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Quantification, Time, and Necessity

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The fundamental assumption of a logic of actual and possible objects is that the concept of *existence* is not the same as the concept of *being*.[1] Thus, even though necessarily whatever exists has being, it is not necessary in such a logic that whatever has being exists; that is, it can be the case that there *be* something that does not exist. No occult doctrine is needed to explain the distinction between existence and being, for an obvious explanation is already at hand in a framework of tense logic in which being encompasses past, present, and future objects (or even just past and present objects) while existence encompasses only those objects that presently exist. We can interpret modality in such a framework, in other words, whereby it *can* be true to say that some things do not exist. Indeed, as indicated in Section 3, infinitely many different modal logics can be interpreted in the framework of tense logic. In this regard, we maintain, tense logic provides a paradigmatic framework in which *possibilism* (i.e., the view that existence is not the same as being, and that therefore there can *be* some things that do not exist) can be given a logically perspicuous representation.

Tense logic also provides a paradigmatic framework for *actualism* as the view that is opposed to possibilism; that is, the view that denies that the concept of existence is different from the concept of being. Indeed, as we understand it here, actualism does not deny that there can be names that have had denotations in the past but that are now denotationless, and hence that the statement that some things do not exist can be true in a semantic metalinguistic sense (as a statement about the denotations, or lack of denotations, of singular terms). What is needed, according to actualism, is not that we should distinguish the concept of existence from the concept of being, but only that we should modify the way that the concept of existence (being) is represented in *standard* first-order predicate logic (with identity). A first-order logic of existence should allow for the possibility that some of our singular terms might fail to denote an existent object, which, according to actualism, is only to say that those singular terms are denotationless rather than that what they denote are objects (beings) that do not exist. Such a logic for actualism amounts to what nowadays is called *free logic*.

In what follows we shall first formulate a logic of actual and possible objects in which existence and being are assumed to be distinct second-level concepts represented by the different quantifiers $\forall^a$ and $\forall$, respectively (with $\exists^a$ and $\exists$ defined in terms of $\forall^a$ and $\forall$ in the usual way).[2] The free logic of actual objects, where existence is not distinguished from being—but also where it is not assumed that all singular terms denote—is then described as a certain subsystem of the logic of actual and possible objects. Of course, it is only from the perspective of possibilism that the logic of actual objects is to be viewed as a *proper* subsystem of the logic of being, since, according to possibilism, the logic of being includes the logic of possible objects as well. From the perspective of actualism, the logic of actual objects is all there is to the logic of being.

Both the free logic of actual objects and the logic of actual and possible objects are formulated in Section 1 in such a way as not to presuppose any further encompassing framework, such as tense or modal logic, where these logics find their most natural applications. We describe a framework for tense logic in Section 2, where we distinguish an application of the logic of actual and possible objects from an application of the free logic of actual objects *simpliciter*. In Section 3 we indicate how different modal logics can be interpreted in terms of tense logic, and, in that regard, how an application of the logic of actual and possible objects in modal logic can be distinguished from an application of the free logic of actual objects *simpliciter*. In Section 4 we indicate the kinds of qualifications that are required in the statement of the laws involving the interplay of quantifiers, tenses, and modal operators. The three tense-logical frameworks for which these laws are stated provide logically perspicuous representations of the differences between actualism and possibilism (including a restricted version of temporal possibilism where determinate being includes only what did or does exist, leaving the future as an indeterminate realm of nonbeing). Tense logic, as these developments indicate, is indeed a paradigmatic framework in which to formally represent the differences between actualism and possibilism.

1. A Logic of Actual and Possible Objects

We take a *language* to be a set of predicate and operation expressions of arbitrary (finite) degrees. The set of *terms* and *atomic formulas* of a language are understood to be in accordance with their usual definitions within standard first-order logic with identity. We use two quantifiers—though only one style of individual variable—one for quantification over possible objects, or *possibilia*, and the other for quantification over actual objects. The *formulas* of a language L are those objects that belong to every set K containing the atomic formulas of L and such that $\sim\phi$, $(\phi \rightarrow \psi)$, $\forall x\phi$, $\forall^a x\phi \in K$ whenever $\phi, \psi \in K$ and x is a variable.

By a *model* suited to a language L we understand a 3-tuple $\langle A, B, R\rangle$, where (1) A, called the *universe* of the model, is included in B, the domain or set of possibilia of the model, (2) B is nonempty, and (3) R is a function with L as domain and such that for all n, π, δ, (i) if n is a natural number and π is an n-place predicate expression in L, then $R(\pi) \subseteq B^n$, and (ii) if n is a natural number and δ is an n-place operation expression in L, then $R(\delta) \in B^{B^n}$. Satisfaction and truth are defined in the usual Tarski manner, except that the satisfaction clause for the actual quantifier applies only to the universe of the model in question, whereas the satisfaction clause for the possible quantifier covers the entire domain of discourse, that is, the set of *possibilia* of the model. Finally, ϕ is said to be *logically true* if for some language L of which ϕ is a formula, ϕ is true in every model suited to L.

