Effect of Light Fermions on the Confinement Transition in QCD-Like Theories

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The dependence of the confinement transition parameters on the fermion content provides information on the mechanism of confinement. Recent progress in lattice gauge theories has allowed us to study it for a light flavor number $N_f \sim O(10)$ and found this transition to shift toward significantly stronger coupling. We propose an explanation for that: light fermions can occupy the chromomagnetic monopoles, via zero modes, making them "distinguishable" and unsuitable for Bose-Einstein condensation. Such dilution of unoccupied monopoles is compensated by stronger coupling that makes them lighter and more numerous. We also suggest that flavor-carrying quark-monopole objects account for the density beyond the quark Fermi sphere seen in the cold dense phase of $N_c = 2$ lattice QCD.

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1. The color confinement of quantum chromodynamics (QCD) remains one of the most outstanding puzzles of the standard model, in spite of intense studies via lattice QCD simulations [1]. By virtue of asymptotic freedom, at high temperature, the effective coupling is weak and the matter is in a deconfined phase known as the quark-gluon plasma, or QGP [2]. When T is lowered, as happened after the Big Bang or in heavy ion collisions, the effective coupling grows. At certain critical temperature T_c, a transition into the confined hadronic world occurs [3,4]. Physics had many examples in which the dependence of T_c on some parameters had offered crucial insights, e.g., when the T_c of superconductivity had shown an isotope mass dependence. Our strategy is similar: examining how this transition depends on the fermion representation and flavor number N_f may lead to insights about the mechanism of confinement.

Our phenomenological input comes from the lattice studies. A (very incomplete) list of those ranges from the well studied region of $N_f = 2, 3$ (see, e.g., Refs. [5,6]) to the recent extension toward $N_f = 8$ [7,8] and even $N_f = 12$ [9], which attracted special attention in connection with the search for a conformal regime [10]. There are also studies with adjoint [11–13] and tensor-symmetric quarks [14] in similar regime, see, e.g., reviews [15,16]. Starting from the pure gauge theory ($N_f = 0$) and increasing N_f, one finds a monotonic and persistent shift toward the stronger coupling at T_c. (The value of T_c itself is usually expressed via units customary in lattice community, which fixes the $T = 0$ string tension to the same real world value. We will not use such units as they confuse the comparison across different QCD-like theories.) The absolute magnitude of the gauge coupling constant β_c at the transition temperature T_c, instead, bears more direct information. For doing so, we evolve the critical lattice coupling $\beta_L = 2Nc/g^2$ at the lattice scale a (by two-loop running) to the scale $1/T_c = N_\tau a$:

$$\left(\frac{2N_c}{\beta_c}\right)^{-(b_1/b_0^2)} e^{-[(4\pi)^2/2N_c b_0]\beta_c} = N_\tau^2 \left(\frac{2N_c}{\beta_L}\right)^{-(b_1/b_0^2)} e^{-[(4\pi)^2/2N_c b_0]\beta_L}, \quad (1)$$

where b_0, b_1 are the usual β-function coefficients. In Fig. 1, we've collected such β_c values for theories with varied N_f at $N_c = 3$ from various lattice simulations. Although the qualitative trend is clear, quantitatively, it is still hard to compare the works of different lattice groups even for the same theory due to, e.g., difference in the actions used. Only the recent data from $N_f = 0$ to $N_f = 12$ from the same group [9] (shown as blue boxes in Fig. 1) allow for direct comparison. (These results are for chiral restoration, which for the fundamental quarks is believed to trace the deconfinement rather closely.) The main observation from Fig. 1 is that the critical coupling β_c changes by a substantial factor when the flavor number increases from $N_f = 0$ to $N_f = 12$. Similarly, the $N_c = 2$ theory with two adjoint fermions flows into the coupling at the infrared fixed point $1/g_*^2 = 0.20(4)(3)$ [13], about a factor 4 stronger than the critical one at the deconfinement of the $N_c = 2$ pure gauge theory.

