Environmental Instrument Choice in a Nonlinear World

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Abstract

The substantial literature on environmental instrument choice under uncertainty has provided valuable insights using simplifying assumptions of linear marginal cost and benefit curves and additive error terms to determine when and why a price or quantity instrument should be preferred. But empirical analysis has shown that linearity and additivity are not the norm. This paper explores the formal properties of instrument choice when the marginal benefit and marginal cost curves are non-linear; the error term is multiplicative and assumed to be exponential; and the choice is expanded to include comparisons among different price and different quantity regimes, as well as choices between these types. In this more realistic environment, we show that small changes in variance and parameter values can have a significant impact on optimal instrument choices.

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1. Introduction

Weitzman's (1974) seminal paper "Prices v. Quantities" stimulated a substantial literature on the choice of environmental instruments for pollution control. Weitzman concluded that the relative elasticities of the social cost and benefit curves will point to a choice of either a price-based instrument or a quantity-based one. That is, the nature of the curves determined whether a price-based instrument, such as an effluent tax, is likely to function more efficiently than a quantity-based one, such as cap-and-trade.

As instrument choice must be made under some degree of ex ante uncertainty, policymakers necessarily seek to maximize the expected net benefits of instrument selection. Weitzman's model raised--but did not resolve – many questions related to uncertainty and risk in this process, but others elaborated on that point. Yohe (1978, 238) explicitly extended Weitzman's model to include uncertainty in production and informational problems, but came to the conclusion that Weitzman's model "survives extensive generalization completely intact." Adar and Griffin (1976) examined the effects of both uncertainty and risk aversion on instrument choice. They considered uncertainty of both marginal control costs and marginal damage costs and determined that only the former "will yield different expected welfare losses", but they also concluded that the elasticity of the marginal damage function made a "strong case" for the choice of type one instrument rather than another (Adar and Griffin 1976, 188).

These models, as well as dozens of others that have followed, have had similar assumptions: the purpose of instrument choices is to maximize expected social welfare or minimize expected social loss; uncertainty exists with respect to ex post costs, benefits and net social welfare; marginal cost and marginal benefit functions are linear (as a simplification); errors are additive; and instrument choice means the selection of either a price or a quantity instrument. Given this standard set-up, considerable attention has been focused on the optimal instrument choice itself, including the use of both command-and-control (e.g., Fishelson, 1976; Hoel and Karp, 2001; Hoel and Karp, 2002; Stavins, 1996) and market-based instruments (e.g. Stavins, 1995), as well as the adoption of more complex hybrid instruments (e.g. Ambec and Coria, 2013; Mandell, 2008; Roberts and Spence, 1976; Xepapadeas, 1995). In this context, who has the decision-making power to select the instrument, whether it be the firm or a central authority, can play a critical role (e.g. Krysiak and Oberauner, 2010; Malueg and Yates, 2009). Extending these often static models to a more dynamic framework, a variety of modifications have been made, including the explicit incorporation of correlated cost shocks, discounting, and stock decay (e.g. Newell and Pizer, 2003). The strategic response of firms to specific policies has also been examined in a dynamic framework, emphasizing the sometimes unintended incentives created once policies are implemented (e.g. Montero, 2000; Moledin et al, 2003).

This paper is not intended as a rejection of this earlier theoretical work but rather an extension in a hitherto overlooked direction. We use as a starting point the same basic assumption—ex ante uncertainty in pollution control. But we show that changes in just two of the assumptions found in most of the literature can profoundly affect the choice of instrument. Moreover, we go beyond previous models in that the choice may no longer be just a binary prices versus quantities, but can, and really must, include a larger universe of possibilities including a price instrument (an effluent tax) versus another price instrument (an output tax), or a quantity instrument (cap-and-trade) versus another quantity instrument (emissions quota). In both the quantity/quantity and price/price cases just noted, the second choice (inferior from idealized models) might well be the less costly to administer and potentially less costly overall.

First, similar to Paulson Gjerde and Grossman (2015), we assume that the marginal cost and benefit curves are nonlinear. Because of nonlinearity the slopes of the curves may vary from the near vertical to the near horizontal. As we will show, the value of the slope is crucial in a nonlinear case. Of course, the value of the slope also matters in the linear case. But because the curve is nonlinear, the slope is no longer a constant, and ultimately depends *greatly on the variance* of the error term. Consequently, the preferred instrument choice will be more sensitive to model parameters. The implication is that small changes in parameter values will have a significant impact on the preferred instrument choice, substantially adding to the complexity of the optimal instrument problem.

The choice of nonlinear curves departs from most of the theoretical literature but it is in accord with empirical efforts to measure costs and benefits of pollution control technologies. For example, Watson and Ridker (1984, 310) found that by using actual pollution control data they "empirically derived nonlinear" marginal cost and benefit functions. A similar finding was reported by Cofala and Syri (1998) with respect to sulfur dioxide abatement technologies. Indeed, while it is understandable that essentially all instrument choice theory to date has started with a simplification of linearity, the real world of pollution control appears to be distinctly nonlinear. This paper provides a theoretical perspective from the more realistic assumption of nonlinearity.

Second, and most importantly, we assume an error term that is exponential and multiplicative, not linear and additive. Our choice for the error term may seem like an especially strong assumption, but in fact we regard the assumption most often found in the literature to be far more restrictive and, also unrealistic. It can as easily be argued that just as with marginal costs or benefits, error may expand (exponentially) with the desired level of pollution control. Again, we note that our assumption has empirical support. According to Watson and Ridker (1984), control and damage cost functions derived from their data are consistent with a multiplicative error term which is exponential.

A multiplicative error has appeared before[1] in theoretical work on instrument choice but has been incorporated in a *linear* fashion. The effect of this assumption (though common in the economic literature generally) is that both additive and multiplicative models show the optimal level of pollution control is independent of the error term in any quantity instrument case. The assumption of additive error eliminates the term in the result for the price case as well, such that the optimal instrument choice is

[1] For example, Adar and Griffin (1976); Fishelson (1976); Paulson Gjerde and Grossman (2015).

necessarily independent of the error term. The error does remain in existing multiplicative models for the price instrument, and because of this difference, the optimal instrument choice is no longer independent of the error term, such that the results may be skewed in favor of a quantity instrument as uncertainty increases (Fishelson, 1976).