Where ϕ, ψ, χ are formulas, x, y are variables, and ζ, η are terms, universal generalizations of all instances of the following axiom schemas, together with modus ponens as the only inference rule, yield all and only the logical truths:

(A1) $\phi \rightarrow (\psi \rightarrow \phi)$

(A2) $[\phi \rightarrow (\psi \rightarrow \chi)] \rightarrow [(\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \chi)]$

(A3) $(\sim\phi \rightarrow \sim\psi) \rightarrow (\psi \rightarrow \phi)$

(A4) $\forall x(\phi \rightarrow \psi) \rightarrow (\forall x\phi \rightarrow \forall x\psi)$

(A5) $\forall^a x(\phi \rightarrow \psi) \rightarrow (\forall^a x\phi \rightarrow \forall^a x\psi)$

(A6) $\phi \rightarrow \forall x\phi$, where x is not free in ϕ

(A7) $\forall x\phi \rightarrow \forall^a x\phi$

(A8) $\exists x(\zeta = x)$, where x does not occur in ζ

(A9) $\forall^a x \exists^a y(x = y)$

(A10) $\zeta = \eta \rightarrow (\phi \rightarrow \psi)$, where ϕ, ψ are atomic formulas and ψ is obtained from ϕ by replacing an occurrence of η by ζ

This sound and complete axiom set, it should be noted, does not involve the notion of the proper substitution of a term for a variable (and even the notion of bondage and freedom in (A6) can be replaced by the notion of occurrence *simpliciter*). If we restrict ourselves to *standard formulas* (i.e., those in which the actual quantifier does not occur), then (A1)–(A4), (A6), (A8), and (A10) yield all and only the standard logical truths. (The completeness of the latter system is due to R. Montague and D. Kalish, their result being obtained by a modification of an original formulation by A. Tarski.) If we restrict ourselves to *E-formulas* (i.e., those in which the possible quantifier does not occur), then (A1)–(A3), (A5), (A9), (A10), together with the schemas $\phi \rightarrow \forall^a x\phi$, where x is not free in ϕ, and $(\zeta = \zeta)$, where ζ is an arbitrary term, yield all and only those logical truths that are E-formulas. Thus, whereas the standard formulas that are logically true constitute the *logic of possible objects simpliciter*, the E-formulas that are logically true constitute the *logic of actual objects simpliciter*. All of the formulas together (i.e., the standard formulas, the E-formulas, and the formulas that contain both the possible and the actual quantifiers) that are logically true constitute *the logic of actual and possible objects.*

Note that whereas by (A8) every term denotes a possible object (i.e., that there *be* an object denoted by that term), it is not true that every term denotes an actual (existent) object. In the logic of actual objects *simpliciter* (i.e., as interpreted from the perspective of actualism), this means that some terms may be denotationless. Also, whereas the law of universal instantiation,

$$\forall x\phi \rightarrow \phi(\zeta/x)$$

is logically true for the possible quantifier without any qualification, only the following qualified version is logically true for the actual quantifier:

$$\exists^a y(\zeta = y) \rightarrow [\forall^a x\phi \rightarrow \phi(\zeta/x)]$$

where y does not occur in ζ and ζ is free for x in ϕ. In addition to the second-level concept of existence represented by $\forall^a$ as a primitive logical constant, the first-level concept of existence is definable as follows (where x and y are distinct variables):

$$E!(x) =_{df} \exists^a y(x = y)$$

2. A Completeness Theorem for Tense Logic

In applying the logic of actual and possible objects to the framework of tense logic we add to our logical constants the unary formula operators P and F (read, respectively, as 'It was the case that' and 'It will be the case that') and define the *tensed formulas* of a language L to be those objects that belong to every set K containing the atomic formulas of L and such that $\sim\phi$, $(\phi \rightarrow \psi)$, $\mathrm{P}\phi$, $\mathrm{F}\phi$, $\forall x\phi$, $\forall^{a}x\phi \in K$ whenever $\phi, \psi \in K$ and x is a variable. (We assume & and $\vee$ to be defined in terms of $\sim$ and $\rightarrow$ in the usual way.)

Where $\mathfrak{B}$ is a model (as defined earlier), we set $\mathfrak{U}_{\mathfrak{B}}$ as the universe of $\mathfrak{B}$ and $\mathfrak{P}_{\mathfrak{B}}$ as the set of possibilia of $\mathfrak{B}$. If L is a language and R is a relation (set of ordered pairs), then we say that $\mathfrak{A}$ is an R-history with respect to L if there are a nonempty index set I included in the field of R and an I-termed sequence $\mathfrak{B}$ of models suited to L such that (i) $\mathfrak{A} = \langle R, \mathfrak{B}\rangle$; (ii) I is identical with the field of R if I has more than one element; (iii) $\bigcup_{j \in I} \mathfrak{U}_{\mathfrak{B}_j} \subseteq \mathfrak{P}_{\mathfrak{B}_i}$, for all $i \in I$; and (iv) $\mathfrak{P}_{\mathfrak{B}_i} = \mathfrak{P}_{\mathfrak{B}_j}$, for all $i, j \in I$. Where $\langle R, \mathfrak{B}\rangle$ is such a history, we take the members of the set I indexing $\mathfrak{B}$ to be the *moments* of the history and R to be the *earlier-than* relation ordering those moments. The structure of R is the temporal structure of the history; for example, it may have a beginning, or an end, both, or neither, and it may be discrete, dense, or continuous, and so on. Condition (iii) stipulates that whatever is actual at one time or another in a history is a possible object of that history. Condition (iv) states the requirement that whatever is a possible object at one moment of a history is a possible object at any other moment of that history.

