THINK-PAIR-THINK-SHARE AND LANGUAGE IN THE HIGH SCHOOL GEOMETRY CLASSROOM

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I wish to dedicate this work to my three children, Mary Beth, Karen Ann, and Matthew Charles Balle, whose learning differences illuminate all that I do.

Acknowledgements

"At times our own light goes out and is rekindled by a spark from another person. Each of us has cause to think with deep gratitude of those who have lighted the flame within us."

Albert Schweitzer

For those many times when my light was dim, I wish to thank those who kept the flame glowing. The chair of my committee, Dr. Peter Kloosterman, spent untold hours reading and commenting on manuscripts. His words of wisdom and unfaltering encouragement kept my eyes focused and pen poised as he smoothed the path toward achieving the goal. I am indebted to my committee members, Dr. Mary McMullen, Dr. Craig Wiley, and Dr. Enrique Galindo for their patience and passions.

This has been a long journey. A professor first suggested a doctoral degree for me nearly half a century ago. It seemed implausible, impossible, that a girl from a small Township school could do such a thing – especially in math. The professor, perhaps unknowingly, struck a spark that took years to flame. Friends added kindling and my husband, Rick, dutifully tended the fire.

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THINK-PAIR-THINK-SHARE AND LANGUAGE IN THE HIGH SCHOOL GEOMETRY CLASSROOM

This study was the first formal look at a cooperative learning strategy, Think-Pair-Think-Share (TPTS), designed to aid learners who struggle with academic language. TPTS allowed quiet time for individual thought when the memory of those with language differences was heavily taxed. This qualitative interview study with a descriptive issue-focused analysis described the experiences of six students in a private Indiana high school geometry classroom over a three-month period. I interviewed each of these subjects one month prior to the first class session that involved TPTS, following one class session using video-stimulated recall, the day following the last observed session, and one month later in a focus group. The four additional students in the classroom were included in field notes and videotapes to document their interactions with the six chosen students. The analysis focused on the students' usage of mathematics vocabulary, cultural connections, response variety, and gestures showing cognitive shifts and observable activity during quiet times.

Students who struggled with language comprehension or vocabulary found additional quiet time useful when they knew they would be required to explain thinking to the class. Discussing mathematics tasks with peers increased their understanding above teacher lecture alone. Fluency with mathematics vocabulary and variation of solutions increased. Finally, over the course of the TPTS intervention, students made better use of homework time, student discourse increased during problem solving, and the teacher's views about the effectiveness of cooperative learning became more positive.

Chairperson

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Chapter 1: Introduction

The learning of academic mathematics relies not only on foundations in the mathematical concepts themselves but also on the language and the level of understanding of a language for the learner. This dependence upon language can manifest itself in many ways. The depth of vocabulary in a native tongue when listening and speaking to a person in power, the ability to have reflective self-talk, rich discourse with others, and the complexity of the language and text during the initial training in academic mathematics all contribute to success in mathematics classes. Each of these elements is strongly affected by environment, mediated by executive function and restrained by working memory.

This study was the first phenomenological study of the phases of a cooperative learning strategy, Think-Pair-Think-Share (TPTS), designed to aid learners who struggled with academic language. Central to the TPTS model was a Private and Individual reflexive stage of sign appropriation and use in learning where self-talk and sub-vocalizations allowed external text to be assimilated into internal structures of cognition. In contrast to the well-known Think-Pair-Share (TPS) teaching strategy, TPTS allowed a significant amount of quiet time for individual thought at moments when the memory of language learners was most heavily taxed. I used a qualitative interview methodology with a descriptive issue-focused analysis to describe the experiences of six students purposefully chosen to represent differences in language ability in a private Indiana high school classroom using TPTS. Though six students participated fully, I videotaped all ten students in the classroom and included them in field notes. One of the four students that was not involved with the interviews left the school before the end of the study but data from that individual were included in the analyses to supplement observations of the chosen students. The classroom teacher, trained in the strategy, used six tasks involving TPTS in his tenth-grade geometry classes. A pre-observation interview with the six chosen subjects in each class provided information about attitudes concerning group or individual work, word usage in class, use of written text, and the availability of quiet time for thinking.

I took field notes during 18 class periods spaced over three months. The class was audio and videotaped on eight occasions to provide data on usage of mathematics vocabulary, cultural connections, response variety, and gestures showing cognitive shifts, and observable activity during individual quiet times. I interviewed each of the six selected subjects four times to provide additional data on these issues.

Language differences as used in this study

Language differences in this study included students whose *mother tongue* differed from the language used in the mathematics classroom, students who had a *language disorder* such as dyslexia, and students who differed from classroom peers in the understanding of *academic words*. *Mother tongue* referred to the first language spoken at home by family members. *Language disorder* was a significant discrepancy in language skills compared to the normative standards for the speaker's age or developmental level. *Academic words* referred to formal words used in the context of mathematics as opposed to words commonly used with peers in informal settings.

These groups of students were considered together in this study because I wanted to gather baseline information on links between language and mathematics. I recognized that the strengths and needs of these three groups might vary widely and sought to understand each of their experiences while using a strategy designed to aid working memory.

Need for the study

Students with language differences such as language learners and those with dyslexia or other reading disorders frequently struggle in high school mathematics classrooms. Although the National Council for Teachers of Mathematics (National Council of Teachers of Mathematics, 2014) advocates the use of mathematical discourse for fostering student understanding of mathematical concepts, there is a scarcity of research in mathematics education on the particular relationship between mathematical discourse and mathematics learning of adolescent students with language differences.

Kenney (2005) reported that practitioners had little research to draw on with respect to the process of learning the language of mathematics in relation to the learning of any other second language. She contended that mathematical language could be considered a foreign language because, other than the speaking of the names of small numbers, mathematics was not a "first language" and was often not spoken at home. Both formal (language used in academic writing and serious situations with unfamiliar people) and informal (language used in relaxed situations with familiar people) expressions were evident and the "street math" language did not always map directly or correctly onto the mathematical syntax (the way words were placed together to form phrases and sentences). Language learners heard spoken words in the instructor's manner of speaking, decoded them into their own manner of speaking, then decoded them again to attach mathematical meaning. This double decoding was further complicated when the mathematical context was not prior knowledge. Decoding was complex for most students but even more difficult for second language learners. Because of the decoding issue, Short and Fitzsimmons (2006) contended that English Language Learners (ELLs) performed double the work of native English speakers at the middle and high school levels. Because the cognitive work doubled, processing needed more time.

A 2005 project at the secondary level in New Zealand was undertaken to better understand the relationship between English language and mathematics learning for students for whom English was an additional language (Neville-Barton & Barton, 2005). All five studies reviewed by the project found evidence that language learners suffered a disadvantage of about 10-15 percent in mathematics learning due to language difficulties. Four of the five studies found that students did not realize the extent of their difficulties.

Many of the issues that have an impact on bilingual education also have an impact on special education. Raborn (1995) suggested that students who had deficits in receptive or expressive language skills profited from learning the language of mathematics. The predictable patterns in math could foster a natural acquisition of social and academic language proficiency. However, these students needed time to pose their own questions and to explore ways of answering them. Rabon indicated that it was important to give students the opportunity to talk with peers and adults so they could experiment with and validate their own ideas.

Purpose Statement

I used a phenomenological inquiry as a means to uncover meaning of the phases of TPTS in a high school geometry classroom. The inquiry sought to express insights from high school geometry students' lived experiences as they participated in the Think-

Pair-Think-Share protocol. Using the lens of a classroom teacher, I focused on student perceptions of their own experiences. This perspective facilitated giving voice for students with language differences as it articulated and amplified students' stories of experiences with self-talk and oral or written mathematical text.

It has long been my frustration that researchers from education, psychology, neuroscience, and the cognitive sciences did not give credence to each other's work. It sometimes seemed that there was a pervasive belief that the brain and learning had nothing to do with each other. Studies that focused on quantitative scores from formalized assessments had no need to view internal functions of the brain. Psychological studies often focused on social interactions and meaning making rather than biological functions. Neuroscience studies recorded brain activity with new technology but their work was constrained by laboratory settings rather than mirroring classroom environments. This study included interdisciplinary methods of inquiry and phenomenological reflection on data elicited by an existential investigation of students' experiences and investigation of the phenomenon in educational, neurological, and cognitive literatures.