We regard, however, that the choice of a *linear* multiplicative error term is, in fact, a special case. Our choice of a more general exponential random variable for the error/disturbance term, u^x, where x > 1, is in marked contrast, and is both more in accord with the empirical literature and is also more intuitively likely. That is, the standard assumption implies that the level of uncertainty (for which the error term is proxy (Watson and Ridker 1984) remains constant even as pollution control intensity increases, presumably even as marginal costs would approach vertical. Our less restrictive assumption is that the variance of the expected error grows exponentially with the amount of pollution control. Thus the expected net gain in social welfare may be greater with a given instrument where the variance of the error is lower *even if the expected marginal cost is greater*. Moreover, the error term does not drop out of the formulation of either the price or the quantity instrument.

The critical nature of the slope and the variance emerge directly from the choice of a nonlinear function. In his paper, Weitzman (1974) had speculated on the effects of non-linearity, particularly on how it might affect point elasticities, which could determine the optimal instrument choice depending on where the marginal benefit and cost curves intersected. But he did not model nonlinearity. Watson and Ridker (1984) and Cole and Grossman (2002) examined possible effects of nonlinearity graphically but did not address its formal properties, which we do here.

Multiplicative error and nonlinearity are particularly important on the cost side, because another simplifying assumption in the literature is to focus exclusively on the marginal cost of compliance and/or damage. But marginal costs must include a much larger range of costs than those of control and damage. As Goulder and Parry (2008, 171) note in their survey of the instrument choice literature, "Many analyses disregard administrative, legal or institutional issues relevant to policy costs...[or] ignore details about market structure or producers' objectives that can influence the relative effectiveness of various instruments."

In fact, it is likely that such "administrative, legal or institutional" costs and particular ex post costs of monitoring, measurement and enforcement (usually grouped together under the heading of "transaction costs" (Krutilla 2010)) can make any actual instrument choice relatively inefficient (Cole and Grossman 1999, 2002).[2] As Goulder and Perry (2008, 160) observe, monitoring costs alone can be so high as to make an instrument's utilization "virtually infeasible."[3] Idealized instrument types may be efficient in theory but may provide a poor guide to actual performance. This paper, which uses more realistic assumptions, clearly has implications that are important in the design of environmental control regimes.

In the next section, we develop a model of instrument choice, using nonlinear benefit and cost functions with (exponential) multiplicative error terms, toward an objective of net social welfare maximization. We apply the model, first, to a quantity instrument and then to a price instrument. Then we compare both price and quantity instruments to alternatives of the same type as well as to each other, and derive general conditions about instrument choice. In the third section, we flesh out the problem with a numerical simulation comparing different quantity instruments, different price instruments as well as a price versus a quantity instrument. A discussion of the implications of this model concludes.

2. The Model

2.1 A Quantity Instrument

[2] A number of papers have discussed the impacts of transaction costs on environmental instrument choice and performance. For example, Stavins 1995; Woerdman 2001; Stranlund et al. 2002.
[3] See for example, Cole (1998); Russell and Powell (1996); and Blackman and Harrington (2000).

We assume that the policymakers need to find the instrument that will provide the degree of pollution control, q, that will maximize expected social welfare, EW_q. In general we follow the approach found in Adar and Griffin (1976), (as well as Fishelson (1976), Baumol and Oates (1988), and Shrestha (2001)), explicitly modeling social marginal benefit and marginal cost functions. The cost function is given as $C(q, u)$, and marginal cost can be written

$$MC(q, u) = \alpha + \beta(qu)^2, \quad (1)$$

where $\alpha > 0$ and $\beta > 0$ are constants, and u is a random variable with expected value μ, variance σ_u^2 and known density dF(u).[4] Note that this formulation of marginal cost differs from the multiplicative case found in Adar and Griffin (1976), which incorporates uncertainty in both the vertical intercept and the slope of a linear marginal cost function, and Paulson Gjerde and Grossman (2015), which incorporates a linear error term. In more general terms, for a given marginal cost function MC(q,u), the majority of previous models assume $MC_{11}(q,u)=0$, $MC_{12}(q,u)=0$, and $MC_{22}(q,u)=0$. More recent studies suggest that the first two assumptions are not overly restrictive. In other words, relaxing these assumptions does not significantly change the results. Our model, however, demonstrates that modifying the third assumption, such that $MC_{22}(q,u) > 0$, introduces new complexity into the optimal instrument choice problem. In contrast, consistent with Cole and Grossman (2002), we assume a fixed vertical intercept of a nonlinear marginal cost function, i.e. parameter uncertainty is modeled here only with respect to the slope of the marginal cost function. This approach allows us to more closely match the shape of the marginal cost function observed in the real world, as empirically estimated by Watson and Ridker (1984), but also

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[4] The underlying total cost function is, thus, $C(q, u) = A + \alpha q + \frac{1}{3}\beta u^x q^3$, which is a special case of the more general cubic function $C(q, u) = A + \alpha q + Dqu^2 + \frac{1}{3}\beta u^x q^3$. Using the specialized case in which D=0 and x=2 allows to improve the tractability of the model while still retaining the general shape of the cubic total cost function.

isolate and, hence, more clearly understand the ramifications of the nonlinear multiplicative error assumption under these conditions.