Satisfaction and *truth in* a history $\langle R, \mathfrak{B}\rangle$ *at* a given moment i of $\langle R, \mathfrak{B}\rangle$ is understood, except where the tense operators are involved, as satisfaction and truth in the model $\mathfrak{B}_i$. The satisfaction clauses for the tense operators have the obvious references to the models associated with the moments before and after the moment i. *Validity in* a history is defined as truth at all times in that history.

If R is a relation, then ϕ is said to be *R-valid* if ϕ is a tensed formula of some language L such that for each R-history $\mathfrak{A}$ with respect to L, ϕ is valid in $\mathfrak{A}$. A tensed schematic formula ϕ is understood to *characterize* a class K of relations if for each relation R, ϕ is R-valid if, and only if, $R \in K$. Special schematic formulas can be shown to characterize various classes of relations.

In regard to the characterization of logical truth as extended to all tensed formulas, we restrict our considerations—in deference to this fundamental feature of time (or rather of *local* time)—to *serial* histories (i.e., histories whose temporal ordering is a series) and say that ϕ is *tense-logically* true if for some language L of which ϕ is a tensed formula, ϕ is valid in every serial history suited to L. Equivalently, a tensed formula ϕ is tense-logically true if, and only if, for every series R, ϕ is R-valid. Given as inference rules modus ponens, universal generalization (for $\forall$), and

(i) if $\vdash_t \phi$, then $\vdash_t \sim P \sim \phi$
(ii) if $\vdash_t \phi$, then $\vdash_t \sim F \sim \phi$

then all instances of the following schemas, together with all instances of (A1)–(A10) of Section 1 (applied now to tensed formulas), yield all and only the tense-logical truths:[3]

(A11) $\sim P \sim (\phi \rightarrow \psi) \rightarrow (P\phi \rightarrow P\psi)$

(A12) $\sim F \sim (\phi \rightarrow \psi) \rightarrow (F\phi \rightarrow F\psi)$

(A13) $\phi \rightarrow \sim P \sim F\phi$

(A14) $\phi \rightarrow \sim F \sim P\phi$

(A15) $PP\phi \rightarrow P\phi$

(A16) $FF\phi \rightarrow F\phi$

(A17) $P\phi \,\&\, P\psi \rightarrow P(\phi \,\&\, \psi) \vee P(\phi \,\&\, P\psi) \vee P(\psi \,\&\, P\phi)$

(A18) $F\phi \,\&\, F\psi \rightarrow F(\phi \,\&\, \psi) \vee F(\phi \,\&\, F\psi) \vee F(\psi \,\&\, F\phi)$

(A19) $P(\psi \,\&\, F\phi) \rightarrow P(\phi \,\&\, P\psi) \vee (\phi \,\&\, P\psi) \vee (F\phi \,\&\, P\psi)$

(A20) $F(\psi \,\&\, P\phi) \rightarrow F(\phi \,\&\, F\psi) \vee (\phi \,\&\, F\psi) \vee (P\phi \,\&\, F\psi)$

(A21) $(x = y) \rightarrow \sim P \sim (x = y) \,\&\, \sim F \sim (x = y)$, where x, y are variables

If we restrict ourselves to *tensed standard formulas* (i.e., those tensed formulas in which the actual quantifier does not occur), then (A1)–(A4), (A6), (A8), (A10)–(A21) yield all and only those tense-logical truths that are tensed standard formulas. These tense-logical truths constitute the logic of possible objects *simpliciter* as applied in the framework of tense logic (for local time). On the other hand, if we restrict ourselves to *tensed E-formulas* (i.e., those in which the possible quantifier does not occur), then (A1)–(A3), (A5), (A9), (A10)–(A21), together with the schemas $\phi \rightarrow \forall^a x\phi$, where x is not free in ϕ, and $(\zeta = \zeta)$, where ζ is an arbitrary term, as well as the following inference rules (for each natural number n)

added to those already noted above (but with universal generalization for $\forall^a$ instead of $\forall$):

(iii) if $\vdash_{t^e} \sim P \sim (\phi_0 \to \sim P \sim [\phi_1 \to \cdots \to \sim P \sim (\phi_{n-2} \to \sim P \sim \phi_{n-1}) \cdots])$ and x is not free in $\phi_0, \ldots, \phi_{n-2}$, then

$$\vdash_{t^e} \sim P \sim (\phi_0 \to \sim P \sim [\phi_1 \to \cdots \to \sim P \sim (\phi_{n-2} \to \sim P \sim \forall^a x \phi_{n-1}) \cdots])$$

(iv) if $\vdash_{t^e} \sim F \sim (\phi_0 \to \sim F \sim [\phi_1 \to \cdots \to \sim F \sim (\phi_{n-2} \to \sim F \sim \phi_{n-1}) \cdots])$ and x is not free in $\phi_0, \ldots, \phi_{n-2}$, then

$$\vdash_{t^e} \sim F \sim (\phi_0 \to \sim F \sim [\phi_1 \to \cdots \to \sim F \sim (\phi_{n-2} \to \sim F \sim \forall^a x \phi_{n-1}) \cdots])$$

yield all and only those tense-logical truths that are tensed E-formulas. These tense-logical truths constitute the logic of actual objects *simpliciter* (i.e., the logic of actualism), as applied to the framework of tense logic (for local time).