2. Let us now turn to the mechanism of confinement. 't Hooft and Mandelstam suggested a "dual superconductor" model [17] relating it to the Bose-Einstein condensation (BEC) of chromomagnetic monopoles. For reviews of those ideas at $T = 0$ in lattice context see, e.g., Ref. [1]. Only recently, it was realized that if monopoles are indeed the emergent excitations one should better study them in a "normal" phase. Furthermore, in the region right above the transition $T > T_c$, such monopoles should be the dominant

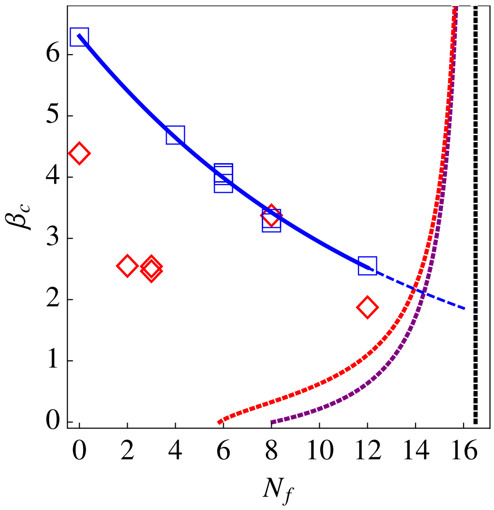

FIG. 1 (color online). Dependence of the critical lattice coupling β_c at scale T_c versus the number of fundamental quark flavors N_f in QCD-like theories. Blue boxes are from Ref. [9] with near-coincident boxes being lattice data for the same N_f with different N_τ which demonstrate lattice spacing consistency. Red diamonds are from various other literature. The thick blue line is the fitting curve, extended as dashed blue line beyond $N_f = 12$. The black, purple, and red dashed curves on the right are lines for vanishing beta function at one, two, three-loop levels.

thermal degrees of freedom "ready" for BEC [18,19]. Lattice and model studies have shown that the effective coupling of the "magnetic plasma" does run as $1/g$, inversely to the electric [20,21]. Consequences of the magnetic scenario help to understand other lattice results [22], as well as, heavy ion experiments [23,24]. This study is based on the scenario that confinement occurs as Bose condensation of monopoles that are the dominant physical degrees of freedom in the plasma near T_c.

How can the light fermions affect the monopoles? It is known that the light fermions can become attached to them. The so-called "fermionic zero modes" are specific bound states in which positive kinetic energy of localization exactly cancels the magnetic-moment-field interaction. (Their existence and number N_M are required by the topological index theorems, and thus, insensitive to any perturbative monopole deformations.) While such states may still be bosons, they carry flavor indices due to the fermions and are "distinguishable", thus, not contributing to the BEC of "unoccupied" monopoles.

These zero modes are known explicitly for 't Hooft–Polyakov monopoles [25] which are present in gauge theories with adjoint scalars, such as in $\mathcal{N} = 2$ supersymmetric gauge theories. Since flipping the charge and the spin leads to the same Dirac equation, antiquarks also have the same zero modes. Furthermore, for each of these zero mode states, it can be either populated or not, so the number of totally available states grows exponentially $\sim 2^{2N_f N_M}$. Spectroscopy of those states in the supersymmetric setting was developed in 1990's, see, e.g., Refs. [26,27]. "Magnetic supermultiplets" have been explicitly checked for two famous conformal theories, the $\mathcal{N} = 4$ SYM and the $\mathcal{N} = 2$ SQCD with $N_f = 4$: in both cases, one finds exactly the same set of spins or multiplicities as that in the original electric one. (Since the electric and magnetic formulations of those theories are the same, except $g \to 1/g$, their coupling cannot run at all.) There is one zero mode for the fundamental while two for the adjoint (Dirac) fermions in $SU(2)$ case: so these monopole-single-fermion states cast into spin 0 and $\frac{1}{2}$ objects, respectively. While in the static case zero mode, states are degenerate with the pure monopole, it is not so for the dynamical lattice monopoles with nonstatic paths.

