
| N. of subjects | 52 | 52 |

Probit regression with subject random effects. Robust standard errors in parentheses. Significance is denoted * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The variable *alliance formed* is an indicator variable which takes value 1 if an alliance is formed.

and low-status uncommitted companies are significantly more likely to contribute when an alliance is formed than when an alliance was not formed ($\beta = 2.380, p < 0.001$ for high-status companies; $\beta = 1.461, p < 0.001$ for low-status companies). Third, the three types of alliance are equally effective in inducing contribution by uncommitted companies. The average contribution by the two uncommitted companies is 5.2 points when the alliance is of type HH, 7.97 points for HL/LH, and 8.75 points for LL (Kruskal-Wallis equality-of-populations rank test $p = 0.2991$).

**Lab Result 7.** *The total contribution by uncommitted companies in the contribution stage is significantly higher when an alliance is formed. Every alliance type is equally effective in inducing the uncommitted companies to contribute.*

The fact that all alliance types are equally effective in inducing contributions by uncommitted companies suggests that uncommitted high-status companies do contribute in the contribution stage when an alliance was formed. This is an unexpected result not predicted by our theory, as high-status companies do not derive any additional utility from status and therefore have no incentive to contribute at the last stage of the game. In the next subsections we explore this result more in-depth and we uncover the behavioral driver of this result.

**Effect of Alliance Type on Individual Contribution.** Proposition 3.4 predicts that with moderate concern for status (case MCS) the low-status uncommitted company will contribute only to an alliance of the type HH, and with high concern for status (case HCS) the low-status uncommitted

Table 3.7: Treatment F: Contribution by Uncommitted Company with Alliance

| | | Status of uncommitted company | | |
|---|---|---|---|---|
| | | High | Low | |
| | | (1) | (2) | Test 2 ($p$-value) |
| Alliance type | HH | | 5.20<br>(50) | |
| | HL/LH | 9.37<br>(79) | 6.58<br>(79) | 0.075 |
| | LL | 8.75<br>(16) | | |
| | Test 1 ($p$-value) | 0.822 | 0.407 | |

Average contribution by an uncommitted company when each alliance type is in place. Number of observations reported in parentheses. Test 1: Wilcoxon rank-sum test on the hypothesis that the uncommitted company contributes equally likely to the two types of alliance the uncommitted company may face. Test 2 Wilcoxon rank-sum test on the hypothesis that high- and low-status uncommitted companies are equally likely to contribute when an HL/LH alliance is in place.

company will contribute both to an alliance of the type HH and HL/LH (since either alliance type has at least one high-status member). A high-status uncommitted company is not expected to contribute regardless of the alliance type.

Table 3.7 shows the average contribution by high- and low-status uncommitted companies when the different types of alliance were formed. Low-status uncommitted companies contribute equally to the two types of alliance, consistent with a high concern for status (5.20 points for HH vs. 6.58 points for HL/LH; Wilcoxon rank-sum test $p = 0.407$). We confirm the result by regressing an indicator variable that takes value one if the uncommitted company contributes and zero otherwise on an indicator variable that takes value one if the alliance is of the type HH (see Table 3.14, $\beta = -0.144, p = 0.714$).

A result not predicted by the theory is that high-status uncommitted companies also contribute in the contribution stage. Table 3.7 shows that a high-status uncommitted company's average contribution is 8.75 points when an LL alliance is formed vs. 9.37 points when an HL/LH alliance is formed (Wilcoxon rank-sum test $p = 0.822$). We confirm the result with a regression of an indicator variable that takes value one if the uncommitted company contributes and zero otherwise on an indicator variable that takes value one if the alliance is of the type LL (see Table 3.14, $\beta = -0.244; p = 0.538$). This result shows that high-status uncommitted companies are equally likely to contribute to either an alliance LL or LH/HL, which is consistent with the fact that a high-status company's decision is not driven by status-seeking preferences.

Table 3.8: Treatment F—Uncommitted Company Contribution by Status

| | Contribution by uncommitted company (when alliance HL/LH was formed) |
|---|---|
| Own status high | 0.747$^{*}$ |
| | (0.410) |
| Round | -0.149$^{***}$ |
| | (0.053) |
| Constant | -0.125 |
| | (0.353) |
| Observations | 158 |
| N. of subjects | 73 |

Probit regression with subject random effects. Robust standard errors in parentheses. Significance is denoted $^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$. The variable *Own status high* is an indicator variable which takes value 1 if the company has high status.

Table 3.7 shows that high-status uncommitted companies not only contribute in the contribution stage, but also that they contribute even *more* than low-status uncommitted companies when they face the same type of alliance. To study this, we focus on the alliance HL/LH, which may be faced by both high- and low-status uncommitted companies. Test 2 in Table 3.7 shows that the average contribution by a high-status uncommitted company is 9.37 points vs. 6.58 points by the low-status company (Wilcoxon rank-sum test $p = 0.075$). In Table 3.8 we regress an indicator variable that takes value one if the uncommitted company contributes (considering only cases where an alliance of the type HL/LH was formed) and zero otherwise on an indicator variable that takes value one if the company has high status and zero otherwise. The regression confirms that a high-status company contributes significantly more often than a low-status company ($\beta = 0.747, p = 0.068$).

**Lab Result 8.** *When an alliance of the type HL/LH has been established the high-status uncommitted company is significantly more likely to contribute than the low-status uncommitted company.*

The experimental results show that all alliance types are equally effective at inducing later contributions by uncommitted companies in the contribution stage. In fact, while the theory predicts that only low-status uncommitted companies contribute in the contribution stage, the results show that high-status uncommitted companies contribute as well (and even *more* than low-status companies). This explains why the high-status initiator has no reason to favor inviting one type of company over the other. Both high- and low-status invitees are equally likely to accept an invitation from a high-status initiator (consistent with our theoretical prediction when concern for status is high), and both HL and HH alliances induce the same expected contribution from uncommitted companies.

Table 3.9: Treatment F—Effect of Pro-sociality on Uncommitted Companies' Contribution

| | Contribution by uncommitted companies (when alliance was formed) | |
|---|---|---|
| | High status | Low status |
| | (1) | (2) |
| Pro-social | 1.100** | 1.038 |
| | (0.550) | (0.773) |
| Round | -0.160** | -0.210*** |
| | (0.070) | (0.068) |
| Constant | 0.422 | -0.463 |
| | (0.350) | (0.483) |
| Observations | 95 | 129 |
| N. of subjects | 39 | 46 |

Probit regression with subject random effects. Robust standard errors in parentheses. Significance is denoted * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$. *Pro-social* is an indicator variable taking value 1 if the company is pro-social and zero otherwise.

The question remaining unanswered is: Why do high-status uncommitted companies contribute in the contribution stage? A high-status company does not gain utility from status by contributing along with the alliance members since she already has the highest status in the group. We conjecture that having a high status may be perceived as having a higher "moral responsibility" to contribute along with others. If this is the case, we would expect that more *pro-social* individuals will be more likely to experience this sense of higher moral responsibility of joining others' efforts. To explore the idea, we next study whether pro-social preferences provide a good explanation for the high-status companies' behavior.

**Effect of Pro-sociality.** We elicit a measure of subjects' pro-sociality based on their decision in the additional pro-social orientation task. Specifically, we create a measure of pro-sociality by taking the amount of points a subject chooses to give the other subject in the dictator game. We define that a subject is "pro-social" if the amount she sends in the dictator game is greater than or equal to the 75th percentile (= 4 points) of the pro-sociality measure, and we create a binary variable, *Pro-social*, which takes value one if the subject is pro-social and zero otherwise.[19]

The experimental results show that pro-sociality is a good predictor of contributions by high-status uncommitted companies when an alliance was formed, but not of contributions by low-status uncommitted companies. In Table 3.9 we consider uncommitted companies in the contribution stage

[19]The results are qualitatively the same if we take the 65th and 85th percentiles.

when an alliance has been established, and we regress an indicator variable that takes value one if the uncommitted company contributes and zero otherwise, on the indicator variable Pro-social. The regression shows that pro-sociality drives contribution by a high-status uncommitted company (column (1), $\beta = 1.100, p = 0.045$) but not by a low-status uncommitted company (column (2), $\beta = 1.038, p = 0.179$). This result suggests that, when an alliance is in place, high-status uncommitted companies' pro-sociality gets "triggered" leading to high contributions. On the other hand, low-status uncommitted companies' decisions appear to be less driven by their inherent pro-sociality (possibly overshadowed by the status-seeking behavior). Note that in the same way that high-status uncommitted companies are *overall* more likely to contribute when an alliance is in place (Table 3.6), *pro-social* high-status uncommitted companies also contribute more when an alliance was established than when it was not ($\beta = 2.025$, $p = 0.002$ in Table 3.15). This suggests that pro-social companies may also be driven by a desire to reciprocate the initiator's and invitee's contributions.

To further explore whether high-status inherently reinforces the effect of pro-sociality on companies' decisions (or whether the invitation stage also plays a role) we test whether the effect is also present in treatment S, where there is no invitation stage and alliances cannot be established. We find that in treatment S, pro-sociality is not correlated with higher contributions by either a high- or a low-status company ($\beta = 0.104$, $p = 0.826$ for high status; $\beta = 0.408$, $p = 0.154$ for low status—see Table 3.16). This suggests that experiencing the invitation stage elicits the sense of "moral responsibility" of the high-status company. When the invitation stage is not present, pro-sociality does not have an effect on companies' decisions.

**Lab Result 9.** *When an alliance is in place, pro-sociality drives contribution by a high-status uncommitted company. Pro-sociality does not have a significant effect on high-status companies' contributions in treatment S, which has no invitation stage and therefore no alliance.*

Our analysis sheds light on the question of why high-status uncommitted companies contribute. Given that other companies have committed to contribute, a sense of "moral responsibility" drives pro-social high-status uncommitted companies to contribute. This is consistent with what we observe in the IAF example: the list of contributors on the IAF's website shows that a number of companies with a high brand value also contribute *after* the alliance has been established. Our results suggests that they do so out of a sense of higher moral responsibility as a high-status company, after having seen that other companies contributed to the IAF.

**Effect of Alliance on Profit.**

We review average round profit by role and status, separately when an alliance is formed and when an alliance is not formed. Considering all roles combined, the results confirm that establishing an alliance leads to significantly higher profits (28.2 points vs. 20.4 points for high- and low-status combined, $p < 0.001$—see Table 3.17). Interestingly, those who benefit the most from establishing an alliance are low-status initiators (column (2)). According to the theory, alliances initiated by a high-status company should be more profitable as they lead to four companies contributing rather than three when the initiator is a low-status company. However, since in the lab uncommitted high-status (pro-social) companies also contribute when an alliance is established, low-status initiators can benefit highly from forming an alliance.

**Discussion of Alternative Explanations**

While subjects' behavior throughout the experiment clearly demonstrates the influence of status in their decision making, we also elicited subjects' own perception of whether status played a role in their decisions. A significant number of subjects in treatment F perceived that status influenced their decisions. Specifically, in the debriefing questionnaire at the end of the session, 27.2% of the subjects in treatment F said they agreed or strongly agreed with the statement "The brand values influenced my decision-making" (46.4% of them had high status and 53.6% had low status; binomial probability test $p = 0.851$). This suggests that status not only affected subjects' decisions (as is reflected in the experimental results), but it was also one of the factors that subjects *perceived* as a behavioral driver. In addition, this result provides further support for the status inducement procedure used in the experiment.

While the theory predicts that both high- and low-status companies can initiate profitable alliances (as long as the concern for status is sufficiently high), strategic uncertainty may render it risky to form an alliance. In this case, risk aversion may affect initiators' and invitees' decisions. To test this, we elicit a measure of subjects' risk aversion based on their decisions in the additional risk aversion task.[20] We regress companies' decisions in each role on the risk aversion metric and

[20]We measure a subject's risk aversion by the number of scenarios in which the subject chooses the fixed

observe that the coefficient of the risk aversion metric is not significant for any role and any status companies (see Table 3.18). This result suggests that risk aversion does not seem to be a main behavioral driver in this setting.

### 3.7 Conclusion and Managerial Implications

We study the formation of an industrial alliance to fund a corporate social responsibility project that benefits a group of companies which differ in their brand value. Recognizing the non-exclusivity of the benefits from the project, we model the problem as a public goods game. We incorporate two salient features from the IAF motivational example. First, we consider an invitation stage (where an industry alliance can be formed) to be followed by a contribution stage where non-alliance members can contribute as well. Second, we assume that companies are heterogeneous in their status (capturing different brand values) and that low-status companies gain utility from being associated with high-status companies by contributing along with them. Our model shows that both the invitation mechanism and the heterogeneity in status and status-seeking behavior of low-status companies are necessary for the successful formation of an alliance and for contributions to arise in equilibrium. Whether and how the group of companies form an alliance and contribute to the project depend critically on the extent to which the companies are concerned about status and the marginal benefit each company derives from the project. In particular, within a focal range of the marginal benefit, higher concern for status enables the companies to form an alliance in more scenarios and increases the equilibrium group contribution to the project.

We conduct a laboratory experiment to test the predictions of our model. We find that the key to high contribution is the opportunity to form an alliance, whether there is heterogeneity in status or not. When both the invitation stage and heterogeneity in status are present, we confirm the status-seeking behavior of low-status companies. A low-status initiator strongly prefers to invite a high-status company than to invite a low-status company, and a low-status invitee is directionally more likely to accept an invitation from a high-status initiator than from a low-status initiator. A result not predicted by our theory is that, when an alliance has been established, not only low-status companies contribute in a later contribution stage (a result consistent with status-seeking behavior)

payoff over the random payoff. We create a binary variable for risk aversion for analysis: a subject is "risk averse" if the number of fixed options she chooses is greater than or equal to the 75th percentile of the distribution (= 8.5).

but high-status companies contribute as well. This is surprising because high-status companies do not derive utility from status association (they already *have* high status) and contributing in the final contribution stage cannot result in monetary gains. Our experimental design allows us to identify the underlying behavioral driver behind this result: pro-social high-status companies develop a sense of moral responsibility that drives them to contribute when an alliance is in place. The finding is consistent with the contributions to the IAF—a number of high-brand-value companies also contribute to the IAF *after* the initial round of contribution.

Our results have important managerial implications for companies seeking to address a common social responsibility problem. Both our theoretical model and our experiment confirm that forming an alliance of initial contributors is key to reach an overall high contribution. Then a follow-up question is, how can companies successfully build such alliances? While in the case of the IAF all the alliance members were companies with high-brand value ("status"), our theory and our experimental results suggest that a low-brand-value company can also start an alliance that will be equally effective in inducing contribution by non-alliance members. To increase the chances of forming an alliance the low-brand-value company should approach some high-brand-value companies, who are more likely to agree to jointly initiate the social responsibility project. In addition, while the IAF was established by a high brand value company inviting other companies with high brand value to join the alliance, our experimental results indicate that companies with high brand value can also initiate a successful alliance by inviting companies with low brand value to join. Establishing the alliance will induce other pro-social companies with high brand value to contribute out of a sense of moral responsibility to be part of the initiative.

## Appendix 3.A Brand Value and Ranking of Contributors to IAF

Table 3.10: Contributors to IAF in Ranking of World's Most Valuable Brands by *Forbes*

| *Forbes* Ranking | Contributor to IAF | Brand Value ($b) |
|---|---|---|
| | Initial Contributors | |
| 1 | Apple | 104.3 |
| 7 | General Electric | 34.2 |
| 31 | Hewlett-Packard | 15.3 |
| 8 | Intel | 30.9 |
| 2 | Microsoft | 56.7 |
| | Subsequent Contributors | |
| $> 100$ | 3M | - |
| $> 100$ | ABB | - |
| $> 100$ | Baxter Healthcare Corporation | - |
| $> 100$ | Becton, Dickinson and Company | - |
| $> 100$ | Best Buy | - |
| 84 | Dell Inc. | 6.4 |
| 84 | EMC Corporation | 6.4 |
| 44 | Ford Motor Company | 10.6 |
| 5 | Google | 47.3 |
| $> 100$ | GM | - |
| 20 | Honda | 21.1 |
| 4 | IBM | 50.7 |
| $> 100$ | Juniper Networks | - |
| $> 100$ | Lenovo | - |
| $> 100$ | Lockheed Martin | - |
| 72 | Nokia | 7.0 |
| $> 100$ | NXP Semiconductors | - |
| $> 100$ | On Semiconductor | - |
| $> 100$ | Qualcomm | - |
| 80 | Sony Corporation | 6.6 |
| $> 100$ | Tenneco | - |
| $> 100$ | Texas Instruments Inc. | - |

The ranking and estimated brand value of every contributor to the IAF in the World's Most Valuable Brands ranking by *Forbes* magazine in 2013 (Forbes 2013), the year the IAF was launched. The ranking includes only the top one hundred brands. Dell acquired EMC Corporation in 2016.

## Appendix 3.B Proofs

*Proof of Proposition 3.1.* For company $i \in N$, given $c_{-i}$,

$$\pi_i(0, c_{-i}) = w + b \sum_{k \in N-i} c_k \tag{3.4}$$

$$\pi_i(w, c_{-i}) = bw + b \sum_{k \in N-i} c_k. \tag{3.5}$$

By Assumption 3.1a, $\pi_i(0, c_{-i}) > \pi_i(w, c_{-i})$. Hence $c_i = 0, \forall i \in N$ in equilibrium. □

*Proof of Proposition 3.2.* For high-status company $i \in N_H$, $f_{ik}(c_i, c_k) = \gamma \cdot 0 \cdot \mathbb{1}_{\{c_i c_k > 0\}} = 0, \forall k \in N - i$. Hence high-status company $i$ maximizes $\pi_i(c_i, c_{-i})$ and as in the proof of Proposition 3.1 will not contribute. For low-status company $i \in N_L$, $f_{ik}(c_i, c_k) = \gamma \cdot (s_H - s_L)^+ \cdot 0 = 0, \forall k \in N_H$ (because neither high-status company contributes) and $f_{ik}(c_i, c_k) = \gamma \cdot 0 \cdot \mathbb{1}_{\{c_i c_k > 0\}} = 0, \forall k \in N_L - i$. Therefore low-status company $j$ also maximizes $\pi_i(c_i, c_{-i})$ and will not contribute. In equilibrium no company contributes. □

*Proof of Proposition 3.3.* As in the proof of Proposition 3.1 an uncommitted company will not contribute in the contribution stage. In the invitation stage, if an initiator $i$ invites a company $j$, the invitee $j$'s profit is

$$\pi_j(c_j, c_{-j}) = \begin{cases} w, & \text{if invitee } j \text{ declines so that } c_j = 0 \\ 2bw, & \text{if invitee } j \text{ accepts so that } c_j = w \end{cases} \tag{3.6}$$

By Assumption 3.2, $w > 2bw$, so invitee $j$ will decline the invitation. Therefore the initiator's profit is equal to $w$ regardless of whether she invites some company (in which case the invitee will decline the invitation) or not to invite any one. In equilibrium no company will contribute to the project. □

*Proof of Proposition 3.4.* We first look at the low-status uncommitted company's problem. For $i \in N_L$, company $i$'s utility is

$$u_i(c_i, c_{-i}) = w - c_i + b \sum_{k \in N} c_k + \gamma(s_H - s_L) \sum_{k \in N_H} \mathbb{1}_{\{c_i c_k > 0\}}. \tag{3.7}$$

If both high-status companies contribute, i.e., $c_j = w, \forall j \in N_H$, then

$$u_i(0, c_{-i}) = w + b \sum_{k \in N-i} c_k \tag{3.8}$$

$$u_i(w, c_{-i}) = b\left(w + \sum_{k \in N-i} c_k\right) + 2\gamma(s_H - s_L). \tag{3.9}$$

The low-status company $i$ contributes (i.e., $c_i = w$) if and only if $u_i(w, c_{-i}) > u_i(0, c_{-i})$, or $\gamma > \frac{1}{2}\frac{(1-b)w}{s_H - s_L}$, i.e., case HCS or MCS. If exactly one high-status company contributes, i.e., without loss of generality $c_1 = w, c_2 = 0$, then

$$u_i(0, c_{-i}) = w + b \sum_{k \in N-i} c_k \tag{3.10}$$

$$u_i(w, c_{-i}) = b\left(w + \sum_{k \in N-i} c_k\right) + \gamma(s_H - s_L) \tag{3.11}$$

The low-status company $i$ contributes if and only if $u_i(w, c_{-i}) > u_i(0, c_{-i})$, or $\gamma > \frac{(1-b)w}{s_H - s_L}$, i.e., case HCS. If neither high-status company contributes, i.e., $c_j = 0, \forall j \in N_H$, then

$$u_i(0, c_{-i}) = w + b \sum_{k \in N-i} c_k \tag{3.12}$$

$$u_i(w, c_{-i}) = b\left(w + \sum_{k \in N-i} c_k\right). \tag{3.13}$$

By Assumption 3.1a $b < 1$, so $u_i(w, c_{-i}) < u_i(0, c_{-i})$. The low-status company $i$ will not contribute.

The high-status uncommitted company's problem is identical to the problem in the proof of Proposition 3.1. Hence a high-status uncommitted company will not contribute. □

*Proof of Proposition 3.5.* By Proposition 3.4 with low concern for status (case LCS) no uncommitted company will contribute in the contribution stage. A high-status invitee will get a utility of $2bw$ by accepting any invitation and a utility of $w$ by declining. By Assumption 3.2 she will decline the invitation. A low-status invitee will get a utility of $2bw$ by accepting an invitation from a low-status initiator and get $w$ by declining it. By Assumption 3.2 she will decline the invitation. If the initiator is of high status, a low-status invitee will get $2bw + \gamma(s_H - s_L)$ by accepting the invitation and get $w$ by declining it. Therefore the low-status invitee will accept the invitation if $2bw + \gamma(s_H - s_L) > w_L$

and decline it otherwise. Nevertheless if $2bw + \gamma(s_H - s_L) > w_L$ so that the low-status invitee will accept an invitation from a high-status initiator, a high-status initiator will get $2bw$ by inviting a low-status company and get $w$ by not inviting any one. By Assumption 3.2 $w > 2bw$ so the high-status initiator will never invite a low-status company. In summary, no alliance will be formed and no contribution will be made in equilibrium. □

*Proof of Proposition 3.6.* The invitee makes the decision to accept or to decline an invitation based on the utility that each choice results in. By Proposition 3.4, a high-status uncommitted company will not contribute whether an alliance is formed or not. Next, for each of the four types of invitation, we analyze the effect of the formation of the alliance on the low-status uncommitted companies' decision in the contribution stage and whether the invitee should thus accept or decline the invitation.

**L→L** By Proposition 3.4, if the alliance is not formed, the low-status companies will not contribute. Therefore the utility of the low-status invitee if she accepts the invitation will be $2bw$ and if she declines will be $w$. By Assumption 3.2 she will decline the invitation.

**L→H** Under case MCS by Proposition 3.4 a low-status uncommitted company will not contribute whether the alliance of the type LH is formed or not. Therefore the utility of the high-status invitee if she accepts the invitation will be $2bw$ and if she declines will be $w$. By Assumption 3.2 she will decline the invitation.

Under case HCS by Proposition 3.4 a low-status uncommitted company will contribute if and only if the alliance of the type LH is formed. Therefore the utility of the high-status invitee if she accepts the invitation will be $3bw$ and if he declines will be $w$. Therefore she accepts the invitation under case HMB and declines under case LMB.

**H→L** Under case MCS by Proposition 3.4 a low-status uncommitted company will not contribute whether the alliance of the type H→L is formed or not. Therefore the utility of the low-status invitee if she accepts the invitation will be $2bw + \gamma(s_H - s_L)$ and if she declines will be $w$. Therefore she accepts if and only if $2bw + \gamma(s_H - s_L) \geqslant w_L$, or $\gamma \geqslant \frac{(1-2b)w}{s_H - s_L}$.

Under case HCS by Proposition 3.4 a low-status uncommitted company will contribute if and only if the alliance of the type H→L is formed. Therefore the utility of the low-status invitee if she accepts the invitation will be $3bw + \gamma(s_H - s_L)$ and if she declines will be $w$. Therefore she accepts if and only if $3bw + \gamma(s_H - s_L) \geqslant w$, or $\gamma \geqslant \frac{(1-3b)w}{s_H - s_L}$, which case HCS implies. Hence she will accept the invitation.

**H→H** Under both cases MCS or HCS the low-status uncommitted companies will contribute if and only if the alliance of the type LL is formed. The utility of the high-status invitee if she accepts the invitation will be $4bw$ and if she declines will be $w$. By Assumption 3.1b the high-status invitee will accept the invitation. □

*Proof of Theorem 3.1.* Under case MCS or LMB by Proposition 3.6 an invitee of either status will decline an invitation from the low-status initiator. Therefore the initiator is indifferent between inviting any company or not inviting any company. If she invites a company, the invitee will decline. By Proposition 3.4 no company will contribute in the contribution stage.

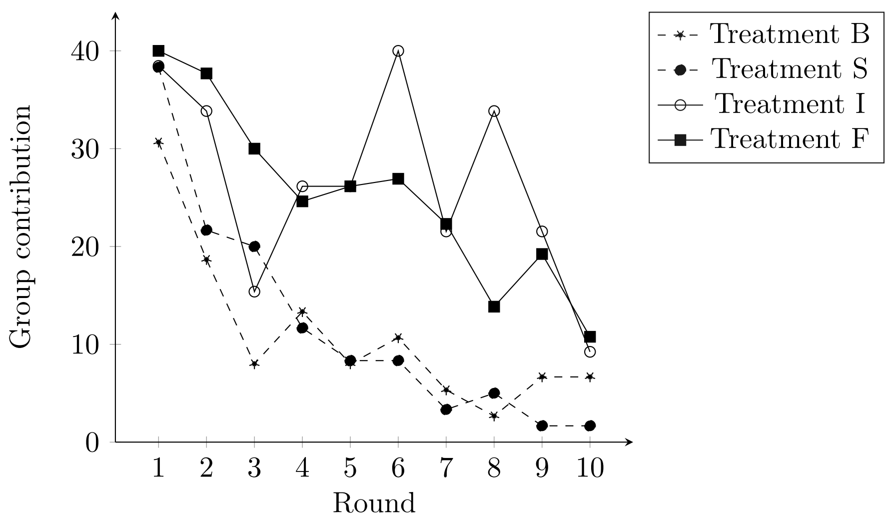
Under the intersection of cases HCS and HMB by Proposition 3.6, while a low-status invitee will still decline an invitation from the low-status initiator, a high-status invitee will accept. In the latter case by Proposition 3.4 the low-status uncommitted company will contribute as well. Therefore the low-status initiator's utility if she invites a high-status company will be $3bw + \gamma(s_H - s_L)$ and if she does not invite any company or invites a low-status company (who will decline) the low-status initiator's utility will be $w$. In case HMB the initiator will invite a high-status company. □

*Proof of Theorem 3.2.* Under case HCS by Proposition 3.6 any invitee will accept an invitation from the high-status initiator. By Proposition 3.4 regardless of whether the initiator invites a high- or low-status company to form the alliance, a low-status uncommitted company will contribute in the contribution stage. On the other hand by Proposition 3.4 if the initiator does not invite any company, no company will contribute. Therefore the high-status initiator's utility will be $4bw$ if she invites the other high-status company, $3bw$ if she invites a low-status company, and $w$ if she does not invite any company. By Assumption 3.1b in equilibrium the high-status initiator will invite the other high-status company.

Under case MCS by Proposition 3.6 a high-status invitee will always accept an invitation from the high-status initiator and a low-status invitee will accept if and only if $\gamma \geqslant \frac{(1-2b)w}{s_H - s_L}$. If the initiators invites the other high-status company, the equilibrium in the subgame will be identical to that under case HCS and the initiator's utility will be $4bw$. If the initiator invites a low-status company, and if the invitee accepts, by Proposition 3.4 the low-status uncommitted company will contribute but the high-status one will not. The initiator's utility will be $3bw$. On the other hand if the low-status invitee declines, by Proposition 3.4 no company will contribute, so the high-status initiator's utility will be $w$. If the initiator does not invite any company, by Proposition 3.4 the initiator's utility will be $w$. By Assumption 3.1b in equilibrium the high-status initiator will invite the other high-status company. □

## Appendix 3.C Additional Laboratory Results

Figure 3.5: Group Contribution Over Time




Group contribution average by the round across all sessions and treatments.

Table 3.11: Group Contribution Across Treatments

| | Group contribution |
|---|---|
| Treatment S | 0.8 |
| | (1.9) |
| Treatment I | 15.4 *** |
| | (2.8) |
| Treatment F | 14.0 *** |
| | (2.3) |
| Round | -2.6*** |
| | (0.3) |
| Constant | 25.4 *** |
| | (2.3) |
| Observations | 660 |
| N. of subjects | 182 |
| Test 1: S = F | $p < 0.001$ |
| Test 2: I = F | $p = 0.666$ |

GLS regression with subject random effects. Robust standard errors in parentheses. Significance is denoted * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$. Tests 1 & 2: Wald $\chi^2$-test on the hypothesis that the average group contribution is equal between in the two treatments in comparison.

Table 3.12: Comparison of Treatments I and F

| | Frequency of alliance | Profit with alliance | Test: Profit> 20? ($p$-value) | Profit without alliance |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Treatment I | 0.431 | 28.5 | <0.001 | 20.6 |
| Treatment F | 0.431 | 28.2 | <0.001 | 20.4 |
| Rank-sum test ($p$-value) | 1.000 | 0.701 | - | 0.285 |

Table 3.13: Treatment F—Effect of Status on Acceptance

| | Accept | | | |
|---|---|---|---|---|
| | High-status invitee | Low-status invitee | High-status initiator | Low-status initiator |
| | (1) | (2) | (3) | (4) |
| Initiator status high | 0.129 | 0.735 | | |
| | (0.533) | (0.584) | | |
| Invitee status high | | | -0.107 | 0.445 |
| | | | (0.302) | (0.330) |
| Round | -0.0980 | -0.170 | -0.0663 | -0.123* |
| | (0.0719) | (0.117) | (0.0463) | (0.0650) |
| Constant | 1.06** | 0.620 | 0.683* | 0.446 |
| | (0.490) | (0.761) | (0.368) | (0.475) |
| Observations | 122 | 64 | 89 | 97 |
| N. of subjects | 49 | 39 | 40 | 44 |

Probit regression with subject random effects. Robust standard errors in parentheses. Significance is denoted * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$. Variables: *Accept* is an indicator variable which takes value 1 if an invitation is accepted; *initiator status high* is an indicator variable which takes value 1 if the invitation is from a high-status initiator; *invitee status high* is an indicator variable which takes value 1 if the initiator invites a high-status company.

Table 3.14: Treatment F—Contribution by Uncommitted Company to an Alliance

| | Contribution by uncommitted company | |
|---|---|---|
| | High status | Low status |
| | (1) | (2) |
| Alliance type LL | -0.244 | |
| | (0.396) | |
| Alliance type HH | | -0.144 |
| | | (0.393) |
| Round | -0.162** | -0.207*** |
| | (0.073) | (0.065) |
| Constant | 0.692* | -0.095 |
| | (0.393) | (0.452) |
| Observations | 95 | 129 |
| N. of subjects | 39 | 46 |

Probit regression with subject random effects. Robust standard errors in parentheses. Significance is denoted * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$. The variable *Alliance type LL* is an indicator variable which takes value 1 if the alliance is of the type LL and zero otherwise; *Alliance type HH* is an indicator variable which takes value 1 if the alliance formed is of the type HH and zero otherwise. The baseline for the type of alliance is HL/LH.

Table 3.15: Treatment F—Contribution by Uncommitted Companies when Alliance Is Formed

| | Contribution by uncommitted company |
|---|---|
| | High-status—Pro-social |
| Alliance formed | 2.025*** |
| | (0.648) |
| Round | -0.045 |
| | (0.090) |
| Constant | -1.206*** |
| | (0.416) |
| Observations | 74 |
| N. of subjects | 10 |

Probit regression with subject random effects. Robust standard errors in parentheses. Significance is denoted * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The variable *alliance formed* is an indicator variable which takes value 1 if an alliance is formed.

Table 3.16: Treatment S—Effect of Pro-sociality on Contributions

| | Contribution in treatment S | |
|---|---|---|
| | High status | Low status |
| | (1) | (2) |
| Pro-social | 0.104 | 0.408 |
| | (0.474) | (0.286) |
| Round | -0.294*** | -0.238*** |
| | (0.054) | (0.059) |
| Constant | 0.029 | -0.271 |
| | (0.315) | (0.299) |
| Observations | 240 | 240 |
| N. of subjects | 24 | 24 |

Probit regression with subject random effects. Robust standard errors in parentheses. Significance is denoted * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$. The variable *Pro-social* is an indicator variable which takes value 1 if the company is pro-social and zero otherwise. The data includes treatment S only.

Table 3.17: Treatment F—Effects of Alliance on Profit

| | Average profit (in points) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Initiator | | | Invitee | | | All roles | | |
| | High status | Low status | All | High status | Low status | All | High status | Low status | All |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Alliance formed | 21.4 | 22.4 | 21.9 | 21.6 | 22.7 | 21.9 | 26.5 | 30.0 | 28.2 |
| | (53) | (59) | (112) | (76) | (36) | (74) | (224) | (224) | (448) |
| Alliance not formed | 20.1 | 19.7 | 19.9 | 21.2 | 21.7 | 21.4 | 20.5 | 20.3 | 20.4 |
| | (75) | (73) | (148) | (46) | (28) | (112) | (296) | (296) | (592) |
| Test 1 ($p$-value) | 0.570 | 0.012 | 0.021 | 0.856 | 0.382 | 0.744 | <0.001 | <0.001 | <0.001 |

Average round profit per role. Number of observations reported in parentheses. "All roles" include all companies in the roles of initiator, invitee, and uncommitted company. Test 1: Wilcoxon rank-sum test on the hypothesis of equality of profits when an alliance is and is not formed.