Similarly, we assume the marginal benefit function $MB(q)$ is nonlinear, capturing the idea that as the level of abatement increases, the marginal benefit declines at an increasing rate. In other words, the marginal benefit associated with abatement tend to fall off more quickly at high levels of abatement compared to low levels of abatement. For analytical ease, we assume the marginal benefit function is known with certainty.[5] Thus, the marginal benefit associated with pollution control can be written as:

$$MB(q) = a - bq^2 . \tag{2}$$

The policymaker's goal is to set q in order to maximize expected social welfare, where:

$$EW_q = \underset{u}{E} \int_0^q [MB(q) - MC(q,u)]dq \tag{3}$$

Let q* be the solution to this problem, which must satisfy the first-order condition:

$$\underset{u}{E}[MB(q^*)] = \underset{u}{E}[MC(q^*, u)] \tag{4}$$

Substituting (1) and (2) into (4), we obtain:

$$a - bq^{*2} = \alpha + \beta q^{*2}\, E[u^2] \tag{5}$$

or

[5] A common approach used in the literature to examine optimal instrument choice under uncertainty is to divide the discussion into two parts: comparison of the relative efficiency of quantity and price instruments when the marginal benefit function is subject to uncertainty and comparison when the marginal cost function is subject to uncertainty. (e.g. Adar and Griffin, 1976; Fishelson,1976). Introducing uncertainty in the marginal benefit function and assuming a known marginal cost function yields a result consistent with that of these linear models (i.e. no difference between the expected social welfare under quantity and price instruments). Thus, we focus our attention on the case in which there is uncertainty only in the marginal cost function.

$$a - bq^{*2} = \alpha + \beta q^{*2}\left(\sigma_u^2 + \mu^2\right) \tag{6}$$

$$q^* = \left(\frac{a - \alpha}{b + \beta\left(\sigma_u^2 + \mu^2\right)}\right)^{\frac{1}{2}} \tag{7}$$

This is an important result, namely, the condition indicates that the *slope* of the marginal cost function will depend on the mean and the variance of the error term.

The expected social welfare under a quantity instrument is:

$$EW_q = \underset{u}{E}\int_0^{q^*}\left[\left(a - bq^2\right) - \left(\alpha + \beta q^2 u^2\right)\right]dq \tag{8}$$

After integrating we find

$$EW_q = \underset{u}{E}\left[(a - \alpha)q^* - \frac{\left(b + \beta u^2\right)}{3}q^{*3}\right] \tag{9}$$

Substituting (7) into (9) and simplifying, it is straightforward to show:

$$EW_q = \frac{2(a - \alpha)^{\frac{3}{2}}}{3\left(b + \beta\left(\sigma_u^2 + \mu^2\right)\right)^{\frac{1}{2}}} \tag{10}$$

Thus, the expected gain in social welfare (along with the slope of the marginal cost curve) will be affected by the variance and the mean of the error/disturbance term. Overall, the expected social welfare from a quantity instrument will depend on parameter values of a, α, b, β, and it also depends on the variance, σ_u^2, and mean, μ, of the disturbance term. We note that in the linear case with multiplicative error, the expected social welfare associated with a quantity instrument depends only on the values of a, α, b, β, and μ (Adar and Griffin, 1976). Similarly, it is straightforward to show that in the non-linear case with additive error, the expected social welfare under a quantity instrument is independent of the variance of the disturbance term (e.g. Paulson Gjerde and Grossman 2015). Thus, it is the non-linear nature of the marginal cost function combined with the exponential multiplicative error term that drives this result.

2.2 Quantity instrument 1 vs. Quantity Instrument 2

Suppose you have two quantity instruments, such that:

$$MC_1(q_1, u_1) = \alpha + \beta_1 q_1^2 u_1^2 \text{ where } E[u_1] = \mu_1 \text{ and } var[u_1] = \sigma_1^2 \tag{11}$$

$$MC_2(q_2, u_2) = \alpha + \beta_2 q_2^2 u_2^2 \text{ where } E[u_2] = \mu_2 \text{ and } var[u_2] = \sigma_2^2 \tag{12}$$

Note the marginal cost functions have the same (vertical) intercept but different slopes and error terms.

Using (10), we can express the corresponding level of the expected gain in social welfare under each quantity instrument as:

$$EW_{q_1} = \frac{2(a-\alpha)^{\frac{3}{2}}}{3\left(b + \beta_1\left(\sigma_1^2 + \mu_1^2\right)\right)^{\frac{1}{2}}} \tag{13}$$

$$EW_{q_2} = \frac{2(a-\alpha)^{\frac{3}{2}}}{3\left(b + \beta_2\left(\sigma_2^2 + \mu_2^2\right)\right)^{\frac{1}{2}}} \tag{14}$$

Quantity instrument 1 is preferred to quantity instrument 2 if:

$$\frac{2(a-\alpha)^{\frac{3}{2}}}{3\left(b + \beta_1\left(\sigma_1^2 + \mu_1^2\right)\right)^{\frac{1}{2}}} > \frac{2(a-\alpha)^{\frac{3}{2}}}{3\left(b + \beta_2\left(\sigma_2^2 + \mu_2^2\right)\right)^{\frac{1}{2}}} \tag{15}$$

Or to simplify further, if:

$$\beta_1\left(\sigma_1^2+\mu_1^2\right)<\beta_2\left(\sigma_2^2+\mu_2^2\right) \qquad (16)$$

That is, if the expected slope of the marginal cost function for quantity instrument 1 is less than the expected slope of the marginal cost function for quantity instrument 2, then quantity instrument 1 is preferred to quantity instrument 2. The simplification illustrates how the mean and variance of the error term affects that slope of the curve.

2.3 A Price Instrument

When a price instrument is used, the quantity $\hat{q}$ of pollution control is determined by a given P (i.e. a tax rate), which is a random variable. In particular, for a given P, a profit maximizing competitive firm will choose to operate where $P = MC$. Taking into account (1), this condition can be written as:

$$P=\alpha+\beta(\hat{q}u)^2 \qquad (17)$$

Solving for $\hat{q}$, the level of pollution control under a price instrument P, we obtain:

$$\beta\hat{q}^2u^2=P-\alpha \qquad (18)$$

$$\hat{q}=\left(\frac{P-\alpha}{\beta u^2}\right)^{\frac{1}{2}} \qquad (19)$$

Where P > 0, α > 0, and β > 0 are constants and u is random variable with expected value μ, variance σ_u^2 and known density $dF(u)$.