In assuming that being and existence are not the same concept, possibilism does not also assume that whatever *is* (i.e., whatever has being) either did exist, does exist, or will exist, a thesis we shall call *temporal possibilism.* Formally, this thesis is stated as follows: $\forall x[PE!(x) \vee E!(x) \vee FE!(x)]$. If we add this formula as a new axiom, then to render it tense-logically true we need only require that the condition stated in clause (iii) of the definition of an R-history be an identity rather than just an inclusion. A somewhat stronger assumption than the thesis of temporal possibilism—but one that still falls short of actualism as a claim about existence and being—can be made to the effect that whatever *is* either did exist or does exist (i.e., that being covers only past or present existence), leaving future existence to the realm of nonbeing (apparently because the future is indeterminate metaphysically and not just epistemically, whereas the past and the present are at least metaphysically determinate). Quantification over past objects, as well as quantification over past and present objects, can be defined in the tense logic of actual and possible objects as follows (where $\exists^P$ and $\exists^P_P$ are defined in the usual way as the duals of $\forall^P$ and $\forall^P_P$, respectively):

$$\forall^P x\phi =_{df} \forall x[PE!(x) \to \phi]$$

$$\forall^P_P x\phi =_{df} \forall x[PE!(x) \vee E!(x) \to \phi]$$

Thus, the metaphysical thesis in question, namely that what *is* either did exist or does exist, can be stated as follows: $\forall x \exists^P_P y(x = y)$. Alternatively, instead of having the concept of being in such a framework represented by the possibilist quantifier $\forall$, we can take it to be represented directly by $\forall^P_P$ as a primitive quantifier together with $\forall^a$ for the concept of existence. A sound and complete axiom set for this system is then given by (A1)–(A3), (A5), (A9), (A10), together with the schemas

$$\forall^P_P x(\phi \to \psi) \to (\forall^P_P x\phi \to \forall^P_P x\psi)$$

$$\phi \to \forall^P_P x\phi, \text{ where } x \text{ is not free in } \phi$$

$$\forall^P_P x\phi \to \forall^a x\phi$$

$$\forall^a x \exists^a y(x = y), \text{ where } x \text{ and } y \text{ are distinct variables}$$

$$\forall^P_P x[\exists^P_P y(x = y) \ \& \ {\sim}F{\sim}\exists^P_P y(x = y)]$$

$$\forall^P_P x {\sim}P{\sim}\phi \to {\sim}P{\sim}\forall^P_P x\phi$$

$$\sim F {\sim}\forall^P_P x\phi \to \forall^P_P x{\sim}F{\sim}\phi$$

$$(\zeta = \zeta), \text{ where } \zeta \text{ is an arbitrary term}$$

and the inference rules modus ponens, universal generalization (for $\forall^P_P$), rules (i), (ii) as described above, and the counterpart of rule (iv) above using $\forall^P_P$ in place of $\forall^a$.[4]

3. Modality Within Tense Logic

It is significant that the first modal concepts to be discussed and analyzed in the history of philosophy are concepts based on the distinction between the past, the present, and the future, that is, concepts that can be analyzed in terms of the temporal modalities that are represented by the standard tense operators. Diodorus, for example, is reported as having argued that the possible is that which either is or will be the case, and that the necessary is that which is and always will be the case. Formally, the Diodorean modalities can be defined as follows:

$$\Diamond^f\phi =_{df} (\phi \lor F\phi)$$

$$\Box^f\phi =_{df} {\sim}\Diamond^f{\sim}\phi$$

Aristotle, on the other hand, included the past as part of what is possible; that is, for Aristotle the possible is that which either was, is, or will be the case (in what he assumed to be the infinity of time), and therefore the necessary is what is always the case:

$$\Diamond\phi =_{df} (P\phi \vee \phi \vee F\phi)$$

$$\Box\phi =_{df} \sim\Diamond\sim\phi$$

Where L is a language, let $\Diamond FM_L$ be the intersection of all sets K containing the atomic formulas of L and such that $\sim\phi$, $\Diamond\phi$, $(\phi \rightarrow \psi)$, $\forall x\phi$, $\forall^a x\phi \in K$ whenever $\phi, \psi \in K$ and x is a variable. We call ϕ an *S5-formula* if $\phi \in \Diamond FM_L$, for some language L. We also say that ϕ is *S5-valid* if ϕ is an S5-formula that is tense-logically true. We obtain the system we call $S5^t$ if to the axioms (A1)–(A10) of the logic of actual and possible objects we add all instances of schemas of the following forms:

($S5^t$-1) $\Box\phi \rightarrow \phi$

($S5^t$-2) $\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$

($S5^t$-3) $\Diamond\phi \rightarrow \Box\Diamond\phi$

($S5^t$-4) $(x = y) \rightarrow \Box(x = y)$, where x, y are variables

and take in addition to modus ponens and universal generalization the following inference rule:

if $\vdash_{S5^t}\phi$, then $\vdash_{S5^t}\Box\phi$

It can be shown that for each S5-formula ϕ, ϕ is a theorem of $S5^t$ if, and only if, ϕ is S5-valid—which is our completeness theorem for $S5^t$. For the logic of actual objects as applied to S5-formulas, we need only restrict the latter to those that are E-formulas (i.e., formulas in which the possible quantifier does not occur), and use only the logic of actual objects as described in Section 1 together with the axiom schemas ($S5^t$-1)–($S5^t$-4) and one new inference rule added to those of $S5^t$. That is, where $S5^t_e$ is that subsystem of $S5^t$ that is the result of replacing (A1)–(A10) of the logic of actual and possible objects by the axioms for the logic of actual objects *simpliciter* and adding to the inference rules of $S5^t$ the following,

if $\vdash_{S5^t}\Box(\phi_0 \rightarrow \Box[\phi_1 \rightarrow \cdots \rightarrow \Box(\phi_{n-2} \rightarrow \Box\phi_{n-1})\cdots])$

and x is not free in $\phi_0, \ldots, \phi_{n-2}$, then

$$\vdash_{S5^t}\Box(\phi_0 \rightarrow \Box[\phi_1 \rightarrow \cdots \rightarrow \Box(\phi_{n-2} \rightarrow \Box\forall^a x\phi_{n-1})\cdots])$$

then for each S5-formula ϕ that is also an E-formula, ϕ is a theorem of $S5^t_e$ if, and only if, ϕ is S5-valid, which is our completeness theorem for $S5^t_e$.

For the Diodorean modalities, let $\Diamond^f FM_L$, where L is a language, be the intersection of all sets K containing the atomic formulas of L and such that $\sim\phi$, $\Diamond^f\phi$, $(\phi \rightarrow \psi)$, $\forall x\phi$, $\forall^a x\phi \in K$ whenever $\phi, \psi \in K$ and x is a variable. We say that ϕ is an S4.3-formula if $\phi \in \Diamond^f FM_L$, for some language L. Also, we say that ϕ is *S4.3-valid* if ϕ is an S4.3-formula that is tense-logically true. We obtain the system we call $S4.3^t$ if we add to the axioms (A1)–(A10) of the logic of actual and possible objects all instances of schemas of the following forms:

($S4.3^t$-1) $\Box^f\phi \rightarrow \phi$

($S4.3^t$-2) $\Box^f(\phi \rightarrow \psi) \rightarrow (\Box^f\phi \rightarrow \Box^f\psi)$

($S4.3^t$-3) $\Box^f\phi \rightarrow \Box^f\Box^f\phi$

($S4.3^t$-4) $\Diamond^f\phi \,\&\, \Diamond^f\psi \rightarrow \Diamond^f(\phi \,\&\, \psi) \vee \Diamond^f(\phi \,\&\, \Diamond^f\psi) \vee \Diamond^f(\psi \,\&\, \Diamond^f\phi)$

($S4.3^t$-5) $\Diamond^f(x = y) \rightarrow \Box^f(x = y)$, where x, y are variables

($S4.3^t$-6) $\forall x\Box^f\phi \rightarrow \Box^f\forall x\phi$

and take in addition to modus ponens and universal generalization the same modal inference rules already described for $S5^t$ except for having $\Box^f$ where $\Box$ occurs in those rules. It can be shown that for each S4.3-formula ϕ, ϕ is a theorem of $S4.3^t$ if, and only if, ϕ is S4.3-valid, which is our completeness theorem for $S4.3^t$.

For the logic of actual objects as applied to S4.3-formulas (i.e., to obtain the subsystem $S4.3^t_e$ of $S4.3^t$ when the latter is restricted to E-formulas), we must first delete the axiom schema ($S4.3^t$-6), which is not an E-formula, and then replace (A1)–(A10) of the logic of actual and possible objects by the axioms of the logic of actual objects simpliciter and adopt the same modal inference rules as already described for $S5^t_e$ except for using $\Box^f$ instead of $\Box$ in those rules. Then, it can be shown that for each S4.3-formula ϕ that is also an E-formula, ϕ is a theorem of $S4.3^t_e$ if, and only if, ϕ is S4.3-valid, which is our completeness theorem for S4.3-formulas when the latter are restricted to E-formulas.

Infinitely many other modal logics can be generated in ways similar to the above by various combination of tenses (e.g., such as merely iterating new occurrences of F in the definition of the Diodorean modalities). In addition to these temporal notions of modality, the semantics for yet another can be given corresponding roughly to the idea that a formula is (conditionally) necessary (in a given history at a given moment of that history) because of the way the past has been. The semantics for this notion also yields a completeness theorem for an S5 type modal structure, and it may be used for a partial or full explication of the notions of causal modality and counterfactuals.