Table 3.18: Treatment F—Effect of Risk Aversion on Alliance Formation and Contributions

| | Invite | | Accept | | Contribution by uncommitted company with alliance | | Contribution when alliance is not formed | |
|---|---|---|---|---|---|---|---|---|
| | High status | Low status | High status | Low status | High status | Low status | High status | Low status |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Risk averse | -0.069 | 0.147 | 0.014 | -0.068 | -0.176 | 0.853 | 0.196 | -0.416 |
| | (0.382) | (0.510) | (0.714) | (0.798) | (0.334) | (0.696) | (0.556) | (0.432) |
| Round | -0.123** | -0.228*** | -0.095 | -0.146 | -0.158** | -0.199*** | -0.103 | -0.111** |
| | (0.058) | (0.083) | (0.071) | (0.106) | (0.071) | (0.069) | (0.072) | (0.054) |
| Constant | 1.425*** | 2.173*** | 1.084** | 1.033 | 0.651* | -0.461 | -2.433*** | -1.403*** |
| | (0.447) | (0.629) | (0.503) | (0.676) | (0.382) | (0.496) | (0.683) | (0.426) |
| Observations | 128 | 132 | 122 | 64 | 95 | 129 | 296 | 296 |
| N. of subjects | 48 | 49 | 49 | 39 | 39 | 46 | 52 | 52 |

Probit regression with subject random effects. Robust standard errors reported in parentheses. Significance is denoted * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

# 4 Social Responsibility Auditing in Supply Chain Networks

*Abstract.* We study a buyer's problem of auditing suppliers within a network to ensure social responsibility compliance. The buyer suffers economic damages if a violation is exposed at a supplier (whether by the media, regulator, or NGO). To avoid damages the buyer may audit them to identify non-compliance. If a supplier fails an audit, the buyer must take one of two costly actions: either rectify the supplier or drop the supplier (along with dependent suppliers). Dropping a supplier reduces competition and therefore increases the buyer's input cost. The network topology evolves as the buyer conducts audits. We build a two-phase model: an auditing phase followed by a production phase. The unique production phase equilibrium arising from the competition in the supply network establishes the buyer's production profit and each supplier's value to the buyer. We then identify the buyer's optimal dynamic auditing policy: the buyer will first audit and drop some suppliers, before either auditing and rectifying suppliers, or proceeding directly to production. Within the audit-and-drop subphase, when auditing only in the upper tier, the buyer always audits the least valuable unaudited supplier, yielding greater balance. We also establish the condition under which the buyer may truncate auditing altogether, carrying unaudited suppliers to production ("hear no evil, see no evil"). When the buyer audits more broadly, any supplier (not necessarily the least valuable) may be chosen. A supplier in a pivotal position may be chosen to help the buyer ascertain the viability of a portion of the network ("litmus test").

## 4.1 Introduction

Violations of social responsibility norms by suppliers are widespread. Such violations appear in domains as varied as infringement on human rights (Teixeira 2019, Segal 2019), animal abuse (Phillips 2016, Elejalde-Ruiz 2019), or environmental harm (Rana 2018). A common trait of these examples is that they are *process compliance* violations, which involve issues with production processes and typically require on-site vetting to detect. Some companies have taken proactive steps to audit their suppliers to ensure compliance on social responsibility issues. Apple Inc. regularly audits suppliers and describes the results in its annual Supplier Responsibility Progress Report (Apple Inc. 2018b). In the 2018 Report, Apple uncovered debt-bonded labor among its suppliers, which the U.N. deems a form of modern slavery (Zeldin 2016) and for which Apple has "zero tolerance" (Apple Inc. 2018b). For each labor violation an audit uncovered, Apple put the supplier on immediate probation, requiring it to undergo rectification, including financial remedies for every affected employee. Another area covered by Apple's report is the sourcing of minerals that may originate from mines that finance armed conflicts (Zhang et al. 2017). Apple reports in a specialized disclosure that it dropped ten smelters and refiners from its supply chain in 2017 for non-compliance on the issue of conflict minerals (Apple Inc. 2018a).

As the example of Apple shows, once an audit uncovers a violation, the buyer may decide to *rectify* the supplier or to *drop* the supplier from the supply network. Such audits can be costly; for large international buyers, indiscriminately auditing all suppliers would be practically impossible (Rowe 2013). Instead, how should a buyer choose the suppliers to audit, then decide to *rectify* or to *drop* a supplier in the event of non-compliance? Three layers of tradeoff complicate the auditing problem. (1) By not auditing a supplier, the buyer faces the potential penalty when an NGO, law enforcement, or the media exposes a violation at the supplier; by auditing a supplier, the buyer incurs the cost to conduct the audit and, if the supplier turns out to be non-compliant, subsequent costs to address the problem, as discussed next. (2) Once an audit reveals non-compliance, the buyer chooses either to incur the cost of rectifying the supplier or to drop it (along with others depending on it) forfeiting profit from production activity attributable to the suppliers dropped: the fewer the suppliers in the upstream markets, the less competitive the markets will be, and thus the higher the input cost for the buyer. (3) The buyer also faces a tradeoff in choosing a supplier to audit, which is influenced by the supplier's location in the network. For example, dropping a supplier with many questionable upstream sources may be a less expensive approach than to audit those sources individually, but potentially carves away a profitable part of the supply network.

We build a two-phase model for a three-tier supply network: an auditing phase followed by a production phase. Each stage of the auditing phase consists of the following: the buyer selects an unaudited supplier to audit; the audit reveals whether the supplier is compliant; in the event of non-compliance, the buyer decides to rectify the supplier or to drop it. Dropping a supplier has the effect of also dropping any other firms relying solely on that dropped supplier. Once the buyer decides to conclude the auditing phase, the remaining supply network is carried to the production phase, where every firm competes with its peers to determine the equilibrium quantities and prices. The equilibrium determines the buyer's profit from production activity.

Equipped with the model, we investigate the following interdependent decisions: (1) Which supplier to audit? Should the buyer prioritize suppliers in an upper tier or a lower tier? More central or more peripheral? (2) When is it optimal to drop a non-compliant supplier, along with its dependents? When is it optimal to rectify the supplier? (3) In which sequence should the buyer conduct audits? The choice of the next supplier to audit may depend on the outcome of an earlier audit. (4) When is it optimal to cease auditing and go straight to production? In other words, when is it in the best interest of the buyer to "see no evil, hear no evil"? Sometimes the buyer may prefer not to learn of non-compliance among some suppliers rather than to uncover the non-compliance and then be obligated to address the problem.

### 4.2 Literature Review

The literature on socially responsible supply chain management covers various aspects, including supplier selection (Guo et al. 2016, Agrawal and Lee 2019), unauthorized subcontracting (Caro et al. 2016), network-wide effects of violation penalty (Zhang et al. 2017), consumer motives (Kraft et al. 2018), disclosure of the supplier list (Chen et al. 2018, Kalkanci and Plambeck 2019b), and investment in supplier social responsibility capacity (Kraft et al. 2019). Huang et al. (2017) and Feng et al. (2019) study when the buyer should directly implement social responsibility standards at upstream suppliers versus delegating the task to midtier suppliers. In particular Feng et al. (2019) considers supply networks with fixed material flow structure and finds that the buyer gains more by engaging directly with upstream suppliers when the network has a complex structure. We focus on directly auditing upstream suppliers in our model.

Within this literature our work is most closely related to the stream on auditing practices. Plambeck and Taylor (2016) shows that under certain conditions increasing auditing effort on a single supplier motivates the supplier to hide misconduct. Through a comparison of alternative contracting arrangements, Chen and Lee (2016) establishes that process auditing effectively reduces the risk of non-compliance by a supplier. Focusing on a single supplier, Caro et al. (2018) and Fang and Cho (2019) compare the mechanism of independent, joint, and shared auditing by multiple buyers. Also on the subject of information sharing, Ha et al. (2018) studies how sharing audit information between competitors interacts with sourcing decisions. Chen et al. (2019b) studies the collusion between the supplier and the auditor. We complement the literature by studying the dynamic auditing policy in a supply network. Closest to our work, Chen et al. (2019a) studies the auditing behavior of two buyers situated in a W-shaped supply network (i.e., each buyer has one exclusive supplier and the two buyers share a third supplier). It shows that without coordination each buyer chooses to audit its exclusive supplier, actions which are suboptimal for the combined profit of the buyers; with joint auditing the buyers choose the shared supplier and avoids the inefficiency. While Chen et al. (2019a) focuses on the coordination between buyers auditing a static supply network, our work considers a single buyer auditing a three-tier network which evolves dynamically throughout the auditing process: the buyer may decide to remove a non-compliant supplier from the network, changing the network topology.

To understand the dynamic auditing decision we also need to model how the topology of a network impacts the competition that determines the quantities and prices. This aspect of our model connects our work to the diverse literature on supply networks, e.g., Acemoglu et al. (2012), Ang et al. (2017), Belavina (2017), Bimpikis et al. (2018), Korpeoglu et al. (2018). The research closest to ours adopt a model of Cournot competition that endogenously determines the equilibrium prices and quantities. The foundational work of Corbett and Karmarkar (2001), along with others including Adida and DeMiguel (2011) and Bimpikis et al. (2019), studies supply networks with a complete market between adjacent tiers: every buyer in a downstream tier procures from every supplier in an upstream tier. While we limit our model to two suppliers in the midstream tier, our work contributes to the literature by allowing arbitrary sourcing relationships between firms in adjacent tiers.

## 4.3 Model Description

The model consists of two phases, an auditing phase followed by a production phase, each consisting of multiple stages. The supply network has three tiers with a single buyer in the most downstream tier. Each supplier in the network can be compliant or non-compliant, which the buyer may discover with an audit. In each stage of auditing, the buyer chooses which supplier to audit, if any. If the audit finds non-compliance at the supplier, then the buyer either rectifies the supplier so that it becomes compliant, or drops it from the supply network. Dropping a supplier may involve dropping dependent suppliers, as discussed in the following section. The buyer may terminate the auditing phase at any point and carry the remaining network to the production phase, in which each firm chooses its supply quantities in competition with one another.

We next describe in detail the supply network model, the auditing phase, and the production phase. We list the notation in Appendix 4.A.

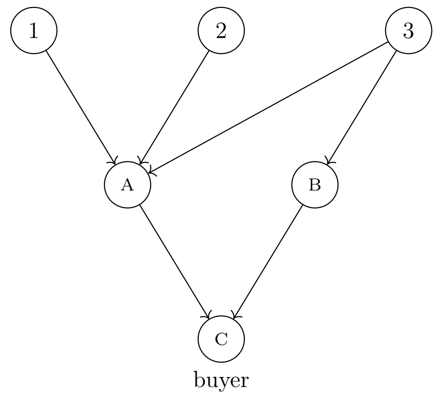
### 4.3.1 Supply Network

We model a three-tier supply network with a single buyer in tier 0, two suppliers in tier 1, and any number of suppliers in tier 2. Suppliers in the same tier manufacture a *perfectly substitutable* product.

We denote the *buyer* as C and the two *tier-1 firms* A and B. Denote the set of tier-1 firms $S(1) = \{\text{A}, \text{B}\}$. Let $S(2)$ denote all tier-2 firms, which we partition into three subsets: $S_{\text{A}}, S_{\text{B}}, S_{\text{AB}}$. The subset $S_{\text{A}}$ is the set of *exclusive suppliers to firm* A, each of which sells to A but not to B. Similarly, the subset $S_{\text{B}}$ is the set of *exclusive suppliers to firm* B. The subset $S_{\text{AB}}$ is the set of *shared suppliers*, each selling to both firms A and B. We represent the *supply network* by the tuple $g = (S(1), S_{\text{A}}, S_{\text{B}}, S_{\text{AB}})$. We denote by $S_g = S(1) \cup S(2)$ the set of all suppliers in $g$. Throughout the auditing phase the buyer may drop tier-1 firms resulting in $S(1)$ having fewer than two firms. In particular when $S(1) = \emptyset$ (or $S(2) = \emptyset$) we denote the resulting *null supply network* $g_{\emptyset}$. We denote $G$ the set of all supply networks.

Given supply network $g \in G$ we denote $t_{\text{A}} = |S_{\text{A}}|$ the number of exclusive suppliers to firm A, $t_{\text{B}} = |S_{\text{B}}|$ the number of exclusive suppliers to firm B, and $t_{\text{AB}} = |S_{\text{AB}}|$ the number of shared suppliers. We call the tier-1 firm with more tier-2 suppliers the *majority tier-1 firm* and the other the *minority*

Figure 4.1: Example of Supplier Dependence




Supply network $g = (\{\text{A}, \text{B}\}, \{1, 2\}, \emptyset, \{3\})$. Here, suppliers 1 and 2 are dependents of firm A, but A is not a dependent of 1 or 2; firm B is a dependent of supplier 3, but 3 is not a dependent of B.

*tier-1 firm*. We call an exclusive supplier to the majority tier-1 firm a *majority-exclusive supplier* and an exclusive supplier to the minority tier-1 firm a *minority-exclusive supplier*.

The model uses the concept of *dependent suppliers*, as illustrated in Figure 4.1. Given a supplier $i \in S_g$ in network $g$, denote $D_g(i)$ the set of *dependents* of $i$ in $g$, each solely relying on $i$ to sell to the buyer. Specifically a supplier is always a dependent of itself. If $i$ is a tier-1 firm, its dependents also include all its exclusive suppliers; i.e., if $i \in S(1)$, $D_g(i) = S_i \cup \{i\}$. For example in Figure 4.1, $D_g(\text{A}) = \{1, 2, \text{A}\}$. If $i$ is a tier-2 supplier, then its dependents also include any tier-1 firm whose sole supplier is firm $i$; i.e., if $i \in S(2)$, $D_g(i)$ includes a tier-1 firm $j \in \{\text{A}, \text{B}\}$ if and only if $S_j \cup S_{\text{AB}} = \{i\}$. In Figure 4.1 we see $D_g(3) = \{3, \text{B}\}$.

### 4.3.2 Auditing Phase

We now describe the auditing phase. Each supplier is either compliant (with probability $1 - u$) or non-compliant (with probability $u$). Through an audit the buyer accurately discovers whether the supplier is compliant or non-compliant. Whether a supplier is compliant or not is independent across suppliers. In each stage of the auditing phase, the buyer decides whether to audit a supplier or to conclude the auditing phase and proceed to the production phase. If the buyer decides to audit (at cost $a \geqslant 0$), it selects an unaudited supplier. If the audit of that supplier reveals non-compliance, the buyer decides either to rectify the non-compliant supplier (at cost $r \geqslant 0$) or to drop it from the supply network. We assume that a supplier undergoing rectification becomes compliant. When a supplier is dropped, its dependents are dropped as a consequence. For example in Figure 4.1, dropping firm A would result in firms 1 and 2 being dropped, too. Consequently, the buyer need not be concerned with non-compliance at those dropped dependent suppliers.

We model the auditing phase as a Markov decision process for the buyer. A *state* consists of a supply network (tier-1 suppliers, their exclusive suppliers, and their shared suppliers) and the auditing status of each supplier (*unaudited* or *vetted*). A supplier is vetted if it passed an audit or underwent rectification upon failing an audit; in either case the buyer knows the supplier is compliant. Specifically a state is a tuple $\gamma = (g_\gamma, U_\gamma)$ where $g_\gamma = (S(1), S_{\text{A}}, S_{\text{B}}, S_{\text{AB}})$ is a supply network and $U_\gamma \subseteq S_{g_\gamma}$ is the set of suppliers that are currently *unaudited*. We omit the subscript "$\gamma$" whenever doing so causes no confusion. Any supplier $i \in S_g \backslash U$ is *vetted*. The *state space* is $\Gamma = \{(g, U) : g \in G, U \subseteq S_g\}$. The terminal states $\Gamma_T$ are the supply networks with no more unaudited suppliers, $\Gamma_T = \{\gamma = (g, U) \in \Gamma : U = \emptyset\}$. In the terminal states the auditing phase necessarily concludes and the production phase begins. However, the buyer may choose to enter the production phase prior to reaching a terminal state (i.e., to proceed to production with unaudited suppliers).

To facilitate the formulation of the dynamic program, we define two operators that will be used when updating the state. Let $Z = \{(\gamma, i) : \gamma \in \Gamma, i \in U_\gamma\}$ be the set of pairs of a state and an unaudited supplier (in that state). The first mapping $\oplus : Z \to \Gamma$ changes a supplier from an unaudited to a vetted status, i.e., given state $\gamma$ and unaudited supplier $i$ in $\gamma$, $\gamma \oplus i$ is the state otherwise identical to $\gamma$ but with a vetted $i$.[1] The operator $\oplus$ will be used when a supplier passes an audit or is rectified upon failing an audit. The second mapping $\ominus : Z \to \Gamma$ removes a supplier along with its dependents from a state, i.e., given state $\gamma$ and unaudited supplier $i$ in $\gamma$, $\gamma \ominus i$ is the state otherwise identical to $\gamma$ but with $i$ and all its dependents removed.[2] The operator $\ominus$ is used when a supplier has failed an audit and will be dropped from the network.

The buyer's set of *admissible actions* at state $\gamma \in \Gamma$ is $X_\gamma = \{\text{PP}\} \cup \left(\bigcup_{i \in U_\gamma} \{\text{AR}(i), \text{AD}(i)\}\right)$. The action PP represents concluding the auditing phase and Proceeding to the Production phase. The action AR$(i)$ represents Auditing supplier $i$ and Rectifying $i$ if the audit uncovers non-compliance. Following AR$(i)$, regardless of whether the supplier passes the audit, the state transits from $\gamma$ to $\gamma \oplus i$.

[1]Given $\gamma = (g, U) \in \Gamma$ and $i \in U$, $\gamma \oplus i = (g, U')$ where $U' = U \backslash \{i\}$.
[2]Given $\gamma = (g, U) \in \Gamma$ where $g = (S(1), S_{\text{A}}, S_{\text{B}}, S_{\text{AB}})$ and $i \in U$, write $\gamma \ominus i = (g', U')$. Then $U' = U \backslash D_g(i)$. If $i \in S(1)$, let $-i \in \{\text{A}, \text{B}\} \backslash \{i\}$, $S'_i = S_i$, and $S'_{-i} = S_{-i} \cup S_{\text{AB}}$, then $g' = (S(1) \backslash D_g(i), S'_{\text{A}} \backslash D_g(i), S'_{\text{B}} \backslash D_g(i), \emptyset)$; otherwise $g' = (S(1) \backslash D_g(i), S_{\text{A}} \backslash D_g(i), S_{\text{B}} \backslash D_g(i), S_{\text{AB}} \backslash D_g(i))$.

Similarly, the action AD($i$) represents <u>A</u>uditing supplier $i$ and <u>D</u>ropping $i$ (and its dependents) if the audit uncovers non-compliance. Following AD($i$) the state transits from $\gamma$ to $\gamma \oplus i$ if $i$ is compliant (passes the audit) and to $\gamma \ominus i$ if $i$ is non-compliant (fails the audit). In the notation for AD and AR, besides using the specific index for a supplier (e.g., $i$), we also use $\mathbb{e}_{\text{A}}$ to represent a generic unaudited exclusive supplier to firm A, $\mathbb{e}_{\text{B}}$ a generic exclusive supplier to firm B, and $\mathbb{s}$ a generic shared supplier (e.g., AD($\mathbb{e}_{\text{A}}$) represents the decision to audit and drop (if non-compliant) an exclusive supplier to firm A). Let $\overline{U} = \bigcup_{\gamma \in \Gamma} U_\gamma$ and $\overline{X} = \{\text{PP}\} \cup \left(\bigcup_{i \in \overline{U}} \{\text{AR}(i), \text{AD}(i)\}\right)$. An *auditing policy* is a mapping $\xi : \Gamma \to \overline{X}$ such that $\xi(\gamma) \in X_\gamma, \forall \gamma \in \Gamma$. Let $\Xi$ be the set of all auditing policies.

Given $\gamma = (g, U) \in \Gamma$, let $\pi(\gamma)$ be the buyer's *production profit*, which is a result of the equilibrium production activity on supply network $g$ in the production phase (we uniquely determine this equilibrium in section 4.4.1). We use $\nabla(\gamma, i) = \pi(\gamma) - \pi(\gamma \ominus i)$ to denote the loss in production profit due to the removal of a supplier $i \in U_\gamma$ (in state $\gamma$) and call it the *production value* of supplier $i$ in state $\gamma$. A violation by a non-compliant supplier will be exposed in the production phase with probability $w \in [0, 1]$; for example, this revelation may arise from an investigation led by an NGO or a regulatory body. The possible exposure of violation is independent across suppliers. The buyer incurs a cost of $z \geqslant 0$ upon the exposure of a violation at each non-compliant supplier. Let $\zeta(\gamma)$ be the expected total penalty from violations on state $\gamma$. (If any penalty arises at all, it does so during the production phase.) By the independence of non-compliance and exposure of violations across suppliers, $\zeta(\gamma) = |U_\gamma| uwz$, since $U_\gamma$ is the set of unaudited suppliers, each of which is non-compliant with probability $u$ and costs the buyer a penalty $z$ if exposed with probability $w$.

We define $V^* : \Gamma \to \mathbb{R}$ as the optimal value function. Let $\widetilde{V}^*(\gamma, x)$ be the expected value of choosing action $x \in X_\gamma$ when in state $\gamma \in \Gamma$ and following an optimal policy thereon. Then

$$V^*(\gamma) = \max_{x \in X_\gamma} \widetilde{V}^*(\gamma, x), \quad \forall \gamma \in \Gamma \tag{4.1}$$

where

$$\widetilde{V}^*(\gamma, \text{PP}) = \pi(\gamma) - \zeta(\gamma) \tag{4.2}$$

and for $i \in U_\gamma$,

$$\widetilde{V}^*(\gamma, \text{AD}(i)) = -a + (1-u)V^*(\gamma \oplus i) + uV^*(\gamma \ominus i) \tag{4.3}$$

$$\widetilde{V}^*(\gamma, \text{AR}(i)) = -a + (1-u)V^*(\gamma \oplus i) + u(-r + V^*(\gamma \oplus i)) \tag{4.4}$$

$$= -a - ur + V^*(\gamma \oplus i). \tag{4.5}$$

Equation (4.2) reflects the buyer's value when it takes action PP, which consists of the production profit minus the expected penalty of violation. Equations (4.3) and (4.4) are the buyer's values when the AD($i$) and AR($i$) actions are taken, respectively, which consist of an auditing cost $a$ and the weighted average of values in consequent states when the supplier passes (with probability $1-u$) or fails (with probability $u$) the audit. In a terminal state $\gamma \in \Gamma_T$, $X_\gamma = \{\text{PP}\}$, so $V^*(\gamma) = \pi(\gamma) - \zeta(\gamma)$. An optimal auditing policy $\xi \in \Xi$ solves $\xi(\gamma) \in \arg\max_{x \in X_\gamma} \widetilde{V}^*(\gamma, x)$ for any $\gamma \in \Gamma$.

Throughout the auditing phase the topology of the supply network and the status of each remaining supplier evolve with the progression of auditing activities. Once the auditing phase concludes (either the buyer decides to proceed to production or the Markov decision process enters a terminal state) the remaining supply network is carried to the production phase in which the buyer's production profit is determined.

### 4.3.3 Production Phase

In the production phase, each firm in the network chooses its production quantity to maximize its profit, given the unit input price and anticipating the downstream demand. The chosen quantities of the upstream firms determine a downstream firm's input price, as we describe below. All firms in $S_\text{A}$ and $S_\text{AB}$ compete to supply firm A; all firms in $S_\text{B}$ and $S_\text{AB}$ compete to supply firm B; and firms A and B compete to supply the buyer. This is similar to Corbett and Karmarkar (2001) other than we limit ourselves to three tiers and two tier-1 firms but allow more general relationships between firms in adjacent tiers. Specifically, in Corbett and Karmarkar (2001) all firms in an upstream tier are shared suppliers of all downstream firms; using our notation they have $S(2) = S_\text{AB}$ and $S_\text{A} = S_\text{B} = \emptyset$.

Corresponding to the three tiers in the supply network, there are three stages in the production phase. In the first stage, the firms in $S_\text{A}$ and $S_\text{AB}$ choose the quantities they will supply to firm A which establishes $p_{(2)\text{A}}$, the selling price of those tier-2 suppliers to firm A (whenever a number appears in parentheses in a subscript, it is referring to the tier in the network). In parallel, the firms in $S_{\text{B}}$ and $S_{\text{AB}}$ choose their quantities to establish $p_{(2)\text{B}}$, their selling price to firm B. In particular a shared supplier in tier 2 may sell to firms A and B at different prices. In the second stage, firms A and B choose quantities which result in $p_{(1)}$, the selling price of the tier-1 firms to the buyer. In the third stage the buyer chooses a quantity to establish $p_{(0)}$, the selling price of the buyer to downstream customers. Let $q_j$ be the total quantity produced by firm $j$. The unit production cost of a tier-$k$ firm is $v_k$; denote $v_T = \sum_{k=0}^{2} v_k$ which is the total production cost embedded in each unit of the final product. Next we describe these three stages in detail, in reverse order.

**Third Stage: Buyer's Problem.** In the third stage of the production phase, the buyer C faces an exogenous linear aggregate demand from downstream customers characterized by inverse demand function

$$p_{(0)}(q_{\text{C}}) = \alpha - \beta q_{\text{C}} \tag{4.6}$$

where $\beta > 0$. Assume $\alpha \geqslant v_T$ to ensure the supply chain is profitable. The buyer takes the price of the input $p_{(1)}$ as given and chooses production quantity $q_{\text{C}}$ in decision space $C_{\text{C}} = \mathbb{R}$ to maximize profit:

$$(P_0) \qquad \max_{q_{\text{C}} \in C_{\text{C}}} \left\{ \pi_{\text{C}}(q_{\text{C}}) \equiv (p_{(0)}(q_{\text{C}}) - v_0 - p_{(1)}) q_{\text{C}} \right\}. \tag{4.7}$$

**Proposition 4.1.** *Given the buyer's input price $p_{(1)}$, there exists a unique optimal quantity $q_{\text{C}}^*$ which solves the buyer's problem $P_0$. Moreover, the resulting inverse demand function faced by the tier-1 firms is*

$$p_{(1)}^*(q_{\text{C}}) = \alpha - v_0 - 2\beta q_{\text{C}}. \tag{4.8}$$

We relegate all proofs to the appendices.

**Second Stage: Tier-1 Firms' Game.** In the second stage of the production phase, tier-1 firm $i \in S(1)$, anticipating inverse demand $p_{(1)}^*(q_{\text{C}})$ and taking the input prices $p_{(2)i}$ as given, chooses production quantity $q_i$ in strategy space $C_i = \mathbb{R}$ to maximize its profit

$$\pi_i = (p_{(1)}^*(q_{\text{C}}) - v_1 - p_{(2)i}) q_i \tag{4.9}$$

subject to the market clearing condition

$$q_{\text{C}} = \sum_{i' \in S(1)} q_{i'}. \tag{4.10}$$

In other words, when there are two tier-1 firms, they engage in Cournot competition for the buyer's business. Denote a strategy profile of the tier-1 firms $\mathbf{q}_{(1)} = (q_i)_{i \in S(1)} \in \prod_{i \in S(1)} C_i$ (i.e., a vector of tier-1 supply quantities). We substitute (4.10) into (4.9) to write tier-1 firm $i$'s payoff function as

$$\pi_i(\mathbf{q}_{(1)}) = \left( p^*_{(1)} \left( \sum_{i' \in S(1)} q_{i'} \right) - v_1 - p_{(2)i} \right) q_i. \tag{4.11}$$

Let the game in the second stage be the strategic-form game $P_1 = (S(1), (C_i)_{i \in S(1)}, (\pi_i)_{i \in S(1)})$.

**Proposition 4.2.** *Given the tier-1 vector of input prices $\mathbf{p}_{(2)} = (p_{(2)i})_{i \in S(1)}$, there exists a unique equilibrium in pure strategies $\mathbf{q}^*_{(1)}$ of the game $P_1$. Moreover, the resulting inverse demand function faced by the tier-2 firms supplying firm $i$ is (for $i \in S(1)$)*

$$p^*_{(2)i}(\mathbf{q}_{(1)}) = \alpha - v_0 - v_1 - 4\beta q_i - 2\beta \sum_{i' \in S(1) \setminus \{i\}} q_{i'}. \tag{4.12}$$

**First Stage: Tier-2 Suppliers' Game.** In the first stage of the production phase, each tier-2 supplier $j$ chooses $s_{j,i}$, the quantity it produces for its tier-1 customer firm $i$. A tier-2 supplier $j$ anticipates inverse demand $p^*_{(2)i}(\mathbf{q}_{(1)})$ for $i \in S(1)$. An exclusive supplier $j$ to tier-1 firm $i \in S(1)$ chooses $\mathbf{s}_j = s_{j,i} \geqslant 0$ to maximize its profit

$$\pi_j = \left( p^*_{(2)i}(\mathbf{q}_{(1)}) - v_2 \right) s_{j,i}. \tag{4.13}$$

A shared supplier $j$ chooses the vector of supply quantities $\mathbf{s}_j = (s_{j,i})_{i \in S(1)} \geqslant 0$ to maximize its profit

$$\pi_j = \sum_{i \in S(1)} \left( p^*_{(2)i}(\mathbf{q}_{(1)}) - v_2 \right) s_{j,i}. \tag{4.14}$$

The tier-2 suppliers' decisions are subject to the market clearing condition

$$q_i = \sum_{j' \in S_i \cup S_{\text{AB}}} s_{j',i}, \quad \forall i \in S(1). \tag{4.15}$$

In other words, for each firm $i$ in tier 1, the tier-2 suppliers in $S_i$ and $S_{\text{AB}}$ engage in Cournot competition to supply firm $i$. When there are two tier-1 firms, a shared tier-2 supplier competes simultaneously for the business of each tier-1 firm.

To make the strategy space of a tier-2 supplier compact for proving existence and uniqueness of the equilibrium, we assume there exists (arbitrarily large) theoretical maximum capacity $M > 0$ such that $s_{j,i} \leqslant M$ for any tier-2 supplier $j \in S(2)$ and its tier-1 customer $i$. That is, the strategy space of supplier $j \in S(2)$ is $C_j = \{\mathbf{s}_j : 0 \leqslant s_{j,i} \leqslant M\}$. Denote a strategy profile of tier-2 suppliers $\mathbf{s}_{(2)} = (\mathbf{s}_j)_{j \in S(2)} \in \prod_{j \in S(2)} C_j$. We substitute (4.15) into (4.13) to write the payoff function of exclusive supplier $j$ to tier-1 firm $i$ as

$$\pi_j(\mathbf{s}_{(2)}) = \left[ p^*_{(2)i} \left( \left( \sum_{j' \in S_{i'} \cup S_{\text{AB}}} s_{j',i'} \right)_{i' \in S(1)} \right) - v_2 \right] s_{j,i}. \tag{4.16}$$

Similarly, we substitute (4.15) into (4.14) to write the payoff function of shared supplier $j$ as

$$\pi_j(\mathbf{s}_{(2)}) = \sum_{i \in S(1)} \left[ p^*_{(2)i} \left( \left( \sum_{j' \in S_{i'} \cup S_{\text{AB}}} s_{j',i'} \right)_{i' \in S(1)} \right) - v_2 \right] s_{j,i}. \tag{4.17}$$

Let the game in the first stage be the strategic-form game $P_2 = (S(2), (C_j)_{j \in S(2)}, (\pi_j)_{j \in S(2)})$. We denote an equilibrium in pure strategies $\mathbf{s}^*_{(2)} = (\mathbf{s}^*_j)_{j \in S(2)} = (((s^*_{j,i})_{j \in S_i})_{i \in S(1)}, ((s^*_{j,i})_{i \in S(1)})_{j \in S_{\text{AB}}})$. Let $\mathbf{p}^*_{(2)} = (p^*_{(2)i})_{i \in S(1)}$. Let $q^*_j = s^*_{j,i}$ if $j \in S_i$, $i \in S(1)$, and $q^*_j = \sum_{i \in S(1)} s^*_{j,i}$ if $j \in S_{\text{AB}}$.

**Production Phase Equilibrium.** Let $\mathbf{s}^*_{(2)}$ be an equilibrium of the game $P_2$ in the first stage and $\mathbf{p}^*_{(2)}$ the resulting selling prices of the tier-2 suppliers. Given $\mathbf{p}^*_{(2)}$, let $\mathbf{q}^*_{(1)}$ be an equilibrium of the game $P_1$ in the second stage and $p^*_{(1)}$ the resulting selling price of the tier-1 firms. Given $p^*_{(1)}$, let $q^*_{\text{C}}$ be an optimal solution to the buyer's problem $P_0$ in the third stage and $p^*_{(0)}$ the resulting selling price of the buyer. We call the tuple of prices and quantities $(p^*_{(0)}, p^*_{(1)}, \mathbf{p}^*_{(2)}, q^*_{\text{C}}, \mathbf{q}^*_{(1)}, \mathbf{s}^*_{(2)})$ a *production phase equilibrium*. In the next section we show this equilibrium is unique and fully characterize the equilibrium.

## 4.4 Production Phase Results

In the spirit of backward induction, we present the results for the two phases in the reverse order: the results for the production phase in this section are followed by the results for the auditing phase in section 4.5.

### 4.4.1 Production Phase Equilibrium

We are now ready to present the existence and uniqueness of the production phase equilibrium.

**Theorem 4.1.** *There exists a unique production phase equilibrium* $(p^*_{(0)}, p^*_{(1)}, \mathbf{p}^*_{(2)}, q^*_{\text{C}}, \mathbf{q}^*_{(1)}, \mathbf{s}^*_{(2)})$ *in every supply network* $g \in G\backslash\{g_\emptyset\}$.

Let $\pi^*_{\text{C}}$ be the buyer's profit in the unique equilibrium in Theorem 4.1, obtained by substituting the equilibrium quantities and prices in (4.7). For any state in the auditing phase $\gamma = (g, U) \in \Gamma$ the buyer's *production profit* $\pi(\gamma)$ is given by $\pi^*_{\text{C}}$ in the equilibrium that arises in network $\gamma$. At the state $\gamma_\emptyset$ which corresponds to the null supply network $g_\emptyset$, we set the buyer's production profit to zero: when the buyer has access to no supplier, there is no production activity, and thus no production profit.