Policymakers set P in order to maximize expected social welfare under a price instrument:

$$EW_t=\underset{u}{E}\int_0^{\hat{q}(P,u)}\left[\left(a-bq^2\right)-\left(\alpha+\beta q^2u^2\right)\right]dq \qquad (20)$$

The optimal price, P*, can be derived by solving the following condition:

$$\frac{d}{dP} EW_t = 0 \tag{21}$$

$$\frac{d}{dP} \underset{u}{E} \int_0^{\hat{q}(P,u)} \left[\left(a - bq^2\right) - \left(\alpha + \beta q^2 u^2\right) \right] dq = 0 \tag{22}$$

Integrating and then differentiating with respect to P, the optimal $P^*$is found where:

$$\frac{\left(P^* - \alpha\right)^{-\frac{1}{2}}}{2} \underset{u}{E} \left[\frac{a - \alpha}{\beta^{\frac{1}{2}} u} \right] - \frac{3\left(P^* - \alpha\right)^{\frac{1}{2}}}{2} \underset{u}{E} \left[\frac{b + \beta u^2}{3\beta^{\frac{3}{2}} u^3} \right] = 0 \tag{23}$$

Rearranging terms we can derive the condition that must be satisfied for the optimal price:

$$\underset{u}{E} \left[\frac{(a - \alpha)}{u} \right] = \underset{u}{E} \left[\hat{q}(P^*, u)^2 \cdot \frac{\left(b + \beta u^2\right)}{u} \right] \tag{24}$$

Taking into account the optimal P*, we can rewrite expected social welfare under a price instrument as:

$$EW_t = \underset{u}{E} \int_0^{\hat{q}(P^*,u)} \left[\left(a - bq^2\right) - \left(\alpha + \beta q^2 u^2\right) \right] dq \tag{25}$$

Integrating (25) and substituting (24) into the result, yields, after some algebraic manipulation:

$$EW_t = \frac{\alpha b}{3\beta} \underset{u}{E} \left[\frac{\hat{q}(P^*, u)}{u^2} \right] + \frac{2a - \alpha}{3} \underset{u}{E} \left[\hat{q}(P^*, u) \right] - \frac{\alpha(a - \alpha)}{3\beta} \underset{u}{E} \left[\frac{1}{u^2 \hat{q}(P^*, u)} \right] \quad . \tag{26}$$

This result, the many factors notwithstanding, shows that expected welfare under a price instrument depends on parameters a, α, b, β, as well as the frequency distribution of the error term u.[6]

[6] This is a similar result to the one found by Adar and Griffin (1976) under the assumptions of linear marginal cost and linear multiplicative error, and is also true under the assumptions of non-linear marginal cost and both additive error and linear multiplicative error (Paulson Gjerde and Grossman 2015).

2.4 Price Instrument 1 vs. Price Instrument 2

Assume two price instruments such that:

$$MC_1(q_1, u_1) = \alpha + \beta_1 q_1^2 u_1^2 \text{ where } E[u_1] = \mu_1 \text{ and } var[u_1] = \sigma_1^2 \quad (27)$$

$$MC_2(q_2, u_2) = \alpha + \beta_2 q_2^2 u_2^2 \text{ where } E[u_2] = \mu_2 \text{ and } var[u_2] = \sigma_2^2 \quad (28)$$

Thus, the marginal cost functions once more have the same vertical intercept, but different slopes and error terms. As in 2.2, the process simply recapitulates the preceding to obtain the expected welfare under each price instrument, which can then be compared. That is, price instrument 1 is preferred to price instrument 2 if:

$$\begin{aligned} &\frac{\alpha b}{3\beta_1} \underset{u}{E}\left[\frac{\hat{q}_1(P_1^*, u_1)}{u_1^2}\right] + \frac{2a-\alpha}{3} \underset{u}{E}\left[\hat{q}_1(P_1^*, u_1)\right] - \frac{\alpha(a-\alpha)}{3\beta_1} \underset{u}{E}\left[\frac{1}{u_1^2 \hat{q}_1(P_1^*, u_1)}\right] \\ &> \frac{\alpha b}{3\beta_2} \underset{u}{E}\left[\frac{\hat{q}_2(P_2^*, u_2)}{u_2^2}\right] + \frac{2a-\alpha}{3} \underset{u}{E}\left[\hat{q}_2(P_2^*, u_2)\right] - \frac{\alpha(a-\alpha)}{3\beta_2} \underset{u}{E}\left[\frac{1}{u_2^2 \hat{q}_2(P_2^*, u_2)}\right] \end{aligned} \quad (29)$$

In contrast to (16), the condition under which price instrument 1 is preferred to price instrument 2 cannot be reduced to a simple, intuitively obvious statement. While the result appears complex, however, the basic point is readily apparent. In particular, which price instrument is preferred depends on parameters a, α, b, β_1, and β_2, as well as the expected value of the level of pollution control under each instrument, $\hat{q}_1(P_1^*, u_1)$ and $\hat{q}_2(P_2^*, u_2)$, which is a function of the error term u and optimal price, P^*. We will highlight the sensitivity of the optimal price instrument choice to two of these parameter values, namely the slope parameter of the marginal cost function, β, as well the variance of the error term, $\sigma^{2,}$, in order to better understand the tradeoffs inherent in the model.

2.5 Price Instrument vs. Quantity Instrument

Our model still allows us consider the standard quantity versus price instrument comparison often used by others in the literature. From (10) and (26), a price instrument is preferred to a quantity instrument if $EW_t > EW_{q.}$, or:

$$\frac{\alpha b}{3\beta} \underset{u}{E}\left[\frac{\hat{q}(P^*,u)}{u^2}\right] + \frac{2a-\alpha}{3} \underset{u}{E}\left[\hat{q}(P^*,u)\right] - \frac{\alpha(a-\alpha)}{3\beta} \underset{u}{E}\left[\frac{1}{u^2 \hat{q}(P^*,u)}\right] > \frac{2(a-\alpha)^{\frac{3}{2}}}{3\left(b+\beta\left(\sigma_u^2+\mu^2\right)\right)^{\frac{1}{2}}} \qquad (30)$$

Note that within this result (and thus determining which instrument is preferred) are the terms for the expected level of pollution control under each instrument $\hat{q}(P^*,u)$ and q*as well as parameters a, α, b, β, and the distribution of the disturbance terms. Again, in contrast to (16), the condition under which a price instrument is preferred to a quantity instrument cannot be reduced to a straightforward, easy-to-check inequality statement. We use a set of specific examples in Section 3 to illustrate this point.