Finally, the semantics for yet another temporal notion of modality is available parallel to the Diodorean approach, except that the role the future plays in the Diodorean concept is altered to that of the causal future as determined by a *relativistic system* of R-histories, where R is a series. This temporal notion of modality results in an S4 type structure.[5]

4. Some Observations on Quantifiers in Tense and Modal Logic

In describing some of the theorem schemas involving quantifiers, tenses and modal operators in these different logics, we shall use $\vdash_t$, $\vdash_{t^e}$, and $\vdash_{t^P}$ to stand for *being a theorem of tense logic (for local time) with quantification over* (a) *both actual and possible objects*, (b) *over just actual objects*, and (c) *over past and present objects*, respectively. As already indicated,

$$\{\phi: \vdash_{t^e}\phi\} \subseteq \{\phi: \vdash_{t^P}\phi\} \subseteq \{\phi: \vdash_t\phi\}$$

and, therefore, we may use $\vdash_{t^e}$ to state what is provable in all three systems (a)–(c), and $\vdash_{t^P}$ for what is provable in (a) and (c). In stating some of these theorem schemas we shall also use the following counterparts of notions already defined:

$$\forall^f x\phi =_{df} \forall x[FE!(x) \rightarrow \phi]$$

$$\Diamond^P\phi =_{df} (\phi \vee P\phi)$$

$$\Box^P\phi =_{df} \sim\Diamond^P\sim\phi$$

Leibniz's Law. We assume that ζ, η are terms, ϕ is a formula, and that ψ is obtained from ϕ by replacing one or more free occurrences of ζ by free occurrences of η. Then,

(1) $\vdash_{t^e}\Box(\zeta = \eta) \rightarrow (\phi \leftrightarrow \psi)$

(2) $\vdash_{t^e}(\zeta = \eta) \rightarrow (\phi \leftrightarrow \psi)$ if ζ, η are variables

(3) $\vdash_{t^e}(\zeta = \eta) \rightarrow (\phi \leftrightarrow \psi)$ if ζ does not occur in ϕ within the scope of a past or future tense operator

(4) $\vdash_{t^e}\Box^P(\zeta = \eta) \rightarrow (\phi \leftrightarrow \psi)$ if ζ does not occur in ϕ within the scope of a future tense operator

(5) $\vdash_{t^e}\Box^f(\zeta = \eta) \rightarrow (\phi \leftrightarrow \psi)$ if ζ does not occur in ϕ within the scope of a past tense operator

Identity and Nonidentity. Although identity and nonidentity as expressed in terms of individual variables is always necessary, that is,

$$\vdash_{tc}(x = y) \to \Box(x = y), \qquad \vdash_{tc}(x \neq y) \to \Box(x \neq y)$$

the same is not true for other singular terms. The relevant qualifications are as follows, where it is assumed that x and y do not occur in ζ and η, respectively:

(6) $$\vdash_t \exists x\Box(x = \zeta) \;\&\; \exists y\Box(y = \eta) \to [(\zeta = \eta) \leftrightarrow \Box(\zeta = \eta)] \;\&\; [(\zeta \neq \eta) \leftrightarrow \Box(\zeta \neq \eta)]$$

$$\vdash_{tc} \exists^a x\Box(x = \zeta) \;\&\; \exists^a y\Box(y = \eta) \to [(\zeta = \eta) \leftrightarrow \Box(\zeta = \eta)] \;\&\; [(\zeta \neq \eta) \leftrightarrow \Box(\zeta \neq \eta)]$$

$$\vdash_{tP} \exists^P_P x\Box(x = \zeta) \;\&\; \exists^P_P y\Box(y = \eta) \to [(\zeta = \eta) \leftrightarrow \Box(\zeta = \eta)] \;\&\; [(\zeta \neq \eta) \leftrightarrow \Box(\zeta \neq \eta)]$$

Similar theorems hold when $\Box$ is uniformly replaced throughout (6) by $\Box^P$ or $\Box^f$, respectively.

Universal Instantiation. The law of universal instantiation does not hold in general in these logics without qualification. The different qualifications are as follows, where x and y are variables, ζ is a term in which y does not occur, and ζ is free for x in ϕ:

(7) $$\vdash_t \exists y\Box(y = \zeta) \to [\forall x\phi \to \phi(\zeta/x)]$$

$$\vdash_{tc} \exists^a y\Box(y = \zeta) \to [\forall^a x\phi \to \phi(\zeta/x)]$$

$$\vdash_{tP} \exists^P_P y\Box(y = \zeta) \to [\forall^P_P x\phi \to \phi(\zeta/x)]$$

(8) If either ζ is a variable or x does not occur in ϕ within the scope of a past or future tense operator, then

$$\vdash_t \forall x\phi \to \phi(\zeta/x)$$

$$\vdash_{tc} \exists^a y(y = \zeta) \to [\forall^a x\phi \to \phi(\zeta/x)]$$

$$\vdash_{tP} \exists^P_P y(y = \zeta) \to [\forall^P_P x\phi \to \phi(\zeta/x)]$$