We begin with a simple relationship between the buyer's equilibrium production quantity and profit:

**Proposition 4.3.** *The buyer's equilibrium profit is* $\pi^*_{\text{C}} = \beta \cdot (q^*_{\text{C}})^2$.

In equilibrium the buyer's production profit depends only on the buyer's quantity of production $q^*_{\text{C}}$, which is also the total quantity produced by the supply network.

Recall that $t_{\text{A}} = |S_{\text{A}}|$, $t_{\text{B}} = |S_{\text{B}}|$, and $t_{\text{AB}} = |S_{\text{AB}}|$. Without loss of generality, we index the majority tier-1 firm as A, i.e., $t_{\text{A}} \geqslant t_{\text{B}}$. We define the following functions $L, s_e, s_s, \bar{q} : \mathbb{R}^3 \to \mathbb{R}$ to facilitate representation of the equilibrium quantities.

$$
\begin{aligned}
L(x_1, x_2, x_3) &= 4x_1 + 4x_2 + 8x_3 + 3x_1x_2 + 4x_1x_3 + 4x_2x_3 + 4x_3^2 + 4 && (4.18)\\
s_e(x_1, x_2, x_3) &= \frac{1}{2}\frac{\alpha - v_T}{\beta}\left(\frac{x_2 + 2x_3 + 2}{L(x_1, x_2, x_3)}\right) && (4.19)\\
s_s(x_1, x_2, x_3) &= \frac{1}{3}\frac{\alpha - v_T}{\beta}\left(\frac{-x_1 + 2x_2 + 2x_3 + 2}{L(x_1, x_2, x_3)}\right) && (4.20)\\
\bar{q}_{(1)}(x_1, x_2, x_3) &= \frac{1}{6}\frac{\alpha - v_T}{\beta}\left(\frac{4x_3^2 + 4x_1x_3 + 4x_2x_3 + 4x_3 + 6x_1 + 3x_1x_2}{L(x_1, x_2, x_3)}\right) && (4.21)\\
\bar{q}(x_1, x_2, x_3) &= \frac{1}{3}\frac{\alpha - v_T}{\beta}\left(\frac{3x_1 + 3x_2 + 4x_3 + 4x_1x_3 + 4x_2x_3 + 3x_1x_2 + 4x_3^2}{L(x_1, x_2, x_3)}\right). && (4.22)
\end{aligned}
$$

The following proposition provides closed-form expressions for the equilibrium quantities.

**Proposition 4.4.** *(a) If $t_{\text{A}} \leqslant 2t_{\text{B}} + 2t_{\text{AB}} + 2$, in equilibrium:*

*i. Exclusive supplier $j \in S_i$ to tier-1 firm $i \in S(1)$ chooses supply quantity $s^*_{j,i} = s_e(t_i, t_{-i}, t_{\text{AB}})$ where $-i \in \{\text{A}, \text{B}\}\backslash\{i\}$;*

*ii. Shared supplier $j \in S_{\text{AB}}$ chooses supply quantities $s^*_{j,\text{A}} = s_s(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}})$ and $s^*_{j,\text{B}} = s_s(t_{\text{B}}, t_{\text{A}}, t_{\text{AB}})$;*

*iii. Tier-1 firm $i \in S(1)$ chooses supply quantity $q^*_i = t_i s_e(t_i, t_{-i}, t_{\text{AB}}) + t_{\text{AB}} s_s(t_i, t_{-i}, t_{\text{AB}}) = \bar{q}_{(1)}(t_i, t_{-i}, t_{\text{AB}})$ where $-i \in \{\text{A}, \text{B}\}\backslash\{i\}$;*

*iv. The total quantity the supply network produces is*

$$
q^*_{\text{C}} = t_{\text{A}} s_e(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}) + t_{\text{B}} s_e(t_{\text{B}}, t_{\text{A}}, t_{\text{AB}}) + t_{\text{AB}}(s_s(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}) + s_s(t_{\text{B}}, t_{\text{A}}, t_{\text{AB}})) = \bar{q}(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}). \quad (4.23)
$$

*(b) If $t_{\text{A}} \geqslant 2t_{\text{B}} + 2t_{\text{AB}} + 2$, in equilibrium:*

*i. Firm A's exclusive supplier $j \in S_{\text{A}}$ chooses supply quantity $s^*_{j,\text{A}} = s_e(t_{\text{A}}, t_{\text{B}} + t_{\text{AB}}, 0)$;*

*ii. Firm B's exclusive supplier $j \in S_{\text{B}}$ chooses supply quantity $s^*_{j,\text{B}} = s_e(t_{\text{B}} + t_{\text{AB}}, t_{\text{A}}, 0)$;*

*iii. Shared supplier $j \in S_{\text{AB}}$ chooses supply quantities $s^*_{j,\text{A}} = 0$ and $s^*_{j,\text{B}} = s_e(t_{\text{B}} + t_{\text{AB}}, t_{\text{A}}, 0)$;*

*iv. Firm A chooses supply quantity $q^*_{\text{A}} = t_{\text{A}} s_e(t_{\text{A}}, t_{\text{B}} + t_{\text{AB}}, 0) = \bar{q}_{(1)}(t_{\text{A}}, t_{\text{B}} + t_{\text{AB}}, 0)$;*

*v. Firm B chooses supply quantity $q^*_{\text{B}} = (t_{\text{B}} + t_{\text{AB}}) s_e(t_{\text{B}} + t_{\text{AB}}, t_{\text{A}}, 0) = \bar{q}_{(1)}(t_{\text{B}} + t_{\text{AB}}, t_{\text{A}}, 0)$;*

Table 4.1: Comparative Statics in Production Phase

| Variables | (I) | (II) | (III) |
|---|---|---|---|
| $q^*_{\text{C}}, m^*_{\text{C}}, \pi^*_{\text{C}}$ | + | + | + |
| $q^*_{\text{A}}, m^*_{\text{A}}, \pi^*_{\text{A}}$ | + | − | +/− |
| $q^*_{\text{B}}, m^*_{\text{B}}, \pi^*_{\text{B}}$ | − | + | + |
| $p^*_{(1)}$ | − | − | − |
| $p^*_{(2)\text{A}}$ | − | − | − |
| $p^*_{(2)\text{B}}$ | − | − | − |
| $\rho^*_{\text{A}}$ | + | − | − |
| $\rho^*_{\text{B}}$ | − | + | + |

How the equilibrium value of each variable changes as the number of tier-2 suppliers increases by one. (See Theorem 4.2 for details.) "+" indicates the variable increases; "−" indicates the variable decreases; "+/−" indicates there exist both instances of the variable increasing and decreasing depending on the specific topology of the supply network (captured by $t_{\text{A}}$, $t_{\text{B}}$, $t_{\text{AB}}$).

*vi. The total quantity the supply network produces is*

$$q^*_{\text{C}} = t_{\text{A}} s_e(t_{\text{A}}, t_{\text{B}} + t_{\text{AB}}, 0) + (t_{\text{B}} + t_{\text{AB}}) s_e(t_{\text{B}} + t_{\text{AB}}, t_{\text{A}}, 0) = \bar{q}(t_{\text{A}}, t_{\text{B}} + t_{\text{AB}}, 0). \tag{4.24}$$

The closed-form expressions for supply quantities in Proposition 4.4 allow us to express the equilibrium prices and profits of all players in closed form. From the proposition, beyond market demand parameters ($\alpha$, $\beta$) and the total production cost ($v_T$), the only determinant of the equilibrium profit is the topology of the supply network captured by $t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}$.

In Proposition 4.4b where $t_{\text{A}} \geqslant 2t_{\text{B}} + 2t_{\text{AB}} + 2$, the majority tier-1 firm A has so many exclusive tier-2 suppliers (relative to shared suppliers and firm B's exclusive suppliers) that its upstream market is too competitive for any shared supplier to profitably participate in it. Each shared supplier chooses to behave as an exclusive supplier to minority tier-1 firm B.

### 4.4.2 Comparative Statics

We now seek to understand the value of the contribution by each supplier to the buyer's production profit, which guides the buyer's decisions in the auditing phase. Let $m^*_{\text{C}}$, $m^*_{\text{A}}$, and $m^*_{\text{B}}$ be the margins of the buyer, firm A, and firm B in equilibrium, i.e., $m^*_{\text{C}} = p^*_{(0)} - p^*_{(1)} - v_0$, $m^*_{\text{A}} = p^*_{(1)} - p^*_{(2)\text{A}} - v_1$, and $m^*_{\text{B}} = p^*_{(1)} - p^*_{(2)\text{B}} - v_1$. Let $\rho^*_{\text{A}} = \frac{q^*_{\text{A}}}{q^*_{\text{C}}}$ and $\rho^*_{\text{B}} = \frac{q^*_{\text{B}}}{q^*_{\text{C}}}$ be the market shares of the two tier-1 firms in equilibrium. As in section 4.4.1 we index the majority tier-1 firm as A, i.e., $t_{\text{A}} \geqslant t_{\text{B}}$.

**Theorem 4.2.** *Given a supply network $g \in G$, adding a tier-2 supplier changes the equilibrium values of the variables in Table 4.1 as follows:*

*(a) Adding a majority-exclusive supplier changes the equilibrium values as in column (I);*

*(b) Adding a minority-exclusive supplier changes the equilibrium values as in column (II);*

*(c) Adding a shared supplier changes the equilibrium values as in column (III) if $t_{\text{A}} \leqslant 2t_{\text{B}}+2t_{\text{AB}}+2$, and as in column (II) otherwise.*

Columns (I) and (II) of Table 4.1 reflect the following intuitive observation: adding an exclusive supplier to a tier-1 firm gives that tier-1 firm greater advantage in competition, increasing its quantity, margin, profit, and market share, and decreasing the same metrics for the other tier-1 firm. If $t_{\text{A}} > 2t_{\text{B}} + 2t_{\text{AB}} + 2$, firm A dominates the supply network so much that the shared suppliers behave as exclusive suppliers to firm B, in which case adding a shared supplier has the same effect as adding an exclusive supplier to firm B, as shown in Column (II). The only ambiguity arises when $t_{\text{A}} \leqslant 2t_{\text{B}} + 2t_{\text{AB}} + 2$, and we add a shared supplier. In this case, as shown in Column (III), the effect on firm B is clear, but the effect on firm A's quantity, margin, and profit is not. First, consider firm B, which has less market power: adding a shared supplier boosts firm B's power more than it boosts that of firm A, which enjoyed greater power to begin with. In fact firm B's quantity, margin, profit, and market share all improve, at the expense of firm A's market share. The following result resolves the ambiguity of the effect on firm A's quantity, margin, and profit.

**Proposition 4.5.** *Given supply network $g \in G$ where $t_{\text{A}} \leqslant 2t_{\text{B}} + 2t_{\text{AB}} + 2$, adding a shared supplier to $g$ increases equilibrium variables $q^*_{\text{A}}$, $m^*_{\text{A}}$, and $\pi^*_{\text{A}}$ if and only if*

$$t_{\text{A}} < \frac{\sqrt{33t_{\text{B}}^2 + 72t_{\text{B}}t_s + 108t_{\text{B}} + 48t_s^2 + 144t_s + 100}}{4} - t_s - \frac{t_{\text{B}}}{4} - \frac{3}{2}. \tag{4.25}$$

When $t_{\text{A}}$ is relatively small, adding a shared supplier benefits both the minority tier-1 firm B (as we have seen in Theorem 4.2) and the majority tier-1 firm A. When $t_{\text{A}}$ is relatively large, adding a shared supplier benefits the minority tier-1 firm B but hurts the majority tier-1 firm A. The reason is that while adding the shared supplier makes firm A's input market more competitive, directly benefiting firm A, the addition benefits the minority tier-1 firm B even more. In fact, the boost in firm B's power improves its position so much in the downstream competition with firm A, that it rebounds to hurt firm A by overwhelmingly slashing its margin.

Despite the ambiguity of the effect on the majority tier-1 firm, Theorem 4.2 shows that regardless of where we add the tier-2 supplier, the addition always decreases the input prices of both tier-1 firms and that of the buyer, and increases the quantity, margin, and profit of the buyer. The next proposition ranks the buyer's gain from the addition of a supplier based on the supplier's location in the network.

**Proposition 4.6.** *Given supply network $g \in G$:*

*(a) Adding a shared supplier to $g$ induces a strictly greater increase in $q^*_{\text{C}}, m^*_{\text{C}}, \pi^*_{\text{C}}$ than adding a minority-exclusive supplier;*

*(b) Adding a minority-exclusive supplier to $g$ induces a greater or equal increase in $q^*_{\text{C}}, m^*_{\text{C}}, \pi^*_{\text{C}}$ than adding a majority-exclusive supplier.*

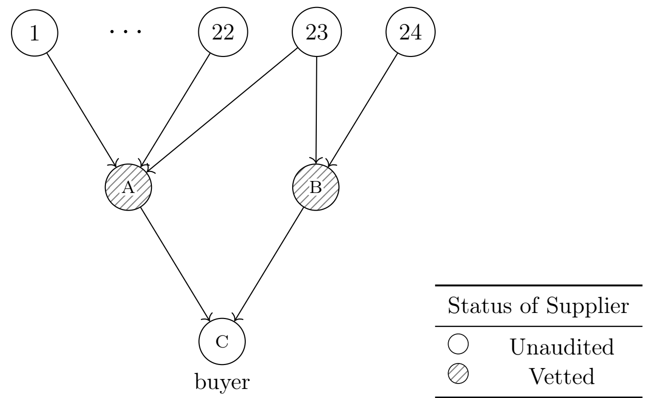
For the buyer, a shared supplier is more valuable than a minority-exclusive supplier (Proposition 4.6a), which in turn is more valuable than a majority-exclusive supplier (Proposition 4.6b). Generally the greater the upstream competition, the better off the buyer. Adding a shared supplier intensifies the competition in both tier-1 firms' input markets, which then intensifies the competition in the buyer's input market, more than adding an exclusive supplier. Adding an exclusive supplier to the minority tier-1 firm helps to counter the disadvantageous position of the minority tier-1 firm in the competition for the buyer's business, bringing down the buyer's input cost more than adding an exclusive supplier to the majority tier-1 firm.

## 4.5 Auditing Phase Results

We begin with two examples that illustrate intriguing properties of the optimal auditing behavior, which we later explain in the following sections.

*Example* 4.1. Let the parameters be $\alpha = 190, \beta = 1.4, v_T = 1.5, a = 75, r = 860, z = 357.88, u = 0.47, w = 0.46$. In the state $\gamma$ shown in Figure 4.2: tier-1 firm A has 22 exclusive suppliers (firms 1 to 22); tier-1 firm B has one exclusive supplier (firm 24); firms A and B share one supplier (firm 23). Both tier-1 firms are vetted. All tier-2 suppliers are unaudited.

Figure 4.2: State $\gamma$ in Example 4.1




The majority tier-1 firm A has 22 exclusive suppliers.

In this example (and the next), the unit rectification cost, $r$, is so high that the buyer bypasses any rectification actions, instead auditing suppliers and dropping them if they are non-compliant. Therefore, the prevailing tradeoff is whether to audit a supplier and potentially drop it from the network versus carrying an unaudited supplier to production and possibly incurring a violation penalty. The buyer is then faced with the question: which supplier to audit first?

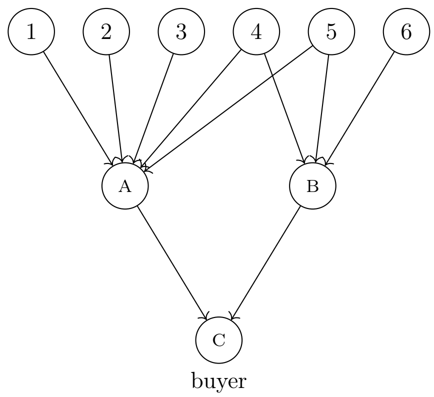
As Proposition 4.6 suggests, firms 1 through 22 are less valuable for the buyer than firms 23 or 24. Therefore we may suspect the more appealing option for the buyer would be to audit, say, firm 1 rather than firm 24. If the buyer starts with AD(1) (i.e., audit firm 1 and drop it if non-compliant) and firm 1 fails the audit, in this example we find that the next optimal action would be one of AD(2), . . . , AD(22) (i.e., audit another exclusive supplier to firm A and drop it if non-compliant). The consequence of this could be that the buyer ends up dropping multiple exclusive suppliers to firm A. However, if the buyer starts with AD(24), the minority-exclusive supplier, and firm 24 fails the audit, we find that the next optimal action in this example would be to stop auditing activities altogether and directly proceed to the production phase (PP).

Which of these two paths, with the initial firms failing their audits, brings the buyer more profit overall: start with an audit of the less valuable firm 1 and risk losing additional suppliers, or start with an audit of the more valuable firm 24, ensuring the retention of all other suppliers?

*Example* 4.2. We consider how the optimal first auditing decision varies with the value of penalty $z$. Still using the same values of parameters in Example 4.1, we consider the state shown in Figure 4.4(a) where all suppliers are unaudited. Figure 4.4(b) presents the optimal first decision as penalty $z$

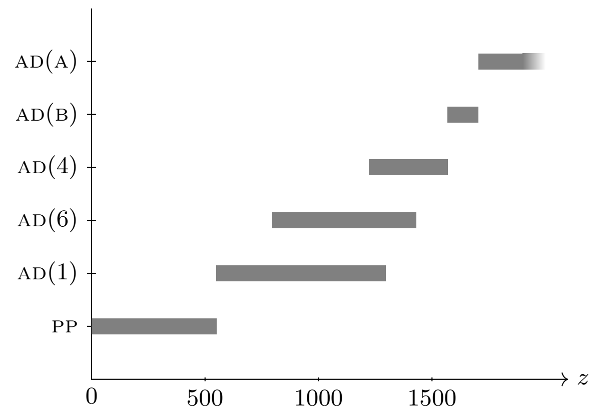
Figure 4.3: State $\gamma$ and Optimal First Decision in $\gamma$ in Example 4.2

(a) State $\gamma$




Every supplier is unaudited in $\gamma$.

(b) Optimal First Decision in State $\gamma$




As penalty $z$ increases the optimal decision at $\gamma$ shifts in order from PP, to AD(1) (or any other exclusive supplier to firm A), to AD(6), to AD(4) (or the other shared supplier 5), to AD(B), and eventually to AD(A).

varies.

With low $z$ ($z < 550$) the potential penalty is too low to justify any audit by the buyer; the buyer proceeds to the production phase directly.

With high $z$ ($z > 1570$) the penalty is so great that the buyer turns its attention to the tier-1 firms. It is optimal for the buyer to start with auditing a tier-1 supplier and dropping it if non-compliant (AD(B) for $1570 < z < 1704$ and AD(A) for $z > 1704$). Dropping a tier-1 firm enables the buyer to drop all dependent tier-2 suppliers, thus avoiding both the expense of auditing them and any associated violation penalties. In short, the penalty is so high that the buyer is willing to remove an entire side of the supply network rather than risk carrying unaudited suppliers through to production.

With intermediate $z$ ($550 < z < 1570$) the buyer focuses the AD effort on the tier-2 suppliers. As $z$ increases from 550, the optimal first audit is AD(1) (or any majority-exclusive supplier). As $z$ further increases, AD(6) (the minority-exclusive supplier) also becomes an optimal first audit. Similarly AD(4) (or any shared supplier) becomes an optimal first audit as $z$ increases further up to 1570. As seen in Figure 4.4(b) there are ranges within $550 < z < 1570$ in which multiple auditing decisions are optimal: for example, for values of $z$ between 1223 and 1297, AD(1), AD(4), and AD(6) are all equally good auditing decisions. For values of $z$ between 1297 and 1431, AD(4) (or AD(5)) and AD(6) are the only optimal first auditing decisions; for values of $z$ between 1431 and 1570, AD(4) (or the other shared supplier) becomes the unique optimal auditing decision. It is peculiar that the buyer would optimally choose supplier 6, a minority-exclusive supplier, or supplier 4, a shared supplier, as the first firm to audit. After all, Proposition 4.6 identifies supplier 1 (or 2 or 3) as the least valuable supplier within tier 2 to the buyer. One may therefore intuit that among tier-2 suppliers, the buyer would prioritize auditing supplier 1 (or any other majority-exclusive supplier) which is less valuable to the buyer when it reaches the production phase, but carries as much penalty as any other supplier in the event of a violation. Yet, as the example shows, the buyer may find it optimal to start with an audit of a minority-exclusive or a shared supplier with greater direct damage to the buyer's production profit when dropped. Why would the buyer start its audit with a minority-exclusive or shared supplier and risk losing more production profit in the event it is dropped?

### 4.5.1 Two Subphases of Auditing

We identify a surprisingly simple property of an optimal policy. The buyer will first audit and drop some suppliers; we call this the AD *subphase*. Only after the buyer ceases the AD subphase will it then proceed to what we refer to as the RP *subphase*. In the RP subphase the buyer either proceeds to the production phase directly, or audits and rectifies all remaining suppliers in an arbitrary sequence.

**Theorem 4.3.** *There exists an optimal policy $\xi^* \in \Xi$ with the property that auditing decisions are divided into two subphases:*

*(a)* AD subphase: *To audit and drop (* AD*) some suppliers (or no supplier); followed by*

*(b)* RP subphase: *To audit and rectify (* AR*) all remaining unaudited suppliers in an arbitrary sequence if $a + ur \leqslant uwz$; or to proceed to production (* PP*) if $a + ur \geqslant uwz$.*

Under the optimal policy in Theorem 4.3 any auditing and dropping activity (AD subphase) will precede any rectification activity (in the RP subphase). Once the buyer starts rectifying suppliers it will not return to dropping suppliers again. The theorem is, however, silent on the length of the AD subphase and when the RP subphase will commence. Specifically, the commencement of the RP subphase depends upon the outcome of the various audits conducted in the AD subphase.

To explain the separation of the AD and AR activities into subphases, we make the following observation: given any policy in which AR($i$) immediately precedes AD($j$) for two unaudited suppliers $i$ and $j$, we can change the sequence of the two actions (so that AD($j$) precedes AR($i$)) and the outcome will either strictly improve or remain the same. When $i$ is a dependent of $j$, the reversal will make the buyer strictly better off (in expectation): in the event $j$ is dropped (and takes $i$ along with it), by putting AD($j$) before AR($i$) the buyer avoids the cost of auditing and rectifying $i$. On the other hand, when $i$ is not a dependent of $j$, the outcome will remain the same. Auditing and rectifying supplier $i$ will make it a vetted supplier, regardless of its initial compliance status, leaving the remainder of the supply network unchanged. Therefore, neither the costs incurred nor the resulting state will be affected by the sequence of the two actions.

Once in the RP subphase, now that the AD activity is over, the topology of the supply network will not change hereafter. This sets in stone the production profit, determined solely by the supply network, thereby leaving the buyer with the following decision for each unaudited supplier: whether to audit and rectify it or to allow it to enter the production phase unaudited. In deciding this, the buyer compares the cost of auditing the supplier and rectifying it if it fails the audit ($a+ur$) against the penalty that arises when an unaudited supplier is exposed to be non-compliant ($uwz$). Since all unaudited suppliers are identical (other than their location in the network), this comparison is identical for all suppliers. Hence in the RP subphase the buyer either proceeds to production directly or audits and rectifies *all* remaining suppliers. In the scenario that the buyer stops short of auditing all suppliers in the AD subphase and proceeds to the production phase directly in the RP subphase, the optimal policy manifests as "see no evil, hear no evil": the buyer conducts no further audits. We revisit this scenario in section 4.5.2.

The optimal behavior in the RP subphase leads to the following result:

**Corollary 4.1.** *At state $\gamma \in \Gamma$, if the optimal policy $\xi^*$ is already in the* RP *subphase,*

$$V^*(\gamma) = \pi(\gamma) - [(uwz) \wedge (a+ur)]|U_\gamma|. \tag{4.26}$$

In the rest of the paper, we focus on policies which consist of the two subphases, described in Theorem 4.3. In particular, in each stage of the auditing phase, we only need to consider the actions to audit and drop a supplier (AD) and the action to proceed to the RP subphase. We introduce a new action RP that is a shorthand for "audit and rectify (AR) all remaining unaudited suppliers if $a + ur < uwz$ and proceed to production (PP) otherwise." Following Corollary 4.1 we denote the cost associated with each unaudited supplier in the RP subphase $c_{\text{RP}} \equiv (uwz) \wedge (a + ur)$.

### 4.5.2 Optimal Auditing Sequence in Tier 2: Toward a Balanced Supply Network

In this subsection we consider the sequence of suppliers the buyer will audit and drop in the AD subphase. As Example 4.2 shows, any supplier may emerge as the buyer's optimal choice for the first audit as a single parameter of the model is varied. Despite this fickle behavior, once we limit the auditing to the tier-2 suppliers, we demonstrate the optimal auditing sequence is determined by the value of the firm, which in turn depends on its location in the network. With this result we address the question posed in Example 4.1, where the buyer faces the quandary of which tier-2 supplier to audit when the tier-1 firms are already vetted. Later in section 4.5.3 we consider the scenario where all suppliers are unaudited to shed light on the question from Example 4.2.

Recall that we define $\nabla(\gamma, i)$ as the production value of supplier $i$ for the buyer at state $\gamma$: $\nabla(\gamma, i) = \pi(\gamma) - \pi(\gamma \ominus i)$. For the purposes of this section, it is useful to separately identify a boundary element of the state space, denoted $\gamma_1 = (g, U)$, the state in which the buyer is served by two separate linear branches (i.e., $g = (\{\text{A}, \text{B}\}, \{1\}, \{2\}, \emptyset)$), and both tier-1 firms are vetted and both tier-2 suppliers are unaudited (i.e., $U = \{1, 2\}$). We treat $\gamma_1$ separately.

We now introduce an intuitive property of the supply network, specifically a condition on production profit $\pi$.

**Condition 1** (decreasing differences of production profit)**.** For any $\gamma \in \Gamma \backslash \{\gamma_1\}$ in which every tier-1 firm is vetted (i.e., $S(1) \cap U_\gamma = \emptyset$) and for any $i, i' \in U_\gamma$ such that $i' \notin D_\gamma(i)$,

$$\nabla(\gamma, i') \leqslant \nabla(\gamma \ominus i, i'). \tag{4.27}$$

Condition 1 says that the production value of a supplier is greater in smaller supply networks. This is intuitive since each additional supplier adds to the buyer's profit (due to increased competition leading to lower input prices) but to a lesser extent than the previous one. Using the closed-form expressions derived in Proposition 4.4, we have algebraically verified Condition 1 for any network with up to 100 tier-2 suppliers.[3]

We now define a concept which is then used in the subsequent result.

**Definition 4.1.** Let $\gamma \in \Gamma$ and $i \in U_\gamma$. If $\nabla(\gamma, i) \leqslant \nabla(\gamma, i')$ for any $i' \in U_\gamma$, then we call $i$ a *least valuable unaudited supplier (LVUS)* in $\gamma$.[4]

An LVUS is the unaudited supplier that carries the least value to the buyer's production profit. We now show that the LVUS is the next firm to audit when auditing tier-2 firms only.

**Theorem 4.4.** *Under Condition 1 the following policy $\xi^{**}$ is optimal at any state $\gamma$ in which every tier-1 firm is vetted: for any nonterminal state $\gamma \neq \gamma_1$, let $i$ be an LVUS in $\gamma$, then*

$$\xi^{**}(\gamma) = \begin{cases} \text{AD}(i), & \text{if } u\nabla(\gamma, i) + a < c_{\text{RP}} \\ \text{RP}, & \text{if } u\nabla(\gamma, i) + a \geqslant c_{\text{RP}} \end{cases}. \tag{4.28}$$

According to Theorem 4.4, the buyer's optimal sequence follows a simple rule: identify an LVUS $i$ and audit it if $u\nabla(\gamma, i) + a < c_{\text{RP}}$. The buyer should always audit and drop an LVUS, a supplier that carries the least production value in the current supply network, until the stage at which even an LVUS carries more production value than the costs and risk associated with the supplier in the RP subphase (captured by $c_{\text{RP}}$). Once at that stage the buyer should proceed to the RP subphase to either rectify all non-compliant suppliers identified by exhaustive auditing or just proceed to production and brace itself for any damages from the potential exposure of violation from unaudited suppliers. Suppose every supplier, even an LVUS, carries sufficient production value to justify retention ($u\nabla(\gamma, i) + a \geqslant c_{\text{RP}}$ for LVUS $i$) and the penalty is not high enough to warrant rectification ($a + ur \geqslant uwz$ as in Theorem 4.3). In such an instance the buyer will optimally "see no evil, hear no evil" and not audit any supplier.

To assist in identifying the LVUS, we list the following result, based on Proposition 4.6.

**Corollary 4.2.** *Any unaudited majority-exclusive supplier will be an LVUS.*

---

[3]Network $\gamma_1$ does not satisfy Condition 1 (hence, we treat it separately) because it is a special case: the removal of any supplier will prune an entire branch of the network, which eliminates the competition between firms A and B. Fortunately, we know the optimal auditing policy at $\gamma_1$, which we fully describe in Theorem 4.4′ in Appendix 4.C.2.

[4]"LVUS" is pronounced |ˈɛlvəs|, the same as the "King of Rock and Roll."

Equipped with Theorem 4.4 and Corollary 4.2, we can return to considering Example 4.1, in which the buyer will identify firm 1 (or any of the majority-exclusive suppliers to firm A) as an LVUS. Firm 1 will be audited and dropped (if it fails) since it is a majority-exclusive supplier. Before considering firm 24, the minority-exclusive supplier, which is not LVUS, the buyer will continue with firms 2, 3, ..., 22, which are LVUS. (Of course, the buyer may choose to truncate the AD subphase and proceed to RP, at any time.) Within the context of Example 4.1, an LVUS is a majority-exclusive supplier to firm A, as the potential for dropping one of many of these suppliers only marginally affects the competition compared to possibly dropping firm 24 which would have a more dramatic effect upon the competition on that side of the network.

In summary by following the optimal policy in Theorem 4.4, the buyer will target whichever side of the supply network has a greater number of exclusive tier-2 suppliers and will continue trimming that side whenever those firms fail their audits. We observe that as firms are dropped through failing audits, the network evolves towards a more balanced shape, where $t_A$ and $t_B$ become more similar.

*Remark* 4.1. Since the value of each state $\gamma$ critically depends on the status (unaudited or vetted) of every supplier, one may expect that the choice of the supplier to audit in the AD subphase should depend not only on the supplier's value to the buyer's production profit, but also on the status of other suppliers. For example, if a tier-2 supplier has many vetted peers (e.g., an unaudited majority-exclusive supplier when most other majority-exclusive suppliers are vetted), shouldn't the buyer be less inclined to audit and drop that supplier and instead focus on less vetted regions of the supply network? Surprisingly under the conditions of Theorem 4.4 the choice of the supplier to audit is independent of the status of any other supplier in the supply network. The value the supplier brings to the buyer's production profit (i.e., $\nabla(\gamma, i)$ for supplier $i$ in state $\gamma$) solely determines whether to audit and drop any supplier and which supplier to audit.

### 4.5.3 Supplier Choice When Auditing One Firm

While we have fully characterized the optimal policy when all tier-1 firms are vetted, the problem gets substantially more complicated if the tier-1 firms are among the choices to audit. We illustrate the complexity of this problem in section 4.5.4. We are able to prove the optimal policy of auditing for any state $\gamma \in \Gamma$ if we allow the buyer to audit at most one supplier (e.g., due to a limited auditing budget).

**Proposition 4.7.** *At nonterminal state $\gamma$, suppose the buyer can audit (* AD *or* AR*) at most one supplier, before proceeding to production (* PP*). There exist two (possibly coinciding) thresholds $\underline{z} \leqslant \bar{z}$ for penalty $z$ such that*

*(a) If $z \leqslant \underline{z}$ the optimal decision is* PP*;*

*(b) If $\underline{z} < z \leqslant \bar{z}$ the optimal decision is* AR$(i)$ *for any $i \in U_\gamma$;*

*(c) If $z > \bar{z}$ the optimal decision is* AD$(i)$ *where $i$ solves*

$$\max_{i \in U_\gamma} \{uwz(|U_\gamma| - |U_{\gamma \ominus i}|) - \nabla(\gamma, i)\} \tag{4.29}$$

We identify the thresholds $\underline{z}$ and $\bar{z}$ in the proof of Proposition 4.7 in Appendix 4.C.3.

With low penalty $z$ ($z \leqslant \underline{z}$) the buyer has no incentive to make any effort to audit and proceeds directly to the production phase. The buyer is willing to suffer the minor penalty if a supplier violates. With intermediate penalty $z$ ($\underline{z} < z \leqslant \bar{z}$) retaining maximal profit from production activity remains the dominating consideration. The buyer audits a supplier but refrains from dropping it if the audit reveals it as non-compliant; instead the buyer rectifies the supplier to keep it in the supply network. With high penalty $z$ ($z > \bar{z}$) the buyer's priority shifts to auditing and then dropping the risky suppliers. When dropping a supplier $i$, the buyer has two effects to consider: the loss in production profit, $\nabla(\gamma, i)$, versus the avoidance of violation penalties. The latter is given by $uwz(|U_\gamma| - |U_{\gamma \ominus i}|)$, notably because $\gamma \ominus i$ will involve not only the removal of $i$ but also any of $i$'s dependents from the network $\gamma$. Hence, the buyer chooses whichever supplier maximizes the difference between the violation penalty avoidance and loss in production profit, as shown in (4.29).