In general, the determination of which instrument is likely to give the greatest net social benefit must take into account the variance of the error term, as well as the expected level of pollution control under any given instrument. Of course the slopes of the marginal cost and benefit curves are important but in our model, the slope of the marginal cost curve is a consequence of the variance. Given two instruments, the social gains associated with using an instrument with a lower expected marginal cost, and a greater expected level of pollution control, must be balanced against the potential losses associated with the greater uncertainty of this instrument. The conclusion is that the instrument that actually maximizes social welfare is not necessarily the instrument that has *potentially* the lowest marginal compliance costs (e.g. effluent taxes over output taxes; cap-and-trade over technical standards). In particular, in some cases, the choice of the instrument with the higher mean marginal cost but the significantly lower variance will be preferred; in other cases the choice is reversed. The choice must then depend on an overall appraisal of the total costs and the level of uncertainty. We explore these circumstances in Section 3.

2.6 Graphical Representation of Gain and Loss

As derived in the previous section, the expected welfare gain under a quantity instrument is characterized by (10). Recall that q* represents the a priori optimal level of pollution control. The gain can be represented by the area between the marginal benefit and expected marginal cost functions at q* (see green shaded area in Figure 1). Given uncertainty, however, the realized marginal cost function is likely to differ from the expected marginal cost function. Thus, with hindsight the optimal level of pollution control $q^* \varepsilon [q_1, q_2]$. We illustrate the social loss associated with this difference between the ex post optimal q and the a priori optimal q*, as the red shaded area of Figure 1. The net welfare gain is the difference between the green shaded area and the red shaded area.

Similarly, the expected welfare gain under a price instrument is characterized by (26). Recall that P* represents the a priori optimal price, and $E[\hat{q}(P^*, u)]$ is the expected level of pollution control under this instrument. The gain can be represented by the area between the marginal benefit and expected marginal cost functions at $E[\hat{q}(P^*, u)]$ (see green shaded area in Figure 2). If marginal cost is greater than expected, however, P* will result in a level of pollution control which is less than optimal. If marginal cost is less than expected, P* will result in a level of pollution control which is greater than optimal. We illustrate the social loss associated with this difference between the ex post optimal q and the a priori optimal q, $E[\hat{q}(P^*, u)]$, as the red shaded area in Figure 2. Again, the net welfare gain is the difference between the green shaded area and the red shaded area.

Note that when the error is additive, which is the standard assumption in the literature, $q^* = E[\hat{q}(P^*, u)]$], such that the expected welfare gain under the quantity and price instruments is identical, as illustrated in Figure 1 and Figure 2. In this case, maximizing the net welfare gain is equivalent to minimizing the social loss. It is the latter criterion which is typically used in the literature, and this approach is valid given the additive error assumption (e.g. Fishelson, 1976). Introducing a nonlinear multiplicative error, however, changes this result (i.e., $q^* \neq E[\hat{q}(P^*, u)]$), such that it is necessary to consider both expected welfare gain as well as social loss when comparing instruments (Figure 1 versus Figure 3) This consideration of both expected welfare gain and social loss is also necessary when comparing two different quantity instruments or two different price instruments. Thus, we are restricted in the case of a nonlinear multiplicative error to maximize net welfare gain, and not simply minimize the social loss. In the next section, we present a numerical simulation which is consistent with the model assumptions in Section 2 in order to illustrate our key takeaways.