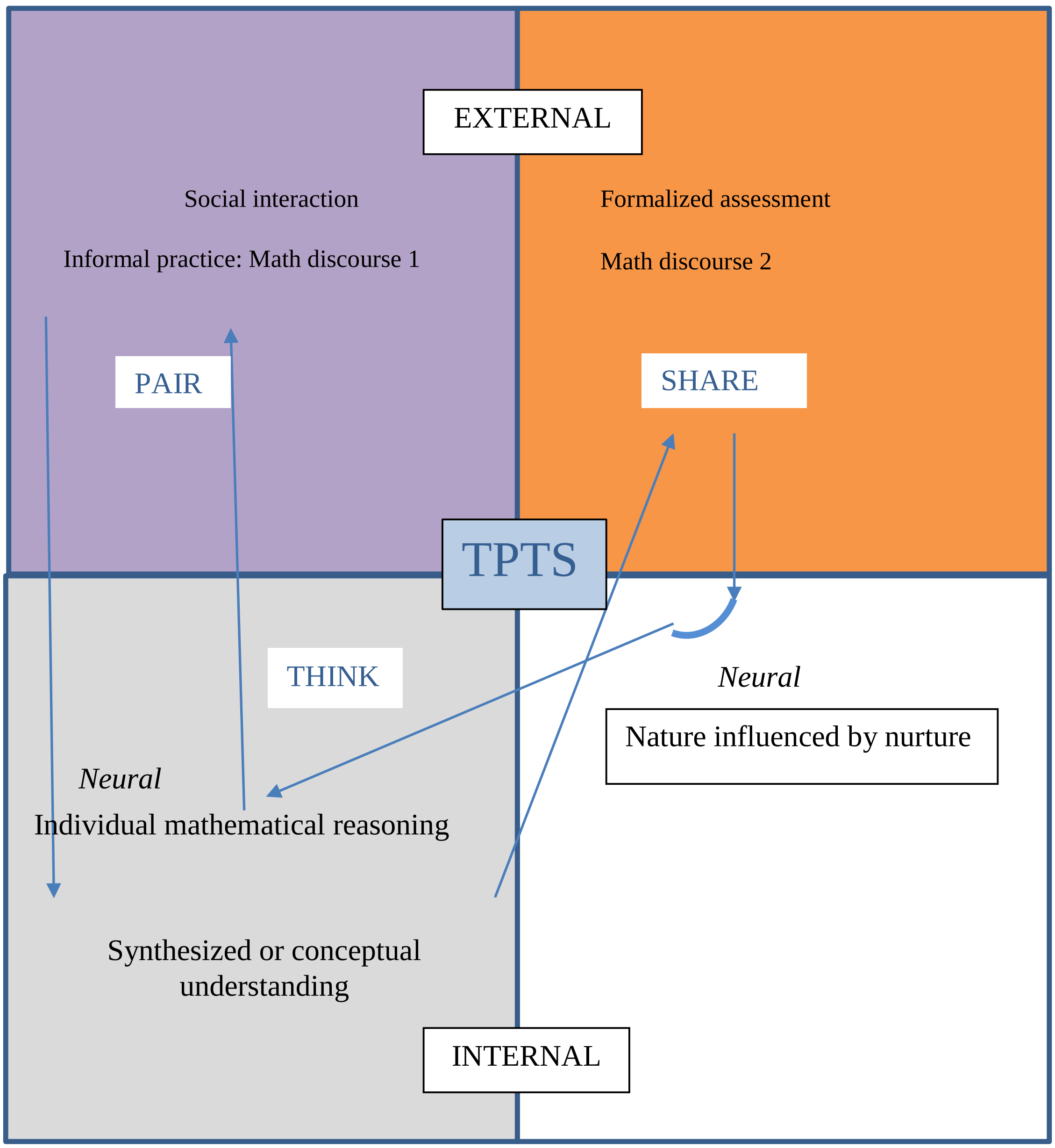
Design of the Study

Bronfenbrenner (2014) proposed that cognitive development theories could reach a measure of unification by using an ecological paradigm with an evolving process of organism-environment interaction. I assumed that studies in mathematics education could reach a measure of unification with cognitive studies by using a process that allowed organisms (students) with neurological/biological needs (time for language processing) to interact with the microenvironment of a classroom that allowed significant think time.

In accordance with Cresswell's (2009) comments indicating that qualitative studies can provide insight into phenomena that are not well understood, I used a natural setting and collected, examined, and analyzed my own data. Multiple forms of data (interviews, observations, and documents) were synthesized to make sense of the phenomenon of the phases of TPTS. I organized data and inductively built themes across data, collaborating with participants interactively to shape abstractions as they evolved from the process. The focus was on the participants' meaning rather than that expressed in literature. My interpretations could not be separated from my lens as a pragmatic classroom teacher and my prior understandings about how the brain works. To aid in identification of key structures in this multidisciplinary report, a visual model of the phases of the phenomenon was used to establish a holistic picture.

Background. I taught high school mathematics for over 20 years. Both biology and environment, nature and nurture, were evident in individuals sitting in my classrooms and TPTS developed from a combination of studies from nurture and nature in an attempt to help those students. The framework for this study began with Ernest's (2008) semiotic model of mathematical text as a basis for discussion about communicating mathematics in the classroom. His cyclical model included four quadrants where mathematical text was privately appropriated, privately transformed, published in personal terminology, and finally conventionalized in formal language for public acceptance.

Figure 1.1 is a map of the TPTS framework. This model could be used as a basis for designing studies and analyzing data related to studies of (1) the need for mathematical discourse to develop mathematical understanding, (2) the need for extra time for language learners to process new information and to speak about those understandings, (3) the need for wait and think time for all learners to clarify and correct responses, (4) the need for constructive cooperative learning strategies to facilitate social interactions



within the classroom, and (5) models of neurological processing for language and mathematics. I reviewed several of these types of studies for use in the current study.

Figure 1.1 Conceptual map for TPTS

Studies in mathematical discourse (Cross, 2009; Goos, 2004) supported the importance of creating a classroom environment that allowed talking. Studies in language learning (Cummins, 1979; Short & Fitzsimmons, 2006; Skutnabb-Kangas & Toukomaa, 1976), showed links between language and mathematical understanding and the need for extra time. Studies were plentiful on the manipulation of environment through wait-time (Rowe, 1968, 1969a, 1969b, 1974, 1975, 1986), think-time (Stahl, 1994), and cooperative and collaborative learning (Baleghizadeh, 2010; Johnson & Johnson, 1991; Lyman, 1981; Marzano, Pickering, & Pollock, 2001). Finally, neurological findings (De Smedt & Boets, 2010; Dehaene, 1997; Dehaene, Piazza, Pinel & Cohen, 2003; Krajewski & Schneider, 2009; Lee, et al., 2007) provide biological rationale for links between mathematics and language. My rationale for developing TPTS came from personal experience, so I began with personal experience and then further explained the conceptual framework built from Ernest's model.

My worldview: that of a teacher. A long history of teaching at the middle school (English), high school (mathematics) and college (education) levels followed by completion of the Masters level Mind, Brain and Education program at Harvard University molded my decision to pursue this study. I spent over twenty years teaching mathematics in a high school known for its academic and behavior troubles. Fewer than 30% of the adults in the community had a high school diploma even though it was home to a university. It was common to see fights and overt sexual activity in the hallways and the teen pregnancy rate was among the highest in the nation. Community pride centered on basketball, not academics, and teachers made national headlines when imprisoned for going on strike. The community was the site of the last lynching in the United States and racial riots broke out frequently. Mexican migrant workers came because of the local tomato industry and few students of Mexican background passed either algebra or geometry without great struggle. Many of these students were semilinguistic – they were not proficient in either of the two languages spoken at home and in school. I began to suspect that language difficulties factored into their struggles with mathematics. Professional development in learning styles and my late husband's brain tumor led me to look for biological links between language and mathematics within the brain. The focus on language, mathematics, and the brain eventually prompted the development of the TPTS teaching strategy used for this investigation.

I worked with students whose grandparents came from Mexico to be migrant workers and settled in the community. The grandparents spoke no English but their children developed what was known as Spanglish, a combination of Spanish and English with proficiency in neither. When those children became parents, their children spoke no Spanish and the English was comprised mainly of one- and two-syllable words as evidenced by records kept by the English teacher who teamed with me. It was a challenge to teach the population of that community but professional development was plentiful during the 1980s and teachers were encouraged to experiment to meet the needs of the students. It was here that I experimented with learning styles and heard about brain research that might affect education. My husband, a teacher of special education, had brain cancer and we saw first-hand the impact that lesions in the left hemisphere had on problem solving. I continued to run experiments in my classroom and found that lecture alone did not work as well as differentiated instruction. I used problem-solving tasks, projects, and cooperative learning to engage those who would otherwise be disinterested, including those considered gifted.

Although difficult for many students, I required the use of mathematical discourse and heard struggles with language. In Family Math afterschool sessions, parents and children came together and found that 'big math words' were confusing. I formed a company, the How-to House, to teach students and parents how to use math as they remodeled houses. I located, bought, remodeled and sold "homes in need in communities that care" then hired neighbors and students during remodeling to learn mathematical and technical skills that they could transfer into their own homes and lives. The neighbors, my students and their parents and siblings each used their own expertise to teach each other different strategies for solving math problems.

This experience allowed me to understand some of the struggles and barriers to learning for these populations. One of those barriers was the link between language and mathematics. As part of their jobs, I required all workers to calculate their times and wages, sometimes without pencil or paper. Occasionally I had them talk about how they solved their time sheets, expected pay, or construction measurement to the entire group. As parents and children stood in a circle, explaining how they had solved some mathematics task, I witnessed parent-child interactions and saw parents interrupting when their children presented ways to solve the problems that were different from their own. Tensions rose among parents and their children and I had to negotiate. It became imperative to keep the group quiet so each individual could develop a strategy for solving the problem and then find the words to express thought. Analytical thought such as that found in algebraic reasoning or problem solving, interrupted by human speech, came to a halt.

Frustrated that so many people struggled with language and mathematics, I returned to graduate school to look for neurological intersections of language and mathematics. In my first class at Harvard, the professor spoke about neuroscience with a Boston accent; it was here that I understood the dilemma of the language learner. His words were the same as mine but accent, intonations, and pacing were different. He used 'big words' that I did not understand, academic language, just like the 'big math words' that the students and parents did not understand. I needed time to interpret the words so I asked him to slow down because I spoke "Hoosier." I needed time to make connections between his words and my prior knowledge and then to find the words for coherent thought and concise sentences for response.

As the year passed, I learned about complex neuronal circuitry that must be activated so sensory input could be processed and transformed into understanding and knowledge. Listening to a new language, even a new academic language, required more neuronal activation. There were networks for problem solving and fact retrieval and a whole host of activations for formulating verbal or written responses. Neuroscience supported and expanded my training in teacher education; it gave me biological reasons for why the use of wait-time increased performance after asking questions. This was important. Researchers from the fields of education, neuroscience, and socio-cultural psychology had denigrated each other's work in public forums that I had attended. Educators, cognitive scientists, and neuroscientists asked me, "What does the brain have to do with learning?" and I heard keynote speakers from each of the fields question the validity of research of other areas. As a teacher, I knew that the condition of the brain, the environment of the classroom, and the pedagogy of the teacher all made a difference and it was frustrating to hear the researchers clashing rather than collaborating to offer help to the practitioner. I concurred with William (2008) who stated that knowledge generated through a clinical approach to education research should be regarded alongside more traditional forms of inquiry. Research on brain function provided evidence that language and mathematics were biologically related and that extra time was needed for learning mathematics when language differences increased working memory load. I supposed that language differences impeded participation in mathematical discourse when time was constrained, perhaps because the neurological processing in working memory was overloaded.

In my last class at Harvard, the professor asked a question to which no one could immediately respond. He gave us think-time. There was still no response so we asked for discussion time with others. Following this, there was still deathly silence. We asked for more private think-time to assimilate ideas. Within a matter of seconds, hands started to raise and ideas emerged from all around the room. This was the birth of TPTS. By adding time for private thought to the commonly used Think-Pair-Share framework first published by Lyman in 1981, we assimilated the thoughts of others into our own notions and were able to respond in a manner acceptable to that environment.