(9) If x does not occur in ϕ within the scope of a future tense operator, then

$$\vdash_t \exists y\Box^P(y = \zeta) \to [\forall x\phi \to \phi(\zeta/x)]$$

$$\vdash_{tc} \exists^a y\Box^P(y = \zeta) \to [\forall^a x\phi \to \phi(\zeta/x)]$$

$$\vdash_{tP} \exists^P_P y\Box^P(y = \zeta) \to [\forall^P_P x\phi \to \phi(\zeta/x)]$$

(10) If x does not occur in ϕ within the scope of a past tense operator, then

$$\vdash_t \exists y \Box^f (y = \zeta) \rightarrow [\forall x\phi \rightarrow \phi(\zeta/x)]$$

$$\vdash_{te} \exists^a y \Box^f (y = \zeta) \rightarrow [\forall^a x\phi \rightarrow \phi(\zeta/x)]$$

$$\vdash_{tP} \exists^P_P y \Box^f (y = \zeta) \rightarrow [\forall^P_P x\phi \rightarrow \phi(\zeta/x)]$$

Laws of Commutation. The possible quantifier $\exists$ commutes with both the past and future tense operators and therefore with $\Diamond^f$, $\Diamond^P$, and $\Diamond$ as well. Dually, $\forall$ commutes with $\sim P \sim$ and $\sim F \sim$ and therefore with $\Box^f$, $\Box^P$, and $\Box$ as well:

(11)

$$\vdash_t P\exists x\phi \leftrightarrow \exists x P\phi, \qquad \vdash_t \sim P \sim \forall x\phi \leftrightarrow \forall x \sim P \sim \phi$$

$$\vdash_t F\exists x\phi \leftrightarrow \exists x F\phi, \qquad \vdash_t \sim F \sim \forall x\phi \leftrightarrow \forall x \sim F \sim \phi$$

$$\vdash_t \Diamond^f \exists x\phi \leftrightarrow \Diamond^f \exists x\phi, \qquad \vdash_t \Box^f \forall x\phi \leftrightarrow \forall x \Box^f \phi$$

$$\vdash_t \Diamond^P \exists x\phi \leftrightarrow \Diamond^P \exists x\phi, \qquad \vdash_t \Box^P \forall x\phi \leftrightarrow \forall x \Box^P \phi$$

$$\vdash_t \Diamond \exists x\phi \leftrightarrow \exists x \Diamond \phi, \qquad \vdash_t \Box \forall x\phi \leftrightarrow \forall x \Box \phi$$

The actual quantifier $\exists^a$ does not commute with the past or future tense operators except under special conditions, and even then different conditions are required for each direction—unless it is assumed that nothing ever comes to exist or ceases to exist (in symbols, $\Box\forall^a x \Box E!(x)$), in which case $\exists^a$ commutes with $\Diamond^f$, $\Diamond^P$, and $\Diamond$ as well (and therefore $\forall^a$ commutes with $\sim P \sim$, $\sim F \sim$, $\Box^f$, $\Box^P$, and $\Box$):

(12)

$$\vdash_{te} \forall^a x \sim P \sim E!(x) \rightarrow (\exists^a x P\phi \rightarrow P\exists^a x\phi)$$

$$\vdash_{te} \sim P \sim \forall^a x \sim F \sim E!(x) \rightarrow (P\exists^a x\phi \rightarrow \exists^a x P\phi)$$

$$\vdash_{te} \forall^a x \sim F \sim E!(x) \rightarrow (\exists^a x F\phi \rightarrow F\exists^a x\phi)$$

$$\vdash_{te} \sim F \sim \forall^a x \sim P \sim E!(x) \rightarrow (F\exists^a x\phi \rightarrow \exists^a x F\phi)$$

$$\vdash_{te} \Box\forall^a x \Box E!(x) \rightarrow (\exists^a x P\phi \leftrightarrow P\exists^a x\phi) \;\&\; (\exists^a x F\phi \leftrightarrow F\exists^a x\phi)$$
$$\&\; (\exists^a x \Diamond^f \phi \leftrightarrow \Diamond^f \exists^a x\phi) \;\&\; (\exists^a x \Diamond^P \phi \leftrightarrow \Diamond^P \exists^a x\phi)$$
$$\&\; (\exists^a x \Diamond \phi \leftrightarrow \Diamond \exists^a x\phi)$$

Assumptions weaker than the condition that nothing ever comes into or goes out of existence—such as that everything presently existing always has existed and always will exist, or that everything now existing will never cease to exist, or that everything now existing always has existed—yield commutations in only one direction:

$$\vdash_{te} \forall^a x \Box E!(x) \rightarrow (\Box \forall^a x\phi \rightarrow \forall^a x \Box\phi)$$

$$\vdash_{te} \forall^a x \Box^f E!(x) \rightarrow (\Box^f \forall^a_x \phi \rightarrow \forall^a_x \Box^f \phi)$$

$$\vdash_{te} \forall^a x \Box^P E!(x) \rightarrow (\Box^P \forall^a x\phi \rightarrow \forall^a_x \Box^P \phi)$$