3. The BEC criteria for an interacting boson ensemble was proposed by Feynman 50 years ago for the study of liquid ^{4}He [28] and was recently generalized by the analysis of Cristoforetti and Shuryak [29]. In the finite-T description with periodic paths, there appear "k-clusters" of bosons interchanging their initial (at Matsubara time 0) and final (at Matsubara time $\beta = 1/T$) positions. Those clusters can be depicted as "Feynman polygons" with k points. Their probability depends on two competing factors, suffering from suppression due to the extra action $\exp(-kS_{\rm ex})$, while benefiting from large combinatory number of k-polygons. The balance point marks the onset of condensation by divergence of the sum over the k-clusters. This method has been used in Ref. [30] where it was shown that lattice monopoles do have the BEC transition exactly at T_c. See also other studies of macroscopic clusters (or "percolation") such as Refs. [19,20]. According to Feynman, quantitative BEC condition is a *universal* critical value of the extra exchange action per particle, which in three spatial dimensions is given by

$$S_{\rm ex} \leq S_c \approx 1.655. \tag{2}$$

Upon fulfilling Eq. (2), long sequences of "hopping" bosons will occur, creating a macroscopic "supercurrent". Its validity for interacting systems is demonstrated in Ref. [29].

Let us now apply the above criteria to the monopole condensation in QCD-like theories. The minimal exchange action $S_{\rm ex}$ for two nearest-neighbor bosons that are separated by a typical distance $d = n^{-1/3}$ (with n the number density) during the Matsubara time from $\tau = 0$ to $\tau = \beta = 1/T$ could be estimated as

$$S_{\rm ex} = m^*\sqrt{\beta^2 + d^2} - m^*\beta + S_V \tag{3}$$

with an explicitly written kinetic term, containing an effective mass m^*, and the implicit potential term S_V. When close to condensation, the monopoles are very dense, with typical spatial separation d comparable or smaller than the inverse temperature β, therefore. justifying a further approximation of the above expression: $S_{ex} \approx \frac{1}{2} m^* T d^2 + S_V = \frac{1}{2}(\frac{m^*}{T})(\frac{n}{T^3})^{-(2/3)} + S_V$. The term $S_V = \int_0^{1/T} V[r(\tau)] d\tau$ is related to the ratio of the intermonopole interaction potential and T, also known as classical plasma coupling $\Gamma_M \sim \langle V \rangle / T \sim S_V$. As shown in Fig. 3(b) of Ref. [18], at high T where the "magnetic scaling" $d \sim 1/(g^2 T)$ and $g_{\text{magnetic}} \sim 1/g$ works, this ratio Γ_M does not depend on the coupling or T and is a constant ≈ 5, while close to condensation, it decreases to a value of about 1 as $T \to T_c$ for $N_f = 0$. The following onset condition will then be applied for the monopole condensation in QCD-like theories for the rest of our analysis:

$$\left(\frac{m^*}{T}\right)\left(\frac{n}{T^3}\right)^{-(2/3)} \le \tilde{S}_c \equiv 2(S_c - S_V). \quad (4)$$

The constant $\tilde{S}_c$ is of order one and its precise value is not needed as long as its N_f-dependence is negligible.

4. Now, how would the transition get affected by adding light fermions? As already pointed out, the fermions can be attached to some of the monopoles via zero modes and effectively reduce the number of identical monopoles. Consider a monopole with one flavor of light quark added: for each of its allowed zero modes, there is a probability for it to be occupied by a fermion or not. Let us assume the ratio of the probabilities (occupied/unoccupied) to be f (a kind of zero-mode fugacity), we then see that the overall probability for a monopole (with N_M number of zero modes for *each fermion flavor*) to stay as a "pure" monopole is simply $1/(1+f)^{2N_M}$ (with the factor of 2 accounting for both quark and anti-quark contributions for Dirac fermions). So, effectively, the available density for BEC condensation will be $n/(1+f)^{2N_f N_M}$. Combined with the BEC condition in Eq. (4), we obtain

$$\left(\frac{m^*}{T}\right)\frac{(1+f)^{4N_f N_M/3}}{(n/T^3)^{2/3}} \le \tilde{S}_c. \quad (5)$$

This implies that with increasing N_f, the density n has to increase and mass m^* to decrease, correspondingly, so as to reach the same condensation condition. This pushes the transition to stronger coupling, therefore, explaining the N_f-dependence of the critical coupling in Fig. 1.