As a pragmatist, finding the dynamic Truth (things that were useful-to-believe) about language and mathematics took into account social as well as academic contexts for me. This study was a phenomenological investigation of the phases of TPTS in a high school geometry classroom. I focused on the use of language in a social setting as students explained academic understandings of mathematical concepts. The TPTS model used in this study was an adaptation of the Think-Pair-Share (TPS) cooperative learning strategy (Kagan, 2001), framed by Ernest's model of sign acquisition (2008).

Ernest’s model, TPS, and TPTS

Semiotics of mathematical text. Ernest's (2008) semiotics model of mathematical text lends a framework to think about how to structure lessons that include private thought time to assist those who struggle with language so that they might engage in mathematical discourse. His model defines *text* broadly as written, spoken, and multi-modal presentations, including gestures, facial expressions, and even clothing when culturally understood to convey a meaning.

Complex relationships exist between mathematical *texts*, the people that create those texts, and the learners who use those texts. According to Ernest (2008), social norms about how mathematical texts must be written, styled, structured and presented serve a gatekeeper function with filters created by persons or institutions that have power over the acceptance of texts for mathematical representations. The learners may not understand or interpret these conventions as the creators intended. Every mathematics task begins with text, usually in symbolic or written language form, that externally imposes constraints on a situation. The learner performs a series of transformations to the text, which normally require reading and writing and end in a final answer (Ernest, 2008), *shared* in what may be considered a formal assessment. Ernest (2008) explained that the *person in power* (usually the classroom teacher but the *person* could be a standardized test) then judges the final answer. Students may not understand the rhetoric of text or the idiosyncratic elements such as the style of answer desired by the teacher (Ernest, 2008). This is particularly difficult for a student who is learning a new language.

Ernest (2008) analyzed the complexity of the language of mathematics through the lens of a semiotic system with three components: a set of signs, a set of rules for sign use and production, and an underlying meaning structure that incorporated a set of relationships between signs and rules. The signs in school algebra include constants (usually numbers), variables (usually letters), a 1-place function sign (exponents), 2-place function signs (+, -, x, /), 2-place relation signs (=, <, >), and punctuation signs (parentheses, comma, full-stop). He notes that these algebraic notations change throughout the school years. For instance, an x begins as multiplication and later changes to a variable while the symbol for multiplication changes to a dot.

Decoding this complex set of signs is difficult for all students but especially the second language or academic language learner. Algebra's highly defined symbolic structure also complicates language in beginning algebra classrooms. Letters such as a, b, c, x, and y suddenly become variables rather than pieces of the alphabet or placeholders for some mystery number. The processing of algebraic expressions uses a syntax that guides the parsing of expressions (Jansen, Marriott, & Yelland, 2007). In a compound expression such as $\frac{5x-2}{2y+7} - \frac{3y-1}{4}$, subtraction signs and division signs can be viewed as continuous lines and scanned left to right and then top to bottom as if reading a sentence in a book (National Mathematics Advisory Panel, 2007). This eye flow differs from that of an expert mathematician who scans for subexpressions from top to bottom and then left to right (National Mathematics Advisory Panel, 2007). Learning to read mathematical signs may require direct instruction to help learners understand the difference in novel forms of text. Teachers may have difficulty recognizing which text needs interpretation for all of the learners.

Ernest (2008) stated that a cyclic model adapted from Vygotsky's (1978) two levels of cultural development for children: socio-cultural and psychological can be used to model the learning of mathematical signs. The model shown in Figure 1.2 presents a beginning discussion on semiotic mathematics models (Ernest, 2008). Ernest stated that the cycle can begin in any quadrant.

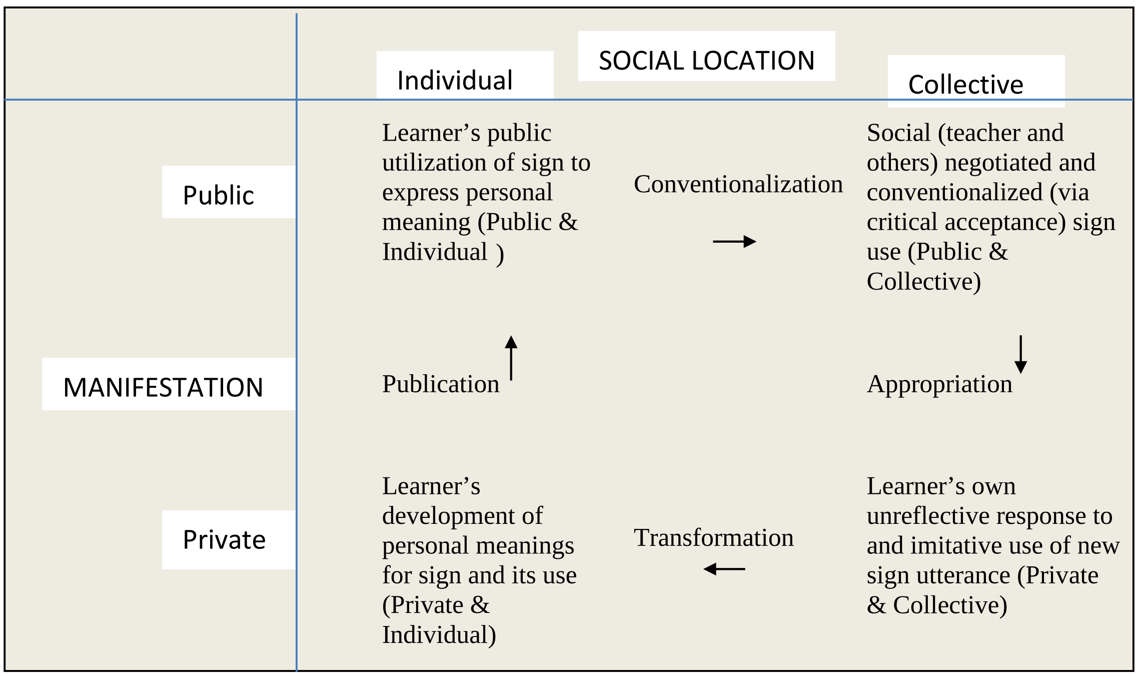


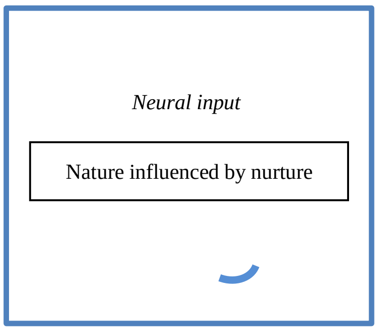
Figure 1.2: Model of sign appropriation and use in learning (Ernest, 2008)

I compared Think-Pair-Share (TPS) to this model, and started in the upper right quadrant where Social Location is Collective and the Manifestation is Public. Because I studied a classroom, the Person in Power was the teacher. Following is an explanation of Ernest's model as it relates to my conceptual framework based on a classroom using TPS and TPTS learning strategies. Icons guide navigation through the framework.

Conceptual Framework

I used the four quadrants of the TPTS model, adapted from Ernest's semiotic model, as a conceptual framework to design and analyze this study. Icons from each of the quadrants depict the processes being studied at each stage. Here, I explain more fully each of those four quadrants.

To begin the process in a classroom, the teacher provides a task presented in a

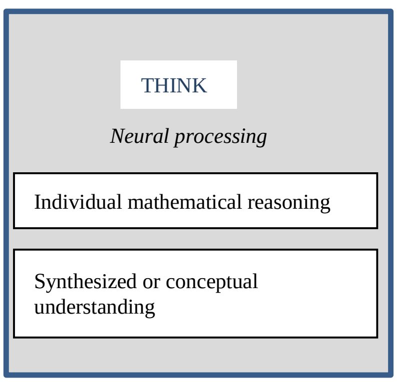


conventionalized manner, using mathematical terminology and symbols agreed upon within the field. Mathematics educators understand these and use them conventionally in mathematics textbooks but they may be entirely new to the students. Text is received through *neutral input* at first by the learner (collectively in the classroom but then perceived privately) without understanding or connection. In the conceptual map, I signified this with an arc  to highlight the fact that, in education, we often overlook the biological need for time to process new learning. This moment takes place internally, without conscious perception.

Ernest (2008) stated that vocabulary and symbols appropriate unreflectively through imitative use of signs. I questioned the use of the term *unreflectively* until I considered the neurological implications. Sensory input travels through neural circuitry, activating neurons, creating synapses across dendrites, and creating new connections. These biological functions occur so quickly that we do not perceive their natural occurrences. We do them internally and unreflectively.

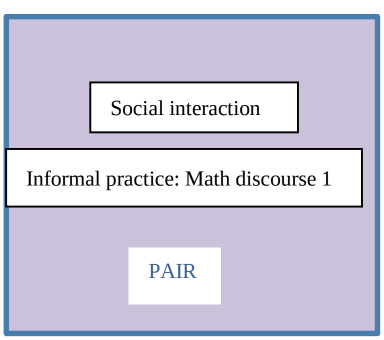
Neurologically, the knowledge of number requires a widely distributed network with specialized functions (Dehaene, 1997). Language in problem solving requires several other circuits with their own specializations for symbols and verbal counting. Still another circuit must be enlisted if there multiplication tables that must be recalled from memory. If judgment must be withheld to form a critical thinking task, the anterior portion of the brain called the prefrontal cortex must be used. Finally, to make a final response to the teacher, yet other circuits are activated. Listening, reading, interpreting within context, computing, writing and speaking each require separate neural resources. When the learner has a shallow vocabulary, there are fewer resources available for these processes to succeed.

The next quadrant involves the first think time, or quiet time, in TPTS. The



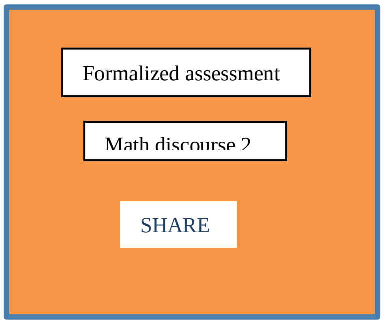
learner transforms information as he **thinks** so that it can have personal meaning. This is private and individual (Ernest, 2008) and may differ for each learner because each person has different prior knowledge and experience. He uses his own individual reasoning abilities to try to understand the problem. In the brain, signals broadcast across neural networks, looking for connections by identifying patterns.

