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Measurement and Limits in the Principia, Section 10
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This paper has two related aims: first, to elucidate the methodological sophistication displayed in Newton's treatment of constrained motion in Section 10 of the Principia, and second, to discuss measurement and limitcase reasoning more generally, drawing on our reading of Newton.
Section 10 of Book 1 of the Principia includes 10 theorems regarding constrained motion – motion on inclined planes, more general constraint surfaces, and the oscillating motion of pendulums – that significantly extend earlier results obtained by Galileo, Huygens, and others. These topics were of central importance to the history of mechanics before and after the Principia. Newton's treatment differs from the earlier mechanical tradition in two striking ways: first, pathwise independence of acquired velocity follows directly from the Laws of Motion, rather than being assumed as a separate principle; second, Galilean gravity, a force with constant magnitude directed along parallel lines, is replaced with gravity treated as a centripetal force. Newton proved a more general version of the key result of Huygens's Horologium: he stated a necessary and sufficient condition for isochrony, and established that oscillations along a (generalized) cycloid are isochronous for a force law varying as f (r)/r .
The results of Section 10 are particularly striking when one considers why Newton pursued them and how they relate to the Principia as a whole. This section is where Newton considers limiting cases most carefully, and thus it illustrates an under-appreciated aspect of his methodology. We will argue that Newton's work in Section 10 establishes that the central results of the Horologium survive the conceptual transition from Huygensian mechanics to Newton's more general framework. The fact that Huygens's results can be recovered as well-defined limit cases justifies Newton's reliance on these results, most prominently Huygens's measurement of surface gravity. In addition, Newton's careful treatment insures that evidence in favor of Galilean mechanics carries over to the more general theoretical framework of the Principia, and there will be no grounds for objecting to the new physics on the basis of earlier results regarding constrained motion.
Turning to more general questions, our discussion of Newton's method addresses two philosophical issues. First, Newton's approach to measurement requires that a real system used to measure a quantity can approach
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ideal precision in specific circumstances. Assessing whether the requirement holds depends on the theory itself, not on external stipulations regarding observability and measurement. In this case, Newton showed that the actual motions of a cycloidal pendulum bob approximate a motion that would be exactly periodic in specific circumstances, and he further quantified departures from isochronism. We will illustrate the importance of considerations along these lines by contrasting this case with an example in which this requirement does not hold. Second, Newton's results in Section 10 make it possible to characterize the relationship between Galilean and Newtonian mechanic quite clearly. Galileo's theory is a suitable approximation of Newtonian theory in a specific circumstance, namely near the surface of the earth (on the assumption that it has uniform density). The limiting relationship between theories is not simply characterized as holding between fundamental equations, but instead depends on the descriptions of particular situations. Furthermore, various

lawlike relationships in Galilean mechanics retain their lawlike force, as Newton's limit-case reasoning shows. More generally, we will argue that such limit-case reasoning provides a convincing reply to (one aspect of) Kuhnian worries regarding incommensurability.