To make a semiquantitative estimate, we use the following magnetic scaling relations that connect the monopole mass and density with gauge coupling: $m^*/T \sim 1/g$ and $n^{1/3}/T \sim g^2$ [18,20,31]. Combined with the above condition, we obtain the critical gauge coupling for monopole condensation to be: $g(N_f) = g_0(1+f)^{4N_f N_M/15}$, where g_0 is the corresponding critical coupling for pure gauge case. This can be further converted to the lattice coupling $\beta = 2N_c/g^2$:

$$\beta_c(N_f) = \beta_0(1+f)^{-8N_f N_M/15}. \quad (6)$$

As a concrete example, let us focus on the case of fundamental fermions (with $N_M = 1$). The observable we examine is the critical (lattice) gauge coupling, $\beta_c \equiv \beta(T = T_c) = 2N_c/g^2(T_c)$ as a function of N_f, as shown in Fig. 1. In particular, we concentrate on the data for $N_f = 0$ to $N_f = 12$ from the same group [9], shown in Fig. 1 as blue boxes. We've used the above Eq. (6) to make a fit for these data (blue boxes) and obtained the optimal value $f \approx 0.154$: the fitting curve is shown as the thick blue line in Fig. 1. Our model formula in Eq. (6) with one parameter nicely describes all the data points in Ref. [9] from $N_f = 0$ to $N_f = 12$. The suppression factor f of monopole-quark as compared with pure monopole may be understood as follows: the monopole-quark has color-electric charge, and in the near-T_c plasma, it was known from previous studies [22] that the electric particles are heavier than the magnetic particles by roughly $\Delta M \sim 2T_c$, thus, leading to a suppression factor $\sim e^{-\Delta M/T} \sim 0.135$ that is fairly close to $f \approx 0.154$.

For completeness, we have also displayed in Fig. 1 the lattice results from various other groups [5–8,10]. These are shown as red diamonds. Admittedly, there are uncertainties due to different lattice actions and ambiguities associated with the possible differences between chiral and confinement transitions, which shall all be sorted out in the future lattice simulations. Nevertheless, the qualitative trend of decreasing lattice coupling with N_f is well in line with the data in Ref. [9] and with our model formula.

The proposed mechanism suggests an approximate "$N_f N_M$ scaling", e.g., that the effect of adding N_c fundamental fermions is about as large as adding one adjoint. Essentially, for a given gauge group, the critical couplings shall fall on one single curve when plotted against the combination $N_f N_M$ with various fermion representations and flavor numbers. Such scaling can be readily tested.

5. To further "probe" the fermions' effect on confinement transition, one may turn on a quark chemical potential μ_q and see how the critical coupling changes accordingly. With the presence of a small μ_q, there will be a difference if a zero mode is occupied by a quark or an anti-quark. This effect can be incorporated into the present model by replacing $(1+f)^2$ in Eq. (6) by $(1+fe^z)(1+fe^{-z})$ with $z \equiv \mu_q/T$. We then obtained:

$$\frac{\beta_c(N_f, z)}{\beta_c(N_f, z=0)} = 1 - \frac{4N_f N_M}{15}\frac{f}{(1+f)^2} z^2 + \hat{O}(z^4) \quad (7)$$

for small chemical potential $\mu_q \ll T$. Present lattice simulations, though not capable of handing finite μ directly due to the sign problem, are actually able to extract such dependence (or Taylor coefficients in μ_q/T expansion) [32] and test the above prediction. Similar estimates can be made for the dependence on the isospin chemical potential as well as the axial chemical potential.