I labeled the individual social location/ public manifestation quadrant as **Pair**



because the learner publishes his thinking by making his individual thoughts public so he can utilize the information and express his own personal meaning (Ernest, 2008). If this social interaction is with a peer, I contend that it is an *informal practice* or *mathematical discourse 1*. It is discourse with a goal of advancement of meaning making.

In Ernest's model, the learner then publishes his knowledge or the answer to the task in a public and collective forum. I call this **Share**. This conventionalized report may sometimes be to the teacher and it is sometimes to the whole class (Ernest, 2008). If a discussion results that reveals the student's current understanding of mathematical concepts that involves an explanation of reasoning and/or debate, it can then be termed discourse. There is a long process between the input of text and the output of discourse. By adding the terms Think (Transformation/ Private & Individual), Pair (Publication/ Public & Individual), and Share (Conventionalization/ Public & Collective) to appropriate blocks in Ernest's model, we find that the collaborative structure Think-Pair-Share, introduced by Lyman (1981) and touted by Kagan (2001) fully completes the cycle of the model. I will first explain TPS and then TPTS as a framework for this study.



Think-Pair-Share. In the TPS strategy, students first work (Think) on a task quietly. Teachers often see students drawing diagrams, talking to themselves, subvocalizing the problem, playing with manipulatives to build a model of the task, or looking for patterns in calculations. Some students seem totally lost and get easily distracted so the teacher must be alert and provide scaffolding to help. Students then move to face another student and the Pair shares ideas and works collaboratively to formulate a solution to the task (Lyman, 1981). In practice, in pairs where one student feels less capable either mathematically or verbally, one listens and records while the other speaks. Students expect accountability so understanding at least one solution is important. Individuals then Share orally to the whole class either on a volunteer basis or by teacher selection.

The problem that I experienced in my classroom teaching was that small groups frequently had one student that needed to reflect longer on newly acquired information and immediate reporting was burdensome. I called on other pairs to report first, giving that student time to prepare, but he was then unable to hear arguments of other groups. According to Skutnabb-Kangas (1981) semilinguals can be described in terms of deficits in six language competences: size of vocabulary, correctness of language, unconscious processing of language, language creation, master of language functions, and meanings and imagery. Semilinguals in my class normally had limited vocabularies, used words with no more than two syllables, and unconsciously mouthed the words that they read. Semilingual students had heavier working memory loads, especially during TPS when they were required to report to the class.

To alleviate working memory load for the language learner, I modified Ernest's model to include an additional return to Individual/Private prior to reporting a final answer in Public/Collective (Figure 1.3). This allowed language learners additional time to process new information and signs in informal register, formalize language, and ask questions about idiosyncrasies in final response text expectations.

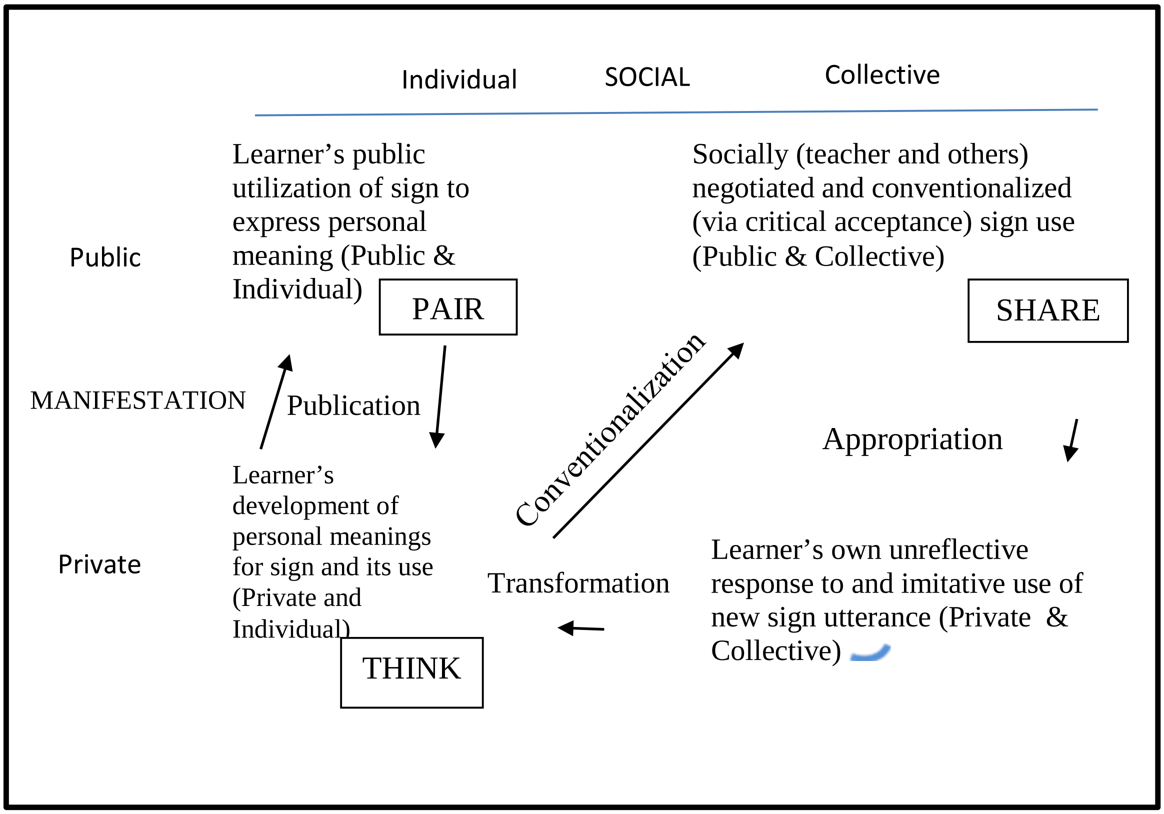


Figure 1.3: Think-Pair-Think-Share model of sign appropriation and use in learning

Think-Pair-Think-Share. Learners with language differences have an excessive number of signs and rules to learn as they integrate the signs of academic language with the signs of academic mathematics. The use of social group settings that allow discourse in a native tongue lessen work load during the Public & Individual phase but it is not always possible to provide this native tongue discourse option in American high schools in the Midwest. As shown in Figure 1.3, TPTS included a return from speaking with a peer (Pair) to individual thought (Think) prior to speaking formally to the group (Share). The informal practice through social interaction preceded extra quiet time for neural processing with the hope that the student could create deeper meaning by synthesizing thoughts. By including an extra think time just prior to presentations, I proposed that students could analyze comments from their partners and decide what salient parts to integrate into their own arguments. It gave time to practice mathematics vocabulary to formalize the presentation and to draw pictures to clarify meaning.

I expected to see connections to prior cultural experience, use of academic words, willingness to speak, shifts in gestures from quick and inward to slow and outward, and a variety of solutions to problem solving tasks. It was assumed that additional think-time in TPTS would decrease working memory load and increase the use of mathematics vocabulary, variety of responses, cultural references and gesturing shifts for academic language learners. I based this expectation on literature compiled across disciplines and on two informal studies that I completed using TPTS.

Informal studies. In the first informal study, a teacher of remedial senior English used the TPTS strategy to teach British poetry. After hearing my observations about a predominance of one- and two-syllable words being used by the workers at my housing projects, papers from earlier in the year were analyzed and found to contain only one- and two-syllable words except for those lifted directly from text or commonly used words such as "graduation." She attended a brain conference where I suggested that she use TPTS in her classes to allow more time for high-risk students to strategically process information. Following use of TPTS in a unit on British poetry, students used more words with multiple syllables in written reports. This group of students did not normally receive British poetry well, but several students remarked in later evaluations that this was the favorite unit of the year. Students remarked that they liked having time to think before speaking. I later used the strategy in a professional development with teachers from multiple content areas and a passage from Beowulf and found similar results.

In the second informal study, one of my fellow graduate students taught a mathematics lesson that used TPTS in a kindergarten classroom at an international school. She formed two groups and used TPS with one group and TPTS with the other. Students who used TPTS were more anxious to give responses than those using TPS and the responses varied more and had fuller explanations.

Significance of the Study

Potentially, any classroom could use TPTS with academic language learners. Students with language differences or weaknesses of some type sit in nearly every classroom in the United States. Rather than providing a structure that is difficult to implement, this model requires only small modifications in classrooms of many disciplines. If students have positive academic experiences with TPTS, it will open areas for new studies in education.

Description

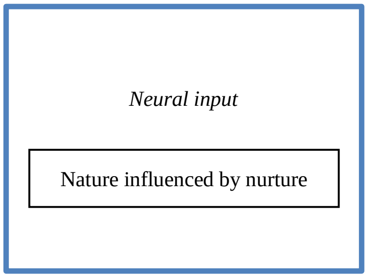
This study describes the phenomenon of students' experiences during each of the phases of TPTS in a geometry classroom from their perspectives. It seeks to illustrate the students' perceptions on whether or not the technique is useful for learning and to describe how they use quiet time. Much of the focus is on the importance of the internal Think phase as evidenced through student interviews and classroom observations. This study provides foundational information for further studies of students with language differences. Mathematics language, considered universal, implies that differences in language have little effect on mathematical understanding. Future studies can compare results from this study to data relating mathematics achievement and language differences.

Teachers unfamiliar with TPS argue that there is not enough time in a curriculum packed with high expectations. It is essential that researchers inform practice by gathering information about how students experience new strategies. Implications from this study may further the conversation that researchers are having about connections between language, mathematics and learning and provide strategies for presentation of text in the classroom. It is not the intent of this study to inspect the architecture of the brain, but it could further discussion for studies on working memory, eye flow, and brain activation by language and mathematics.