The quantifier $\exists^P_P$ commutes with the past and future tense operators in only one direction, each the converse to the other, and therefore it commutes with $\Diamond^P$ and $\Diamond^f$ in only one direction as well. Similarly, $\forall^P_P$ commutes with $\sim P \sim$ and $\sim F \sim$, and therefore with $\Box^P$ and $\Box^f$, in only one direction:

(13) $\vdash_{tP} P\exists^P_P x\phi \rightarrow \exists^P_P x P\phi$, $\qquad \vdash_{tP} \forall^P_P x \sim P \sim \phi \rightarrow \sim P \sim \forall^P_P x\phi$

$\vdash_{tP} \exists^P_P x F\phi \rightarrow F\exists^P_P x\phi$, $\qquad \vdash_{tP} \sim F \sim \forall^P_P x\phi \rightarrow \forall^P_P x \sim F \sim \phi$

$\vdash_{tP} \exists^P_P x \Diamond^f \phi \rightarrow \Diamond^f \exists^P_P x\phi$, $\qquad \vdash_{tP} \Box^f \forall^P_P x\phi \rightarrow \forall^P_P x \Box^f \phi$

$\vdash_{tP} \Diamond^P \exists^P_P x\phi \rightarrow \exists^P_P x \delta^P \phi$, $\qquad \vdash_{tP} \forall^P_P x \Box^P \phi \rightarrow \Box^P \forall^P_P x\phi$

$\forall^P_P$ commutes with $\Box^P$ in both directions if every past and present object always was a past or present object:

$$\vdash_{tP} \forall^P_P x \Box^P [E!(x) \vee PE!(x)] \rightarrow (\Box^P \forall^P_P x\phi \leftrightarrow \forall^P_P x \Box^P \phi)$$

Strong conditions are needed in order to commute $\forall^P_P$ with $\Box$, and in fact only a very strong condition suffices for commutation in both directions:

$$\vdash_{tP} \forall^P_P x \Box [E!(x) \vee PE!(x)] \rightarrow (\Box \forall^P_P x\phi \rightarrow \forall^P_P x \Box\phi)$$

$$\vdash_{tP} \Box \forall^P_P x \Box [E!(x) \vee PE!(x)] \rightarrow (\forall^P_P x \Box\phi \leftrightarrow \Box \forall^P_P x\phi)$$

Concluding Remarks

Tense logic is not the only framework in which both the logic of actual and possible objects and the logic of actual objects *simpliciter* have natural applications and in which the differences between possibilism and actualism can be made perspicuous. There is also, for example, the logic of intentional discourse and the differences between the possible quantifier and the actual quantifier binding variables otherwise occurring free within the scope of operators for propositional attitudes. Still, even these other frameworks, it would seem, must presuppose some account of the logic of tenses, in which case the differences between possibilism and actualism within tense logic becomes paradigmatic. Indeed, as we have indicated, this is certainly the case for the differences between possibilism and actualism in modal logic, since some of the very first modal concepts ever to be discussed in the history of philosophy have been modal concepts that can be analyzed in the framework of tense logic.

Notes

1. Sections 1 through 3 of this paper are slightly revised versions of three abstracts (with the same titles as those sections) from *The Journal of Symbolic Logic*, vol. 31 (no. 4) (1966) pp. 688–91. The abstracts are summaries of lectures given at the December, 1965 meetings of the Association for Symbolic Logic. (A preliminary version of those lectures was given at UCLA in 1963, and a final version was given at UCLA in the spring of 1965 at a public lecture constituting the defense of my doctoral dissertation.) Section 4 is from a handout that was part of a lecture given in 1966 at the Berkeley campus of the University of California.

   The revisions made here have mainly to do with symbolic notation, replacing, for example, the Tarski quantifiers $\bigwedge$ and $\bigvee$ with $\forall$ and $\exists$, and making several other similar changes. Other revisions have to do with maintaining continuity of text, correcting typographical errors, and making explicit certain points that were originally left as implicit. The observation at the end of Section 2 about using a primitive quantifier over past and present objects is from the handout of the 1966 lecture at Berkeley. It is added to Section 2 where its content is more appropriate rather than left in Section 4.
2. We could use $\forall^e$ and $\exists^e$ instead of $\forall^a$ and $\exists^a$, especially in an applied framework in which we do not want existence to have the connotation that actuality has—namely, of existing in time and being a component of causal nexuses—which is exactly the connotation that we want in a framework such as tense logic. In the pure logic of actual and possible objects that we describe in Section 1 we ignore the differences, if any, between existence and actuality and consider them to be the same concept.
3. (A19) and (A20) were subsequently shown by E. J. Lemmon to be redundant.
4. The counterpart of rule (iii) with $\forall^P_P$ in the place of $\forall^a$ is provable, and so it does not need to be assumed as a primitive rule.
5. This notion was later described in more detail in Section 15 of my essay, "Philosophical perspectives on quantification in tense and modal logic," in *Handbook of Philosophical Logic*, vol. II (ed. Gabbay, D. and Guenthner, F.), Reidel, Dordrecht (1984) pp. 309–53.