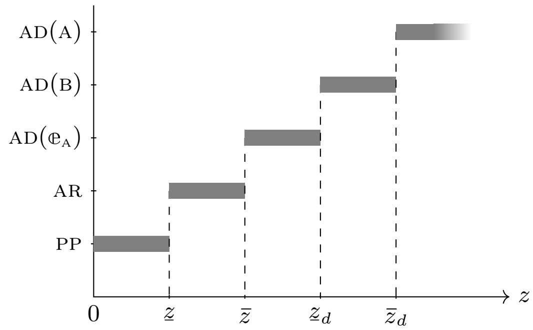
When the buyer's auditing expands beyond tier 2, it is now possible that the buyer's optimal first audit will not be an LVUS (which is the supplier with the lowest $\nabla(\gamma, i)$). Instead, in an effort to quickly eliminate violation penalties due to multiple suppliers, the buyer now considers the possibility of dropping tier-1 suppliers, which will take away their exclusive tier-2 suppliers with them. With this in mind, the next proposition specifies which particular supplier the buyer would audit once $z > \bar{z}$. We consider a state $\gamma_+ = (g, U)$ in which there is at least one supplier in each position in tier 2 (majority-exclusive, minority-exclusive, shared; i.e., $t_{\text{A}}, t_{\text{B}}, t_{\text{AB}} \geqslant 1$), all suppliers (including those in tier 1) are unaudited, and the majority tier-1 firm A has strictly more suppliers than the minority tier-1 firm B (i.e., $t_{\mathrm{A}} > t_{\mathrm{B}}$). This structure allows us to compare all possible auditing choices. We define in Appendix 4.C.3 the thresholds used in the following proposition, $z_{\mathrm{A}|1}$, $z_{\mathrm{B}|1}$, $z_{\mathrm{A}|\mathrm{B}}$, in closed-form expressions.

**Proposition 4.8.** *At state $\gamma_+$ suppose the buyer can audit at most one supplier before proceeding to production (*PP*). Let $\underline{z}_d = (z_{\mathrm{A}|1} \wedge z_{\mathrm{B}|1}) \vee \bar{z}$ and $\bar{z}_d = z_{\mathrm{A}|1} \vee z_{\mathrm{A}|\mathrm{B}} \vee \bar{z}$ where $\bar{z}$ is as in Proposition 4.7. The optimal decision is* AD($\mathbb{e}_{\mathrm{A}}$) *(i.e., auditing and dropping (if non-compliant) an exclusive supplier to firm* A*) if and only if $\bar{z} < z \leqslant \underline{z}_d$,* AD(B) *if and only if $\underline{z}_d < z \leqslant \bar{z}_d$, and* AD(A) *if and only if $z > \bar{z}_d$.*

We illustrate the results of Propositions 4.7 and 4.8 in Figure 4.5. As we have seen in Proposition 4.7, the buyer starts auditing with the intention to drop only if $z$ exceeds $\bar{z}$. In that region, if $z$ is relatively low ($\bar{z} < z < \underline{z}_d$), the buyer will still choose an LVUS — a majority-exclusive supplier — to audit. However, once $z$ exceeds $\underline{z}_d$, the supplier to audit is no longer an LVUS. When penalty $z$ is intermediate ($\underline{z}_d < z \leqslant \bar{z}_d$) the buyer chooses the minority tier-1 firm B (AD(B)). The decision risks losing the production profit attributable to firm B and the entire side of the network dependent on firm B, but simultaneously avoids the potential penalty from all those suppliers. When penalty $z$ is high ($z > \bar{z}_d$) the buyer chooses the majority tier-1 firm A with even higher stakes: the potential of losing all production profit attributable to firm A and its side of the supply network while avoiding penalty from this large group of suppliers.

Let us revisit Example 4.2 in light of Propositions 4.7 and 4.8, by comparing Figure 4.4(b) with Figure 4.5. First, in Example 4.2 the rectification cost was sufficiently high that AR is not utilized at all. Second, in both figures we see as $z$ increases, the first auditing choice moves from a majority-exclusive supplier and then eventually to the minority tier-1 firm B to the majority tier-1 firm A. However, we also observe notable differences, primarily attributable to the fact that the results in this section limit the audit to a single firm while Example 4.2 considers the first auditing choice within a sequence of audits. Notably, in Figure 4.4(b) for certain values of $z$ the buyer will first audit a minority-exclusive or shared supplier in tier 2, something we do not observe in Figure 4.5. In this section where the buyer is restricted to auditing at most one supplier, it is never optimal for the buyer to audit and drop a minority-exclusive or shared supplier: dropping one such supplier

Figure 4.5: Optimal Decision When Buyer Audits At Most One Supplier




Schematic illustration of the optimal decision as penalty $z$ varies and when the buyer is limited to conduct at most one audit. AR represents auditing and rectifying if non-compliant any unaudited supplier.

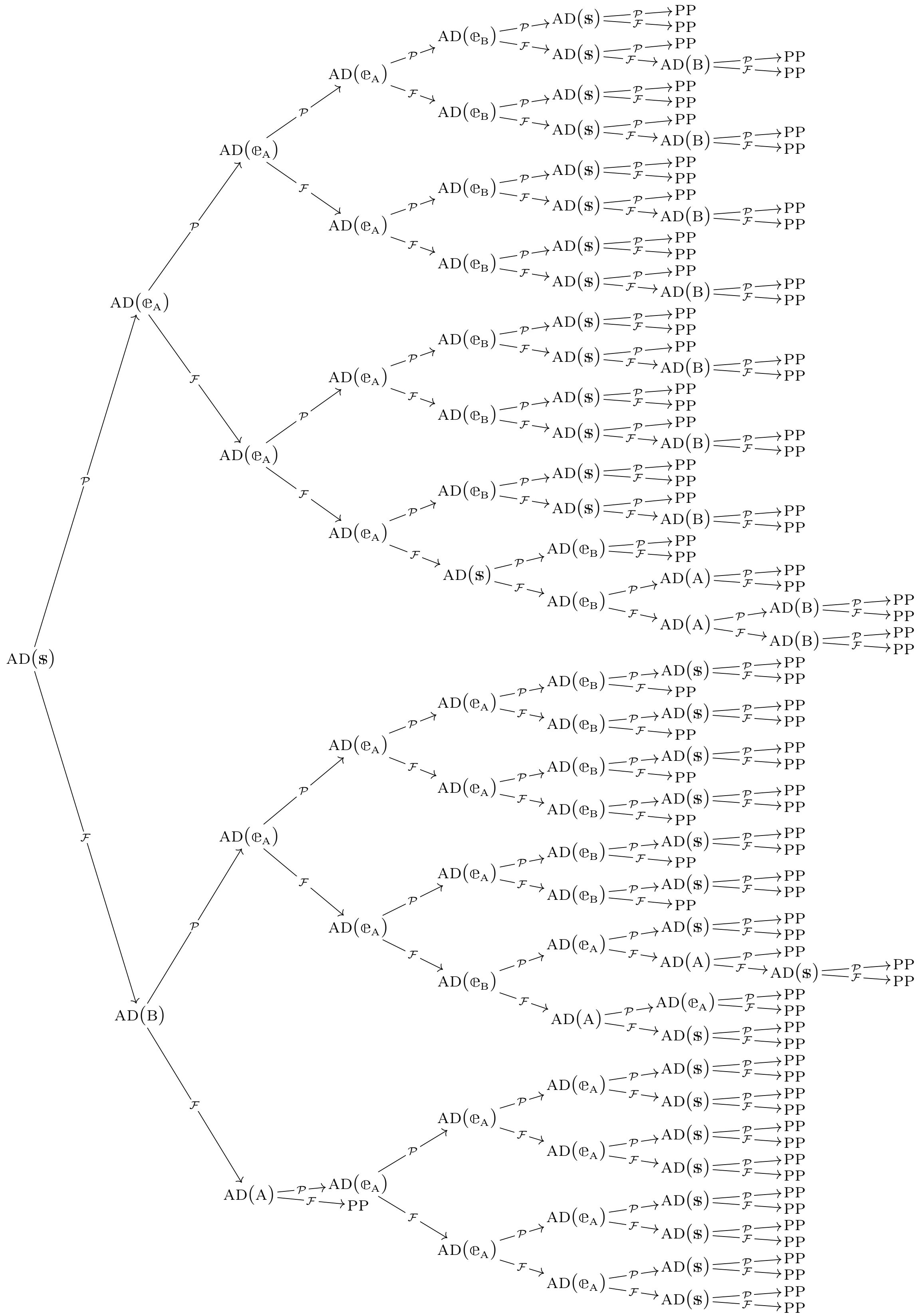
leads to greater loss of profit compared to an LVUS (majority-exclusive supplier) while avoiding only as much potential penalty from violation. However, when the buyer may conduct a sequence of audits, shared or minority-exclusive suppliers may be chosen as we see in Example 4.2, something we explore further in the next section.

### 4.5.4 Patterns in Optimal Auditing Paths

We have shown in section 4.5.2 a complete picture of optimal auditing behavior in the second tier. In particular the buyer will always prioritize an LVUS for AD. On the other hand Example 4.2 shows that, if we include the tier-1 firms as auditing candidates, over a certain range of parameters the optimal first decision can be auditing and dropping (if non-compliant) a minority-exclusive supplier or even a shared supplier, neither of which is an LVUS. What drives such behavior? How can auditing and dropping (if non-compliant), say, a shared supplier benefit the buyer more than auditing a majority-exclusive supplier?

Figure 4.6 presents an event tree of the buyer's optimal auditing paths in Example 4.2. The initial network state $\gamma$ may be seen in Figure 4.4(a). The event tree in Figure 4.6 is attained when penalty $z = 1500$ so that the buyer strictly prefers to first audit and drop a shared supplier, i.e., take action AD($\mathbb{s}$) at $\gamma$. In network $\gamma$, $\mathbb{e}_{\text{A}}$ may correspond to any exclusive supplier to firm A, suppliers 1, 2, or 3; $\mathbb{e}_{\text{B}}$ refers to the exclusive supplier to firm B, supplier 6; and $\mathbb{s}$ refers to either shared supplier, 4 or 5. We use $\mathcal{P}$ to represent the branch for passing an audit and $\mathcal{F}$ for failing. Figure 4.6 shows an example in which the RP subphase consists only of PP activity but not AR, due to the high rectification cost, as discussed in section 4.5.1.

Figure 4.6: An Event Tree Under Optimal Auditing Policy



The first pattern we discuss is what we call the "litmus test." In certain states the buyer opts to audit a supplier in a pivotal position in the network (instead of an LVUS) to gauge the viability of a portion of the supply network or even the entire network. For example, the buyer chooses the shared supplier as the first firm to audit in state $\gamma$, as seen in Figure 4.6. Such a supplier is in a pivotal location in that it serves both firms A and B; knowing the outcome of the audit allows the buyer to deduce how valuable firms A and B will be. By testing the shared supplier early on, the buyer gains information that it can act up on in the early stages of auditing. In the event that the shared supplier passes, the buyer is optimistic about the viability of both sides of the supply network, and as seen in Figure 4.6, it takes a more cautious approach by auditing a sequence of majority-exclusive suppliers, each of which is an LVUS. Notably, however, when the shared supplier fails the audit, the buyer is pessimistic about the value of firm B, because that side of the supply network is not as attractive as it once might have been when the shared supplier was in the network. Hence, the buyer goes on to audit firm B, which is yet another litmus test: that supplier is also in a pivotal position (and not an LVUS), and the buyer takes dramatically different actions depending on the outcome of that audit, as we discuss next.

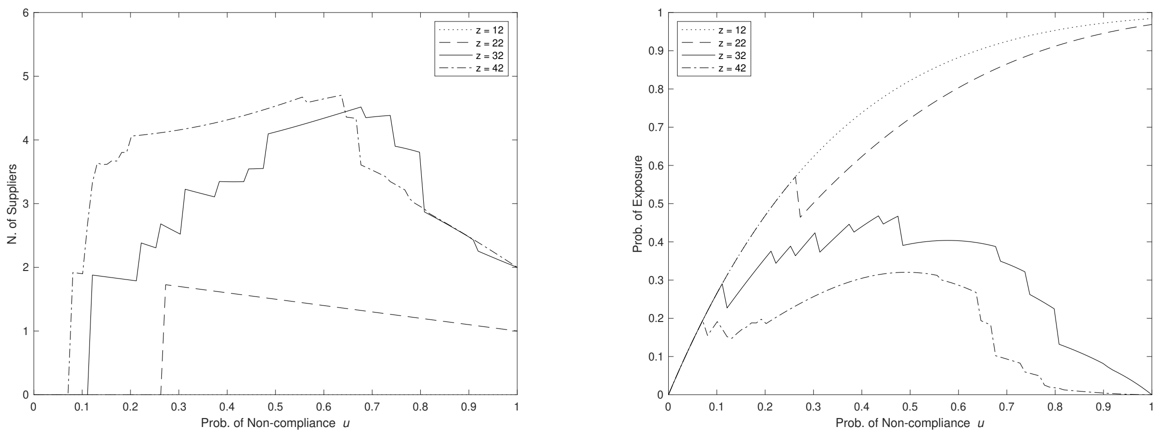
We observe a second pattern, a "rescue operation" versus a "kill mission," that follows the auditing of firm B in the lower half of the event tree (AD(B)), which follows when AD(S) fails. What we label as the "kill mission" follows when firm B fails. In that case the buyer is left with only firm A's side of the supply network with four unaudited tier-2 suppliers (there is no longer a distinction between exclusive and shared suppliers). Given the unattractive production profit of such a limited network, the buyer proceeds to audit firm A, the failing of which will kill the entire supply network, thus avoiding costly audits of all remaining tier-2 suppliers. Even if firm A passes, the buyer keeps auditing all remaining suppliers to eliminate any non-compliance.

In contrast if firm B passes, the buyer conducts what we label as a "rescue operation": after the initial setback of dropping the first shared supplier, the prospect has improved sufficiently with the passing of firm B that the buyer mostly follows a conventional auditing sequence of LVUS suppliers. Thus, the buyer goes on to audit numerous tier-2 suppliers in an attempt to only prune the less valuable suppliers.

In summary as both the production profit and the penalty can be potentially high, the buyer takes great care in auditing. In several states the buyer's auditing choice is intended to test the

Figure 4.7: Auditing and Risk as Probability of Non-compliance Varies

(a) Expected Number of Suppliers Audited (b) Probability of Exposure in Production Phase




State $\gamma = (g, U)$ where $g = (\{\text{A}, \text{B}\}, \{1, 2\}, \{3\}, \{4\})$ and $U = S_g$. Parameters $\alpha = 100$, $\beta = 10$, $v_T = 2$, $a = 1$, $r = 20$, $w = 0.5$. (In the left panel, the graph for $z = 12$ coincides with the horizontal axis.)

waters and determine which part of the network is worthy to protect and which is not. Because the shared supplier occupies the key position of simultaneously influencing both sides of the supply network, it is the perfect candidate for the first audit as a litmus test (with the given parameters). The buyer prefers AD($\mathbb{s}$) over AD($\mathbb{e}_{\text{A}}$) for the information AD($\mathbb{s}$) can provide to guide the buyer's subsequent decision. In particular, if the first audit AD($\mathbb{s}$) ascertains compliance the buyer takes a more optimistic approach by proceeding to audit the other tier-2 suppliers, the dropping of which wouldn't damage the buyer's production profit too much. On the other hand, if the first audit AD($\mathbb{s}$) reveals non-compliance the buyer starts to approach auditing more aggressively by directly turning to the tier-1 firms, starting with firm B. Depending on the outcome of AD(B), the buyer will then conduct either a rescue operation (upon B passing the audit) or a kill mission (upon B failing the audit). The rescue operation intends to preserve the profitability of the network, while the kill mission has a good chance of putting an end to the network and preventing any production.

### 4.5.5 Effect of Probability of Non-compliance

What is the role of the ex ante probability of non-compliance $u$ in the auditing activity and the level of risk in the supply network resulting from the auditing phase? We consider a state with supply network $g = (\{\text{A}, \text{B}\}, \{1, 2\}, \{3\}, \{4\})$ in which all suppliers are unaudited, with values of parameters $\alpha = 100$, $\beta = 10$, $v_T = 2$, $a = 1$, $r = 20$, and $w = 0.5$. Figure 4.8(a) shows the *expected* number of suppliers to be audited throughout the auditing phase as $u$ increases from 0 to 1 for various values of penalty $z$; this expectation is taken over all possible sample paths of whether any supplier passes or fails an audit under the optimal auditing policy. Figure 4.8(b) shows the corresponding *expected* probability of the exposure of violation at any remaining unaudited suppliers in the supply network after the auditing phase. This is calculated by $1 - (1 - uw)^{|U|}$ where $uw$ is the probability of an unaudited supplier being non-compliant and subsequently exposed and $U$ is the set of any unaudited suppliers at the conclusion of the auditing phase (these are the only suppliers which could possibly violate in the production phase). The set $U$ is the culmination of the path-dependent auditing process.

In Figures 4.8(a) and 4.8(b), a jump from one smooth segment on a curve to the next smooth segment represents a shift in the auditing policy. For low values of $z$ (e.g., $z \leqslant 12$) the buyer conducts no audits at all regardless of the probability of non-compliance; the number of suppliers to be audited remains zero and the network's probability of exposure increases monotonically in $u$ without auditing. With higher $z$ the buyer starts to audit once $u$ reaches a threshold, which decreases as $z$ gets larger. For example, compare the $z = 22$ to the $z = 12$ curves: for $z = 22$ when $u \geqslant 0.27$ the buyer is sufficiently concerned about non-compliance that some auditing will occur, resulting in a corresponding drop in the probability of exposure, relative to the $z = 12$ curve. Overall as $u$ increases the expected number of suppliers audited first shows an upward trend, reflecting the buyer's greater concern of the potential penalty from violation, leading to more audits. The expected number of audited firms trends downward, however, as $u$ increases further. With a higher probability of non-compliance the business becomes too risky so the buyer turns to auditing the tier-1 firms directly. The buyer expects to drop these tier-1 firms, along with their tier-2 dependents if the tier-1 firms turn out to be non-compliant, thus avoiding the cost of conducting those tier-2 audits (highly likely to be non-compliant). As $u$ approaches 1, the expected number of audits approaches 2, because the buyer audits firms A and B (which are very likely to fail the audits), thus dropping them and killing the business. Figure 4.8(b) reflects the same effects: the probability of exposure in the network first exhibits an overall upward trend with increasing $u$ but eventually falls to zero as the buyer drops both firms A and B, and consequently the entire supply network, thus avoiding risk entirely.

Neither figure shows a generally monotonic pattern. Higher probability of non-compliance drives auditing. As $u$ increases the buyer adopts an increasingly aggressive approach to auditing, the effectiveness of which is the concomitant reduction in the probability of exposure (Figure 4.8(b)). Ultimately, such auditing may be exhaustive (all firms are audited) to ensure full compliance or to extinguish the business.

### 4.6 Concluding Remarks

We study a buyer's problem of dynamically auditing its suppliers for social responsibility compliance in a three-tier supply network with general sourcing relationships between adjacent tiers. We model the competition in the supply network in the production phase and characterize the unique equilibrium arising from the Cournot competition. In particular, we characterize the buyer's production profit, observing how the configuration of the supply network affects the profit, and the value each supplier carries for the buyer. We find that the buyer profits from a more balanced supply network.

Turning to the auditing phase, we identify two subphases of the optimal policy: (1) the buyer conducts all activities of auditing and dropping (if non-compliant), then (2) proceeds either to the production phase directly or to audit and rectify (if non-compliant) all remaining suppliers. When the buyer focuses exclusively on the second tier, the value each supplier carries in the buyer's production profit entirely determines the buyer's choice of which supplier to audit (and drop if non-compliant): the buyer always chooses a *least valuable unaudited supplier* (LVUS) in the current state. On the other hand, if even the LVUS carries more value for the buyer than the potential penalty associated with the supplier as well as the cost of auditing and rectification, the buyer will skip auditing and proceed directly to production: "see no evil, hear no evil." In contrast if we allow a broader set of firms to be audited — if the tier-1 firms could now be candidates along with the tier-2 firms — we observe that the buyer may wish to audit and drop a non-LVUS supplier. Depending on the danger of violation, the buyer may shift the focus to a shared tier-2 supplier or even a tier-1 firm, which occupies a pivotal location within the network. The result of such audits helps the buyer ascertain the viability (or lack thereof) of the entire supply chain. We think of such audits as "litmus tests" since they deliver valuable information, upon which the buyer may act to "rescue" or "kill" the network.

As our model shows, oftentimes it is not optimal for a buyer to exhaustively audit all suppliers.

Strategic considerations direct the choice of the suppliers to audit. Earlier we describe Apple's policy of dropping a non-compliant supplier or working with the supplier to rectify its practice. Our results show that either choice can be optimal depending on the properties of the supply network and the stage in the auditing process. A buyer like Apple may benefit by prioritizing auditing suppliers that it is prepared to drop if an audit finds non-compliance and postponing auditing suppliers that the buyer is willing to rectify. When focusing on an upstream tier the buyer may want to first audit suppliers in more peripheral positions that carry less weight in the buyer's profit from production activity.

## Appendix 4.A Symbols Used

| | |
|---|---|
| $\nabla(\gamma, i)$ | $= \pi(\gamma) - \pi(\gamma \ominus i)$ for $i \in U_\gamma$ where $\gamma \in \Gamma$, the production value of supplier $i$ in state $\gamma$ |
| $\oplus, \ominus$ | operators that map a state-unaudited-supplier pair to a state; used in defining dynamic program |
| $a$ | cost of auditing a supplier |
| A, B | tier-1 firms |
| $\alpha, \beta$ | parameters of the demand function from customers downstream to the buyer |
| C | the buyer |
| $C_i$ | strategy space of firm $i$ |
| $D_g(i)$ | set of dependents of supplier $i$ in supply network $g$ |
| $ⓔ_{\mathrm{A}}$, $ⓔ_{\mathrm{B}}$ | an exclusive supplier to firm A and that to firm B, used in notation for AD and AR decisions |
| $g$ | supply network |
| $g_\emptyset$ | the null supply network |
| $\gamma = (g, U)$ | state in the auditing phase |
| $\Gamma$ | state space of the auditing phase |
| $\Gamma_T$ | set of terminal states |
| $p_{(0)}$ | selling price of buyer |
| $p_{(1)}$ | selling price of tier-1 firms to the buyer |
| $p_{(2)i}$ | selling price of tier-2 suppliers to tier-1 firm $i$ |
| $\pi(\gamma)$ | buyer's production profit in state $\gamma$ |
| $\pi_i$ | profit of firm $i$ from production activity |
| $q_i$ | total quantity produced by firm $i$ |
| $r$ | cost of rectifying a non-compliant supplier |
| $R^+(\gamma)$ | set of states reachable from state $\gamma$ |

| | |
|---|---|
| $s_{j,i}$ | quantity supplier $j$ produces for downstream firm $i$ |
| $\$$ | a shared supplier, used in notation for AD and AR decisions |
| $S_{\text{A}}$, $S_{\text{B}}$ | set of exclusive suppliers to tier-1 firm A and that to firm B |
| $S_{\text{AB}}$ | set of shared suppliers |
| $S_g$ | set of suppliers in supply network $g$ |
| $S(k)$ | set of suppliers in tier $k = 1, 2$ |
| $u$ | probability that an unaudited supplier is non-compliant |
| $U_\gamma$ | set of unaudited suppliers in state $\gamma$ |
| $\overline{U}$ | union of sets of unaudited suppliers in any state in $\Gamma$ |
| $V$ | value function in auditing phase |
| $V^*$ | optimal value function in auditing phase |
| $\widetilde{V}$ | state value function in auditing phase |
| $\widetilde{V}^*$ | optimal state value function in auditing phase |
| $v_k$ | unit production cost in tier $k$ |
| $v_T$ | sum of production costs per unit across tiers |
| $w$ | probability that violation at a supplier will be exposed, given that it is non-compliant |
| $\overline{X}$ | union of sets of admissible actions in any state in $\Gamma$ |
| $X_\gamma$ | set of admissible actions at state $\gamma$ |
| $\xi$ | auditing policy |
| $\Xi$ | set of all auditing policies |
| $z$ | cost to the buyer of an exposed violation |
| $Z$ | set of state-unaudited-supplier pairs |
| $\zeta(\gamma)$ | expected total penalty from violations on state $\gamma$ |

## Appendix 4.B Proofs for the Production Phase

### 4.B.1 Proofs of Results in Section 4.3.3

*Proof of Proposition 4.1.* Substitute (4.6) into (4.7) and differentiate to get

$$\frac{\partial \pi_{\text{C}}}{\partial q_{\text{C}}} = -\beta q_{\text{C}} + (\alpha - \beta q_{\text{C}} - v_0 - p_{(1)}) \tag{4.30}$$

$$\frac{\partial^2 \pi_{\text{C}}}{\partial q_{\text{C}}^2} = -2\beta. \tag{4.31}$$

Since $\beta > 0$, (4.31) implies that $\pi_{\mathrm{C}}$ is strictly concave. Hence a quantity $q_{\mathrm{C}}$ maximizes $\pi_{\mathrm{C}}$ if and only if it sets $\frac{\partial \pi_{\mathrm{C}}}{\partial q_{\mathrm{C}}} = 0$ in (4.30); the unique such $q_{\mathrm{C}}$ is given by

$$q_{\mathrm{C}}^* = \frac{\alpha - v_0}{2\beta} - \frac{p_{(1)}}{2\beta}. \tag{4.32}$$

We rewrite it in the form of an inverse demand function to obtain (4.8). □

*Proof of Proposition 4.2.* Substitute (4.8) into (4.9) to get tier-1 supplier $i$'s profit

$$\pi_i = (\alpha - v_0 - 2\beta q_{\mathrm{C}} - v_1 - p_{(2)i})q_i. \tag{4.33}$$

Substitute (4.10) into (4.33) to get

$$\pi_i = \left(\alpha - v_0 - v_1 - p_{(2)i} - 2\beta \sum_{j\in S(1)} q_j\right) q_i. \tag{4.34}$$

Then

$$\frac{\partial \pi_i}{\partial q_i} = \alpha - v_0 - v_1 - p_{(2)i} - 4\beta q_i - 2\beta \sum_{j\in S(1)\setminus\{i\}} q_j \tag{4.35}$$

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -4\beta. \tag{4.36}$$

Given $\beta > 0$, (4.36) implies $\pi_i$ is strictly concave in $q_i$. Given any other tier-1 firm's decision, a quantity $q_i$ maximizes $\pi_i$ if and only if it sets $\frac{\partial \pi_i}{\partial q_i} = 0$ in (4.35); the unique such $q_i$ is

$$q_i = -\frac{1}{2}\left(\sum_{j\in S(1)\setminus\{i\}} q_j\right) + \frac{\alpha - v_0 - v_1 - p_{(2)i}}{4\beta}. \tag{4.37}$$

Hence a strategy profile $\mathbf{q}_{(1)} = (q_i)_{i\in S(1)}$ is an equilibrium of the tier-1 firms' game $P_1$ if and only if it solves the system of linear equations (4.37) for all $i \in S(1)$.

If $|S(1)| = 1$, let $i \in S(1)$, then it is clear that $\mathbf{q}_{(1)} = q_i = \frac{\alpha - v_0 - v_1 - p_{(2)i}}{4\beta}$ is the unique (degenerate) equilibrium of the game $P_1$. If $|S(1)| = 2$, i.e., $S(1) = \{\text{A}, \text{B}\}$, we write the system (4.37) as

$$\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} \mathbf{q}_{(1)} = \frac{\alpha - v_0 - v_1}{4\beta} - \frac{1}{4\beta} \begin{bmatrix} p_{(2)\text{A}} \\ p_{(2)\text{B}} \end{bmatrix}. \tag{4.38}$$

Clearly the matrix $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$ is invertible; hence the system (4.37) has a unique solution, which is the unique equilibrium of the game $P_1$. We rewrite (4.37) in the form of an inverse demand function to obtain (4.12). □

### 4.B.2 Existence and Uniqueness of Equilibrium

**Proposition 4.9.** *(a) A tier-2 supplier $j$'s profit $\pi_j$ is strictly concave in $\mathbf{s}_j$.*

*(b) There exists a unique equilibrium of the game $P_2$ of Cournot competition among tier-2 suppliers in the first stage of the production phase.*

*(c) Given the inverse demand function* (4.12) *from tier-1 firm $i$ that its tier-2 suppliers collectively receive, for $j \in S_i$, $i \in S(1)$,*

$$\frac{\partial \pi_j(\mathbf{s}_{(2)})}{\partial s_{j,i}} = -4\beta s_{j,i} + \left[\alpha - v_T - 4\beta \sum_{j' \in S_i \cup S_{\text{AB}}} s_{j',i} - 2\beta \sum_{i' \in S(1)\setminus\{i\}} \left( \sum_{j' \in S_{i'} \cup S_{\text{AB}}} s_{j',i'} \right) \right] \tag{4.39}$$

*and for $j \in S_{\text{AB}}$ and $i \in S(1)$,*

$$\frac{\partial \pi_j(\mathbf{s}_{(2)})}{\partial s_{j,i}} = -4\beta s_{j,i} + \left[\alpha - v_T - 4\beta \sum_{j' \in S_i \cup S_{\text{AB}}} s_{j',i} - 2\beta \sum_{i' \in S(1)\setminus\{i\}} \left( \sum_{j' \in S_{i'} \cup S_{\text{AB}}} s_{j',i'} \right) \right] - 2\beta \sum_{i' \in S(1)\setminus\{i\}} s_{j,i'}. \tag{4.40}$$

*Proof.* (Part c). For exclusive supplier $j \in S_i$, $i \in S(1)$, substitute (4.12) into (4.13), replace $v_0 + v_1 + v_2$ with $v_T$, and then substitute (4.15) to get

$$\pi_j = \left( \alpha - v_T - 4\beta q_i - 2\beta \sum_{i' \in S(1)\setminus\{i\}} q_{i'} \right) s_{j,i} \tag{4.41}$$

$$= \left[\alpha - v_T - 4\beta \sum_{j' \in S_i \cup S_{\text{AB}}} s_{j',i} - 2\beta \sum_{i' \in S(1)\setminus\{i\}} \left( \sum_{j' \in S_{i'} \cup S_{\text{AB}}} s_{j',i'} \right) \right] s_{j,i}. \tag{4.42}$$

Differentiate (4.42) with respect to $s_{j,i}$ to get (4.39). For shared supplier $j \in S_{\text{AB}}$, substitute (4.12) into (4.14), replace $v_0 + v_1 + v_2$ with $v_T$, and then substitute (4.15) to get

$$\pi_j = \sum_{i \in S(1)} \left( \alpha - v_T - 4\beta q_i - 2\beta \sum_{i' \in S(1) \setminus \{i\}} q_{i'} \right) s_{j,i} \tag{4.43}$$

$$= \sum_{i \in S(1)} \left[ \alpha - v_T - 4\beta \sum_{j' \in S_i \cup S_{\text{AB}}} s_{j',i} - 2\beta \sum_{i' \in S(1) \setminus \{i\}} \left( \sum_{j' \in S_{i'} \cup S_{\text{AB}}} s_{j',i'} \right) \right] s_{j,i}. \tag{4.44}$$

Differentiate (4.44) with respect to $s_{j,i}$, $i \in S(1)$, to get (4.40).

(Part a). For exclusive supplier $j \in S_i$, $i \in S(1)$, differentiate (4.39) with respect to $s_{j,i}$ to get

$$\frac{\partial^2 \pi_j}{\partial s_{j,i}^2} = -8\beta. \tag{4.45}$$

Therefore $\pi_j$ is strictly concave in $\mathbf{s}_j = s_{j,i}$. For shared supplier $j \in S_{\text{AB}}$, differentiate (4.40) with respect to $s_{j,i}$ and with respect to $s_{j,i'}$ for $i' \in S(1) \setminus \{i\}$ to get

$$\frac{\partial^2 \pi_j}{\partial s_{j,i}^2} = -8\beta \quad \text{and} \quad \frac{\partial^2 \pi_j}{\partial s_{j,i'} \partial s_{j,i}} = -4\beta. \tag{4.46}$$

Hence the Hessian of $\pi_j$ with respect to $\mathbf{s}_j$ is $\begin{bmatrix} -8\beta & -4\beta \\ -4\beta & -8\beta \end{bmatrix}$ which, given that $\beta > 0$, can be easily verified to be negative definite. Therefore $\pi_j$ is strictly concave in $\mathbf{s}_j = \{s_{j,i}\}_{i \in S(1)}$.

(Part b). (Existence.) The strategy space $C_j$ of tier-2 supplier $j$ is a nonempty compact convex subset of $\mathbb{R}$ (if $j$ is an exclusive supplier) or $\mathbb{R}^2$ (if $j$ is a shared supplier). The payoff function $\pi_j$ of supplier $j$ (4.42) (for exclusive supplier $j$) or (4.44) (for shared supplier $j$) is continuous in the strategy profile $(\mathbf{s}_j)_{j \in S(2)}$ and strictly concave, hence quasi-concave, in supplier $j$'s own strategy $\mathbf{s}_j$. By Proposition 20.3 in Osborne and Rubinstein (1994), there exists a pure-strategy equilibrium of the game among tier-2 suppliers in the first stage.