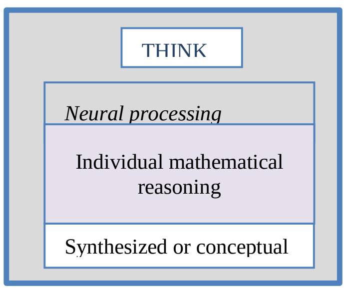
Research Questions

1. How are the phases of TPTS experienced by students with language differences in a high school geometry classroom?
    a. How do students use the first THINK time?
    b. How do students use mathematics vocabulary during the PAIR phase?
    c. How do students use the second THINK time?
    d. How do students use supportive cultural connections and when do they appear?
2. How do students with language differences report their answers to mathematical tasks in a classroom that uses the TPTS cooperative learning strategy?
    a. How do students use mathematics vocabulary during the SHARE phase?
    b. How do explanations vary during the SHARE phase?
    c. How do students use gesturing throughout the TPTS process?

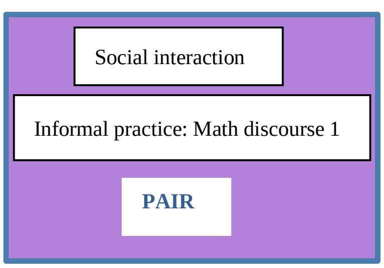
Chapter 2: Literature Review



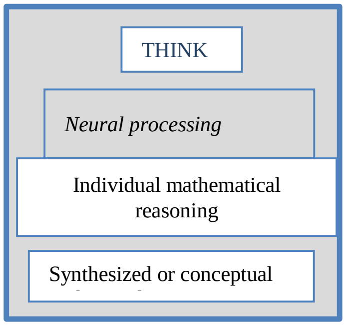
In the first part of the chapter, I review studies involving neural processing and cognitive load during the learning of language and mathematics. Recent technological advances are beginning to make this process less invisible, giving the researcher and the teacher more insights into the learning process. Because TPTS was designed to aid those who struggle with language, neurological research for second language learners is included.



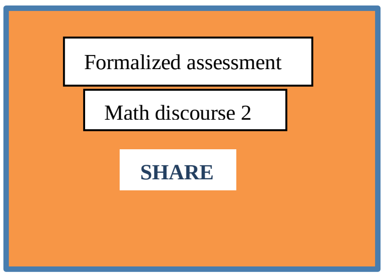
Next, I visit literature about the value of wait- and think-time through the work of Rowe (1966, 1968, 1969, 1974, 1975, 1986) Tobin (1987), and Stahl (1994). Although these studies focus on science and elementary students, their work has been fundamental in the development of teacher wait-time for instructional strategies. This literature gives insight into the experiences described by students in the current study.



Studies of mathematical discourse have their basis in social interaction and rely on the use of language, both formal and informal. Chazan's work illustrates the use of TPS in the high school classroom and Cross discusses argumentation in an algebra classroom. Studies on collaborative learning and TPS describe the affordances of social interaction during learning.

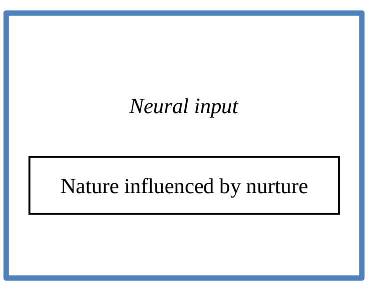


Key to this study is the description of experiences when solving mathematical tasks for students with differing language abilities during a second think time. Studies highlighting the temporal needs of second language learners and students with dyslexia are reviewed.



Although assessment of student understanding is not directly studied in this research, the use of formal mathematical terminology is used as a basis for understanding student thinking throughout the study. The language of mathematics is explored as part of the SHARE phase of TPTS. Finally, I describe research that utilized techniques with focus groups, gesture, and video-stimulated recall that were used to help guide this study.

Language and the Brain



How we learn language. I present here a primer on language and the brain to help explain how language differences might impact the acquisition of mathematical skills. Perception normally involves the integration of each of the five senses: taste, touch, vision, smell, and audition (hearing). Once humans perceive on object, they try to recognize or make sense of the object. Learned words are stored in a mental lexicon – a mental store of information that includes semantic information (the words' meanings), syntactic information (how the words are combined to form sentences), and the details of word forms (their spellings and sound patterns) (Gazzaniga, Ivry, & Mangun, 2009). In the mental lexicon, more frequently used words are accessed more rapidly. In addition, the speed with which the words are accessed is influenced by the neighborhood effect (Gazzaniga, Ivry, & Mangun, 2009). An auditory neighborhood of a word is defined as the number of words that differ from the target word by one phoneme, the smallest unit of sound that makes a difference. For instance, the English language has sounds for L and R, two distinct phonemes but the Japanese language makes no distinction between L and R so they are represented by one phoneme; therefore hate, late, rate, and eight are auditory neighbors. Words having more neighbors are identified more slowly (Gazzaniga, Ivry, & Mangun, 2009). Second language learners potentially have words with many auditory neighbors.

Neurologically, an organism perceives sensory input and then transmits the information throughout the brain through synapses. The process of creating new synapses is called synaptogenesis (Baars & Gage, 2010). These synapses create three times more activity in the brain of a child under the age of three than after puberty. If we stop exercising mental skills, regions of the brain for those skills are reallocated to the skills we practice and a pruning of unused dendrites occurs. Any two cells of the brain that are repeatedly active together at the same time tend to become associated . This proclivity has been popularized with the saying, “cells that fire together, wire together” (Hebb, 1949). Postsynaptic neurons are bound together and signal strength increases. Within language learning, synapses are strengthened in our target language so less input is needed to activate the entire neural network. Eventually, activation and recognition are nearly automatic (Maher, 2013).

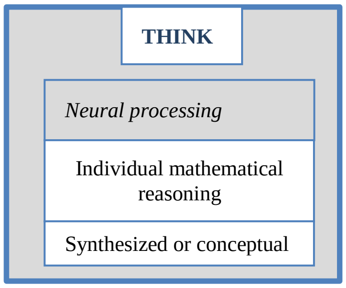
When we learn language. Because this is study of adolescents, it is important to recognize the developmental differences in learning languages. Children acquire language in predictable stages. It becomes more difficult to acquire the skills of a native speaker after childhood (Baars & Gage, 2010). Learning is different for high school students than it is for those in elementary school because neurological mappings are different. When an adult hears a foreign language, it sounds rapidly spoken and it is difficult to determine where one word stops and another starts, like readingacomplexsentencewithoutspaces. Intonations, pauses, and contextual or environmental cues help parse the words. Students with language differences may require extra time to identify these cues.

Neuroplasticity refers to the ability of the brain to structurally change in relation to input from the environment (Shaw & McEachern, 2012). Although the brain is able to form new synapses continuously well into adulthood, neural mappings for language learning occur differently for infants than for adults. Kim et al. (1997) conducted an fMRI study of six early bilinguals who learned their second language (L2) as young children and six late bilinguals who learned their L2 in adulthood (post puberty.) They found that early bilinguals mapped both languages in the same area. However, late bilinguals have a separate space in the brain for L2 grammar, rules and structure. They concluded that the language networks had been mapped out as a young child using only L1 (the initial language).

Anecdotally, students need time to steep in their thinking when quietly pondering a novel or challenging idea. As a teacher, I frequently saw students mumbling to themselves while trying to perform mathematics tasks that were difficult for them. Language learners used the native tongue in sub-vocalizations. This self-talk and self-reflection allowed the learner to personalize new information, and to connect it to established knowledge. The learners prepared for the next stage where they exchanged ideas with a peer, preferably first in a native tongue, where ideas became clarified as text was shared socially.

Think 1 and Studies of Wait and Think Time

One of the most important yet most neglected stages of sign appropriation and use



is the self-reflective stage of private and individual. During this stage, learners develop personal meanings in what Vygotsky (1978) called the intrapsychological stage. The value of metacognition during this stage has been studied with novice and experienced problem solvers. During problem solving, novices spend a minute or two reading the question followed by an unbroken sequence of unreflective exploration of the problem. They try to solve without plan, typically through the generation of specific example texts. This usually leads to failure due to the lack of regulation and monitoring of outputs and the learner's poor match with task goals. Experienced problem solvers typically work through levels of analysis cyclically. They ask themselves questions privately or aloud throughout the process, suggesting metacognitive self-monitoring and self-regulation of their overall problem solving processes (Schoenfeld, 1992).

Prior to these studies on metacognition in problem solving, there were many investigations of the use of wait-time and think-time as teaching strategies. Figure 2.1 provides a framework to compare Ernest's semiotics model, wait-time studies and a neurological model with TPTS.   Columns represent phases in cognitive processing and in the TPTS strategy.

Schoolcraft TPTS Teaching strategy	During teacher task presentation Learner’s appropriation and imitative use	**Think$_1$** Learner’s transformation to personal meaning	**Pair** Learner’s publication to express personal meaning	**Think$_2$** Learner’s transformation to personal meaning	**Share** Learner’s conventionalization and socially negotiated response	Classroom discussion
Paul Ernest (2008) Semiotics model	Appropriation Private/Collective Learner’s unreflective response and imitative use of new sign utterance	Transformation Private /Individual development of personal meanings	Publication Public/Individual Public utilization to express personal meaning		Conventionalization Learner’s Public/ Collective Socially negotiated, conventionalized via critical acceptance	Teacher and Classmates Appropriation Private/ Collective
Mary Budd Rowe Teaching strategies (1969, 1972, 1974, 1986)	Question by the teacher/Learner listens	Wait-time 1 > 3 seconds	Student individual response	Wait-time 2 >3 seconds		Teacher’s reaction
Robert J. Stahl Teaching strategies (1994)	Question/Task presentation by the teacher	Think-time Between 3 seconds and 2 minutes for task completion				
Frank Lyman Teaching strategy (1981)	Task Presentation	Think	Pair		Share	
P. H. Winnie & R. W. Marx (1982) Neural model	Neural system perceives instructional stimuli and notes their occurrences	Understand the cognitive processes required	Use the processes to create or manipulate information	Encode the information for later retrieval		

Figure 2.1. Semiotics model, teaching strategies, neural model and TPTS.

Rowe (1974) described wait-time as the period, seconds in length, between teacher and student response. Studies on wait-time showed that waiting even three seconds substantially improved the quality of student responses (Rowe, 1974) and that a second wait-time provided richer student elaboration (Rowe, 1986). Rowe first studied tape recordings of high school biology students discussing laboratory findings (Rowe & Hurd, 1966) and then later studied conversations during an elementary school science lesson (Rowe, 1968; 1969a,b) and found that in the three anomalous recordings where the pacing seemed slower, the level of student participation was greater (1986). Rowe fed the tapes into a servo-chart plotter that made a graph of the speech patterns and pauses. This revealed that two wait times might be important: one between the teacher question and the beginning of student response and the other one between student utterances and teacher response. There was an accumulation of pauses during student speech and if the teacher reacted quickly, it appeared to cut off student elaboration (Rowe, 1986).