Figure 1: Welfare Gain and Loss with Quantity Instrument

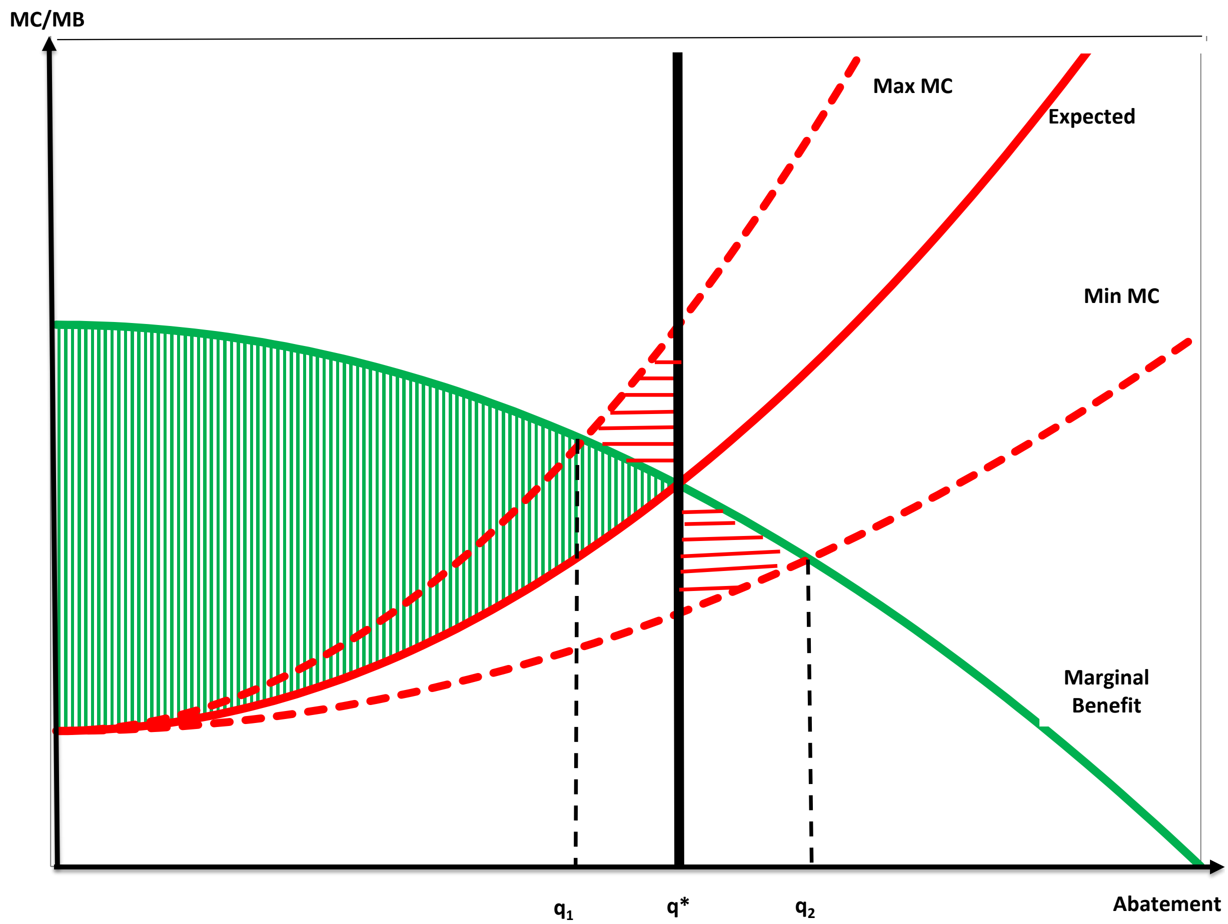


Figure 2: Welfare Gain and Loss with Price Instrument when q* = E[$\hat{q}(P^*, u)$]

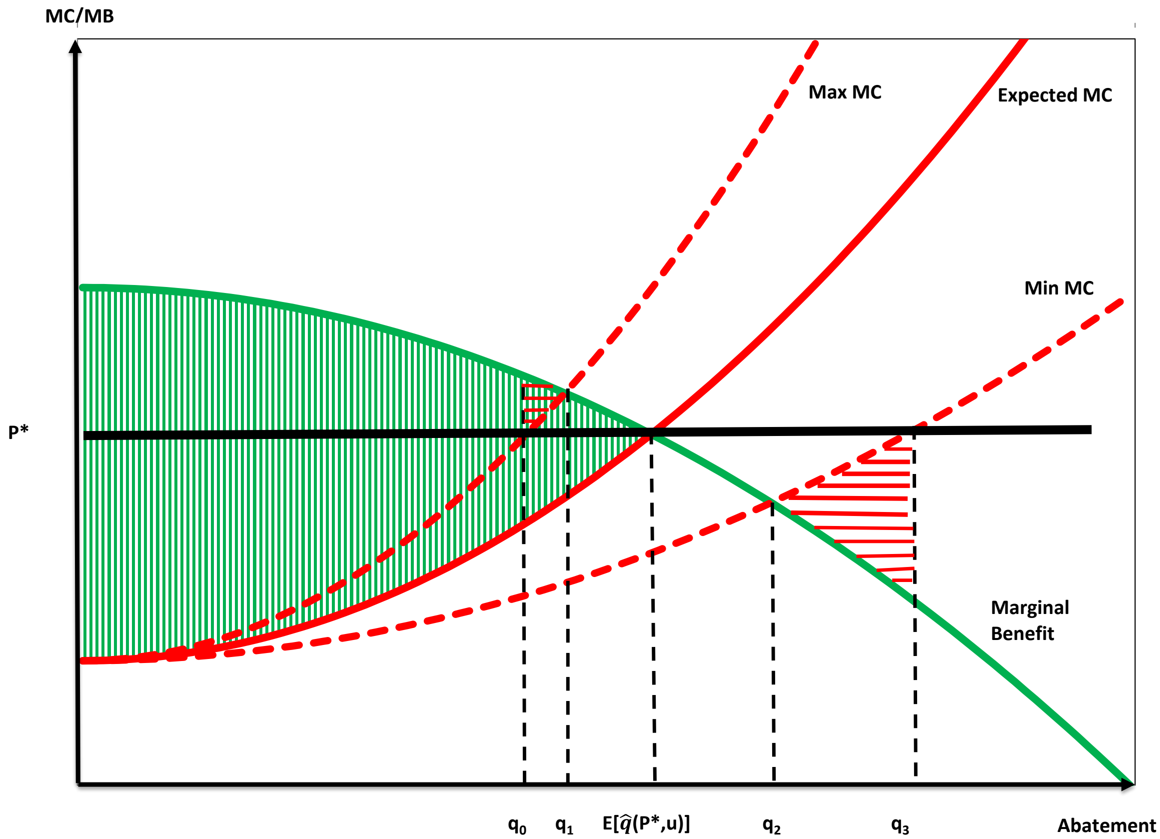
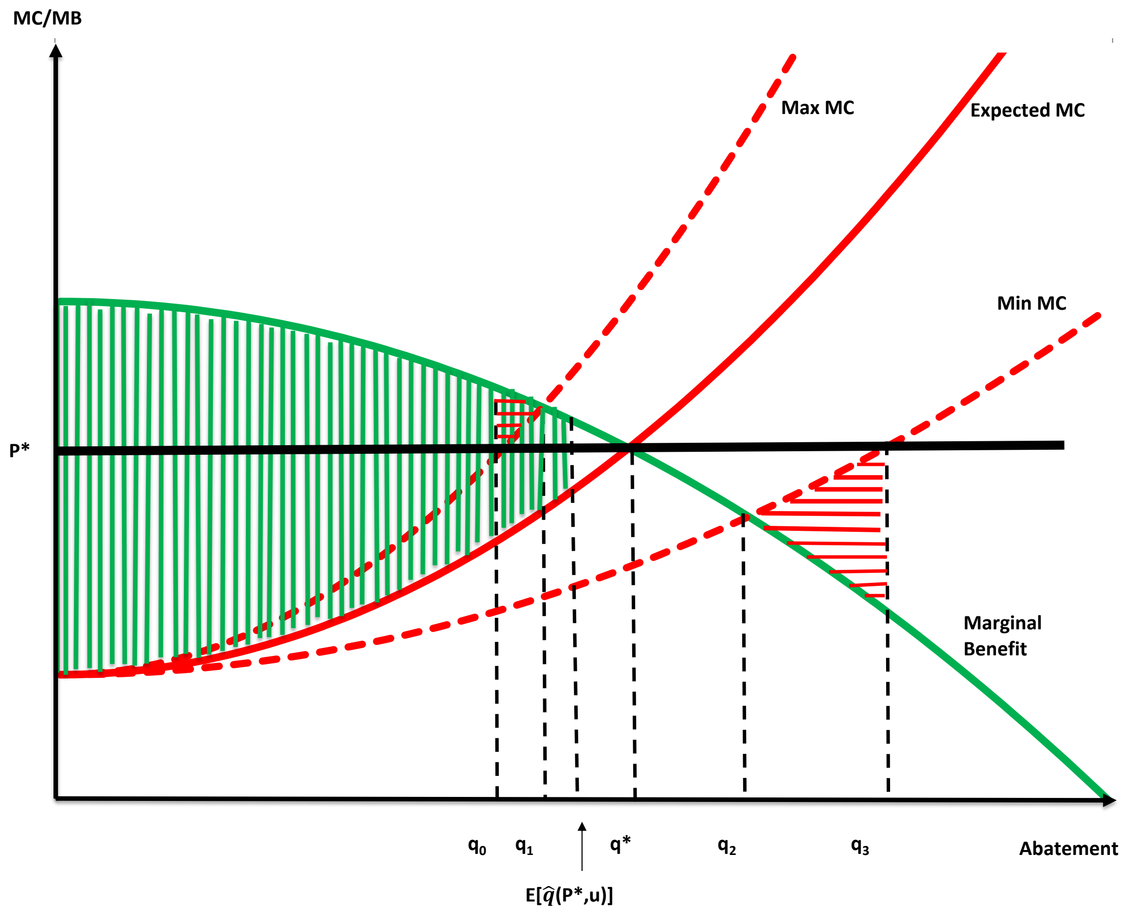