(Uniqueness.) We use the method due to Rosen (1965). As we have seen, the strategy space $C_j$ of supplier $j$ is convex, closed, and bounded. $\pi_j$ is continuous in the strategy profile and concave in supplier $j$'s strategy. Label a tier-1 firm A and, if there is a second one, label it B. Label $S(2) = \{1, \ldots, n\}$, where $n = |S(2)|$, such that $\{1, \ldots, t_{\text{A}}\}$ are tier-1 firm A's exclusive suppliers,

$\{t_{\mathrm{A}}+1,\ldots,t_{\mathrm{A}}+t_{\mathrm{AB}}\}$ are the shared suppliers, and $\{t_{\mathrm{A}}+t_{\mathrm{AB}}+1,\ldots,n\}$ are tier-1 firm B's exclusive suppliers (any of the subsets could be empty, but at least $S(2)$ is nonempty, i.e., $n>0$). Let $\mathbf{x}=(\mathbf{s}_j)_{j\in S(2)}$. We choose $\mathbf{r}=\boldsymbol{\iota}_n=(1,\ldots,1)^{\mathsf{T}}\in\mathbb{R}^n$ as the weights for the payoff functions in $\sigma(\mathbf{x},\mathbf{r})$ in Rosen (1965). Then the pseudogradient of $\sigma(\mathbf{x},\mathbf{r})=\sigma(\mathbf{x},\boldsymbol{\iota}_n)$ is

$$\begin{aligned} &g(\mathbf{x},\boldsymbol{\iota}_n)\\ &=\left(\frac{\partial\pi_1}{\partial s_{1,\mathrm{A}}},\ldots,\frac{\partial\pi_{t_{\mathrm{A}}}}{\partial s_{t_{\mathrm{A}},\mathrm{A}}},\frac{\partial\pi_{t_{\mathrm{A}}+1}}{\partial s_{t_{\mathrm{A}}+1,\mathrm{A}}},\frac{\partial\pi_{t_{\mathrm{A}}+1}}{\partial s_{t_{\mathrm{A}}+1,\mathrm{B}}},\ldots,\frac{\partial\pi_{t_{\mathrm{A}}+t_{\mathrm{AB}}}}{\partial s_{t_{\mathrm{A}}+t_{\mathrm{AB}},\mathrm{A}}},\frac{\partial\pi_{t_{\mathrm{A}}+t_{\mathrm{AB}}}}{\partial s_{t_{\mathrm{A}}+t_{\mathrm{AB}},\mathrm{B}}},\frac{\partial\pi_{t_{\mathrm{A}}+t_{\mathrm{AB}}+1}}{\partial s_{t_{\mathrm{A}}+t_{\mathrm{AB}}+1,\mathrm{B}}},\ldots,\frac{\partial\pi_n}{\partial s_{n,\mathrm{B}}}\right)^{\mathsf{T}}\\ &\qquad\in\mathbb{R}^{t_{\mathrm{A}}+2t_{\mathrm{AB}}+t_{\mathrm{B}}} \end{aligned} \tag{4.47}$$

By (4.45), (4.46), (4.42), and (4.44), the Jacobian $G(\mathbf{x},\boldsymbol{\iota}_n)$ of $g(\mathbf{x},\boldsymbol{\iota}_n)$ with respect to $\mathbf{x}$ is equal to $-2\beta\Lambda(t_{\mathrm{A}},t_{\mathrm{B}},t_{\mathrm{AB}})$, a symmetric matrix. Hence $G(\mathbf{x},\boldsymbol{\iota}_n)+(G(\mathbf{x},\boldsymbol{\iota}_n))^{\mathsf{T}}=-4\beta\Lambda(t_{\mathrm{A}},t_{\mathrm{B}},t_{\mathrm{AB}})$, which is negative definite for any $\mathbf{x}\in\prod_{j\in S(2)}C_j$ by Lemma 4.3 in Appendix 4.D and that $\beta>0$. By Theorem 6 in Rosen (1965), $\sigma(\mathbf{x},\boldsymbol{\iota}_n)$ is diagonally strictly concave. By Theorem 2 in Rosen (1965), the equilibrium of the game among tier-2 suppliers in the first stage of the production phase is unique. □

*Proof of Theorem 4.1.* By Proposition 4.9, there exists a unique equilibrium $(\mathbf{s}_j^*)_{j\in S(2)}$ of the game in the first stage among tier-2 supplier; let $(p_{(2)i}^*)_{i\in S(1)}$ be the resulting selling prices of the tier-2 suppliers. Given $(p_{(2)i}^*)_{i\in S(1)}$, by Proposition 4.2, there exists a unique equilibrium $\mathbf{q}_{(1)}^*$ of the game in the second stage among tier-1 firms; let $p_{(1)}^*$ be the resulting selling price of the tier-1 firms. Given $p_{(1)}^*$, by Proposition 4.1, there exists a unique optimal solution $q_{\mathrm{C}}^*$ to the buyer's problem in the first stage; let $p_{(0)}^*$ be the resulting selling price of the buyer. Hence the tuple of prices and quantities $\left(p_{(0)}^*,p_{(1)}^*,(p_{(2)i}^*)_{i\in S(1)},q_{\mathrm{C}}^*,\mathbf{q}_{(1)}^*,(\mathbf{s}_j^*)_{j\in S(2)}\right)$ is the unique production phase equilibrium. □

### 4.B.3 Equilibrium Characterization

*Proof of Proposition 4.3.* Substitute (4.6) for $p_{(0)}$ and (4.8) for $p_{(1)}$ in (4.7). □

*Proof of Proposition 4.4.* Tier-2 supplier $j$'s problem is

$$(P_{(2)j}) \qquad \max \quad \pi_j(\mathbf{s}_{(2)}) \tag{4.48}$$

$$\text{subject to} \quad \mathbf{s}_j \geqslant 0. \tag{4.49}$$

By Proposition 4.9a, $\pi_j(\mathbf{s}_{(2)})$ is concave in $\mathbf{s}_j$. With merely the nonnegativity constraints, constraint qualification always holds. Therefore the Karush-Kuhn-Tucker (KKT) conditions

$$\frac{\partial \pi_j(\mathbf{s}^*_{(2)})}{\partial s_{j,i}} \leqslant 0, \quad \text{with equality if } s^*_{j,i} > 0, \qquad (\text{if } j \in S_i, i \in S(1); \text{ or for } i \in S(1) \text{ if } j \in S_{\text{AB}}) \tag{4.50}$$

are necessary and sufficient for $\mathbf{s}^*_j \geqslant 0$ to be a global maximizer. Given $(\mathbf{s}^*_{j'})_{j' \in S(2)\setminus\{j\}}$, if $\mathbf{s}^*_j$ solves $P_{(2)j}$, then $\mathbf{s}^*_j$ is a best response to $(\mathbf{s}^*_{j'})_{j' \in S(2)\setminus\{j\}}$. If for every $j \in S(2)$, $\mathbf{s}^*_j$ is a best response to $(\mathbf{s}^*_{j'})_{j' \in S(2)\setminus\{j\}}$, then $\mathbf{s}^*_{(2)}$ is an equilibrium in pure strategies of $P_2$.

(Case a). We note $s_e(x_1, x_2, x_3) > 0, \forall x_1, x_2, x_3 \geqslant 0$; hence the supply quantity of every exclusive supplier, as defined using $s_e$, is positive. If there exists a shared supplier $j$, then by the assumption $t_{\text{A}} \leqslant 2t_{\text{B}} + 2t_{\text{AB}} + 2$ and the expression in (4.20), $s^*_{j,\text{A}} = s_s(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}) \geqslant 0$. Since $t_{\text{A}} \geqslant t_{\text{B}}$, $s^*_{j,\text{B}} = s_s(t_{\text{B}}, t_{\text{A}}, t_{\text{AB}}) > 0$. $\mathbf{s}^*_j$ as defined is nonnegative for every $j \in S(2)$.

Substituting supply quantities in parts i–ii into (4.39) and (4.40), following some algebra, we verify that $\frac{\partial \pi_j\left(\mathbf{s}^*_{(2)}\right)}{\partial s^*_{j,i}} = 0$ for $i \in S(1)$ and $j \in S_i \cup S_{\text{AB}}$. Therefore $\mathbf{s}^*_{(2)}$ satisfies the nonnegativity constraints and the KKT conditions (4.50) for every tier-2 supplier $j \in S(2)$. Hence $\mathbf{s}^*_{(2)}$ is an equilibrium of $P_2$.

We verify part iii by substituting the values of $s^*_{j,i}$ in parts i and ii into (4.15) for the corresponding quantities, and substituting (4.19), (4.20), and (4.21). We verify (4.23) by substituting (4.15) into (4.10), then substituting the values of $s^*_{j,i}$ in parts i and ii for the corresponding quantities.

(Case b). Except for $s^*_{j,\text{A}} = 0$ for a shared supplier $j$, the supply quantities in parts i–iii are defined using $s_e$ and, as such, positive. Therefore $\mathbf{s}^*_j$ as defined is nonnegative for every $j \in S(2)$.

Analogous to case a, substituting supply quantities in parts i–iii into (4.39) and (4.40), following some algebra, we verify that $\frac{\partial \pi_j\left(\mathbf{s}^*_{(2)}\right)}{\partial s^*_{j,i}} = 0$ for exclusive supplier $j \in S_i$, $i \in S(1)$, and for shared supplier $j \in S_{\text{AB}}$ and $i = \text{B}$. We only need to verify that $\frac{\partial \pi_j(\mathbf{s}^*_{(2)})}{\partial s_{j,\text{A}}} \leqslant 0$ for every shared supplier $j \in S_{\text{AB}}$ to verify the KKT conditions (4.50). We substitute the supply quantities $\mathbf{s}^*_{(2)}$ into (4.40) to find

$$\frac{\partial \pi_j(\mathbf{s}^*_{(2)})}{\partial s_{j,\text{A}}} = a - v_T - 4\beta t_\text{A} s_e(t_\text{A}, t_\text{B} + t_\text{AB}, 0) - 2\beta(t_\text{B} + t_\text{AB}) s_e(t_\text{B} + t_\text{AB}, t_\text{A}, 0) - 2\beta s_e(t_\text{B} + t_\text{AB}, t_\text{A}, 0) \tag{4.51}$$

$$= a - v_T - 4\beta t_\text{A} s_e(t_\text{A}, t_\text{B} + t_\text{AB}, 0) - 2\beta(t_\text{B} + t_\text{AB} + 1) s_e(t_\text{B} + t_\text{AB}, t_\text{A}, 0). \tag{4.52}$$

$L(x_1, x_2, x_3)$ is symmetric in $x_1$ and $x_2$ in the sense that $L(x_1, x_2, x_3) = L(x_2, x_1, x_3)$. Let $\widehat{L} = L(t_\text{A}, t_\text{B} + t_\text{AB}, 0) = L(t_\text{B} + t_\text{AB}, t_\text{A}, 0)$. Substitute the definition of $s_e$ into (4.52) to get

$$\frac{\partial \pi_j(\mathbf{s}^*_{(2)})}{\partial s_{j,\text{A}}} = a - v_T - 2(\alpha - v_T)\frac{t_\text{A}(t_\text{B} + t_\text{AB} + 2)}{\widehat{L}} - (\alpha - v_T)\frac{(t_\text{A} + 2)(t_\text{B} + t_\text{AB} + 1)}{\widehat{L}} \tag{4.53}$$

$$= (a - v_T)\left[1 - \frac{1}{\widehat{L}}(3t_\text{A}t_\text{B} + 3t_\text{A}t_\text{AB} + 5t_\text{A} + 2t_\text{B} + 2t_\text{AB} + 2)\right] \tag{4.54}$$

$$= (a - v_T)\frac{1}{\widehat{L}}(-t_\text{A} + 2t_\text{B} + 2t_\text{AB} + 2) \tag{4.55}$$

which is nonpositive by the assumption $\alpha \geqslant v_T$ and the premise $t_\text{A} \geqslant 2t_\text{B} + 2t_\text{AB} + 2$.

We verify parts iv and v by substituting the values of $s^*_{j,i}$ in parts i–iii into (4.15) for the corresponding quantities. We verify (4.24) by substituting (4.15) into (4.10) then substituting the values of $s^*_{j,i}$ in parts i–iii for the corresponding quantities. □

### 4.B.4 Comparative Statics

In this section, we consider only the case $\alpha > v_T$. (If $\alpha = v_T$, by Proposition 4.4, every supply quantity is zero.) All comparative statics results, except those on $q^*_\text{C}$ and $\pi^*_\text{C}$, are applicable only to non-null supply networks. To facilitate the proofs we introduce an alternative notation of a supply network in terms of $t_\text{A}$, the number of exclusive suppliers to firm A, $t_\text{B}$ that of exclusive suppliers to firm B, and $t_{\text{AB}}$ that of shared suppliers. Specifically, we define an operator $\langle\cdot\rangle : \mathbb{N}_0^3 \to G$ by

$$\langle t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}\rangle = \begin{cases} \begin{array}{c}(\{\text{A},\text{B}\},\{1,\ldots,t_{\text{A}}\},\{t_{\text{A}}+1,\ldots,t_{\text{A}}+t_{\text{B}}\},\\ \{t_{\text{A}}+t_{\text{B}}+1,\ldots,t_{\text{A}}+t_{\text{B}}+t_{\text{AB}}\})\end{array}, & \text{if } t_{\text{A}}+t_{\text{AB}}>0 \text{ and } t_{\text{B}}+t_{\text{AB}}>0 \\ (\{\text{A}\},\{1,\ldots,t_{\text{A}}+t_{\text{AB}}\},\emptyset,\emptyset), & \text{if } t_{\text{A}}+t_{\text{AB}}>0 \text{ and } t_{\text{B}}+t_{\text{AB}}=0 \\ (\{\text{B}\},\emptyset,\{1,\ldots,t_{\text{B}}+t_{\text{AB}}\},\emptyset), & \text{if } t_{\text{A}}+t_{\text{AB}}=0 \text{ and } t_{\text{B}}+t_{\text{AB}}>0 \\ g_{\emptyset}, & \text{if } t_{\text{A}}+t_{\text{AB}}=0 \text{ and } t_{\text{B}}+t_{\text{AB}}=0 \end{cases} \cdot \tag{4.56}$$

Let $f$ be a variable that arises from the production phase equilibrium (quantity, margin, profit, price, market share). We denote by $f(g) = f\langle t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}\rangle$ the value of this variable in supply network $g = \langle t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}\rangle$. We further denote $\Delta_1 f\langle t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}\rangle = f\langle t_{\text{A}}+1, t_{\text{B}}, t_{\text{AB}}\rangle - f\langle t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}\rangle$, $\Delta_2 f\langle t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}\rangle = f\langle t_{\text{A}}, t_{\text{B}}+1, t_{\text{AB}}\rangle - f\langle t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}\rangle$, and $\Delta_3 f\langle t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}\rangle = f\langle t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}+1\rangle - f\langle t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}\rangle$.

*Proof of Theorem 4.2.* We begin by noting a few relationships between the equilibrium variables. By the definition of $p_{(0)}$ and (4.8),

$$m_{\text{C}}^* = (\alpha - \beta q_{\text{C}}^*) - (\alpha - v_0 - 2\beta q_{\text{C}}^*) - v_0 = \beta q_{\text{C}}^*. \tag{4.57}$$

By Proposition 4.3, $\pi_{\text{C}}^* = \beta (q_{\text{C}}^*)^2$. Therefore, $q_{\text{C}}^*$, $m_{\text{C}}^*$, and $\pi_{\text{C}}^*$ always change in the same direction, which is opposite to the change in $p_{(1)}^*$ by (4.8). Hence the direction of the change in any one of the four variables determines those of the other three. Let $i \in S(1)$. By (4.12),

$$p_{(2)i}^* = \alpha - v_0 - v_1 - 4\beta q_i^* - 2\beta(q_{\text{C}}^* - q_i^*) = \alpha - v_0 - v_1 - 2\beta q_i^* - 2\beta q_{\text{C}}^*. \tag{4.58}$$

Then by (4.8),

$$m_i^* = (\alpha - v_0 - 2\beta q_{\text{C}}^*) - v_1 - (\alpha - v_0 - v_1 - 2\beta q_i^* - 2\beta q_{\text{C}}^*) = 2\beta q_i^*. \tag{4.59}$$

By (4.9),

$$\pi_i^* = m_i^* q_i^* = 2\beta (q_i^*)^2. \tag{4.60}$$

Therefore, $q_i^*$, $m_i^*$, and $\pi_i^*$ always change in the same direction. Finally, $\rho_{\text{A}}^* = 1 - \rho_{\text{B}}^*$.

Given the characterization of the equilibrium in Proposition 4.4, we directly calculate the change $\Delta_k\, f(g)$ in each equilibrium variable $f$, factor the expression when appropriate, and then check its sign, for $k = 1, 2, 3$. We illustrate the procedure for $\Delta_1\, f(g)$ only, which involves incrementing $t_{\text{A}}$. We consider cases which satisfy $t_{\text{A}} \leqslant 2t_{\text{B}}+2t_{\text{AB}}+2$ and $t_{\text{A}}+1 \leqslant 2t_{\text{B}}+2t_{\text{AB}}+2$ so that Proposition 4.4a is applicable before and after incrementing $t_{\text{A}}$. We also elaborate on $\Delta_3\, f(g)$ for $f \in \{q^*_{\text{A}}, m^*_{\text{A}}, \pi^*_{\text{A}}\}$ when $t_{\text{A}} \leqslant 2t_{\text{B}} + 2t_{\text{AB}} + 2$ that results in the peculiar case of the cell with "+/−" in column (III) of Table 4.1 in the proof of Proposition 4.5 that follows.

With some algebraic computation we find

$$\Delta_1\, q^*_{\text{C}}(g) = \frac{(\alpha - v_T)(t_{\text{B}} + 2t_s + 2)^2}{\beta L(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}) L(t_{\text{A}} + 1, t_{\text{B}}, t_{\text{AB}})} > 0 \tag{4.61}$$

since $\alpha \geqslant v_T$ and $L(x_1, x_2, x_3) > 0, \forall x_1, x_2, x_3 \geqslant 0$. Since $q^*_{\text{C}}$, $m^*_{\text{C}}$, and $\pi^*_{\text{C}}$ always change in the same direction, opposite to the change in $p^*_{(1)}$, we have the results for $m^*_{\text{C}}, \pi^*_{\text{C}}$, and $p^*_{(1)}$ as well.

$$\Delta_1\, q^*_{\text{A}}(g) = \frac{2(\alpha - v_T)(t_{\text{AB}} + t_{\text{B}} + 1)(2t_{\text{AB}} + t_{\text{B}} + 2)}{\beta L(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}) L(t_{\text{A}} + 1, t_{\text{B}}, t_{\text{AB}})} > 0. \tag{4.62}$$

This gives the results for $q^*_{\text{A}}, m^*_{\text{A}}$, and $\pi^*_{\text{A}}$. Similarly

$$\Delta_1\, q^*_{\text{B}}(g) = -\frac{(\alpha - v_T)t_{\text{B}}(2t_{\text{AB}} + t_{\text{B}} + 2)}{\beta L(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}) L(t_{\text{A}} + 1, t_{\text{B}}, t_{\text{AB}})} < 0 \tag{4.63}$$

This gives the results for $q^*_{\text{B}}, m^*_{\text{B}}$, and $\pi^*_{\text{B}}$.

$$\Delta_1\, p^*_{(2)\text{A}}(g) = -\frac{2(\alpha - v_T)(2t_{\text{AB}} + t_{\text{B}} + 2)(4t_{\text{AB}} + 3t_{\text{B}} + 4)}{L(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}) L(t_{\text{A}} + 1, t_{\text{B}}, t_{\text{AB}})} < 0 \tag{4.64}$$

$$\Delta_1\, p^*_{(2)\text{B}}(g) = -\frac{4(\alpha - v_T)(t_{\text{AB}} + 1)(2t_{\text{AB}} + t_{\text{B}} + 2)}{L(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}}) L(t_{\text{A}} + 1, t_{\text{B}}, t_{\text{AB}})} < 0. \tag{4.65}$$

Finally,

$$\Delta_1\, \rho^*_{\text{A}}(g) = \frac{3(2t_{\text{AB}} + t_{\text{B}} + 2)(2t_{\text{AB}} + 3t_{\text{B}})}{2(4t^2_{\text{AB}} + 4t_{\text{A}}t_{\text{AB}} + 4t_{\text{B}}t_{\text{AB}} + 4t_{\text{AB}} + 3t_{\text{A}} + 3t_{\text{A}}t_{\text{B}} + 3t_{\text{B}})} \times \frac{1}{4t^2_{\text{AB}} + 4t_{\text{A}}t_{\text{AB}} + 4t_{\text{B}}t_{\text{AB}} + 8t_{\text{AB}} + 3t_{\text{A}} + 3t_{\text{A}}t_{\text{B}} + 6t_{\text{B}} + 3} \geqslant 0 \tag{4.66}$$

("$> 0$" if $t_{\text{B}} + t_{\text{AB}} > 0$) which gives the results for $\rho^*_{\text{B}}$ as well. □

*Proof of Proposition 4.5.* As we have shown in the proof of Theorem 4.2, $q_{\text{A}}^*, m_{\text{A}}^*$, and $\pi_{\text{A}}^*$ always change in the same direction. Hence it suffices to show the effect on $q_{\text{A}}^*$. Using the result from Proposition 4.4, we find

$$\Delta_3\, q_{\text{A}}^*(g) = -\frac{2(\alpha - v_T)(4t_{\text{A}}t_{\text{AB}} - 4t_{\text{AB}}^2 - 8t_{\text{B}}t_{\text{AB}} - 12t_{\text{AB}} + 2t_{\text{A}}^2 + t_{\text{B}}t_{\text{A}} + 6t_{\text{A}} - 4t_{\text{B}}^2 - 12t_{\text{B}} - 8)}{3\beta L(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}})L(t_{\text{A}}, t_{\text{B}}, t_{\text{AB}} + 1)}. \tag{4.67}$$

Since $L(x_1, x_2, x_3) > 0, \forall x_1, x_2, x_3 \geqslant 0$,

$$\text{sgn}(\Delta_3\, q_{\text{A}}^*(g)) = \text{sgn}(-(4t_{\text{A}}t_{\text{AB}} - 4t_{\text{AB}}^2 - 8t_{\text{B}}t_{\text{AB}} - 12t_{\text{AB}} + 2t_{\text{A}}^2 + t_{\text{B}}t_{\text{A}} + 6t_{\text{A}} - 4t_{\text{B}}^2 - 12t_{\text{B}} - 8)) \tag{4.68}$$

Note that what is inside the sgn operator on the right-hand side of (4.68) is quadratic in $t_{\text{A}}$ with coefficient $-2$ on $t_{\text{A}}^2$ and two roots in $\mathbb{R}$ as follows:

$$t_{\text{A}-} = \frac{1}{4}(-6 - t_{\text{B}} - 4t_{\text{AB}} - \sqrt{72t_{\text{B}}t_{\text{AB}} + 48t_{\text{AB}}^2 + 144t_{\text{AB}} + 33t_{\text{B}}^2 + 108t_{\text{B}} + 100}) \tag{4.69}$$

$$t_{\text{A}+} = \frac{1}{4}(-6 - t_{\text{B}} - 4t_{\text{AB}} + \sqrt{72t_{\text{B}}t_{\text{AB}} + 48t_{\text{AB}}^2 + 144t_{\text{AB}} + 33t_{\text{B}}^2 + 108t_{\text{B}} + 100}) \tag{4.70}$$

It is clear that $t_{\text{A}-} < 0$. We next show that $0 < t_{\text{A}+} < 2t_{\text{B}} + 2t_{\text{AB}} + 2$. Note

$$\begin{aligned}(\sqrt{72t_{\text{B}}t_{\text{AB}} + 48t_{\text{AB}}^2 + 144t_{\text{AB}} + 33t_{\text{B}}^2 + 108t_{\text{B}} + 100})^2 - (-6 - t_{\text{B}} - 4t_{\text{AB}})^2 \\ = 32(t_{\text{B}}(2t_{\text{AB}} + 3) + t_{\text{AB}}^2 + 3t_{\text{AB}} + t_{\text{B}}^2 + 2) > 0\end{aligned} \tag{4.71}$$

so

$$\sqrt{72t_{\text{B}}t_{\text{AB}} + 48t_{\text{AB}}^2 + 144t_{\text{AB}} + 33t_{\text{B}}^2 + 108t_{\text{B}} + 100} > |-6 - t_{\text{B}} - 4t_{\text{AB}}| \tag{4.72}$$

which is equivalent to $t_{\text{A}+} > 0$. On the other hand,

$$(2t_{\text{B}} + 2t_{\text{AB}} + 2) - t_{\text{A}+} = \frac{1}{4}\left(12t_{\text{AB}} + 9t_{\text{B}} + 14 - \sqrt{36t_{\text{B}}(2t_{\text{AB}} + 3) + 4(12t_{\text{AB}}^2 + 36t_{\text{AB}} + 25) + 33t_{\text{B}}^2}\right). \tag{4.73}$$

Now

$$(12t_{\text{AB}} + 9t_{\text{B}} + 14)^2 - (\sqrt{36t_{\text{B}}(2t_{\text{AB}} + 3) + 4(12t_{\text{AB}}^2 + 36t_{\text{AB}} + 25) + 33t_{\text{B}}^2})^2$$
$$= 8[37 + 6t_{\text{B}}^2 + 60t_{\text{AB}} + 24t_{\text{AB}}^2 + 9t_{\text{B}}(5 + 4t_{\text{AB}})] > 0 \quad (4.74)$$

which implies (4.73) is negative. Therefore, when $t_{\text{A}} \leqslant 2t_{\text{B}} + 2t_{\text{AB}} + 2$, $\Delta_3(q_{\text{A}}^*) > 0$ if $t_{\text{A}} < t_{\text{A}+}$ and $\Delta_3(q_{\text{A}}^*) < 0$ if $t_{\text{A}} > t_{\text{A}+}$. □

*Proof of Proposition 4.6.* Similar to the proof of Theorem 4.2; by calculating, factoring, and observing the sign of the relevant difference. □

## Appendix 4.C Proofs for the Auditing Phase

We define $V : \Xi \times \Gamma \to \mathbb{R}$ as the value function. Let $\widetilde{V}(\xi, \gamma, x)$ be expected value of choosing $x \in X_\gamma$ when in state $\gamma \in \Gamma$ and following policy $\xi \in \Xi$ thereon. Therefore, given auditing policy $\xi \in \Xi$ and state $\gamma \in \Gamma$, $V(\xi, \gamma) = \widetilde{V}(\xi, \gamma, \xi(\gamma))$, and

$$\widetilde{V}(\xi, \gamma, \text{PP}) = \pi(\gamma) - \zeta(\gamma) \quad (4.75)$$

and given $i \in U_\gamma$,

$$\widetilde{V}(\xi, \gamma, \text{AD}(i)) = -a + (1-u)V(\xi, \gamma \oplus i) + uV(\xi, \gamma \ominus i). \quad (4.76)$$

$$\widetilde{V}(\xi, \gamma, \text{AR}(i)) = -a + (1-u)V(\xi, \gamma \oplus i) + u(-r + V(\xi, \gamma \oplus i)) \quad (4.77)$$

$$= -a - ur + V(\xi, \gamma \oplus i). \quad (4.78)$$

Recall from section 4.5.1 that RP is a shorthand for “audit and rectify (AR) all remaining unaudited suppliers if $a + ur < uwz$ and proceed to production (PP) otherwise” and $c_{\text{RP}} \equiv (uwz) \wedge (a + ur)$ is the cost associated with each unaudited supplier in the RP subphase. For any $\gamma \in \Gamma$ and $\xi \in \Xi$ we write $\widetilde{V}^*(\gamma, \text{RP}) = \widetilde{V}(\xi, \gamma, \text{RP}) = \pi(\gamma) - c_{\text{RP}}|U_\gamma|$.

Given state $\gamma \in \Gamma$, let $R^+(\gamma) \subseteq \Gamma$ be the set of states reachable from $\gamma$ (including $\gamma$ itself): $\gamma' \in R^+(\gamma)$ if and only if there exists a policy $\xi \in \Xi$ such that $\gamma'$ is reached from $\gamma$ with strictly positive probability by following $\xi$.

### 4.C.1 Two Subphases of Auditing

**Proposition 4.10.** *The buyer can be at least as well off by postponing all audit and rectify (* AR*) actions to after all audit and drop (* AD*) actions.*

*Proof.* Let $\xi \in \Xi$ be such that there exists $\gamma = (g, U) \in \Gamma$, $i \in U_\gamma$, and $j \in U_{\gamma \oplus i}$ such that

$$\xi(\gamma) = \textsc{ar}(i) \quad \text{and} \quad \xi(\gamma \oplus i) = \textsc{ad}(j). \tag{4.79}$$

(If there does not exist such a triple of $\gamma$, $i$, and $j$ then in $\xi$ already all AR actions come after all AD actions.) We specify a policy $\xi' \in \Xi$ otherwise identical to $\xi$ but with the sequence of the above two actions swapped, namely,

$$\xi'(\gamma') = \xi(\gamma'), \quad \forall \gamma' \in \Gamma \backslash \{\gamma, \gamma \oplus j, \gamma \ominus j\} \tag{4.80}$$

$$\xi'(\gamma) = \textsc{ad}(j) \tag{4.81}$$

$$\xi'(\gamma \oplus j) = \textsc{ar}(i) \tag{4.82}$$

$$\xi'(\gamma \ominus j) = \begin{cases} \textsc{ar}(i), & \text{if } i \notin D_g(j) \\ \xi(\gamma \ominus j), & \text{if } i \in D_g(j) \end{cases}. \tag{4.83}$$

It suffices to show $V(\xi', \gamma) \geqslant V(\xi, \gamma)$.

Now

$$V(\xi, \gamma) = \widetilde{V}(\xi, \gamma, \textsc{ar}(i)) \tag{4.84}$$

$$= -a - ur + V(\xi, \gamma \oplus i) \tag{4.85}$$

$$= -a - ur + \widetilde{V}(\xi, \gamma \oplus i, \textsc{ad}(j)) \qquad \text{(by (4.79))} \tag{4.86}$$

$$= -a - ur - a + (1-u)V(\xi, \gamma \oplus i \oplus j) + uV(\xi, \gamma \oplus i \ominus j) \qquad \text{(by (4.76))} \tag{4.87}$$

and

$$V(\xi', \gamma) = \widetilde{V}(\xi', \gamma, \textsc{ad}(j)) \tag{4.88}$$

$$= -a + (1-u)V(\xi', \gamma \oplus j) + uV(\xi', \gamma \ominus j). \tag{4.89}$$

There are two cases of $i$:

- *Case 1:* $i \notin D_g(j)$. Then

$$V(\xi', \gamma) = -a + (1-u)\widetilde{V}(\xi', \gamma \oplus j, \text{AR}(i)) + u\widetilde{V}(\xi', \gamma \ominus j, \text{AR}(i)) \tag{4.90}$$

$$= -a + (1-u)(-a - ur + V(\xi', \gamma \oplus j \oplus i))$$

$$+ u(-a - ur + V(\xi', \gamma \ominus j \oplus i)) \tag{4.91}$$

$$= -a - a - ur + (1-u)V(\xi', \gamma \oplus j \oplus i) + uV(\xi', \gamma \ominus j \oplus i) \tag{4.92}$$

Note that $\xi'|_{R^+(\gamma \oplus j \oplus i)} = \xi|_{R^+(\gamma \oplus i \oplus j)}$, so $V(\xi', \gamma \oplus j \oplus i) = V(\xi, \gamma \oplus i \oplus j)$. Since $i \notin D_g(j)$, $\gamma \ominus j \oplus i = \gamma \oplus i \ominus j$. Also, $\xi'|_{R^+(\gamma \ominus j \oplus i)} = \xi|_{R^+(\gamma \oplus i \ominus j)}$. Hence, $V(\xi', \gamma \ominus j \oplus i) = V(\xi, \gamma \oplus i \ominus j)$. Therefore by comparing (4.87) and (4.92) we conclude $V(\xi', \gamma) = V(\xi, \gamma)$.

- *Case 2:* $i \in D_g(j)$. Immediately, $\gamma \oplus i \ominus j = \gamma \ominus j$.

$$V(\xi', \gamma) = -a + (1-u)\widetilde{V}(\xi', \gamma \oplus j, \text{AR}(i)) + uV(\xi', \gamma \ominus j) \tag{4.93}$$

$$= -a + (1-u)(-a - ur + V(\xi', \gamma \oplus j \oplus i)) + uV(\xi', \gamma \ominus j) \tag{4.94}$$

$$= -a - (1-u)(a + ur) + (1-u)V(\xi', \gamma \oplus j \oplus i) + uV(\xi', \gamma \ominus j). \tag{4.95}$$

Same as above, since $\xi'|_{R^+(\gamma \oplus j \oplus i)} = \xi|_{R^+(\gamma \oplus i \oplus j)}$,

$$V(\xi', \gamma \oplus j \oplus i) = V(\xi, \gamma \oplus i \oplus j). \tag{4.96}$$

Since $i \in D_g(j)$, $\xi'(\gamma \ominus j) = \xi(\gamma \ominus j)$. Hence $\xi'|_{R^+(\gamma \ominus j)} = \xi|_{R^+(\gamma \ominus j)}$. By $\gamma \oplus i \ominus j = \gamma \ominus j$,

$$V(\xi, \gamma \oplus i \ominus j) = V(\xi, \gamma \ominus j) = V(\xi', \gamma \ominus j). \tag{4.97}$$

Substitute (4.96) and (4.97) into (4.95), then subtract (4.87) to obtain

$$V(\xi', \gamma) - V(\xi, \gamma) = a + ur - (1-u)(a + ur) = u(a + ur) \geqslant 0. \tag{4.98}$$

This completes the proof. Note that the buyer is strictly better off by swapping the actions AR($i$)

and $\text{AD}(j)$ (as in $\xi'$) if and only if $i \in D_g(j)$ and $u(a+ur) > 0$. $\square$

**Proposition 4.11.** *Limit the buyer's actions to audit and rectify (* AR*) unaudited suppliers and proceed to production (* PP*). The optimal auditing policy is to audit and rectify all unaudited suppliers in any sequence if $a + ur \leqslant uwz$ and to proceed to production if $a + ur \geqslant uwz$. Furthermore, given $\gamma \in \Gamma$,*

$$V^*(\gamma) = \pi(\gamma) - [(uwz) \wedge (a+ur)]|U_\gamma|. \tag{4.99}$$

*Proof.* Given $\gamma \in \Gamma$ and $i \in U_\gamma$, by (4.2) and the definition of $\zeta$,

$$\widetilde{V}^*(\gamma, \text{PP}) = \pi(\gamma) - uwz|U_\gamma|. \tag{4.100}$$

We prove the result by mathematical induction on the number of unaudited supplier in the state, $|U_\gamma|$. If $|U_\gamma| = 1$, let $i \in U_\gamma$, then $\gamma \oplus i$ is a terminal state. By (4.5),

$$\widetilde{V}^*(\gamma, \text{AR}(i)) = -a - ur + V^*(\gamma \oplus i) = -a - ur + \pi(\gamma \oplus i) = -a - ur + \pi(\gamma) \tag{4.101}$$

since states $\gamma \oplus i$ and $\gamma$ have the same underlying supply network, which determines the production profit. Note $\widetilde{V}^*(\gamma, \text{AR}(i))$ is independent of $i$. PP is preferred to $\text{AR}(i)$ iff $\widetilde{V}^*(\gamma, \text{PP}) \geqslant \widetilde{V}^*(\gamma, \text{AR}(i))$, or $\pi(\gamma) - uwz \geqslant -a - ur + \pi(\gamma)$, or $a + ur \geqslant uwz$. Hence,

$$V^*(\gamma) = \pi(\gamma) - (uwz) \wedge (a+ur). \tag{4.102}$$

By mathematical induction, suppose if $|U_\gamma| = m$,

$$V^*(\gamma) = \pi(\gamma) - m[(uwz) \wedge (a+ur)]. \tag{4.103}$$

Now if $|U_\gamma| = m+1$, pick arbitrary $i \in U_\gamma$, then $|U_{\gamma \oplus i}| = m$. By (4.5) and (4.103),

$$\widetilde{V}^*(\gamma, \text{AR}(i)) = -a - ur + V^*(\gamma \oplus i) = -a - ur + \pi(\gamma) - m[(uwz) \wedge (a+ur)]. \tag{4.104}$$

PP is preferred to $\text{AD}(i)$ iff $\widetilde{V}^*(\gamma, \text{PP}) \geqslant \widetilde{V}^*(\gamma, \text{AR}(i))$, or $\pi(\gamma) - (m+1)uwz \geqslant -a - ur + \pi(\gamma) - m[(uwz) \wedge (a+ur)]$, or $a + ur + m[(uwz) \wedge (a+ur)] \geqslant (m+1)uwz$, which holds iff $a + ur \geqslant uwz$, as we wanted to show. Finally, to complete the induction step, note

$$V^*(\gamma) = \begin{cases} \widetilde{V}^*(\gamma, \text{PP}), & \text{if } a + ur \geqslant uwz \\ \widetilde{V}^*(\gamma, \text{AR}(i)), & \text{if } a + ur \leqslant uwz \end{cases} \tag{4.105}$$

$$= \begin{cases} \pi(\gamma) - (m+1)uwz, & \text{if } a + ur \geqslant uwz \\ -a - ur + \pi(\gamma) - m[(uwz) \wedge (a + ur)], & \text{if } a + ur \leqslant uwz \end{cases} \tag{4.106}$$

$$= \pi(\gamma) - (m+1)[(uwz) \wedge (a + ur)]. \tag{4.107}$$

□

*Proof of Theorem 4.3.* The result is a direct consequence of Propositions 4.10 and 4.11. □

*Proof of Corollary 4.1.* The result follows (4.107) in the proof of Proposition 4.11. □

### 4.C.2 Optimal Auditing Sequence

We first identify an optimal policy in a general class of supply networks in Theorem 4.5, the proof of which serves as the basis for our proof of Theorem 4.4′, an expanded version of Theorem 4.4.