Rowe then conducted a series of studies that lasted a number of years involving alternately small groups in a laboratory setting and whole classes of elementary students. She manipulated the wait times separately and then together and monitored the consequences of longer wait time schedules for both students and teachers (Rowe, 1972; 1974; 1975). Rowe frequently found that with three or more seconds of uninterrupted times to consider and then respond to the query, there were three positive outcomes: 1) the length and correctness of responses increased, 2) the number of volunteered, appropriate answers by larger numbers of students increased and 3) the scores on academic achievement tests tended to increase (Rowe, 1974). Rowe's findings as early as 1969 spawned a large number of studies across the world over the next decades (Tobin, 1987). In particular, Stahl (1994) built on the work of Rowe and defined think-time as a distinct period of uninterrupted silence by the teacher and all students so that they could complete appropriate information processing tasks, feelings, oral responses, and actions.

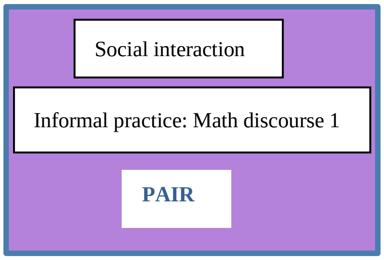
Stahl outlined eight different categories of periods of silence (Stahl, 1994). The period of silence he called student-task-completion-work-time was most similar to that used in TPTS. That think-time occurred when a period of 3-5 seconds, several seconds (e.g.15, 20, 30, or 90) or two or more minutes of uninterrupted silence was provided for students to remain on-task. It allowed students to complete an academic task that demanded undivided attention and the length of each period varied according to the task (Stahl, 1994). While Stahl and Rowe looked at quiet times as teacher wait-time in science classrooms and Rowe viewed it as task completion think-time, other researchers studied the use of quiet time in different settings.

Tobin (1984) conducted a study in 20 intact 6th and 7th grade classes in Australia that investigated changes in discourse attributable to the use of extended teacher wait time. During seven lessons using probabilistic reasoning, ten of the randomly selected classes utilized a normal wait time (control). Audio tapes were analyzed to measure wait time and teacher discourse variables. The results of the study showed that use of an extended wait time in middle school classes could enhance achievement, improve classroom management, and increase student discourse. Variation in formal reasoning ability for feedback group classes was significantly higher than variation in control group classes. The use of extended wait time appeared to contribute to the maintenance of classroom environment and the length of student discourse was significantly greater for wait time feedback group classes. Elaborated responses often came in bursts of speech separated by distinct pauses that exceeded three seconds. By waiting five seconds before asking a new question or interjecting with new information, the teacher allowed students to clarify their statements. Teachers using longer wait times tended to probe for additional information after a student response.

Tobin (1984) was careful to add that the provision of additional silence alone was not sufficient to lead to increased achievement but associated changes in teacher and student discourse appeared to make positive changes. Tobin called for studies of learners of English as a second language that utilize stimulated recall procedures to investigate whether student thoughts are on task during periods of silence.

Pair and Teaching Strategies for Mathematical Discourse

The National Council of Teachers of Mathematics (NCTM) specified promotion



of mathematical discourse as a teaching standard in 1991. The Professional Standards for Teaching Mathematics recommended that students should listen to, respond to, and question the teacher and one another. Classroom environments therefore needed to shift away from memorizing procedures and toward mathematical reasoning (National Council of Teachers of Mathematics, 1991). Reasoning linked to communication (Lampert, Rittenhouse, & Crumbaugh, 1996) and therefore language became a larger factor in the classroom. Lampert challenged that students sometime viewed reasoning and disagreeing as agitation or quarreling, not something you do with a friend. Children may not separate the quality of ideas from the person expressing those ideas while judging the veracity of assertions (Lampert et. al, 1996). Lampert reported her fifth grade classroom's discussion about discussions. While two students seemed to equate reasoning with academic argument, others were concerned about being wrong in front of the group. One student commented that she had trouble keeping track of her thinking when others were trying to prove her wrong. The lack of intellectual personal property when involved in collaborative learning also caused dismay for the students. The authors recounted a situation where the entire group developed an answer but only one student reported to the class.

Studies at the high school level tended to focus on development of curricular materials rather than on the development of teaching practices that use these materials but Chazan (2000) conducted a series of studies that investigated teaching practices, including group learning, at the high school level in mathematics. His experiences were at three very different types of high schools; the first was a Northeastern suburban private Jewish day school, the second was in a Northeastern suburban public high school, and the third was in a rural Midwestern professional development school where he taught lower-track algebra one. His placement in the Northeastern suburban public high school was for one period per day during a three-year study where he worked first with a student teacher, then the department chair, and finally with a teacher who split her time between mathematics and foreign language. All the students were White, academically motivated, and spoke the English language.

His next placement in a public low-track rural high school algebra class was much different from teaching motivated students in a suburban school. Unlike his prior experiences, his students were sometimes passed from grade to grade through a process called social promotion prior to high school even though they did not understand foundational material. This meant that they already had a lack of success in pre-algebra and brought with them a host of faulty algebraic rules. These high-risk students took algebra repeatedly until they either passed or dropped out of school. They frequently believed that math was just a production of answers, completely disconnected from real life (Chazan, 2000). He noted that many of the teaching practices used in schools and classrooms with more motivated students would not work with these non-college-intending students. Only a few students would volunteer to speak in class, homework was seldom finished, students were frequently absent, and the students had low expectations of each other because they came from "Practical Math" or "special ed." or "voc ed." (Chazan, 2000, p. 29). One articulate though confrontational student often had deep mathematical insights; yet he turned in blank test papers. When asked why, he said that he did not understand what the instructions said.

By the beginning of the third year of Chazan's study, his primary use of textbook problems in a teacher-centered classroom had changed to a task-centered design, modeled after designs of instruction published by Lampert (1990) and Ball (1993) where students developed their own connections. They began the period with a discussion of the mathematics task. Students then explored the task on their own or in small groups, sometimes using technology. The class would reconvene to discuss findings. Although not noted in his findings, this format was the familiar TPS strategy. This task-centered design of teaching was advocated by the mathematics education reform movement (*Professional standards for teaching mathematics*, 1991), with the premise that students' ideas were central to their learning and were a key resource in teaching. Tasks played an important role in eliciting student thought; the students had to understand the task before they could develop a strategy to solve it. For this group of Chazan's students, textbook examples of "real-world" problems were found only in school, disconnected from their lives outside of the classroom and they did not understand the tasks. Chazan (2000) stated that this method made the teachers dependent upon the students' motivation, ability to explore, and willingness to talk with peers. If students were not willing to talk with each other, it became a series of individual conversations with the teachers. All too often, students were unable to check their solutions, had no sense of preservation from step to step in equation manipulation, and were unable to diagnose their own learning needs (Chazan, 2000). During review situations, they had trouble seeing how their own work differed from new information. Student engagement with the task fluctuated; it seemed, without rhyme or reason. They would fully engage one day and the next day would be a flop (Chazan, 2000).

According to Chazan (2000), these high-risk students needed to be engaged in the task of developing their own individual connections to mathematics rather than the teachers taking on the sole responsibility for that task. He tried to design ways to have students tell him about algebra in their lives and he described his vision for ideal kinds of conversations in mathematics classrooms. In his guidelines, tasks included contexts that allowed students to discuss their understandings of the material and of other students' understandings of the material. Everyone was viewed as a participant in the unfolding understanding of the whole class and as a resource for learning. All participants carefully allowed members of the class to appreciate ways in which classmates had diverse ideas, understood the same concept or term differently, and had conflicting ideas, and everyone needed the opportunity for extended turns. Students needed to be able to come to the board to draw or to reference objects that could be used to clarify their intents, and students needed to be able to ask for clarification and elaboration when they had not understood a speaker. They were the audience for one another's comments (Chazan, 2000).

Chazan's (2000) guidelines for the success in mathematics of these high-risk high school students depended on the verbalization of language (discuss, participate, express and appreciate diverse/conflicting ideas, extended turns, clarify intent, ask for clarification and elaboration.) Verbalization also helps to develop technical understanding needed in transitions from algebra to geometry, emphasizes attention to argument, and develops the process of defending convictions (Lodholz, 1990).

Cross (2009) conducted a study in an algebra classroom that used verbalization through quiet writing and group argumentation. Analysis of the transcripts showed that writing after discourse allowed students to organize and refine ideas (Cross, 2009). This time for writing was quiet time that I would liken to the second Think time in TPTS. I reviewed the tasks used in that study and found that the teacher used the TPTS strategy to present a slope task. Students were seated in groups during task presentation but were allowed to examine the formulas individually (Think). They then engaged in discourse (Pair but in groups of three). The teacher approached the groups and they shared answers to the teacher voluntarily (Share). In the transcript (Appendix A), the teacher used leading and probing questions followed by a five-second quiet wait-time just prior to a student response that shows a break-through in insight. In a second transcript from the Cross study, one student told the group, "Wait…let me think," but the group continued talking. He eventually found the correct answer but his explanation was confused and the others in the group did not understand. They agreed to accept his response but were confused because it conflicted with the one method taught previously in class. They left in cognitive disequilibrium.

Learners gather in social settings to express personal meaning of the newly acquired signs and rules. The National Council of Teachers of Mathematics lists discourse as one of the main elements that teachers must provide (National Council of Teachers of Mathematics, 1991). Discourse in this context, is a dynamic exchange of dialogue between individuals that focuses on understanding mathematics concepts and solving problems. Humans are social and need dynamic social interactions over a period of time for developing language (Hinton, Miyamoto, & della Chiesa, 2008) and mathematics. This is managed in the classroom through small group settings. The intention is that students will share equally the given task, engage in rich mathematical discourse, and come to a consensus about the posed problem.