The existence of these monopole-quark states also implies a contribution from them to the thermal fluctuations of the conserved charges (e.g., baryon number) they carry, thus, contributing to the quark number susceptibilities [33,34]. A monopole may have one, two, or more zero modes occupied, but since the extracted $f \approx 0.154$ is small, the main contribution would be from the monopoles with one single zero-mode quark. The net baryonic density from these monopole-quark states (with the presence of a small quark chemical potential $z = \mu_q/T$) can be estimated as $n_{m-q}/T^3 \approx (n/T^3)(e^z - e^{-z})f/[(1 + fe^z)(1 + fe^{-z})]^{N_f N_M}$. This yields a contribution to the quark number susceptibilities $\chi_i = \partial^{i-1}(n_{m-q}/T^3)/\partial(\mu_q/T)^{i-1}$ as $\chi_2^{m-q} \approx \frac{n}{T^3} \frac{2f}{(1+f)^{N_f N_M}} \sim 0.4$–$0.8$, where we used lattice results for a total monopole density near T_c to be $n/T^3 \sim 2$–4 and $N_f N_M = 3$. The number obtained makes a significant fraction of the lattice results around T_c [33]. Higher-order susceptibilities can be estimated similarly, and the multi-quark-monopole states with higher charges may be important there. Similar effects can be estimated along this line also for the isospin and electric charge fluctuations.

6. It is well-known that the $N_c = 2$ theory is a very special case, with extra symmetry between quarks-antiquarks and mesons-diquarks. It also allows the finite density lattice simulations without the "sign problem". Lattice study of this theory was recently extended to the low-T finite-μ region with $N_f = 2$, 4 quarks by Hands *et al.* [35]. The quark density (per flavor) shown in their Fig. 3 displays a number of features: (i) a structure at $\mu \approx m_\pi/2$ as predicted by the rotation from the $\bar{\psi}\psi$ to diquark condensate [36,37]; (ii) the usual quark Fermi sphere at higher μ; and (iii) an unexpected growth of quark density at still higher μ to about twice the value as expected from the Fermi sphere. The deconfinement as per the Polyakov loop appears concurrent with (iii).

We now propose that this extra quark density in (iii) is due to the condensate of the *monopole-quark* states. The high quark chemical potential strongly favors states with quark numbers and efficiently converts the pure monopoles into monopole-quark states, thus, explaining the deconfinement at about the same density. At such low T, the dominant monopole-single-quark objects, being bosonic, would appear mostly as a condensate like the diquarks. Assuming standard effective potential with a repulsive binary interaction $V_{\rm int} = \lambda n^2/4$, one gets the condensate density growing linearly with μ, $n_{\rm BEC} \sim (\mu - m)/\lambda$ at $\mu > m$, which is consistent with observations of Ref. [35]. Furthermore, a similar density per flavor for both $N_f = 2$ and $N_f = 4$ is consistent with our view that these objects are dominantly states with only a *single* quark per monopole. States with multiple quark may become relevant at even higher density though. This proposal should and can be checked in many ways. Direct monopole-flavor correlations can be seen in the configurations of these simulations. One may also find the evaporation of this BEC as T is above a certain critical value, into the Bose gas of such monopole-quark states.

7. While we believe the main idea is robust, the model here is, admittedly, crude and intended to lead toward further studies and direct tests by dedicated lattice data. In particular, the correlation between the monopole line with flavor-carrying fermionic operators can be used to measure the probability for quarks "riding" on monopoles. The discussion here is limited to the usual confinement, due to BEC of "empty" monopoles. More exotic objects, bosonic monopoles *with* quarks or even two monopoles bound by quarks, can in principle undergo BEC as well: see recent discussion in Ref. [38].

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Note added in proof.—After submitting the paper we became aware of the paper [39] discussing a similar problem in 2 + 1 dimensions. While the phase transition in this case is not of Bose condensation type and monopoles are substituted by vortices, the role of fermionic zero modes is similar and it also leads to reduced transition temperature.

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