**Assumption 4.1** (decreasing differences of production profit)**.** *For any $\gamma \in \Gamma$ and any $i, i' \in U_\gamma, i' \notin D_\gamma(i)$,*

$$\nabla(\gamma, i') \leqslant \nabla(\gamma \ominus i, i'). \tag{4.108}$$

To state the next assumption, we define a concept of symmetry for suppliers.

**Definition 4.2.** In state $\gamma = (g, U)$ two unaudited suppliers $i, i' \in U$ are *symmetric* (1) if they belong to the same class of tier-2 suppliers $S_{\text{A}}$, $S_{\text{B}}$, or $S_{\text{AB}}$; or, (2) in the case of $|S_{\text{A}}| = |S_{\text{B}}|$ and $|S_{\text{A}} \cap U| = |S_{\text{B}} \cap U|$, (i) if $i \in S_{\text{A}}$ and $i' \in S_{\text{B}}$, or (ii) if $i = \text{A}$ and $i' = \text{B}$.

**Assumption 4.2** (preservation of LVUS)**.** *Let $\gamma \in \Gamma$ and $i$ be an LVUS in $\gamma$. Let $i' \in U_\gamma$ that is not symmetric with $i$. Then $i$ is an LVUS in $\gamma \ominus i'$, i.e.,*

$$\nabla(\gamma \ominus i', i) \leqslant \nabla(\gamma \ominus i', i''), \quad \forall i'' \in U_{\gamma \ominus i'}. \tag{4.109}$$

Under Assumption 4.2, an LVUS remains an LVUS when we remove a nonsymmetric supplier from the supply network.

Given the assumptions we may completely characterize the optimal auditing policy.

**Theorem 4.5.** *Let $\gamma_0 = (g, U) \in \Gamma$ be such that for any $\gamma \in R^+(\gamma_0)$, no unaudited supplier in $\gamma$ is a dependent of another unaudited supplier, i.e., any $i, i' \in U_\gamma$ ($i \neq i'$) satisfy $i \notin D_g(i')$ and $i' \notin D_g(i)$. Under Assumptions 4.1 and 4.2, the following policy $\xi^*$ is optimal in every state $\gamma \in R^+(\gamma_0)$:*

$$\xi^*(\gamma) = \begin{cases} \text{AD}(i), & \text{if } i \in U_\gamma, u\nabla(\gamma, i) + a < c_{\text{RP}}, \text{ and } \nabla(\gamma, i) \leqslant \nabla(\gamma, i'), \forall i' \in U_\gamma \\ \text{RP}, & \text{if } u\nabla(\gamma, i) + a \geqslant c_{\text{RP}}, \forall i \in U_\gamma \end{cases}. \tag{4.110}$$

*Proof.* We prove the result by mathematical induction on the number of unaudited supplier in the state, $|U_\gamma|$. If $|U_\gamma| = 1$, let $i \in U_\gamma$, then $\widetilde{V}^*(\gamma, \text{AD}(i)) > \widetilde{V}^*(\gamma, \text{RP})$ iff

$$-a + (1-u)V^*(\gamma \oplus i) + uV^*(\gamma \ominus i) > \pi(\gamma) - c_{\text{RP}} \tag{4.111}$$

iff

$$-a + (1-u)\pi(\gamma \oplus i) + u\pi(\gamma \ominus i) > \pi(\gamma) - c_{\text{RP}}. \tag{4.112}$$

But $\pi(\gamma \oplus i) = \pi(\gamma)$, so above is equivalent to

$$-a - u(\pi(\gamma \oplus i) - \pi(\gamma \ominus i)) > -c_{\text{RP}} \tag{4.113}$$

equivalent to the condition stipulated by $\xi^*$ for taking action $\text{AD}(i)$. Therefore $\xi^*$ is optimal at $\gamma$.

By way of mathematical induction, suppose $\xi^*$ is optimal for all $\gamma' \in R^+(\gamma_0)$ such that $|U_{\gamma'}| \leqslant k \in \mathbb{N}^+$. Let $\gamma \in R^+(\gamma_0)$ be such that $|U_\gamma| = k + 1$. We divide the proof of the induction step into two cases based on (4.110).

**Case a** ($u\nabla(\gamma, i) + a \geqslant c_{\text{RP}}, \forall i \in U_\gamma$). We show that $\widetilde{V}^*(\gamma, \text{RP}) \geqslant \widetilde{V}^*(\gamma, \text{AD}(i))$ for any $i \in U_\gamma$, thus proving the optimality of the action RP when $u\nabla(\gamma, i) + a \geqslant c_{\text{RP}}, \forall i \in U_\gamma$ as Theorem 4.5 prescribes. Let $i \in U_\gamma$. We first show two equalities: $V^*(\gamma \oplus i) = \widetilde{V}^*(\gamma \oplus i, \text{RP})$ and $V^*(\gamma \ominus i) = \widetilde{V}^*(\gamma \ominus i, \text{RP})$.

First consider the state $\gamma \oplus i$. Note $\pi(\gamma \oplus i) = \pi(\gamma)$ and for any $i' \in U_\gamma \backslash \{i\}$, $\pi(\gamma \oplus i \ominus i') = \pi(\gamma \ominus i')$.

Then for any $i' \in U_{\gamma\oplus i} = U_\gamma \backslash \{i\}$,

$$u(\pi(\gamma \oplus i) - \pi(\gamma \oplus i \ominus i')) + a = u(\pi(\gamma) - \pi(\gamma \ominus i')) + a = u\nabla(\gamma, i') + a \geqslant c_{\text{RP}}. \tag{4.114}$$

Hence by the definition of $\xi^*$, $\xi^*(\gamma \oplus i) = \text{RP}$, i.e., $\xi^*$ prescribes the action RP in state $\gamma \oplus i$. But $|U_{\gamma\oplus i}| = k$. By invoking the induction hypothesis (that $\xi^*$ is optimal at any state $\gamma' \in R^+(\gamma_0)$ such that $|U_{\gamma'}| \leqslant k$), we conclude that the action RP is optimal at state $\gamma \oplus i$. Therefore $V^*(\gamma \oplus i) = \widetilde{V}^*(\gamma \oplus i, \text{RP})$.

Next consider the state $\gamma \ominus i$. Since no unaudited supplier in $\gamma$ is a dependent of another, $U_{\gamma\ominus i} = U_\gamma \backslash \{i\}$. By Assumption 4.1, for any $i' \in U_{\gamma\ominus i}$,

$$u(\pi(\gamma \ominus i) - \pi(\gamma \ominus i \ominus i')) + a \geqslant u(\pi(\gamma) - \pi(\gamma \ominus i')) + a = u\nabla(\gamma, i') + a \tag{4.115}$$

which we know is greater than or equal to $c_{\text{RP}}$ for any $i' \in U_\gamma$. Hence $\xi^*(\gamma \ominus i) = \text{RP}$. But $|U_{\gamma\ominus i}| \leqslant k$. By invoking the induction hypothesis (that $\xi^*$ is optimal at any state $\gamma' \in R^+(\gamma_0)$ such that $|U_{\gamma'}| \leqslant k$), we conclude that the action RP is optimal in state $\gamma \ominus i$. Therefore $V^*(\gamma \ominus i) = \widetilde{V}^*(\gamma \ominus i, \text{RP})$.

Now

$$\begin{aligned}
\widetilde{V}^*(\gamma, \text{RP}) &= \pi(\gamma) - c_{\text{RP}}|U_\gamma| && (4.116)\\
&= \pi(\gamma) - c_{\text{RP}}(|U_\gamma| - 1) - c_{\text{RP}} && (4.117)\\
&\geqslant \pi(\gamma) - c_{\text{RP}}(|U_\gamma| - 1) - [u(\pi(\gamma) - \pi(\gamma \ominus i)) + a] && (4.118)\\
&= -a + (1-u)(\pi(\gamma \oplus i) - c_{\text{RP}}|U_{\gamma\oplus i}|) + u(\pi(\gamma \ominus i) - c_{\text{RP}}|U_{\gamma\ominus i}|) && (4.119)\\
&= -a + (1-u)\widetilde{V}^*(\gamma \oplus i, \text{RP}) + u\widetilde{V}^*(\gamma \ominus i, \text{RP}) && (4.120)\\
&= -a + (1-u)V^*(\gamma \oplus i) + uV^*(\gamma \ominus i) && (4.121)\\
&= \widetilde{V}^*(\gamma, \text{AD}(i)) && (4.122)
\end{aligned}$$

where (4.118) is by the assumption $u(\pi(\gamma)-\pi(\gamma\ominus i))+a \geqslant c_{\text{RP}}$; (4.119) is by $|U_{\gamma\oplus i}| = |U_{\gamma\ominus i}| = |U_\gamma|-1$ (no unaudited supplier in $\gamma$ is a dependent of another so that $\gamma \ominus i$ has exactly one less unaudited supplier than $\gamma$); and (4.121) is by $V^*(\gamma \oplus i) = \widetilde{V}^*(\gamma \oplus i, \text{RP})$ and $V^*(\gamma \ominus i) = \widetilde{V}^*(\gamma \ominus i, \text{RP})$.

**Case b** ($\exists i' \in U_\gamma$ such that $u\nabla(\gamma, i') + a < c_{\text{RP}}$). Let $i \in U_\gamma$ be an LVUS in $\gamma$, i.e., $\nabla(\gamma, i) \leqslant \nabla(\gamma, j), \forall j \in U_\gamma$. We first show that $\widetilde{V}^*(\gamma, \text{AD}(i)) \geqslant \widetilde{V}^*(\gamma, \text{RP})$, then show that $\widetilde{V}^*(\gamma, \text{AD}(i)) \geqslant \widetilde{V}^*(\gamma, \text{AD}(i'))$ for any $i' \in U_\gamma$. With these we prove that if $i$ is an LVUS in $\gamma$ and $u\nabla(\pi, i) + a < c_{\text{RP}}$ then the optimal action to take in state $\gamma$ is $\text{AD}(i)$ as Theorem 4.5 prescribes. Now

$$
\begin{aligned}
\widetilde{V}^*(\gamma, \text{AD}(i)) &= -a + (1-u)V^*(\gamma \oplus i) + uV^*(\gamma \ominus i) && (4.123)\\
&\geqslant -a + (1-u)\widetilde{V}^*(\gamma \oplus i, \text{RP}) + u\widetilde{V}^*(\gamma \ominus i, \text{RP}) && (4.124)\\
&= -a + (1-u)(\pi(\gamma \oplus i) - c_{\text{RP}}|U_{\gamma\oplus i}|) + u(\pi(\gamma \ominus i) - c_{\text{RP}}|U_{\gamma\ominus i}|) && (4.125)\\
&= -a + \pi(\gamma) - c_{\text{RP}}|U_{\gamma\oplus i}| - u(\pi(\gamma) - \pi(\gamma \ominus i)) && (4.126)\\
&> \pi(\gamma) - c_{\text{RP}}|U_\gamma| && (4.127)\\
&= \widetilde{V}^*(\gamma, \text{RP}) && (4.128)
\end{aligned}
$$

where (4.124) is by $V^*$ being optimal; (4.126) is by $\pi(\gamma \oplus i) = \pi(\gamma)$; and (4.127) is by $|U_{\gamma\oplus i}| = |U_\gamma| - 1$ and the premise of case b.

We next show that $\widetilde{V}^*(\gamma, \text{AD}(i)) \geqslant \widetilde{V}^*(\gamma, \text{AD}(i'))$ for any $i' \in U_\gamma$. Let $i' \in U_g$ such that $i'$ is not symmetric with $i$. (If $i'$ is symmetric with $i$, clearly $\widetilde{V}^*(\gamma, \text{AD}(i)) = \widetilde{V}^*(\gamma, \text{AD}(i'))$.) Since $i$ is an LVUS in $\gamma \oplus i'$ and $u(\pi(\gamma \oplus i') - \pi(\gamma \oplus i' \ominus i)) + a = u(\pi(\gamma) - \pi(\gamma \ominus i)) + a < c_{\text{RP}}$ (by the premise of case b), by the induction hypothesis, $\xi^*(\gamma \oplus i') = \text{AD}(i)$. On the other hand, by Assumption 4.2, $i$ is an LVUS in $\gamma \ominus i'$; therefore[5]

$$
\xi^*(\gamma \ominus i') = \begin{cases} \text{AD}(i), & \text{if } u(\pi(\gamma \ominus i') - \pi(\gamma \ominus i' \ominus i)) + a < c_{\text{RP}} \\ \text{AR}(i), & \text{if } u(\pi(\gamma \ominus i') - \pi(\gamma \ominus i' \ominus i)) + a \geqslant c_{\text{RP}} \text{ and } a + ur < uwz \\ \text{PP}, & \text{if } u(\pi(\gamma \ominus i') - \pi(\gamma \ominus i' \ominus i)) + a \geqslant c_{\text{RP}} \text{ and } a + ur \geqslant uwz \end{cases} \cdot \qquad (4.129)
$$

We next look at the three cases in (4.129) separately. In each case we devise a policy $\widehat{\xi}$ so that the buyer's expected profit from first taking the action $\text{AD}(i)$ and following $\widehat{\xi}$ thereafter is at least as good as the expected profit from first taking $\text{AD}(i')$ and following the optimal policy $\xi^*$ thereafter ($\xi^*$ is optimal thereafter by the induction hypothesis). That is, $\widetilde{V}(\widehat{\xi}, \gamma, \text{AD}(i))) \geqslant \widetilde{V}(\xi^*, \gamma, \text{AD}(i')) =$

[5]If $u(\pi(\gamma\ominus i') - \pi(\gamma\ominus i'\ominus i)) + a \geqslant c_{\text{RP}}$ and $a + ur < uwz$, $\xi^*$ prescribes auditing and rectify (if non-compliant) all unaudited suppliers in any sequence; here we choose $i$ to audit next.

$\widetilde{V}^*(\gamma, \text{AD}(i'))$. Since $\widetilde{V}^*(\gamma, \text{AD}(i)) \geqslant \widetilde{V}(\widehat{\xi}, \gamma, \text{AD}(i)))$, we must then have $\widetilde{V}^*(\gamma, \text{AD}(i)) \geqslant \widetilde{V}^*(\gamma, \text{AD}(i'))$ as desired. In each case we consider the following four events that together form a partition of the sample space:

$$H_{11} = \{\text{both } i \text{ and } i' \text{ are compliant}\} \tag{4.130}$$

$$H_{10} = \{i \text{ is compliant and } i' \text{ is not compliant}\} \tag{4.131}$$

$$H_{01} = \{i \text{ is not compliant and } i' \text{ is compliant}\} \tag{4.132}$$

$$H_{00} = \{\text{neither } i \text{ nor } i' \text{ is compliant}\}. \tag{4.133}$$

**Case b(i)** $(u(\pi(\gamma \ominus i') - \pi(\gamma \ominus i' \ominus i)) + a < c_{\text{RP}})$. Let $\widehat{\xi} \in \Xi$ be the policy such that $\widehat{\xi}(\gamma \oplus i) = \widehat{\xi}(\gamma \ominus i) = \text{AD}(i')$ and $\widehat{\xi}(\gamma') = \xi^*(\gamma')$ for any $\gamma' \in \Gamma \backslash \{\gamma \oplus i, \gamma \ominus i\}$.

*Conditional on* $H_{11}$: The path of state transition by taking $\text{AD}(i)$ at state $\gamma$ then following policy $\widehat{\xi}$ is

$$\gamma \xrightarrow{\text{AD}(i)} \gamma \oplus i \xrightarrow{\text{AD}(i')} \gamma \oplus i \oplus i' \tag{4.134}$$

while that by taking $\text{AD}(i')$ at state $\gamma$ then following policy $\xi^*$ is

$$\gamma \xrightarrow{\text{AD}(i')} \gamma \oplus i' \xrightarrow{\text{AD}(i)} \gamma \oplus i' \oplus i. \tag{4.135}$$

Note that $\gamma \oplus i \oplus i' = \gamma \oplus i' \oplus i$ and $\widehat{\xi}|_{R^+(\gamma \oplus i \oplus i')} = \xi^*|_{R^+(\gamma \oplus i' \oplus i)}$, so the expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following policy $\widehat{\xi}$ is the same as that from first taking $\text{AD}(i')$ then following $\xi^*$ conditional on $H_{11}$.

*Conditional on* $H_{10}$: The path of state transition by taking $\text{AD}(i)$ at state $\gamma$ then following policy $\widehat{\xi}$ is

$$\gamma \xrightarrow{\text{AD}(i)} \gamma \oplus i \xrightarrow{\text{AD}(i')} \gamma \oplus i \ominus i' \tag{4.136}$$

while that by taking $\text{AD}(i')$ at state $\gamma$ then following policy $\xi^*$ is

$$\gamma \xrightarrow{\text{AD}(i')} \gamma \ominus i' \xrightarrow{\text{AD}(i)} \gamma \ominus i' \oplus i. \tag{4.137}$$

Note that $\gamma \oplus i \ominus i' = \gamma \ominus i' \oplus i$ and $\widehat{\xi}|_{R^+(\gamma \oplus i \ominus i')} = \xi^*|_{R^+(\gamma \ominus i' \oplus i)}$, so the expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following policy $\widehat{\xi}$ is the same as that from first taking $\text{AD}(i')$ then following $\xi^*$ conditional on $H_{10}$.

*Conditional on* $H_{01}$ *or* $H_{00}$: Similarly we can show that the expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following policy $\widehat{\xi}$ is the same as that from first taking $\text{AD}(i')$ then following $\xi^*$.

Therefore the unconditional expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following policy $\widehat{\xi}$ and that from first taking $\text{AD}(i')$ then following $\xi^*$, which are integrals of the respected conditional expected profits, must be equal; that is $\widetilde{V}(\widehat{\xi}, \gamma, \text{AD}(i)) = \widetilde{V}(\xi^*, \gamma, \text{AD}(i'))$. Therefore

$$\widetilde{V}^*(\gamma, \text{AD}(i)) \geqslant \widetilde{V}(\widehat{\xi}, \gamma, \text{AD}(i)) = \widetilde{V}(\xi^*, \gamma, \text{AD}(i')) = \widetilde{V}^*(\gamma, \text{AD}(i')) \tag{4.138}$$

where the induction hypothesis (that $\xi^*$ is optimal at any state $\gamma' \in R^+(\gamma_0)$ with $|U_{\gamma'}| \leqslant k$) gives the last equality.

**Case b(ii)** $(u(\pi(\gamma \ominus i') - \pi(\gamma \ominus i' \ominus i)) + a \geqslant c_{\text{RP}}$ and $a + ur < uwz)$. Let $\widehat{\xi} \in \Xi$ be the policy such that $\widehat{\xi}(\gamma \oplus i) = \text{AD}(i')$, $\widehat{\xi}(\gamma \ominus i) = \text{AR}(i')$, and $\widehat{\xi}(\gamma') = \xi^*(\gamma')$ for any $\gamma' \in \Gamma \backslash \{\gamma \oplus i, \gamma \ominus i\}$.

*Conditional on* $H_{11}$: Using the same steps as in case b(i) we can show the expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following $\widehat{\xi}$ is the same as that from first taking $\text{AD}(i')$ then following $\xi^*$ conditional on $H_{11}$.

*Conditional on* $H_{10}$: The path of state transition by taking $\text{AD}(i)$ at state $\gamma$ then following policy $\widehat{\xi}$ is

$$\gamma \xrightarrow{\text{AD}(i)} \gamma \oplus i \xrightarrow{\text{AD}(i')} \gamma \oplus i \ominus i' \tag{4.139}$$

while that by taking $\text{AD}(i')$ at state $\gamma$ then following policy $\xi^*$ is

$$\gamma \xrightarrow{\text{AD}(i')} \gamma \ominus i' \xrightarrow{\text{AR}(i)} \gamma \ominus i' \oplus i. \tag{4.140}$$

Note that $\gamma \oplus i \ominus i' = \gamma \ominus i' \oplus i$ and $\widehat{\xi}|_{R^+(\gamma \oplus i \ominus i')} = \xi^*|_{R^+(\gamma \ominus i' \oplus i)}$, so the expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following $\widehat{\xi}$ is the same as that from first taking $\text{AD}(i')$ then following $\xi^*$ conditional on $H_{10}$.

*Conditional on* $H_{01}$: Similarly we can show that the expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following policy $\widehat{\xi}$ is the same as that from first taking $\text{AD}(i')$ then following $\xi^*$.

*Conditional on* $H_{00}$: The path of state transition by taking $\text{AD}(i)$ at state $\gamma$ then following policy $\widehat{\xi}$ is

$$\gamma \xrightarrow{\text{AD}(i)} \gamma \ominus i \xrightarrow{\text{AR}(i')} \gamma \ominus i \oplus i' \tag{4.141}$$

while that by taking $\text{AD}(i')$ at state $\gamma$ then following policy $\xi^*$ is

$$\gamma \xrightarrow{\text{AD}(i')} \gamma \ominus i' \xrightarrow{\text{AR}(i)} \gamma \ominus i' \oplus i. \tag{4.142}$$

Since $u(\pi(\gamma\ominus i'\oplus i)-\pi(\gamma\ominus i'\oplus i\ominus i''))+a = u(\pi(\gamma\ominus i')-\pi(\gamma\ominus i'\ominus i''))+a \geqslant u(\pi(\gamma\ominus i')-\pi(\gamma\ominus i'\ominus i))+a$ for any $i'' \in U_{\gamma\ominus i'\oplus i}$ (the last inequality is because $i$ is an LVUS in $\gamma \ominus i'$, by Assumption 4.2), and $u(\pi(\gamma\ominus i')-\pi(\gamma\ominus i'\ominus i))+a \geqslant c_{\text{RP}}$ (premise of case b(ii)), we have $u(\pi(\gamma\ominus i'\oplus i)-\pi(\gamma\ominus i'\oplus i\ominus i''))+a \geqslant c_{\text{RP}}$. Therefore $\xi^*(\gamma \ominus i' \oplus i) = \text{RP}$. Note that since $\widehat{\xi}|_{R^+(\gamma\ominus i\oplus i')} = \xi^*|_{R^+(\gamma\ominus i\oplus i')}$,

$$V(\widehat{\xi}, \gamma \ominus i \oplus i') = V^*(\gamma \ominus i \oplus i') \geqslant \widetilde{V}^*(\gamma \ominus i \oplus i', \text{RP}). \tag{4.143}$$

On the other hand, since $|U_{\gamma\ominus i\oplus i'}| = |U_{\gamma\ominus i'\oplus i}|$,

$$\widetilde{V}^*(\gamma\ominus i\oplus i', \text{RP}) - \widetilde{V}^*(\gamma\ominus i'\oplus i, \text{RP}) = \pi(\gamma\ominus i\oplus i') - \pi(\gamma\ominus i'\oplus i) = \pi(\gamma\ominus i) - \pi(\gamma\ominus i') \geqslant 0 \tag{4.144}$$

Together they imply

$$V(\widehat{\xi}, \gamma \ominus i \oplus i') \geqslant \widetilde{V}^*(\gamma \ominus i' \oplus i, \text{RP}) = V^*(\gamma \ominus i' \oplus i) \tag{4.145}$$

where the last equality is because $\xi^*(\gamma \ominus i' \oplus i) = \text{RP}$. Therefore the expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following $\widehat{\xi}$ is greater than or equal to that from first taking $\text{AD}(i')$ then following $\xi^*$ conditional on $H_{10}$.

Therefore the unconditional expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following policy $\widehat{\xi}$ is greater than or equal to that from first taking $\text{AD}(i')$ then following $\xi^*$; that is $\widetilde{V}(\widehat{\xi}, \gamma, \text{AD}(i)) \geqslant \widetilde{V}(\xi^*, \gamma, \text{AD}(i'))$. Therefore

$$\widetilde{V}^*(\gamma, \text{AD}(i)) \geqslant \widetilde{V}(\widehat{\xi}, \gamma, \text{AD}(i)) \geqslant \widetilde{V}(\xi^*, \gamma, \text{AD}(i')) = \widetilde{V}^*(\gamma, \text{AD}(i')) \tag{4.146}$$

where the induction hypothesis gives the last equality.

**Case b(iii)** $(u(\pi(\gamma \ominus i') - \pi(\gamma \ominus i' \ominus i)) + a \geqslant c_{\text{RP}}$ and $a + ur \geqslant uwz)$. Let $\widehat{\xi} \in \Xi$ be the policy such that (1) $\widehat{\xi}(\gamma \oplus i) = \text{AD}(i')$, (2) for any $\gamma' \in R^{+}(\gamma \ominus i)$ such that $i' \in U_{\gamma'}$, $\widehat{\xi}(\gamma') = \xi^{*}(\gamma' \ominus i')$, and (3) $\widehat{\xi}(\gamma') = \xi^{*}(\gamma')$ for any other state $\gamma'$ (i.e., $\gamma' \in \Gamma \backslash \{\gamma \oplus i\} \backslash \{\gamma'' \in R^{+}(\gamma \ominus i) : i'' \in U_{\gamma'}\}$).

*Conditional on* $H_{11}$: Using the same corresponding steps as in case b(i) we can show the expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following $\widehat{\xi}$ is the same as that from first taking $\text{AD}(i')$ then following $\xi^{*}$ conditional on $H_{11}$.

*Conditional on* $H_{10}$: Since $i$ is an LVUS in $\gamma \ominus i'$, by the premise of case b(iii), any unaudited supplier $i''$ in state $\gamma \oplus i \ominus i'$ must have $u\nabla(\gamma \oplus i \ominus i', i'') + a = u\nabla(\gamma \ominus i', i'') + a \geqslant c_{\text{RP}}$, then the induction hypothesis implies $\xi^{*}(\gamma \oplus i \ominus i') = \text{PP}$. By the definition of $\widehat{\xi}$, $\widehat{\xi}(\gamma \oplus i \ominus i') = \xi^{*}(\gamma \oplus i \ominus i')$. Therefore $\widehat{\xi}(\gamma \oplus i \ominus i') = \text{PP}$. Then the path of the state transition by taking $\text{AD}(i)$ at state $\gamma$ then following policy $\widehat{\xi}$ is

$$\gamma \xrightarrow{\text{AD}(i)} \gamma \oplus i \xrightarrow{\text{AD}(i')} \gamma \oplus i \ominus i' \xrightarrow{\text{PP}} . \tag{4.147}$$

The path of the state transition by taking $\text{AD}(i')$ at state $\gamma$ then following policy $\xi^{*}$ is

$$\gamma \xrightarrow{\text{AD}(i')} \gamma \ominus i' \xrightarrow{\text{PP}} . \tag{4.148}$$

Note that $\pi(\gamma \oplus i \ominus i') = \pi(\gamma \ominus i')$, so the only difference in the conditional expected profit between the above two paths is the additional cost $a$ of carrying out one more audit in (4.147) (since $i$ is compliant on $H_{10}$ it will not incur any penalty from violation later on).

*Conditional on* $H_{01}$: The path of the state transition by taking $\text{AD}(i)$ at state $\gamma$ then following policy $\widehat{\xi}$ is

$$\gamma \xrightarrow{\text{AD}(i)} \gamma \ominus i \tag{4.149}$$

while the path of the state transition by taking $\text{AD}(i')$ at state $\gamma$ then following policy $\xi^{*}$ is

$$\gamma \xrightarrow{\text{AD}(i')} \gamma \oplus i' \xrightarrow{\text{AD}(i)} \gamma \oplus i' \ominus i. \tag{4.150}$$

Note that the definition of $\widehat{\xi}$ means that the path subsequent to $\gamma \ominus i$ in (4.149) and that subsequent to $\gamma \oplus i' \ominus i$ in (4.150) will be identical except that $i'$ will remain unaudited in all subsequent states in (4.149) while it is vetted in (4.150). Since on $H_{01}$ $i'$ is compliant the only difference in the conditional expected profit between the above two paths is the additional cost $a$ of carrying out one more audit in (4.150) (since $i'$ is compliant on $H_{10}$, even if unaudited, it will not incur any penalty from violation later on).

*Conditional on* $H_{00}$: The path of the state transition by taking $\text{AD}(i)$ at state $\gamma$ then following policy $\widehat{\xi}$ is

$$\gamma \xrightarrow{\text{AD}(i)} \gamma \ominus i \xrightarrow{\text{PP}} \tag{4.151}$$

where $\widehat{\xi}(\gamma \ominus i) = \xi^*(\gamma \ominus i \ominus i') = \text{PP}$ by the premise of the current case and Assumption 4.1 (so that $u(\pi(\gamma \ominus i \ominus i') - \pi(\gamma \ominus i \ominus i' \ominus i'')) + a \geqslant c_{\text{RP}}, \forall i'' \in U_{\gamma \ominus i \ominus i'}$), while the path of the state transition by taking $\text{AD}(i')$ at state $\gamma$ then following policy $\xi^*$ is

$$\gamma \xrightarrow{\text{AD}(i')} \gamma \ominus i' \xrightarrow{\text{PP}} . \tag{4.152}$$

Therefore conditional on $H_{00}$ the expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following $\widehat{\xi}$ is greater than that from first taking $\text{AD}(i')$ then following $\xi^*$ by precisely $\pi(\gamma \ominus i) - \pi(\gamma \ominus i') \geqslant 0$.

Therefore the unconditional expected profit at $\gamma$ from first taking $\text{AD}(i)$ then following policy $\widehat{\xi}$ is greater than or equal to that from first taking $\text{AD}(i')$ then following $\xi^*$; that is $\widetilde{V}(\widehat{\xi}, \gamma, \text{AD}(i)) \geqslant \widetilde{V}(\xi^*, \gamma, \text{AD}(i'))$. Therefore

$$\widetilde{V}^*(\gamma, \text{AD}(i)) \geqslant \widetilde{V}(\widehat{\xi}, \gamma, \text{AD}(i)) \geqslant \widetilde{V}(\xi^*, \gamma, \text{AD}(i')) = \widetilde{V}^*(\gamma, \text{AD}(i')) \tag{4.153}$$

where the induction hypothesis yields the last equality.

To sum up, in all cases b(i)–b(iii), $\widetilde{V}^*(\gamma, \text{AD}(i)) \geqslant \widetilde{V}^*(\gamma, \text{AD}(i'))$.

$\xi^*$ is optimal at $\gamma$.

□

Theorem 4.4 is a shortened version of Theorem 4.4′.

**Theorem 4.4′.** *Under Condition 1 the following policy* $\xi^{**}$ *is optimal at any state* $\gamma$ *in which every* *tier-1 firm is vetted: for any nonterminal state $\gamma \neq \gamma_1$, let $i$ be an LVUS in $\gamma$, then*

$$\xi^{**}(\gamma) = \begin{cases} \text{AD}(i), & \text{if } u\nabla(\gamma, i) + a < c_{\text{RP}} \\ \text{RP}, & \text{if } u\nabla(\gamma, i) + a \geqslant c_{\text{RP}} \end{cases} \tag{4.154}$$

*and for $\gamma_1$ and $i \in U_{\gamma_1}$,*

$$\xi^{**}(\gamma_1) = \begin{cases} \text{AD}(i), & \text{if } \frac{1}{1+u}(a + u\nabla(\gamma_1, i)) + \frac{u}{1+u}(a + u\pi(\gamma_1 \ominus i)) < c_{\text{RP}} \\ \text{RP}, & \text{if } \frac{1}{1+u}(a + u\nabla(\gamma_1, i)) + \frac{u}{1+u}(a + u\pi(\gamma_1 \ominus i)) \geqslant c_{\text{RP}} \end{cases}. \tag{4.155}$$

The policy $\xi^{**}$ differs from $\xi^{*}$ only at state $\gamma_1$ where Assumption 4.1 fails. At state $\gamma_1$, $\xi^{**}$ prescribes AD$(i)$ in a larger region of the parameter space than $\xi^{*}$ does, since $\xi^{**}$ takes into account of the fact that if the buyer drops $i$, the last remaining unaudited supplier will be even less valuable. The buyer has less incentive to keep the supply network operating in state $\gamma_1$ than in states in which decreasing differences hold.