This can be a mixed bag for the student with language differences. If language is inaccessible or if the learner is incapable of joining in the dialogue, there is no time to practice use of the sign in a public setting. Lewis and Hill (1997) argued that when students do pair work exercises they are practicing the language by using the target language (in our case, the language of mathematics) with others. It requires that they become more active with their input processing. Students increase confidence, ability to acquire an L2, and therefore neural networks for language by using routines or gambits (Taylor, 2002) in relation to the target language.

However, if the learner’s personal meaning is not accepted by the group, the rejection can trigger neural responses in the affective network that impede further learning. Information enters our affective network through the senses and proceeds to the thalamus. Two routes, sometimes called the low-road and high-road, allow us to either respond immediately to a threat situation or to allow a longer thought process to occur. The low-road travels from the thalamus directly to the amygdala for quick action, acting on crude sensory information. This short pathway allows us to respond quickly in a fight or flight reaction. When the student becomes repeatedly anxious about the collaborative group experiences, he begins to ‘shut down’ and disengage from the activities (Rose, 2009). Direct instruction must be instituted to promote equality during these stages.

If the collaborative group allows equal opportunity for learners to share in expression of personal meaning and use of signs, the high-road is taken. The affective network high-road has multiple pathways from the thalamus to the sensory cortex, the hippocampus, the amygdala and provides more detailed information with and a more complex analysis (Rose, 2009) and emergent thought may arise.

Following this stage, TPTS institutes an extra step in Ernest’s model of sign appropriation and use is learning. This added step adds extra quiet time prior to whole group reporting in the TPS cooperative learning strategy. Before examining literature for the added quiet time, I present studies on TPS.

TPS studies. A few studies have investigated TPS as a teaching strategy either to foster peer communication or to build word vocabulary. One early childhood, one third grade, and one high school study are examined here. While researching how to support the development of communication in early childhood classrooms, two mathematics educators and two early childhood educators collaborated for a year and developed a modified TPS. The study involved one classroom of kindergarteners where shared peer discourse was used to make sense of individual reasoning (Tyminski, Richardson, & Winarski, 2010). The classroom teacher used a five-stage protocol to increase listening, communicating and mathematical understanding. When I relabeled the five stages - orientation, play-investigate share-reorient, pair play/pair listening, and whole-group share – into TPS terms, the first stage was task presentation followed by Think-Share-Pair-Share. The teacher presented a task then the children thought and played with it individually (Think), shared their thoughts with the class (Share), went into pairs to discuss with a peer (Pair), then shared their results with the whole group (Share). The research team reported that, after consistent use of the protocol, the listeners made sense of the presenter's representation in the context of their own reasoning (Tyminski et al., 2010).

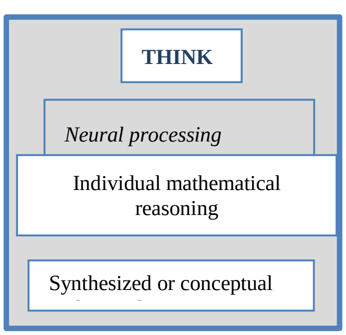
At the third grade level during Calendar Math, Geiger (2010) conducted an eight-session inquiry to investigate supports for student participation. The class was comprised of twenty-four students including two ESL learners and four students receiving Title I math services. There were fourteen male students and ten female. Two students in the class received enrichment in mathematics on a weekly basis. A tally chart of student responses, two participation surveys, and math journal checklists provided data. TPS was the sole intervention for four of the eight days. On the other four days, added wait-time, assigned seating, and hand and arm waving were used as interventions. The instructor chose partners on two of the TPS days and students chose their own partners on the other two. Geiger found that, "allowing students to choose their partner during TPS increased the rate of student participation across a significant majority of the class" (2010, p. 3). Twelve students showed an increase in overall daily participation when paired by different mathematics abilities; twenty students showed an increase in overall participation when allowed to self-choose. However, there was a mismatch between student beliefs about the usefulness of TPS for participation and the collected data on increased student participation. Only ten students stated in the post-survey that the use of TPS helped them participate more during Calendar Math. The researcher postulated that increased participation for some students in TPS might have created more distractions for other children who sat nearby.

One high school study looked at language in a non-mathematical context. A linguistics study with English language learners (mean age 18.5 years) showed TPS to be an effective strategy on word-building tasks in Iran. Baleghizadeh (2010) studied two EFL classes as they worked on a word-building task that required them to fill in the 15 gaps of a newsletter with correct form of the 15 given words. The forty students (28 females and 12 males) either completed the tasks in self-selected dyads based on the TPS technique (treatment) or individually (control). Students in dyads had two copies of the task but they submitted one final copy. Before starting the task, the researcher taught participants in the experimental group TPS and made them aware of collaborative skills such as asking for clarification, listening attentively, giving reasons, and so forth. Participants had as much time to complete the task as needed and the researcher measured the time for completion. The average time spent for the control group was 17 minutes and for the experimental group 23 minutes. Results of the data analysis showed the treatment participants achieved significantly higher scores ($p < .001$) on accuracy of answers for the given task than the participants in the control group. Balezghizadeh (2010) noted that time could be considered a mitigating factor, but results from a study by Storch (2007) found no significant difference in accuracy of language tasks between collaborative versus individual learning even though pairs used more time to complete the task. Instead, Balezghizadeh (2010) argued that better performance of pairs was attributed to the quality of interaction between the students; the students' joint effort while collaborating with each other likely resulted in co-construction of morphological knowledge. The dialogue helped language learners identify, analyze, and use the language of the classroom.

Cooperative learning and TPTS. A meta-analysis of empirical studies on individual and group work and their effect on student achievement by Marzano, Pickering, and Pollock (2001) advocated cooperative learning as one of the strongest methods for increasing academic achievement when tasks were well structured compared to methods where students worked as individuals or in competitive groups. Marzano stated that the use of five tenets of cooperative learning, developed by Johnson and Johnson (1991) provided a well-structured task for productive work in cooperative learning. The five tenets were (1) individual and group accountability, (2) positive interdependence in which all group members participate to achieve a group goal, (3) face-to-face promotive interaction (helping each other learn), (4) group processing, review, reflection, and reevaluation, and (5) interpersonal and small group skills such as communication, trust, leadership and conflict resolution. Tasks that did not include these tenets did not produce the same effects. I used these tenets as one lens to view results in the current study.

Think 2 as a Transitional State

Collaborating with groups of peers allows the learner to practice utilizing signs and rules in public but it does not provide a space for internalizing new information and emergent thought. Working memory and executive functions are active during the Pair stage and emotions may be in an excited state. In order for new ideas synthesize, they need time to think. Neurologically, time is

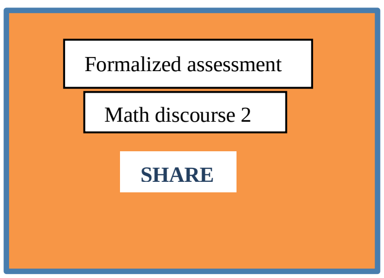


needed for *synaptic consolidation,* a form of memory consolidation found across species that can occur within minutes to hours.

Most people become anxious in anticipation of public speaking. Researchers have studied this phenomenon extensively over the past two decades by administering the Trier Social Stress Test and found that public speaking is a clear, reliable way to elicit a cortical response, a hormonal marker of stress (Jamieson, Nock, & Mendes, 2013). With this anxiety comes a release of neurotransmitters than inhibit the activity of neurons to which they bind (Emotions and the Brain, 2002). The inhibition slows memory retrieval and consolidation of thought. The T2 reflective state allows time to consolidate thought and transition to more formalized speech in preparation for sharing publically.

Share and Language in Mathematics

In the final quadrant of Ernest's model of semiotics, he explains that the learners



publically share formalized information. The sharing can take the form of formalized assessment or reporting through conventionalized signs and rules of language. The language of mathematics is filled with symbols and most are common throughout the civilized world. Children begin using them as soon as they write their first number and teachers use them without thinking about their complexity. Seemingly, minor changes such as the use of commas and periods for place value and decimal demarcation can create misunderstandings. Students may view 2.146 as 2,146 and lose the next ten things said in class as they ponder the incorrect interpretation. Furthermore, Bruner (1966) theorized that symbolic representations were the highest form of language. The use of symbolic language may confound thinking for those students who have not formed beyond the enactive or iconic levels of language representations.

Language influences the processing of mathematics. Language influences the speed for counting, mental representations of fractions and demands on working memry. Arithmetic facts are learned verbatim and therefore become tied to a native language and individuals tend to continue working mathematics in their minds using their native tongue even when they are proficient in a second language (Dehaene, 1997). The complexity of that native language impacts the speed with which the computations can occur. Chinese children can count verbally more quickly and remember more numbers in a span than most other nationalities merely because of their language structure (Dehaene, 1997). Linguistics differences can cause American children to lag behind their Cantonese counterparts by as much as one year. According to Dehaene, this is due in part to the adherence to base ten number names. For instance, American children count to ten and then have eleven, twelve and thirteen, which do not seemingly reflect ten-one, ten-two, ten-three. Anecdotally, I might say “fourteen” and find several exasperated children hurriedly erasing the placement of “4” so they could put a “1” in front of it. “Forty” was often mistaken for “14”, “15” for “50”, and so forth.

Words seem to influence the mental representations formed concerning fractions. Sophian (2007) found that the use of pictorial representations without sufficient emphasis on the wholes in the part-whole relations and the importance of equal-sized parts were an obstacle for mastery of fractional concepts. “Several studies confirmed that being a speaker of English, Croatian, or other languages that do not demarcate parts and wholes in fraction names is an obstacle to mastery of fractions” (Task Group Reports of the

National Mathematics Advisory Panel, 2007, pp.4-51). One-fourth in English is represented in Korean as “of four parts, one” which emphasizes first how may equal parts and then how many of the parts are used. These children are able to correctly associate number fraction words with pictures prior to formal instruction (Task Group, 2007). This is a correlation, however, and could also be indicative of other cultural factors. Paik and Mix (2003) found that Korean and U.S. children made similar errors in fraction-identification tasks when presented with nontransparent whole-number representations. Furthermore, U.S. children’s performance exceeded same-grade Korean children when fraction names that explicitly referred to parts and wholes on analogy with Korean name were used (Paik & Mix, 2003).