Figure 3: Welfare Gain and Loss with Price Instrument when $q^* \neq E[\hat{q}(P^*, u)]$



3. Numerical Simulation

To illustrate the sensitivity of the preferred instrument choice to key model parameters, this section examines the tradeoffs faced when comparing instruments that vary in terms of the slope parameter of the marginal cost function, β, as well the variance of the error term, σ^2. In particular using subscripts to denote different instruments, we assume that in the case of both the of the first two comparisons (that is, quantity versus quantity and price versus price) that Instrument 1 is characterize by a relatively small marginal cost compared to Instrument 2 (i.e. $\beta_1 < \beta_2$) but suffers from greater uncertainty (i.e. $\sigma_1^2 > \sigma_2^2$). We observe that as the degree of differentiation between Instrument 1 and Instrument 2 changes, so does the relative welfare gain, the relative social loss, and, most importantly, the *relative net welfare gain*, the assumed objective of the choice of pollution control instrument.

We set up a sensitivity analysis using the models in Section 2 to compare the expected net welfare gain under two hypothetical quantity instruments (Q1 and Q2), two price instruments (P1 and P2), as well as a price instrument and a quantity instrument (P and Q). For these comparisons we assign values to the intercept and slope parameters of the marginal benefit function (a and b, respectively) and to the intercept and slope parameters of the marginal cost function associated with Instrument 1 (α and β_1, respectively) and Instrument 2 (α and β_2, respectively) which are consistent with our model assumptions in Section 2. For simplicity, the disturbance term is assumed to be uniformly distributed with mean of 1. Holding the variance of Instrument 1 constant, we consider how changes in the variance of Instrument 2 impact the optimal instrument choice under these three scenarios.

3.1 Quantity versus Quantity Instrument

Evaluating (13) and (14) at specific parameter values consistent with model assumptions, it is straightforward to calculate the expected net welfare gain under Instrument Q1 and under Instrument Q2, and, more importantly, examine how the *difference* in these values varies with the relative uncertainty of the instruments. Specifically, let a=2000, b=20, α=500, β_1=10, σ_1^2=0.163, and β_2=11. Note, however, that the particular value of these parameters is not our focus. Rather, our purpose is to demonstrate the sensitivity of the preferred instrument choice to *changes* in key model parameters. Table 1 summarizes the impact of a change in σ_2^2 on the optimal quantity instrument choice.

Table 1: Impact of Change in Variance on Optimal Quantity Instrument Choice

σ_2^2	Optimal Quantity		Expected Welfare Gain		Expected Social Loss		Expected Net Welfare Gain		Percent Difference in Expected Net Welfare Gain
	q_1*	q_2*	Q1	Q2	Q1	Q2	Q1	Q2	
0.083	6.89	6.4327	6,886	6,433	943	808	5,943	5,624	5.7%
0.068	6.89	6.4539	6,886	6,454	943	669	5,943	5,785	2.7%
0.053	6.89	6.4730	6,886	6,473	943	538	5,943	5,935	0.1%
0.041	6.89	6.4900	6,886	6,490	943	419	5,943	6,071	-2.1%
0.030	6.89	6.5049	6,886	6,505	943	312	5,943	6,193	-4.0%
0.021	6.89	6.5175	6,886	6,518	943	219	5,943	6,298	-5.6%

Recall Instrument Q1 is characterize by a relatively small marginal cost compared to Instrument Q2 (i.e. $\beta_1 < \beta_2$) but suffers from greater uncertainty (i.e. $\sigma_1^2 > \sigma_2^2$). Initially, Instrument Q1 has the higher expected net gain in social welfare. That is, the instrument with a relatively small marginal cost compared to Instrument 2 is preferred, the somewhat greater uncertainty notwithstanding. But as the variance of Instrument 2 decreases, its expected net gain increases, such that, for a sufficiently small σ_2^2 , Instrument Q2 becomes the preferred instrument. This simple example highlights the tradeoff that exists between the social gains associated with using an instrument with a lower expected marginal cost (and greater level of abatement) and the potential losses associated with the greater uncertainty of this instrument.

3.2 Price versus Price Instrument

From equation (29), we are able to compare two price instruments, again where Instrument P1 is characterized by a relatively small marginal cost compared to Instrument P2 (i.e. $\beta_1 < \beta_2$) but comes with greater uncertainty (i.e. $\sigma_1^2 > \sigma_2^2$). [7] Table 2 summarizes the impact of a change in σ_2^2 on the optimal price instrument choice.