*Proof of Theorem 4.4′.* Since we limit to states in which all tier-1 firms are vetted, (1) by Proposition 4.6 Assumption 4.2 holds, and (2) no unaudited supplier can be a dependent of another unaudited supplier. Under Condition 1, among all states we consider here the only state at which Assumption 4.1 fails is $\gamma_1$, the induction proof of Theorem 4.5 applies directly by replacing $\xi^{*}$ with $\xi^{**}$, with two exceptions: (1) at $\gamma_1$ itself, at which state we show the optimality of $\xi^{**}$ separately, and (2) at state $\gamma_2 = (g, U)$ where $g = (\{\text{A}, \text{B}\}, \{1\}, \{3\}, \{2\})$ and $U = \{1, 2, 3\}$, and if the LVUS, 1, in $\gamma_2$ satisfies $u\nabla(\gamma_2, i) + a < c_{\text{RP}}$. We will go on to show why the induction proof still applies in the second case.

**$\xi^{**}$ is optimal at $\gamma_1$** Set $\gamma = \gamma_1$. Call the two symmetric tier-2 suppliers in $\gamma_1$ $i$ and $i'$. To analyze the decision at $\gamma$ we first consider the profits in state $\gamma \oplus i$ and $\gamma \ominus i$. In state $\gamma \oplus i$ the only unaudited supplier is $i'$. The decision is between AD$(i')$ (with expected profit $-a + (1-u)\pi(\gamma \oplus i \oplus$

$i') + u\pi(\gamma \oplus i \ominus i'))$ and RP (with expected profit $\pi(\gamma \oplus i) - c_{\mathrm{RP}}$). Therefore

$$V^*(\gamma \oplus i) = \begin{cases} -a + (1-u)\pi(\gamma \oplus i \oplus i') + u\pi(\gamma \oplus i \ominus i'), & \text{if } u(\pi(\gamma \oplus i \oplus i') \\ & -\pi(\gamma \oplus i \ominus i')) + a < c_{\mathrm{RP}} \\ \pi(\gamma \oplus i) - c_{\mathrm{RP}}, & \text{if } u(\pi(\gamma \oplus i \oplus i') \\ & -\pi(\gamma \oplus i \ominus i')) + a \geqslant c_{\mathrm{RP}} \end{cases} \tag{4.156}$$

$$= \begin{cases} -a + (1-u)\pi(\gamma) + u\pi(\gamma \ominus i'), & \text{if } u(\pi(\gamma) - \pi(\gamma \ominus i')) + a < c_{\mathrm{RP}} \\ \pi(\gamma) - c_{\mathrm{RP}}, & \text{if } u(\pi(\gamma) - \pi(\gamma \ominus i')) + a \geqslant c_{\mathrm{RP}} \end{cases}. \tag{4.157}$$

Similary, in state $\gamma \ominus i$ the only unaudited supplier is $i'$. The decision is between AD($i'$) (with expected profit $-a + (1-u)\pi(\gamma \ominus i \oplus i')$) and RP (with expected profit $\pi(\gamma \ominus i) - c_{\mathrm{RP}}$). Therefore

$$V^*(\gamma \ominus i) = \begin{cases} -a + (1-u)\pi(\gamma \ominus i \oplus i'), & \text{if } u\pi(\gamma \ominus i \oplus i') + a < c_{\mathrm{RP}} \\ \pi(\gamma \ominus i) - c_{\mathrm{RP}} & \text{if } u\pi(\gamma \ominus i \oplus i') + a \geqslant c_{\mathrm{RP}} \end{cases} \tag{4.158}$$

$$= \begin{cases} -a + (1-u)\pi(\gamma \ominus i), & \text{if } u\pi(\gamma \ominus i) + a < c_{\mathrm{RP}} \\ \pi(\gamma \ominus i) - c_{\mathrm{RP}} & \text{if } u\pi(\gamma \ominus i) + a \geqslant c_{\mathrm{RP}} \end{cases}. \tag{4.159}$$

By Proposition 4.4 we algebraically verify that $\pi(\gamma) - \pi(\gamma \ominus i') > \pi(\gamma \ominus i') = \pi(\gamma \ominus i)$ (which is how Assumption 4.1 is violated). By (4.157) and (4.159) we obtain

$$\widetilde{V}^*(\gamma, \text{AD}(i)) = -a + (1-u)V^*(\gamma \oplus i) + uV^*(\gamma \ominus i) \tag{4.160}$$

$$= \begin{cases} -2a + (1-u)[(1-u)\pi(\gamma) + u\pi(\gamma \ominus i')] + u[(1-u)\pi(\gamma \ominus i)], & \text{if } u(\pi(\gamma) - \pi(\gamma \ominus i')) + a < c_{\text{RP}} \\ -a + (1-u)[\pi(\gamma) - c_{\text{RP}}] + u[-a + (1-u)\pi(\gamma \ominus i)], & \text{if } \begin{array}{l} u\pi(\gamma \ominus i) + a < c_{\text{RP}} \\ \leqslant u(\pi(\gamma) - \pi(\gamma \ominus i')) + a \end{array} \\ -a + (1-u)[\pi(\gamma) - c_{\text{RP}}] + u[\pi(\gamma \ominus i) - c_{\text{RP}}], & \text{if } u\pi(\gamma \ominus i) + a \geqslant c_{\text{RP}} \end{cases} \tag{4.161}$$

$$= \begin{cases} -2a + (1-u)^2\pi(\gamma) + 2u(1-u)\pi(\gamma \ominus i'), & \text{if } u(\pi(\gamma) - \pi(\gamma \ominus i')) + a < c_{\text{RP}} \\ -a + (1-u)[\pi(\gamma) - c_{\text{RP}}] + u[-a + (1-u)\pi(\gamma \ominus i)], & \text{if } \begin{array}{l} u\pi(\gamma \ominus i) + a < c_{\text{RP}} \\ \leqslant u(\pi(\gamma) - \pi(\gamma \ominus i')) + a \end{array} \\ -a - c_{\text{RP}} + (1-u)\pi(\gamma) + u\pi(\gamma \ominus i), & \text{if } u\pi(\gamma \ominus i) + a \geqslant c_{\text{RP}} \end{cases} . \tag{4.162}$$

On the other hand $\widetilde{V}^*(\gamma, \text{RP}) = \pi(\gamma) - c_{\text{RP}}|U_\gamma| = \pi(\gamma) - 2c_{\text{RP}}$. Hence $\widetilde{V}^*(\gamma, \text{AD}(i)) > \widetilde{V}^*(\gamma, \text{RP})$ if and only if one of the following three (mutually exclusive) conditions holds:

1. $a + u(\pi(\gamma) - \pi(\gamma \ominus i')) < c_{\text{RP}}$ and $-2a + (1-u)^2\pi(\gamma) + 2u(1-u)\pi(\gamma \ominus i') > \pi(\gamma) - 2c_{\text{RP}}$;

2. $a + u\pi(\gamma \ominus i) < c_{\text{RP}} \leqslant a + u(\pi(\gamma) - \pi(\gamma \ominus i'))$ and $-a + (1-u)[\pi(\gamma) - c_{\text{RP}}] + u[-a + (1-u)\pi(\gamma \ominus i)] > \pi(\gamma) - 2c_{\text{RP}}$;

3. $a + u\pi(\gamma \ominus i) \geqslant c_{\text{RP}}$ and $-a - c_{\text{RP}} + (1-u)\pi(\gamma) + u\pi(\gamma \ominus i) > \pi(\gamma) - 2c_{\text{RP}}$.

In 1, the second inequality is equivalent to

$$2[a + u(\pi(\gamma) - \pi(\gamma \ominus i'))] - u^2[(\pi(\gamma) - \pi(\gamma \ominus i')) - \pi(\gamma \ominus i')] < 2c_{\text{RP}} \tag{4.163}$$

which is implied by the first inequality and that $(\pi(\gamma) - \pi(\gamma \ominus i')) > \pi(\gamma \ominus i')$ which we know to be true. So 1 can be simplified to just $a + u(\pi(\gamma) - \pi(\gamma \ominus i')) < c_{\text{RP}}$. In 2, the last inequality is equivalent to

$$[a + u(\pi(\gamma) - \pi(\gamma \ominus i))] + u(a + u\pi(\gamma \ominus i)) < (1 + u)c_{\text{RP}} \tag{4.164}$$

or

$$\frac{1}{1+u}[a + u(\pi(\gamma) - \pi(\gamma \ominus i))] + \frac{u}{1+u}(a + u\pi(\gamma \ominus i)) < c_{\text{RP}}. \tag{4.165}$$

Note that (4.165) and the second inequality $c_{\text{RP}} \leqslant a + u(\pi(\gamma) - \pi(\gamma \ominus i'))$ implies the first inequality $a + u\pi(\gamma \ominus i) < c_{\text{RP}}$. So 2 can be simplified to

$$\frac{1}{1+u}[a + u(\pi(\gamma) - \pi(\gamma \ominus i))] + \frac{u}{1+u}(a + u\pi(\gamma \ominus i)) < c_{\text{RP}} \leqslant a + u(\pi(\gamma) - \pi(\gamma \ominus i')). \tag{4.166}$$

In 3, the second inequality is equivalent to

$$a + u(\pi(\gamma) - \pi(\gamma \ominus i)) < c_{\text{RP}} \tag{4.167}$$

directly contradicting the first inequality; 3 can never hold. Therefore the three conditions above is equivalent to either one of the following two conditions holds:

1. $a + u(\pi(\gamma) - \pi(\gamma \ominus i')) < c_{\text{RP}}$;
2. $\frac{1}{1+u}[a + u(\pi(\gamma) - \pi(\gamma \ominus i))] + \frac{u}{1+u}(a + u\pi(\gamma \ominus i)) < c_{\text{RP}} \leqslant a + u(\pi(\gamma) - \pi(\gamma \ominus i'))$

which is obviously also equivalent to just

$$\frac{1}{1+u}[a + u(\pi(\gamma) - \pi(\gamma \ominus i))] + \frac{u}{1+u}(a + u\pi(\gamma \ominus i)) < c_{\text{RP}} \tag{4.168}$$

since $a + u(\pi(\gamma) - \pi(\gamma \ominus i)) > a + u\pi(\gamma \ominus i)$. This shows the optimality of $\xi^{**}$ at $\gamma_1$.

**$\xi^{**}$ is optimal at $\gamma_2$** The only case to show is when the LVUS $i \in U_{\gamma_2}$ of $\gamma_2$ satisfies $u(\pi(\gamma_2) - \pi(\gamma_2 \ominus i)) + a < c_{\text{RP}}$. The proof is analogous to case b in the proof of Theorem 4.5 by replacing $\xi^*$ with $\xi^{**}$; here we only point out the differences:

- Since now $\gamma = \gamma_2$ and $i'$ is the shared supplier in $\gamma_2$, $\gamma \ominus i' = \gamma_1$. Hence by the induction hypothesis

$$\xi^{**}(\gamma \ominus i') = \begin{cases} \text{AD}(i), & \text{if } F < c_{\text{RP}}; \\ \text{AR}(i), & \text{if } F \geqslant c_{\text{RP}} \text{ and } a + ur < uwz; \\ \text{PP}, & \text{if } F \geqslant c_{\text{RP}} \text{ and } a + ur \geqslant uwz; \end{cases} \tag{4.169}$$

where $F = \frac{1}{1+u}[a + u(\pi(\gamma \ominus i') - \pi(\gamma \ominus i' \ominus i))] + \frac{u}{1+u}(a + u\pi(\gamma \ominus i' \ominus i))$. We redefine the three subcases b(i), b(ii), and b(iii) in the proof by the three cases for $\xi^{**}(\gamma \ominus i')$ in (4.169) (i.e., replace $a + u(\pi(\gamma \ominus i') - \pi(\gamma \ominus i' \ominus i))$ in the original condition for each subcase by $F$).

- In subcases b(ii) and b(iii), owing to (4.170),

$$\nabla(\gamma_1, i') > \nabla(\gamma_1 \ominus i, i'). \tag{4.170}$$

$F \geqslant c_{\text{RP}}$ implies $a + u(\pi(\gamma \ominus i') - \pi(\gamma \ominus i' \ominus i)) \geqslant c_{\text{RP}}$.

- In subcase b(iii) ($F \geqslant c_{\text{RP}}$ and $a + ur \geqslant uwz$) conditional on $H_{00}$ the path of state transition by taking AD($i$) at state $\gamma$ then following policy $\widehat{\xi}$ is

$$\gamma \xrightarrow{\text{AD}(i)} \gamma \ominus i \tag{4.171}$$

while the path of state transition by taking AD($i'$) at state $\gamma$ then following policy $\xi^{**}$ is

$$\gamma \xrightarrow{\text{AD}(i')} \gamma \ominus i' \xrightarrow{\text{PP}} . \tag{4.172}$$

Here by the definition of $\widehat{\xi}$ and the induction hypothesis one of two actions could be taken subsequent to (4.171) ($\pi(\gamma \ominus i \ominus i') = \frac{1}{64}\frac{(\alpha - v_T)^2}{\beta}$ by Proposition 4.4):

i. If $a + u\pi(\gamma \ominus i \ominus i') = a + u\frac{1}{64}\frac{(\alpha - v_T)^2}{\beta} \geqslant c_{\text{RP}}$, then $\widehat{\xi}(\gamma \ominus i) = \xi^{**}(\gamma \ominus i \ominus i') = \text{PP}$;

ii. If $a + u\pi(\gamma \ominus i \ominus i') = a + u\frac{1}{64}\frac{(\alpha - v_T)^2}{\beta} < c_{\text{RP}}$, then $\widehat{\xi}(\gamma \ominus i) = \xi^{**}(\gamma \ominus i \ominus i') = \text{AD}(i'')$ where $i''$ is the only supplier in $U_{\gamma \ominus i \ominus i'}$.

In sub-subcase 2(c)i the original proof applies. In sub-subcase 2(c)ii following (4.171) the action is AD($i''$) with two possible consequences: that $i''$ passes the audit leading to $\gamma \ominus i \oplus i''$ and that $i''$ fails the audit leading to $\gamma \ominus i \ominus i''$. Note that in either case the definition of $\widehat{\xi}$ prescribes PP afterward. Therefore, the expected profit subsequent to $\gamma \ominus i$ in (4.171) is

$$-a+(1-u)\pi(\gamma\ominus i\oplus i'')+u\pi(\gamma\ominus i\ominus i'')-wz=-a+\left[(1-u)\frac{25}{576}+u\frac{1}{36}\right]\frac{(\alpha-v_T)^2}{\beta}-wz \quad (4.173)$$

where the $-wz$ comes from that in event $H_{00}$ we know $i'$ is non-compliant and the equality is by Proposition 4.4. On the other hand the expected profit subsequent to $\gamma \ominus i'$ in (4.172) is

$$\pi(\gamma \ominus i') - uwz - wz = \frac{1}{25}\frac{(\alpha - v_T)^2}{\beta} - uwz - wz \quad (4.174)$$

where the $-uwz$ is due to $i''$ remaining unaudited, the $-wz$ is due to $i$ being non-compliant, and the equality is by Proposition 4.4. We take the difference between (4.173) and (4.174) to get

$$-a + uwz - u\frac{1}{64}\frac{(\alpha - v_T)^2}{\beta} + \frac{49}{14,400}\frac{(\alpha - v_T)^2}{\beta}. \quad (4.175)$$

But the premise of the sub-subcase is that $a + u\frac{1}{64}\frac{(\alpha - v_T)^2}{\beta} < c_{\text{RP}}$ where $c_{\text{RP}} = uwz$ here, so (4.175) is nonnegative.

Therefore the expected profit at $\gamma$ from first taking AD($i$) then following $\widehat{\xi}$ is greater than or equal to that from first taking AD($i'$) then following $\xi^{**}$ conditional on $H_{00}$. This completes the proof.

□

*Proof of Corollary 4.2.* The result follows directly from Proposition 4.6. □

### 4.C.3 Supplier Choice When Auditing One Firm

Let $\gamma \in \Gamma$ and $i \in U_\gamma$. We define two thresholds for $z$:

$$z_p(\gamma, i) = \frac{u\nabla(\gamma, i) + a}{uw[u(|U_\gamma| - |U_{\gamma\ominus i}| - 1) + 1]} \quad \text{and} \quad z_r(\gamma, i) = \frac{\nabla(\gamma, i) - r}{uw(|U_\gamma| - |U_{\gamma\ominus i}| - 1)}. \quad (4.176)$$

**Proposition 4.12.** *At nonterminal state $\gamma$, suppose the buyer can at most audit (*AD *or* AR*) one supplier, then* PP*. The optimal decision is*

*(a)* PP *if and only if $z \leqslant \frac{a+ur}{uw}$ and $z \leqslant z_p(\gamma, i')$ for every $i' \in U_\gamma$;*

*(b)* $\text{AR}(i)$ *(for any* $i \in U_\gamma$*) if and only if* $z > \frac{a+ur}{uw}$*, and* $z \leqslant z_r(\gamma, i')$ *for every* $i' \in U_\gamma$ *with an unaudited dependent in* $\gamma$ *and* $\nabla(\gamma, i') \geqslant r$ *for every* $i' \in U_\gamma$ *without an unaudited dependent in* $\gamma$*.*

*(c)* $\text{AD}(i)$ *if and only if* $z > z_p(\gamma, i)$*,* $z > z_r(\gamma, i)$ *if* $i$ *has an unaudited dependent in* $\gamma$ *and* $\nabla(\gamma, i) < r$ *if* $i$ *does not have an unaudited dependent in* $\gamma$*, and also* $i$ *satisfies* (4.29)*.*

*Proof of Proposition 4.12.* Let $\xi_{\text{PP}}$ be the policy that maps any state in $\Gamma$ to the action PP. Then for $i \in U_g$,

$$\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i)) = -a + (1-u)V(\xi_{\text{PP}}, \gamma \oplus i) + uV(\xi_{\text{PP}}, \gamma \ominus i) \tag{4.177}$$

$$= -a + (1-u)(\pi(\gamma \oplus i) - uwz|U_{\gamma\oplus i}|) + u(\pi(\gamma \ominus i) - uwz|U_{\gamma\ominus i}|) \tag{4.178}$$

$$= -a + (1-u)[\pi(\gamma) - uwz(|U_\gamma| - 1)] + u(\pi(\gamma \ominus i) - uwz|U_{\gamma\ominus i}|) \tag{4.179}$$

$$\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AR}(i)) = -a - ur + \pi(\gamma \oplus i) - uwz|U_{\gamma\oplus i}| \tag{4.180}$$

$$= -a - ur + \pi(\gamma) - uwz(|U_\gamma| - 1) \tag{4.181}$$

and

$$\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{PP}) = \pi(\gamma) - uwz|U_\gamma|. \tag{4.182}$$

Note that $\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AR}(i))$ is independent of $i$.

Therefore $\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i)) > \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AR}(i'))$ for any $i' \in U_\gamma$ iff

$$-a+(1-u)[\pi(\gamma)-uwz(|U_\gamma|-1)]+u(\pi(\gamma\ominus i)-uwz|U_{\gamma\ominus i}|) > -a-ur+\pi(\gamma)-uwz(|U_\gamma|-1) \tag{4.183}$$

which is equivalent to

$$uwz(|U_\gamma| - |U_{\gamma\ominus i}| - 1) > \pi(\gamma) - \pi(\gamma \ominus i) - r. \tag{4.184}$$

If $i$ has no unaudited dependent, i.e., $D_g(i) \cap U_\gamma = \emptyset$ where $g$ is the supply network in state $\gamma$, then $|U_\gamma| - 1 = |U_{\gamma\ominus i}|$; (4.184) is equivalent to $\pi(\gamma) - \pi(\gamma \ominus i) < r$. Otherwise if $i$ has at least one unaudited dependent, then $|U_\gamma| - 1 > |U_{\gamma\ominus i}|$; (4.184) is equivalent to

$$z > \frac{\pi(\gamma) - \pi(\gamma \ominus i) - r}{uw(|U_\gamma| - |U_{\gamma\ominus i}| - 1)}. \tag{4.185}$$

$\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i)) > \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{PP})$ iff

$$-a + (1-u)[\pi(\gamma) - uwz(|U_\gamma - 1|)] + u(\pi(\gamma \ominus i) - uwz|U_{\gamma\ominus i}|) > \pi(\gamma) - uwz|U_\gamma| \tag{4.186}$$

which is equivalent to

$$z > \frac{u(\pi(\gamma) - \pi(\gamma \ominus i)) + a}{uw[u(|U_\gamma| - |U_{\gamma\ominus i}|) + 1 - u]}. \tag{4.187}$$

$\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i)) \geqslant \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i'))$ for $i' \in U_\gamma$ iff

$$\begin{aligned} &-a + (1-u)[\pi(\gamma) - uwz(|U_\gamma| - 1)] + u(\pi(\gamma \ominus i) - uwz|U_{\gamma\ominus i}|) \\ &\qquad \geqslant -a + (1-u)[\pi(\gamma) - uwz(|U_\gamma| - 1)] + u(\pi(\gamma \ominus i') - uwz|U_{\gamma\ominus i'}|) \end{aligned} \tag{4.188}$$

which is equivalent to

$$\pi(\gamma \ominus i) - uwz|U_{\gamma\ominus i}| \geqslant \pi(\gamma \ominus i') - uwz|U_{\gamma\ominus i'}|. \tag{4.189}$$

$\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AR}(i)) > \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{PP})$ iff

$$-a - ur + \pi(\gamma) - uwz(|U_\gamma| - 1) > \pi(\gamma) - uwz|U_\gamma| \tag{4.190}$$

which is equivalent to $z > \frac{a+ur}{uw}$.

The optimal decision is PP iff $\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{PP}) \geqslant \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i))$ and $\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{PP}) \geqslant \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AR}(i))$ for any $i \in U_\gamma$. This gives part a. The optimal decision is AD$(i)$ iff $\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i)) > \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AR}(i'))$, $\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i)) > \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{PP})$, and $\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i)) \geqslant \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i'))$ for any $i' \in U_\gamma$. This gives part c. The optimal decision is AR$(i)$ iff $\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AR}(i)) \geqslant \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AD}(i'))$ for any $i' \in U_\gamma$ and $\widetilde{V}(\xi_{\text{PP}}, \gamma, \text{AR}(i)) > \widetilde{V}(\xi_{\text{PP}}, \gamma, \text{PP})$. This gives part b.

□

*Proof of Proposition 4.7.* By Proposition 4.12 the optimal decision depends on the ordering of three thresholds for $z$:

$$\frac{a+ur}{uw}, \quad \underline{z}_r(\gamma) = \min\{z_r(\gamma, i) : i \in U_\gamma \text{ with unaudited dependent}\}, \quad \underline{z}_p(\gamma) = \min_{i \in U_\gamma} z_p(\gamma, i). \tag{4.191}$$

In the following we enumerate all but one possible orderings of the three thresholds to verify that they are consistent with the property we describe in Proposition 4.7. We then show the remaining one ordering can never arise. In the following the supplier $i$ in AR($i$) can be any $i \in U_\gamma$ and the supplier $i$ in AD($i$) is given by (4.29).[6] We consider two mutually exclusive and collectively exhaustive cases as follows.

1. First we look at the case that either $\nabla(\gamma, i') < r$ for some $i' \in U_\gamma$ without an unaudited dependent, or $\frac{a+ur}{uw} \geqslant \underline{z}_r(\gamma)$. Then by Proposition 4.12 AR($i$) is never optimal. Therefore the optimal decision is either PP or AD($i$). By Proposition 4.12 the optimal decision is PP if and only if $z \leqslant \left(\frac{a+ur}{uw}\right) \wedge \underline{z}_p(\gamma)$, which implies the optimal decision is AD($i$) if and only if $z > \left(\frac{a+ur}{uw}\right) \wedge \underline{z}_p(\gamma)$. Setting $\underline{z} = \bar{z} = \left(\frac{a+ur}{uw}\right) \wedge \underline{z}_p(\gamma)$ establishes the property Proposition 4.7 describes.

2. Second we look at the case that $\nabla(\gamma, i') \geqslant r$ for every $i' \in U_\gamma$ without an unaudited dependent, and $\frac{a+ur}{uw} < \underline{z}_r(\gamma)$. By Proposition 4.12 the optimal decision is AR($i$) if and only if $\frac{a+ur}{uw} < z \leqslant \underline{z}_r(\gamma)$. Suppose $\frac{a+ur}{uw} \leqslant \underline{z}_p(\gamma)$ then the optimal decision is PP if and only if $z \leqslant \frac{a+ur}{uw}$. So setting $\underline{z} = \frac{a+ur}{uw}$ and $\bar{z} = \underline{z}_r(\gamma)$ will establish the property Proposition 4.7 describes. We only need to show that indeed $\frac{a+ur}{uw} \leqslant \underline{z}_p(\gamma)$ under case 2.

By way of contradiction suppose $\frac{a+ur}{uw} > \underline{z}_p(\gamma)$. It implies that there exists $i'' \in U_\gamma$ such that

$$z_p(\gamma, i'') = \frac{u\nabla(\gamma, i'') + a}{uw[u(|U_\gamma| - |U_{\gamma \ominus i''}| - 1) + 1]} < \frac{a+ur}{uw}. \tag{4.192}$$

- If $i''$ does not have an unaudited dependent in $\gamma$, then $|U_{\gamma \ominus i''}| = |U_\gamma| - 1$; then (4.192) implies $\nabla(\gamma, i'') < r$, contradicting that $\nabla(\gamma, i') \geqslant r$ for every $i' \in U_\gamma$ without an unaudited dependent (in the premise of case 2).

[6] The identity of supplier $i$ may change as $z$ changes.

- If $i''$ has an unaudited dependent, then (4.192) implies

$$\nabla(\gamma, i'') < (a + ur)(|U_\gamma| - |U_{\gamma\ominus i''}| - 1) + r. \tag{4.193}$$

On the other hand, given that $i''$ has an unaudited dependent in $\gamma$, $\frac{a+ur}{uw} \leqslant \underaccent{\tilde}{z}_r(\gamma)$ implies

$$\frac{a + ur}{uw} \leqslant z_r(\gamma, i'') = \frac{\nabla(\gamma, i'') - r}{uw(|U_\gamma| - |U_{\gamma\ominus i''}| - 1)} \tag{4.194}$$

which is equivalent to

$$\nabla(\gamma, i'') - r \geqslant (a + ur)(|U_\gamma| - |U_{\gamma\ominus i''}| - 1) \tag{4.195}$$

contradicting (4.193). Therefore under case 2, $\frac{a+ur}{uw} \leqslant \underaccent{\tilde}{z}_p(\gamma)$. □

We define the following thresholds used in Proposition 4.8:

$$z_{\mathrm{A|B}} = \frac{1}{16}\frac{(\alpha - v_T)^2}{uw\beta}\frac{1}{t_{\mathrm{A}} - t_{\mathrm{B}}}\left[\left(\frac{t_{\mathrm{A}} + t_{\mathrm{AB}}}{t_{\mathrm{A}} + t_{\mathrm{AB}} + 1}\right)^2 - \left(\frac{t_{\mathrm{B}} + t_{\mathrm{AB}}}{t_{\mathrm{B}} + t_{\mathrm{AB}} + 1}\right)^2\right] \tag{4.196}$$

$$z_{\mathrm{B|1}} = \frac{(\alpha - v_T)^2}{uw\beta}\frac{1}{t_{\mathrm{B}}}\left[\frac{1}{9}Y - \frac{1}{16}\left(\frac{t_{\mathrm{A}} + t_{\mathrm{AB}}}{t_{\mathrm{A}} + t_{\mathrm{AB}} + 1}\right)^2\right] \tag{4.197}$$

$$z_{\mathrm{A|1}} = \frac{(\alpha - v_T)^2}{uw\beta}\frac{1}{t_{\mathrm{A}}}\left[\frac{1}{9}Y - \frac{1}{16}\left(\frac{t_{\mathrm{B}} + t_{\mathrm{AB}}}{t_{\mathrm{B}} + t_{\mathrm{AB}} + 1}\right)^2\right] \tag{4.198}$$

where

$$Y = \begin{cases} \left[\dfrac{3(t_{\mathrm{A}} - 1) + 3t_{\mathrm{B}} + 4t_{\mathrm{AB}} + 4(t_{\mathrm{A}} - 1)t_{\mathrm{AB}} + 4t_{\mathrm{B}}t_{\mathrm{AB}} + 3(t_{\mathrm{A}} - 1)t_{\mathrm{B}} + 4t_{\mathrm{AB}}^2}{L(t_{\mathrm{A}} - 1, t_{\mathrm{B}}, t_{\mathrm{AB}})}\right]^2, & \text{if } t_{\mathrm{A}} \leqslant 2t_{\mathrm{B}} + 2t_{\mathrm{AB}} + 2 \\ \left[\dfrac{3(t_{\mathrm{A}} - 1) + 3(t_{\mathrm{B}} + t_{\mathrm{AB}}) + 3(t_{\mathrm{A}} - 1)(t_{\mathrm{B}} + t_{\mathrm{AB}})}{L(t_{\mathrm{A}} - 1, t_{\mathrm{B}} + t_{\mathrm{AB}}, 0)}\right]^2, & \text{if } t_{\mathrm{A}} > 2t_{\mathrm{B}} + 2t_{\mathrm{AB}} + 2 \end{cases}. \tag{4.199}$$

*Proof of Proposition 4.8.* By the proof of Proposition 4.12, let $i, i' \in U_\gamma$, then $\widetilde{V}(\xi_{\mathrm{PP}}, \gamma, \mathrm{AD}(i)) \geqslant \widetilde{V}(\xi_{\mathrm{PP}}, \gamma, \mathrm{AD}(i'))$ iff

$$\pi(\gamma \ominus i) - uwz|U_{\gamma\ominus i}| \geqslant \pi(\gamma \ominus i') - uwz|U_{\gamma\ominus i'}|. \tag{4.200}$$

Since no tier-2 supplier has a dependent in $\gamma$, (4.200) implies the buyer should prefer among tier-2 suppliers to AD a supplier $i$ with the highest $\pi(\gamma \ominus i)$. By Proposition 4.6 this supplier is a majority-exclusive supplier. Hence we only need to compare the majority-exclusive supplier 1, firm A, and firm B.

By the structure of supply network $g$,

$$|U_{\gamma\ominus\text{A}}| = |U_\gamma| - t_\text{A} - 1 \tag{4.201}$$

$$|U_{\gamma\ominus\text{B}}| = |U_\gamma| - t_\text{B} - 1 \tag{4.202}$$

$$|U_{\gamma\ominus 1}| = |U_\gamma| - 1. \tag{4.203}$$

By Proposition 4.4,

$$\pi(\gamma \ominus \text{A}) = \frac{1}{16}\frac{(\alpha - v_T)^2}{\beta}\left(\frac{t_\text{B} + t_\text{AB}}{t_\text{B} + t_\text{AB} + 1}\right)^2 \tag{4.204}$$

$$\pi(\gamma \ominus \text{B}) = \frac{1}{16}\frac{(\alpha - v_T)^2}{\beta}\left(\frac{t_\text{A} + t_\text{AB}}{t_\text{A} + t_\text{AB} + 1}\right)^2 \tag{4.205}$$

$$\pi(\gamma \ominus 1) = \frac{1}{9}\frac{(\alpha - v_T)^2}{\beta}Y. \tag{4.206}$$

One may verify that the buyer prefers AD(A) to AD(B) if and only if $z \geqslant z_{\text{A}|\text{B}}$, the buyer prefers AD(B) to AD(1) if and only if $z \geqslant z_{\text{B}|1}$, and the buyer prefers AD(A) to AD(1) if and only if $z \geqslant z_{\text{A}|1}$, by plugging (4.201)–(4.206) into (4.200). By Proposition 4.7 when $z > \widehat{z}$ the optimal decision is to AD some supplier. Proposition 4.8 now follows. □

## Appendix 4.D Technical Lemmas

**Lemma 4.1** (determinant of upper arrowhead matrix)**.** *For $n \in \mathbb{N}_+$ and $n \geqslant 2$, and $a, b, c, d \in \mathbb{R}$, then*

$$\det \begin{pmatrix} a & b & b & \cdots & b \\ c & d & 0 & \cdots & 0 \\ c & 0 & d & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ c & 0 & \cdots & 0 & d \end{pmatrix}_{n\times n} = [ad - (n-1)bc]d^{n-2}. \tag{4.207}$$

*Proof.* Denote the matrix in (4.207) by $\Psi$. If $d = 0$ we expanding $\Psi$ along the first column in the way of Laplace to find $\det(\Psi) = 0$ (each submatrix in the expansion has zero determinant). If $d > 0$

we multiply columns 2 to $n$ each by $-\frac{c}{d}$ and add them all to the first column to get

$$\det(\Psi) = \det \begin{pmatrix} a-(n-1)\frac{bc}{d} & b & b & \cdots & b \\ 0 & d & 0 & \cdots & 0 \\ \vdots & \ddots & d & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & d \end{pmatrix} \tag{4.208}$$

which, by the formula for the determinant of a diagonal matrix, is equal to the right-hand side of (4.207). □

For $m, n, k \in \mathbb{N}_0$, denote by $J_{m,n} = [1]_{m \times n}$ the $m \times n$ matrix of 1's; $J_n = J_{n,n}$. Denote

$$D_n = I_n + J_n = \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \cdots & 1 & 2 \end{bmatrix}_{n\times n} \quad Q_{2m,n} = \begin{bmatrix} 1 & \cdots & 1 \\ 2 & \cdots & 2 \\ \vdots & \vdots & \vdots \\ 1 & \cdots & 1 \\ 2 & \cdots & 2 \\ \vdots & \vdots & \vdots \\ 1 & \cdots & 1 \\ 2 & \cdots & 2 \end{bmatrix}_{2m\times n} \quad R_{2m,n} = \begin{bmatrix} 2 & \cdots & 2 \\ 1 & \cdots & 1 \\ \vdots & \vdots & \vdots \\ 2 & \cdots & 2 \\ 1 & \cdots & 1 \\ \vdots & \vdots & \vdots \\ 2 & \cdots & 2 \\ 1 & \cdots & 1 \end{bmatrix}_{2m\times n}. \tag{4.209}$$

Denote $\widetilde{D}_{2n} = D_n \otimes D_2$, where $\otimes$ denotes the Kronecker product. Finally, define the two $(m+2k+n) \times (m+2k+n)$ symmetric matrices

$$\Lambda(m,n,k) = \begin{bmatrix} 2D_m & R_{2k,m}^{\mathsf{T}} & J_{m,n} \\ R_{2k,m} & \widetilde{D}_{2k} & Q_{2k,n} \\ J_{n,m} & Q_{2k,n}^{\mathsf{T}} & 2D_n \end{bmatrix}, \quad \widetilde{\Lambda}(m,n,k) = \begin{bmatrix} 2D_{m+k} & \begin{bmatrix} J_{m,k} & J_{m,n} \\ D_k & J_{k,n} \end{bmatrix} \\ \begin{bmatrix} J_{k,m} & D_k \\ J_{n,m} & J_{n,k} \end{bmatrix} & 2D_{k+n} \end{bmatrix}. \tag{4.210}$$

**Lemma 4.2.** *For $m, n, k$ such that $m+n+k > 0$, $\widetilde{\Lambda}(m,n,k)$ is positive definite.*

*Proof.* Denote $m' = m + k$ and $n' = n + k$. Denote

$$B = \begin{bmatrix} J_{m,k} & J_{m,n} \\ D_k & J_{k,n} \end{bmatrix} \text{ so that } \widetilde{\Lambda} = \begin{bmatrix} 2D_{m'} & B \\ B^\mathsf{T} & 2D_{n'} \end{bmatrix}. \tag{4.211}$$

The proof consists of two parts:

1. $2D_{m'}$ is positive definite;

2. The Schur complement of $2D_{m'}$ in $\widetilde{\Lambda}$, i.e., $\widetilde{\Lambda}/(2D_{m'}) = 2D_{n'} - B^\mathsf{T}(2D_{m'})^{-1}B$, is positive definite.

Then by the Schur complement condition (Boyd and Vandenberghe 2004, Appendix A.5.5), $\widetilde{\Lambda}$ is positive definite, as we want.

**1** To show that $2D_{m'}$ is positive definite, it suffices to show that every leading principal minor of $D_{m'}$ is positive. For $i \in \mathbb{N}_+$, subtract the first row of $D_i$ from every other row to get

$$\det(D_i) = \det \begin{pmatrix} 2 & 1 & \cdots & \cdots & 1 \\ -1 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -1 & 0 & \cdots & 0 & 1 \end{pmatrix} \tag{4.212}$$

which by Lemma 4.1 equals $(2 \times 1 - (i-1) \times 1 \times (-1)) \times 1^{i-2} = i + 1 > 0$. Therefore $2D_{m+k}$ is positive definite.

**2** We next show that $2D_{n'} - B^\mathsf{T}(2D_{m'})^{-1}B$ is positive definite. Use row reduction to find

$$(2D_{m'})^{-1} = \frac{1}{2(m'+1)} \begin{bmatrix} m' & -1 & \cdots & -1 \\ -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ -1 & \cdots & -1 & m' \end{bmatrix}. \tag{4.213}$$

Then

$$B^{\mathsf{T}}(2D_{m'})^{-1}B = \begin{bmatrix} J_{k,m} & D_k \\ J_{n,m} & J_{n,k} \end{bmatrix} \left( \frac{1}{2(m'+1)} \begin{bmatrix} m' & -1 & \cdots & -1 \\ -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ -1 & \cdots & -1 & m' \end{bmatrix} \right) \begin{bmatrix} J_{m,k} & J_{m,n} \\ D_k & J_{k,n} \end{bmatrix} \tag{4.214}$$

$$= \frac{1}{2(m'+1)} \underbrace{\begin{bmatrix} 1 & \cdots & 1 & 2 & 1 & \cdots & 1 \\ \vdots & & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & \ddots & 1 \\ \vdots & & & & & \ddots & 2 \\ \vdots & & & & & & 1 \\ \vdots & & & & & & \vdots \\ 1 & \cdots & \cdots & \cdots & \cdots & \cdots & 1 \end{bmatrix}}_{n' \times m'} \underbrace{\begin{bmatrix} m' & -1 & \cdots & -1 \\ -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ -1 & \cdots & -1 & m' \end{bmatrix}}_{m' \times m'} \begin{bmatrix} J_{m,k} & J_{m,n} \\ D_k & J_{k,n} \end{bmatrix} \tag{4.215}$$

$$= \frac{1}{2(m'+1)} \underbrace{\begin{bmatrix} 0 & \cdots & 0 & m'+1 & 0 & \cdots & 0 \\ \vdots & & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & m'+1 \\ 1 & \cdots & \cdots & \cdots & \cdots & \cdots & 1 \\ \vdots & & & & & & \vdots \\ 1 & \cdots & \cdots & \cdots & \cdots & \cdots & 1 \end{bmatrix}}_{n' \times m'} \times$$

$$\underbrace{\begin{bmatrix} 1 & \cdots & \cdots & \cdots & \cdots & \cdots & 1 \\ \vdots & & & & & & \vdots \\ 1 & & & & & & \vdots \\ 2 & \ddots & & & & & \vdots \\ 1 & \ddots & \ddots & & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & & & \vdots \\ 1 & \cdots & 1 & 2 & 1 & \cdots & 1 \end{bmatrix}}_{m' \times n'} \tag{4.216}$$

$$
= \frac{1}{2(m'+1)} \underbrace{\begin{bmatrix}
2(m'+1) & m'+1 & \cdots & \cdots & m'+1 & m'+1 & \cdots & m'+1 \\
m'+1 & \ddots & \ddots & & \vdots & \vdots & & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots & \vdots & & \vdots \\
\vdots & & \ddots & \ddots & m'+1 & \vdots & & \vdots \\
m'+1 & \cdots & \cdots & m'+1 & 2(m'+1) & m'+1 & \cdots & m'+1 \\
m'+1 & \cdots & \cdots & \cdots & m'+1 & m' & \cdots & m' \\
\vdots & & & & \vdots & \vdots & & \vdots \\
m'+1 & \cdots & \cdots & \cdots & m'+1 & m' & \cdots & m'
\end{bmatrix}}_{n' \times n'} \tag{4.217}
$$

$$
= \underbrace{\begin{bmatrix}
1 & \frac{1}{2} & \cdots & \cdots & \frac{1}{2} & \frac{1}{2} & \cdots & \frac{1}{2} \\
\frac{1}{2} & \ddots & \ddots & & \vdots & \vdots & & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots & \vdots & & \vdots \\
\vdots & & \ddots & \ddots & \frac{1}{2} & \vdots & & \vdots \\
\frac{1}{2} & \cdots & \cdots & \frac{1}{2} & 1 & \frac{1}{2} & \cdots & \frac{1}{2} \\
\frac{1}{2} & \cdots & \cdots & \cdots & \frac{1}{2} & \frac{m'}{2(m'+1)} & \cdots & \frac{m'}{2(m'+1)} \\
\vdots & & & & \vdots & \vdots & & \vdots \\
\frac{1}{2} & \cdots & \cdots & \cdots & \frac{1}{2} & \frac{m'}{2(m'+1)} & \cdots & \frac{m'}{2(m'+1)}
\end{bmatrix}}_{n' \times n'} . \tag{4.218}
$$

Therefore

$$2D_{n'} - B^{\mathsf{T}}(2D_{m'})^{-1}B = \underbrace{\begin{bmatrix} 3 & \frac{3}{2} & \cdots & \cdots & \frac{3}{2} & \frac{3}{2} & \cdots & \cdots & \frac{3}{2} \\ \frac{3}{2} & \ddots & \ddots & & \vdots & \vdots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & & & \vdots \\ \vdots & & \ddots & \ddots & \frac{3}{2} & \vdots & & & \vdots \\ \frac{3}{2} & \cdots & \cdots & \frac{3}{2} & 3 & \frac{3}{2} & \cdots & \cdots & \frac{3}{2} \\ \frac{3}{2} & \cdots & \cdots & \cdots & \frac{3}{2} & \frac{7m'+8}{2(m'+1)} & \frac{3m'+4}{2(m'+1)} & \cdots & \frac{3m'+4}{2(m'+1)} \\ \vdots & & & & \vdots & \frac{3m'+4}{2(m'+1)} & \ddots & \ddots & \vdots \\ \vdots & & & & \vdots & \vdots & \ddots & \ddots & \frac{3m'+4}{2(m'+1)} \\ \frac{3}{2} & \cdots & \cdots & \cdots & \frac{3}{2} & \frac{3m'+4}{2(m'+1)} & \cdots & \frac{3m'+4}{2(m'+1)} & \frac{7m'+8}{2(m'+1)} \end{bmatrix}}_{n' \times n'}. \quad (4.219)$$

It only remains to show that (4.219) is positive definite. We conduct the following row operations on the matrix so that each newly formed row $i$ only references the rows $j \leqslant i$ in (4.219), ensuring that the leading principal minors are preserved:

$$
(4.219) \xrightarrow[\forall i \geqslant 2]{R_i - R_1}
\begin{bmatrix}
3 & \frac{3}{2} & \cdots & \cdots & \frac{3}{2} & \frac{3}{2} & \cdots & \cdots & \frac{3}{2} \\
-\frac{3}{2} & \frac{3}{2} & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 \\
\vdots & 0 & \ddots & \ddots & \vdots & \vdots & & & \vdots \\
\vdots & \vdots & \ddots & \ddots & 0 & \vdots & & & \vdots \\
-\frac{3}{2} & 0 & \cdots & 0 & \frac{3}{2} & 0 & \cdots & \cdots & 0 \\
-\frac{3}{2} & 0 & \cdots & \cdots & 0 & \frac{4m'+5}{2(m'+1)} & \frac{1}{2(m'+1)} & \cdots & \frac{1}{2(m'+1)} \\
\vdots & \vdots & & & \vdots & \frac{1}{2(m'+1)} & \ddots & \ddots & \vdots \\
\vdots & \vdots & & & \vdots & \vdots & \ddots & \ddots & \frac{1}{2(m'+1)} \\
-\frac{3}{2} & 0 & \cdots & \cdots & 0 & \frac{1}{2(m'+1)} & \cdots & \frac{1}{2(m'+1)} & \frac{4m'+5}{2(m'+1)}
\end{bmatrix} \tag{4.220}
$$

$$
\xrightarrow[\forall i \in [k+2, k+n]]{R_i - R_{k+1}}
\begin{bmatrix}
3 & \frac{3}{2} & \cdots & \cdots & \frac{3}{2} & \frac{3}{2} & \cdots & \cdots & \cdots & \frac{3}{2} \\
-\frac{3}{2} & \frac{3}{2} & 0 & \cdots & 0 & 0 & \cdots & \cdots & \cdots & 0 \\
\vdots & 0 & \ddots & \ddots & \vdots & \vdots & & & & \vdots \\
\vdots & \vdots & \ddots & \ddots & 0 & \vdots & & & & \vdots \\
-\frac{3}{2} & 0 & \cdots & 0 & \frac{3}{2} & 0 & \cdots & \cdots & \cdots & 0 \\
-\frac{3}{2} & 0 & \cdots & \cdots & 0 & \frac{4m'+5}{2(m'+1)} & \frac{1}{2(m'+1)} & \cdots & \cdots & \frac{1}{2(m'+1)} \\
0 & \vdots & & & \vdots & -2 & 2 & 0 & \cdots & 0 \\
\vdots & \vdots & & & \vdots & \vdots & 0 & \ddots & \ddots & \vdots \\
\vdots & \vdots & & & \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\
0 & 0 & \cdots & \cdots & 0 & -2 & 0 & \cdots & 0 & 2
\end{bmatrix}. \tag{4.221}
$$

To show that (4.219) is positive definite, it suffices to show that every leading principal minor $\det(M_i)$ of (4.221) is positive.

For $i = 1, \ldots, k$, by Lemma 4.1,

$$
\det(M_i) = \left(3 \times \frac{3}{2} - (i-1) \times \frac{3}{2} \times (-\frac{3}{2})\right) \times \left(\frac{3}{2}\right)^{i-2} = \left(\frac{3}{2}\right)^{i} (i+1) > 0. \tag{4.222}
$$

For $i = k+1$, (i) expanding

$$M_{k+1} = \begin{bmatrix} 3 & \frac{3}{2} & \cdots & \cdots & \frac{3}{2} & \frac{3}{2} \\ -\frac{3}{2} & \frac{3}{2} & 0 & \cdots & 0 & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & \vdots \\ -\frac{3}{2} & 0 & \cdots & 0 & \frac{3}{2} & 0 \\ -\frac{3}{2} & 0 & \cdots & \cdots & 0 & \frac{4m'+5}{2(m'+1)} \end{bmatrix} \tag{4.223}$$

in the way of Laplace along the last row, then (ii) expanding the $(k+1,1)$th minor of $M_{k+1}$ along the last column and (iii) applying Lemma 4.1 on the $(k+1,k+1)$th minor of $M_{k+1}$, we get

$$\begin{aligned} \det(M_{k+1}) = & (-1)^{k+2}\left(-\frac{3}{2}\right)(-1)^{(k+1)}\frac{3}{2}\left(\frac{3}{2}\right)^{k-1} \\ & + (-1)^{2(k+1)}\frac{4m'+5}{2(m'+1)}\left[(3)\left(\frac{3}{2}\right) - (k-1)\left(\frac{3}{2}\right)\left(-\frac{3}{2}\right)\right]\left(\frac{3}{2}\right)^{k-2}. \end{aligned} \tag{4.224}$$

It simplifies to

$$\det(M_{k+1}) = \left(\frac{3}{2}\right)^{k+1} + \frac{4m'+5}{2(m'+1)}(k+1)\left(\frac{3}{2}\right)^{k} > 0. \tag{4.225}$$

For $i = k+2, \ldots, k+n$, expand $M_i$ along the $(k+1)$th row:

$$\begin{aligned} \det(M_i) = & (-1)^{k+2}\left(-\frac{3}{2}\right)\det(S_1) + (-1)^{2(k+1)}\frac{4m'+5}{2(m'+1)}\det(S_{k+1}) \\ & + \frac{1}{2(m'+1)}\sum_{j=k+2}^{i}(-1)^{k+1+j}\det(S_j) \end{aligned} \tag{4.226}$$

where $S_j$ is the submatrix of $M_i$ formed by deleting row $k+1$ and column $j$. We next evaluate the $\det(S_j)$.

(Evaluate $\det(S_1)$). Denote $S_1^{(l)}$ for $l = 1, \ldots, k-1$ the $(i-1-l) \times (i-1-l)$ submatrix of $S_1$ obtained by deleting rows $2, \ldots, l+1$ and columns with the same indices. In addition, let $S_1^{(0)} = S_1$.

That is

$$S_1^{(l)} = \begin{bmatrix} \frac{3}{2} & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{3}{2} \\ \frac{3}{2} & & & & & & \\ & \ddots & & & & & \\ & & \frac{3}{2} & & & & \\ & & & -2 & 2 & & \\ & & & \vdots & & \ddots & \\ & & & -2 & & & 2 \end{bmatrix} \tag{4.227}$$

where empty space represents zeros and there are $k-1-l$ entries of $\frac{3}{2}$ on the subdiagonal and $i-k-1$ entries of 2 on the diagonal. In particular, there is no $\frac{3}{2}$ on the subdiagonal of $S_1^{(k-1)}$ since all of them are deleted. Then expand $S_1^{(l)}$ successively along the second row:

$$\det(S_1) = \det(S_1^{(0)}) = -\frac{3}{2}\det(S_1^{(1)}) = \cdots = \left(-\frac{3}{2}\right)^l \det(S_1^{(l)}) = \cdots = \left(-\frac{3}{2}\right)^{k-1} \det(S_1^{(k-1)}) \tag{4.228}$$

$$= \left(-\frac{3}{2}\right)^{k-1} \left[\left(\frac{3}{2}\right)(2) - (i-k-1)\left(\frac{3}{2}\right)(-2)\right](2)^{i-k-2} = (-1)^{k-1}\left(\frac{3}{2}\right)^k (i-k)(2)^{i-k-1}. \tag{4.229}$$

(Evaluate $\det(S_{k+1})$).

$$S_{k+1} = \begin{bmatrix} 3 & \frac{3}{2} & \cdots & \cdots & \frac{3}{2} & \frac{3}{2} & \cdots & \cdots & \frac{3}{2} \\ -\frac{3}{2} & \frac{3}{2} & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots & \vdots & & & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & \vdots & & & \vdots \\ -\frac{3}{2} & 0 & \cdots & 0 & \frac{3}{2} & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 2 & 0 & \cdots & 0 \\ \vdots & & & & \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & & & & \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 0 & 2 \end{bmatrix}_{(i-1)\times(i-1)} \equiv \begin{bmatrix} A_{k\times k} & E_{k\times(i-k-1)} \\ 0_{(i-k-1)\times k} & 2I_{i-k-1} \end{bmatrix}. \tag{4.230}$$

The submatrix $A$ in the upper-left block is an arrowhead matrix; by Lemma 4.1,

$$\det(A) = \left[(3)\left(\frac{3}{2}\right) - (k-1)\left(\frac{3}{2}\right)\left(-\frac{3}{2}\right)\right]\left(\frac{3}{2}\right)^{k-2} = (k+1)\left(\frac{3}{2}\right)^{k}. \tag{4.231}$$

Observing that the lower-left block of the matrix is a submatrix of zeros, we find

$$\det(S_{k+1}) = \det(A)\det(2I_{i-k-1}) = (k+1)\left(\frac{3}{2}\right)^{k} 2^{i-k-1}. \tag{4.232}$$

(Evaluate $\det(S_j), \forall j > k+1$). For $j > k+1$,

$$S_j = \begin{bmatrix} A_{k\times k} & E_{k\times(i-k-1)} \\ 0_{(i-k-1)\times k} & Y_j \end{bmatrix}_{(i-1)\times(i-1)} \tag{4.233}$$

where $A$ and $E$ are the same as in (4.230) and

$$Y_j = \begin{bmatrix} -2 & 2 & & & & & \\ \vdots & & \ddots & & & & \\ \vdots & & & 2 & & & \\ \vdots & & & 0 & 0 & & \\ \vdots & & & & 2 & & \\ \vdots & & & & & \ddots & \\ -2 & & & & & & 2 \end{bmatrix}_{(i-k-1)\times(i-k-1)} \tag{4.234}$$

where, in particular, the $(j-k+1)$th row is $(-2, 0, \ldots, 0)$. Expand its determinant along this row:

$$\det(Y_j) = (-1)^{j-k+2}(-2)\det(2I_{i-k-2}) = (-1)^{j-k+3}(2^{i-k-1}). \tag{4.235}$$

Since the lower-left submatrix of $S_j$ is zero, we have

$$\det(S_j) = \det(A)\det(Y_j) = (-1)^{j-k+3}(k+1)\left(\frac{3}{2}\right)^{k}(2^{i-k-1}) \tag{4.236}$$

where (4.231) is substituted for $\det(A)$.

Substitute (4.229), (4.232), and (4.236) into (4.226) to get

$$\det(M_i) = \left(\frac{3}{2}\right)^{k+1} (i-k)(2)^{i-k-1} + \frac{4m'+5}{2(m'+1)}(k+1)\left(\frac{3}{2}\right)^{k} 2^{i-k-1} + \frac{1}{2(m'+1)} \sum_{j=k+2}^{i} (k+1)\left(\frac{3}{2}\right)^{k} (2^{i-k-1}) > 0. \quad (4.237)$$

Therefore (4.221) is positive definite. The proof is complete. □

**Lemma 4.3.** *For $m, n, k$ such that $m+n+k > 0$, $\Lambda(m,n,k)$ is positive definite.*

*Proof.* In the following we omit the arguments $(m,n,k)$ of the matrices $\Lambda$ and $\widetilde{\Lambda}$ as doing so causes no confusion.

Rewrite $\widetilde{\Lambda}$ in (4.210) by breaking up its upper-left and lower-right blocks as

$$\widetilde{\Lambda}(m,n,k) = \begin{bmatrix} 2D_m & 2J_{m,k} & J_{m,k} & J_{m,n} \\ 2J_{k,m} & 2D_k & D_k & J_{k,n} \\ J_{k,m} & D_k & 2D_k & 2J_{k,n} \\ J_{n,m} & J_{n,k} & 2J_{n,k} & 2D_n \end{bmatrix}_{(m+2k+n)\times(m+2k+n)}. \quad (4.238)$$

Now the blocks in the four corners of $\widetilde{\Lambda}$ are identical to the corresponding blocks in $\Lambda$. The idea is to permute the rows and columns of $\Lambda$ in between the corner blocks symmetrically to obtain $\widetilde{\Lambda}$. Specifically, let $P$ be the $(m+2k+n)\times(m+2k+n)$ permutation matrix that leaves rows $1,\ldots,m$ and $m+2k+1,\ldots,m+2k+n$ intact and permutes rows $m+1,\ldots,m+2k$ according to the permutation of the set $\{1,\ldots,2k\}$ (where element $l$ corresponds to row $m+l$) as follows (in Cauchy's two-line notation):

$$\begin{pmatrix} 1 & 2 & \cdots & i & i+1 & \cdots & k-1 & k & k+1 & k+2 & \cdots & j & j+1 & \cdots & 2k-1 & 2k \\ 1 & k+1 & \cdots & i & k+i & \cdots & k-1 & 2k-1 & 2 & k+2 & \cdots & j-k+1 & j+1 & \cdots & k & 2k \end{pmatrix} \quad (4.239)$$

where $i$ is any odd number between 1 and $k-1$ and $j$ is any odd number between $k+1$ and $2k-1$

(all ends inclusive), for $k$ even, and

$$\begin{pmatrix} 1 & 2 & \cdots & i & i+1 & \cdots & k-2 & k-1 & k & k+1 & k+2 & \cdots & j & j+1 & \cdots & 2k-2 & 2k-1 & 2k \\ 1 & k+2 & \cdots & i & k+i+1 & \cdots & k-2 & 2k-1 & k & 2 & k+3 & \cdots & j-k+1 & j+2 & \cdots & k-1 & 2k & k+1 \end{pmatrix} \tag{4.240}$$

where $i$ is any odd number between 1 and $k-2$ and $j$ is any even number between $k+1$ and $2k-2$ (all ends inclusive), for $k$ odd. Then $\widetilde{\Lambda} = P\Lambda P^{\mathsf{T}}$.

By Lemma 4.2, $\widetilde{\Lambda}$ is positive definite. By Theorem 6C(V) in Strang (1980), there exists an invertible matrix $Q$ such that $\widetilde{\Lambda} = Q^{\mathsf{T}}Q$ . Since $P$ is a permutation matrix, the matrix $Q(P^{-1})^{\mathsf{T}}$ is invertible. Therefore, again by Theorem 6C(V) in Strang (1980), $(Q(P^{-1})^{\mathsf{T}})^{\mathsf{T}}Q(P^{-1})^{\mathsf{T}}$ is positive definite. But since $\widetilde{\Lambda} = P\Lambda P^{\mathsf{T}}$,

$$(Q(P^{-1})^{\mathsf{T}})^{\mathsf{T}}Q(P^{-1})^{\mathsf{T}} = P^{-1}Q^{\mathsf{T}}Q(P^{-1})^{\mathsf{T}} = P^{-1}\widetilde{\Lambda}(P^{\mathsf{T}})^{-1} = P^{-1}P\Lambda P^{\mathsf{T}}(P^{\mathsf{T}})^{-1} = \Lambda \tag{4.241}$$

implying $\Lambda$ is positive definite. □

# 5 Conclusion

With extensive and interconnected supply chains, companies, regulators, and NGOs must account for the structure of supply networks when working toward their social responsibility goals. I study the effect of the structure of supply networks on social responsibility outcomes from three perspectives.

In chapter 2 we examine the supply network of conflict minerals that consists of manufacturers, smelters, and mines to understand the effects of legislative and NGO efforts to curb the use of conflict sources. Each manufacturer and smelter makes its compliance and sourcing decisions, together determining the equilibrium. We find that an important predictor of outcomes is the "compliance-prone demand" in the network: the total demand of the manufacturers whose penalty for noncompliance exceeds their cost of compliance. If the compliance-prone demand is below a threshold, then there will be enough certified metal to go around. However, once the compliance-prone demand exceeds the threshold, a shortage of certified metal will arise, forcing some compliant manufacturers to use noncertified metal to make up for the shortage. The results imply that even if policy-makers (NGOs and legislative bodies) imposed increasingly high noncompliance penalties on a broader group of manufacturers, such penalties might not be sufficient to eliminate the use of conflict sources. Once the compliance-prone demand reaches a threshold, further increases in the compliance-prone demand will be countered with less than commensurate increases in certified metal capacity, leading to shortages. We also show that when the certified metal is in shortage the NGO should target the manufacturers whose noncompliance penalty, spread over the volume of the metal they use, is high enough to justify paying the premium for certified metal.

Inspired by how a group of companies launched the Initial Audit Fund to finance the auditing of smelters for conflict minerals, in chapter 3 we study the formation of an industrial alliance to fund a social responsibility project that benefits a group of companies. We model the problem as a public goods game that incorporates two features we observe of the Initial Audit Fund: an invitation stage when an initiating alliance can be formed, and the different brand values of the companies. We conduct laboratory experiments to test the predictions of our model. Both our theoretical model and experiments confirm that forming an initiating alliance is key to reaching overall high contribution. While all initiating members of the Initial Audit Fund had high brand values, our results suggest that a low-brand-value company can also start an alliance that will be equally effective in inducing contribution. To increase the chances of forming an alliance the low-brand-value company should approach some high-brand-value companies, who are more likely to agree to jointly initiate the social responsibility project.

In chapter 4 we focus on a single buyer that dynamically audits a decentralized supply network to assure social responsibility compliance. The buyer picks the suppliers to audit, and if an audit uncovers a violation, decides on the remedial action taken: to rectify the noncompliant supplier or to drop it along with its dependent suppliers. Our results show that oftentimes it is not optimal for a buyer to exhaustively audit all suppliers. Strategic considerations direct the choice of the suppliers to audit. Each of the two types of remedial action can be optimal depending on the properties of the supply network, the stage in the auditing process, and the supplier audited. Our results suggest that a buyer benefits by prioritizing auditing suppliers it is prepared to drop and postponing auditing suppliers that it is willing to rectify. When focusing on an upstream tier the buyer may want to first audit suppliers in more peripheral positions, which carry less weight in the buyer's profit from production activities. Companies may also benefit by auditing suppliers in pivotal positions to gauge the viability of the network. Such a "litmus test" may even compel the shutdown of an entire product line (a "kill mission"), a predicament some companies caught off guard with a supplier violation had to undergo (e.g., Phillips 2016).

Supply networks create unique challenges for social responsibility efforts. This dissertation constitutes a first step in understanding how the structure of supply networks affects social responsibility outcomes through corporate and regulatory decisions. It provides frameworks for understanding the implications of supply network structure on social responsibility from many more perspectives, e.g., competition, visibility, and compliance certification.

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Zhang H, Aydin G, Heese HS (2017) Curbing the usage of conflict minerals: A supply network perspective, Working paper.

Zhang H, Aydin G, Parker RP (2019a) Social responsibility auditing in supply chain networks, Working paper.

Zhang H, Beer R, Cattani K (2019b) Building alliances for corporate social responsibility, Working paper.

# Han Zhang

Indiana University
Kelley School of Business
Department of Operations and Decision Technologies
1309 E 10th St Ste 4100, Bloomington, IN 47405, U.S.A.
hz8@iu.edu
http://pages.iu.edu/~hz8/

## Education

Ph.D., Business, with a concentration in Operations Management, Kelley School of Business, Indiana University Bloomington March 2020

Dissertation: "Social Responsibility in Supply Networks"

Dissertation committee: Goker Aydin (co-chair), Rodney P. Parker (co-chair), Ruth Beer, Kyle D. Cattani, Daniela Puzzello

M.Bus., Operations Management, Kelley School of Business, Indiana University Bloomington 2016

M.A., Mathematics, Indiana University Bloomington 2015

M.A., Economics, Indiana University Bloomington 2013

Passed Ph.D. in Economics qualification exams

Primary field (tested): Advanced Economic Theory (Game Theory)

B.B.A. (First Class Honors), Applied Economics, and Applied Mathematics (*minor*), Hong Kong Baptist University 2011

Rank in program: 1st

Scholastic Award

## Research

### Papers under Review or Revision

Han Zhang, Goker Aydin, and Rodney P. Parker. "Social responsibility auditing in supply chain networks." Under Major Revision at *Management Science*.

Han Zhang, Ruth Beer, and Kyle D. Cattani. "Building alliances for corporate social responsibility." Under revision.

Han Zhang, Goker Aydin, and H. Sebastian Heese. "Curbing the usage of conflict minerals: A supply network perspective." Under revision.

### Work in Progress

Han Zhang and Rodney P. Parker. "Cournot competition in supply networks."

# Presentations

## Invited Seminars

Broad College of Business, Michigan State University, "Social responsibility auditing in supply chain networks," 2019.

School of Business, The University of Kansas, "Social responsibility auditing in supply chain networks," 2019.

Business School, The University of Auckland, "Social responsibility auditing in supply chain networks," 2019 (via Zoom).

Congdon School of Supply Chain, Business Analytics, and Information Systems, Cameron School of Business, University of North Carolina Wilmington, "Social responsibility auditing in supply chain networks," 2019.

Department of Economics, Indiana University Bloomington, "Building alliances for corporate social responsibility," 2018.

## Conference Presentations

"Social responsibility auditing in supply chain networks"

- INFORMS Annual Meeting, Seattle, WA, U.S.A., 2019.
- MSOM Conference, Singapore, 2019.
- POMS Annual Conference, Washington, DC, U.S.A., 2019.
- INFORMS Annual Meeting, Phoenix, AZ, U.S.A., 2018.
- POMS Annual Conference, Houston, TX, U.S.A., 2018.
- INFORMS Annual Meeting, Houston, TX, U.S.A., 2017.

"Building alliances for corporate social responsibility"

- POMS Annual Conference, Washington, DC, U.S.A., 2019.
- INFORMS Annual Meeting, Phoenix, AZ, U.S.A., 2018.
- MSOM Conference, Dallas, TX, U.S.A., 2018.
- POMS Annual Conference, Houston, TX, U.S.A., 2018.

"Curbing the usage of conflict minerals: A supply network perspective"

- POMS Annual Conference, Houston, TX, U.S.A., 2018.
- INFROMS Annual Meeting, Houston, TX, U.S.A., 2017.
- POMS Annual Conference, Seattle, WA, U.S.A., 2017.
- INFORMS Annual Meeting, Nashville, TN, U.S.A., 2016.
- MSOM Conference, Auckland, New Zealand, 2016.
- POMS Annual Conference, Orlando, FL, U.S.A., 2016.

(Each paper presented under various titles.)

# Service

### Conference Sessions Organized

Session on "Socially responsible supply chains: Auditing and competition," INFORMS Annual Meeting, Seattle, WA, U.S.A., 2019.

Session on "Auditing, compliance, and incentives in socially responsible operations," POMS Annual Conference, Houston, TX, U.S.A., 2018.

Session on "Socially responsible supply chain management for natural resources," POMS Annual Conference, Seattle, WA, U.S.A., 2017.

### Referee Experience

*Production and Operations Management*, *Decision Sciences*

# Teaching Experience

Kelley School of Business, Indiana University Bloomington

Associate Instructor (instructor of record), Introduction to Operations Management (undergraduate), Spring 2017, Fall 2017.

Department of Economics, Indiana University Bloomington

Associate Instructor, Theory of Prices and Markets II (Ph.D.-level), Spring 2014.

Associate Instructor, Theory of Prices and Markets I (Ph.D.-level), Fall 2013.

Associate Instructor (instructor of record), Introduction to Microeconomics (undergraduate), Spring 2013.

Graduate Assistant, Introduction to Macroeconomics: Honors (undergraduate), Fall 2012.

Graduate Assistant, International Trade (undergraduate), Fall 2011, Fall 2012.

Graduate Assistant, International Monetary Economics (undergraduate), Spring 2012.

Graduate Assistant, The Economics of Industry (undergraduate), Fall 2011, Spring 2012.

# Honors & Awards

Production and Operations Management Society (POMS) College of Sustainable Operations Doctoral Student Travel Award, 2019.

Doctoral Student Research Productivity Award, Kelley School of Business, Indiana University Bloomington, 2017, 2018, 2019.

Doctoral Student Research Travel Award, Kelley School of Business, Indiana University Bloomington, 2017, 2018, 2019.

Dean's Fellowship, Kelley School of Business, Indiana University Bloomington, 2014.

Omicron Delta Epsilon, 2012.

Department of Economics Fellowship, Indiana University Bloomington, 2011.

Scholastic Award, Hong Kong Baptist University, 2011.

Outstanding Student Award, Hong Kong Baptist University, 2011.

MI Asia Award, 2011.

ETS TOEFL Scholarship, Educational Testing Service, 2011.

The 12th Baptist Convention of Hong Kong Outstanding Student Award, 2011.

President's Honor Roll, Hong Kong Baptist University, 2008–2011 (all undergraduate semesters).

Student Ambassadorship, School of Business, Hong Kong Baptist University, 2009–2011.

Hong Kong Electric Scholarship, 2010, 2011.

Vincent Woo Scholarship for Outstanding Mainland Students, 2011.

Champion, and Audience Award, B.B.A. Singing Contest, Hong Kong Baptist University, 2011.

Champion, Economics Society Singing Contest, Hong Kong Baptist University, 2011.

Beta Gamma Sigma, 2010.

Padma & Hari Harilela Scholarship for Outstanding Students, 2010.

TOEFL High Score Grand Award, The New Oriental School, 2009.

Champion, 10th Putonghua Speech Contest, Hong Kong Baptist University, 2008.

First Place, 7th English Speaking Contest, Hong Kong Baptist University, 2007.

## Computing & Language Skills

Computing Skills: MATLAB, Stata, Mathematica, R, Maple.

Language Skills: English (fluent), Chinese (native).
*TOEFL score:* 120/120 (August 2009).

March 2020