Table 2: Impact of Change in Variance on Optimal Price Instrument Choice

σ_2^2	Optimal Price		Expected Welfare Gain		Expected Social Loss		Expected Net Welfare Gain		Percent Difference in Expected Net Welfare Gain
	P_1*	P_2*	P1	P2	P1	P2	P1	P2	
0.049	697.72	1471.06	14,847	9,484	5,961	683	8,887	8,801	1.0%
0.048	697.72	1473.62	14,847	9,483	5,961	655	8,887	8,828	0.7%
0.046	697.72	1478.60	14,847	9,480	5,961	601	8,887	8,879	0.1%
0.043	697.72	1483.38	14,847	9,477	5,961	552	8,887	8,926	-0.4%
0.041	697.72	1487.98	14,847	9,474	5,961	506	8,887	8,968	-0.9%
0.039	697.72	1492.39	14,847	9,471	5,961	463	8,887	9,007	-1.3%

As in the quantity versus quantity case, again we observe that the optimal price instrument is a function of the relative variance of the error terms associated with the two instruments. Initially, the lower marginal cost associated with Instrument P1 (and greater level of abatement) outweighs the social loss associated with this instrument's relatively greater uncertainty. As the degree of certainty associated with Instrument P2 increases, however, this result no longer holds, such that Instrument P2 becomes the optimal price instrument.

3.3 Price versus Quantity Instrument

[7] Specifically, let a=2000, b=5, α=500, β_1=1, σ_1^2=0.051, and β_2=12

Finally, we consider the classic (Weitzman) comparison, namely, Price Instrument versus Quantity Instrument, characterized by (30).[8] Table 3 summarizes the impact of a change in σ^2 on the optimal instrument choice.

Table 3: Impact of Change in Variance on Price vs Quantity Instrument Choice

σ^2	Optimal Instrument Value		Expected Welfare Gain		Expected Social Loss		Expected Net Welfare Gain		Percent Difference in Expected Net Welfare Gain
	P*	**q***	**P**	**Q**	**P**	**Q**	**P**	**Q**	
0.208	1233.99	3.26	3,674	3,264	6,181	4,375	(2,507)	(1,111)	-125.7%
0.203	1260.22	3.27	3,695	3,270	5,475	4,287	(1,780)	(1,017)	-75.0%
0.198	1285.05	3.28	3,713	3,276	4,856	4,197	(1,143)	(921)	-24.1%
0.193	1308.56	3.28	3,727	3,282	4,313	4,107	(585)	(825)	29.1%
0.188	1330.81	3.29	3,739	3,288	3,835	4,016	(96)	(728)	86.8%
0.183	1351.88	3.29	3,749	3,294	3,414	3,925	334	(631)	152.9%

Initially, neither instrument leads to (expected) net social welfare gains; the best that can be said is that the quantity instrument is less bad, because the expected loss is lower. As the variance decreases, the price instrument gains the advantage, although until we reach $\boldsymbol{\sigma^2 = 0.183}$ there is no instrument expected to lead to positive net gains in social welfare. As the variance shrinks both instruments see lower social losses, but only the price instrument leads to a positive outcome.

Although we once again observe a change in the optimal instrument choice as we vary the level of uncertainty, note that the intuition is a bit different than in the case of the comparing two quantity instruments or two price instruments. The classic price versus quantity instrument comparison assumes a

[8] We assume parameter values a=2,000, b=20, α=500, $\beta=\beta_1=\beta_2$=100, and $\sigma^2 = \sigma_1^2 = \sigma_2^2$.

single marginal cost function, eliminating the tradeoff between lower marginal cost and greater uncertainty inherent in comparing two instruments of the same type. In this context, what is driving the result is the difference between the (expected) level of abatement under the quantity and price instrument, q^* and $E[\hat{q}(P^*, u)]$ respectively. As the level of uncertainty decreases, $|q^* - E[\hat{q}(P^*, u)]|$ increases, such the instrument of choice changes from a quantity instrument to a price instrument.

This result differs from that reported by Weitzman (1974) and others, in that the optimal instrument choice is no longer simply a function of the relative magnitude of b and β. Given our use of a multiplicative error term, however, note that the slope of the marginal cost function in our model is a function of β and the error term, u. Although we distinguish between these two factors, the optimal instrument choice still depends generally on the relative slopes of the marginal benefit and marginal cost functions. Thus, our findings are not a contradiction, but an extension, of the Weitzman (1974) model.

4. Conclusion

Weitzman's famous paper illustrated the problem of environmental instrument choice through a simple case where curves were linear and errors were additive, and from that he was able to make some important generalizations about instrument choice under uncertainty. But his theoretical world of linear cost and benefit curves and additive error terms does not actually provide a realistic picture of the consequences of environmental instrument choice under uncertainty. In reality, costs, benefits, and the effects of uncertainty are nonlinear, and consequently the choice of pollution control regime is more nuanced than the one posited in linear models. In fact, the model and sensitivity analyses here have demonstrated that there is often a very fine line determining the expected net social benefits among pollution control instruments. Changes in the variance of the error term can cause changes in parameter values that alter conclusions as to which instrument will yield the highest expected net benefits. This calls in fact for comparison not just between instruments that are generically based on price versus those that are quantity based, but also among the various alternative versions of price and quantity regimes. There is no simple answer unless the curves are simple, which studies have shown, they are unlikely to be. Nonlinearity of both the underlying marginal cost and benefit curves and of the error term raise important questions, not so much about earlier theory but of applications of that theory to policy.

Of course, we realize that it is often a difficult task to analyze the situational components of a pollution control regime ex ante, to construct the curves that define costs and benefits, and then to compare the social welfare effects of that regime versus alternatives. But to engage in such a comparative institutional analysis seems an inevitable conclusion to the model developed in this paper. If the goal is to maximize social welfare only a realistic appraisal of the net social benefits will produce something close to the optimal choice—or at least, to the best, second-best outcome. As Demsetz (1969,1) observes, "Users of the comparative institution approach attempt to assess which alternative real institutional arrangement seems best able to cope with the economic problem; practitioners of this approach may use an ideal norm to provide standards from which divergences are assessed for all practical alternatives of interest and select as efficient that alternative which seems most likely to minimize the divergence." This seems achievable with respect to environmental instrument choice only if the model one starts with embodies the characteristics of the real world. Our analysis has proposed a model for such comparative analyses to enable policymakers to choose the instrument that is most likely to improve social welfare in the circumstances, and we look to future research and policy analysis to test the efficacy of our approach in the practice of pollution control instrument choice.

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