Demands on working memory also vary for different fractional interpretations. For instance, ratios such as 4:5 are binary relations, the fraction 2/3 is a ternary relation involving the set of two, the whole set of three and another implied set of one. Equivalence relations between two fractions such as 6/8 = 12/16 is a quaternary relation and judging magnitudes of fractions with unequal denominators such as comparing 12/25 and 99/200 complicates further the quaternary relation. Individual differences in working memory have been associated with performance on fraction tasks independently from conceptual understandings of fractions (Task Group Reports of the National Mathematics Advisory Panel, 2007).

Summary of Research Relating to Neuroscience, Teaching Strategies, and Language

Literature from very differing fields of studies show that mathematical understandings deepen through social discourse but students with language differences carry heavier cognitive loads as they simultaneously process language and math. By placing the processes of mathematical communication into Ernest's semiotic framework and considering wait time during discourse and adolescent behavior during cooperative learning, I identify key areas to investigate in this study. Those key areas emerge as the four phases in Think1, Pair, Think2, and Share.

I reviewed studies involving the importance of mathematical discourse and the difficulties associated with communication of mathematical ideas in this chapter. Lampert's work with fifth grade math students gave hints about ways students in the current study perceived discussion. In today's multicultural settings, adolescents who have language differences are leery of conflicts with peers as they try to fit into the new culture.

Ernest (2008) developed a framework to analyze mathematical text. Chazan discussed traits peculiar to poorly motivated high school mathematics students using TPS. Dialogue from the Cross study in algebra provided examples of quiet time during writing that aided organization and refinement of ideas. Rowe (1986), Stahl (1994) and Tobin (1987) provided insights into the advantages of allowing wait- or think-time. Johnson and Johnson gave five tenets for cooperative learning found to be highly effective by Marzano (2001). Lyman (1981) introduced the TPS teaching strategy. Tyminski (2010) modified TPS for early childhood; Geiger (2010) used the modification with third grade. Baleghizadeh (2010) looked at the use of TPS with language learners in Iran in a non-mathematical context.

Methodology of studies

This section provides examples that I chose as models of appropriate methodology for answering the research questions I posed. Video stimulated-recall added student voice to observational data, the use of gestures helped to identify cognitive shifts during quiet times and the focus group interview allowed students to provide final thoughts after completion of the treatment.

Video-stimulated recall. Video-stimulated recall (VSR) is an introspective method that represents a means of eliciting data about thought processes involved in carrying out a task or activity (Gass & Mackey, 2000). It is assumed that it is possible to observe internal processes in much the same way as external events through introspection. It is also assumed that humans have some level of access to their internal thought processes and that they can verbalize these. The theoretical foundation for stimulated recall relies on an information-processing approach where a memory prompt activates memory structures and aids in the recall of information.

In VSR, a stimulus such as a recording or paper artifact is presented to the subject to activate memories of an event. The subject is asked to describe what was happening during that event. Researcher comment is restricted to prevent interference with memory traces. The VSR session is recorded, transcribed, and coded for analysis. Clarke (2001) found that observational data alone sometimes led to misinterpretation. For example, student inactivity might be interpreted off-task when the individual was quietly thinking about the task. On one occasion in Clarke’s study, the teacher stated that the student was inattentive and the student, viewing the tape alone, agreed. However, during a post-lesson video stimulated recall interview, the student provided convincing evidence that she had been engaged with the lesson content. Students sometimes do not openly verbalize unproductive pathways to solutions (Krutetskii, 1976). These pathways may not seem relevant so they are pushed to the background of memories. The limitation caused by false or incomplete memories can be reduced with stimuli like video in post-task interviews where the participant can describe their thoughts. However, the interviewer must be wary of asking questions that include constructs not identified by the students because the student can "generate answers without consulting memory traces" (Ericsson & Simon, 1980, p. 217). Instead, the interviewer should fashion questions that ask the student to attend to the video segment and explain what was happening at that time in his own language and words. Gass and Mackey (2000) recommended developing a detailed research protocol for the stimulated recall procedure. They recommend that the researcher should put the participant at ease, allow him or her to stop the video at any point to comment, and keep researcher comments to a minimum.

The Learner's Perspective Study in Australia gathered data from a sequence of fourteen Year 8 lessons with the intent of exploring the teaching and learning of mathematics as viewed by the learner (Williams, 2007). The methodology included videotaped lessons, post-lesson video-stimulated recall interviews with teachers and students, and collections of tasks and lesson artifacts. Three cameras operated simultaneously to capture the teacher, the focus students, and the whole class. A statement at the beginning of the interview let the student know that answer correctness was not of concern to the researcher; she just wanted to know what the student was thinking and feeling (Williams, 2007). Following the lesson, the researcher stimulated student memory during individual interviews by mixing an image of the video of that student (large image) and the teacher (small insert). Students were assured that there was nothing right or wrong about what they said because the intent was just to see how math looked to a student. During the interview after Lesson 12, one student took control of the remote control and fast-forwarded to parts that were important to him. He talked about how he felt, what he was thinking, and what was happening that made this part important. This allowed the student to use his own language and ideas (Williams, 2007).

Focus groups. Focus groups normally involve a moderator that facilitates a controlled production of discussion among a sample of the population (Lederman, 1990). The moderator uses a nondirective technique that allows between six and 12 participants to express their ideas in a spontaneous manner (Bertrand, Brown, & Ward, 1992).

Zhu and Flaitz (2005) examined learner needs from both student and faculty perspectives as they conducted discussions on international students' academic language needs. The study took place at a public research university located in the southeastern United States. Out of the 35,000 students enrolled, approximately four percent were international students from 118 different countries (particularly Asia and Latin American) with majors in a number of different fields but mainly engineering, business, and computer and information sciences. They used two rounds of focus group discussions with the second round to clarify points raised in the first round. The second round of discussions allowed the investigators to explore issues raised in the initial focus groups.

Zhu and Flaitz asked questions regarding the understanding of academic language similar to the information I sought in the current study such as, "What are some of the particular difficulties that you have experienced in using academic English?” (2005, p. 4). Although a short list of questions was prepared for the first round, participants were encouraged to talk about anything they believed to be relevant.

In the same manner that I used with recorded TPTS sessions, the investigators listened to the tapes to clarify issues or confirm that all main points were included in the notes then used note-expansion (Bertrand et al., 1992). They examined the notes inductively for major themes/points discussed then they coded and categorized the themes/points. After independently analyzing their detailed notes, they met and compared notes, discussed the themes that emerged, and agreed on the major categories.

Students' comments covered five skill areas that I anticipated would surface in this study: listening, reading, speaking, writing and culture (Zhu & Flaitz, 2005). In listening, both graduate and undergraduate students commented on rate of speech, organization of the lecture, the lack of experience with long lectures, simultaneously juggling listening and note-taking, understanding special terminology, and understanding idiomatic expressions for classroom procedures. An example of idiomatic confusion occurred when the instructor said that he wanted to "take up" the assignment. The student thought that "take up" meant to discuss but the instructor meant he wanted to collect the assignment.

There are five registers in linguistics: intimate, casual, consultative, formal, and frozen. Intimate register is highly informal, used among family members and may include private vocabulary known only to two people. Casual register is a broader, but still informal, language that includes slang; it is often used in peer discourse within the classroom. Consultative register is moderately formal such as that normally used between teacher and student. Formal register is spoken between strangers or in technical settings and frozen register is ritualistic such as in religious ceremonies.

Students had trouble following multi-participant conversations with different registers (Zhu & Flaitz, 2005). Students had difficulty switching from a formal or consultative register with the teacher to a casual register with peers. For example, classroom discussions between the instructor and the students and among the students required more sophisticated listening skills than teacher lectures because topics tended to change and speakers used different levels of formality in their speech. It was difficult for them to participate in class discussions because they needed to process questions and others' comments and think of responses. They feared using inaccurate speech and asking inappropriate questions and the lacked experience competing for turns in on-going discussions. Spontaneous speech in oral presentations was also a difficulty in both student groups.

Gestures. Student gestures aid collaborative understanding, particularly when formal language specific to the problem domain is limited and, in some circumstances, gesture may index a state of cognitive uncertainty that signals the onset of changes in understanding (Reynolds & Reeve, 2001). Research suggests that gesture is a source of information about what children themselves understand about mathematical cognition (Steffe, 1992). Children acquiring novel concepts sometimes gestured when explaining their understandings to experimenters (Alibali & Goldin-Meadow, 1993); gesture mismatch sometimes signaled cognitive dissonance (Goldin-Meadow, Alibali, & Church, 1993). Roth (2000) found that middle and high school science students who had weak academic language typically used gestures to explain the phenomena. He found that as students began exploring the phenomena, gestures preceded speech but after students obtained familiarity with the phenomena, gestures co-occurred with academic speech.

Reynolds and Reeve (2001) examined two problem-solving episodes about graphical representations of speed from the database of the Classroom Learning Project (Clarke, 2001). The collaborative learning groups in this study lacked an understanding of formal language. The study (Clarke, 2001) illustrated how students dynamically co-constructed an understanding of mathematical problems. I used their work as an example of writing up data to show how gestures make a difference. In order to identify transitional events that reliably signaled changes in understanding, protocols of two brief exchanges between two students included the students' speech and gestures in two columns with gestures on the same line as the accompanying speech. Speech that was slower than normal was italicized and pauses longer than one second were placed in brackets. Taps on paper directed group attention and representational hand movements indicated acceleration. Slowed speech occurred in conjunction with representational gestures. Reynolds (2001) claimed that gestures indicated aspects to which the students attended and the state of the students' cognitive certainty.

Summary of Literature Relating to Video Stimulated Recall as a Research Method

Interpreting student experiences during quiet times requires analyzing gestures and post-event interviews because there is no real-time external verbalization. Clarke (2001) and Williams (2007) used video-stimulated recall (VSR) to reduce the potential for misinterpretation by allowing the student to use his own language to describe thought in a post-task setting. TPTS utilized two think times, T1 and T2. I believed that it would be difficult to determine what was happening during T2 since external cues might be limited; students frequently sit very still when told to “Think.” The student experience of the T2 phase was illuminated through VSR.

To supplement the interviews and observations I included other techniques. Bertrand (1992) described the use of nondirective techniques to lead focus group interviews and Steffe (1992) suggested that gesture was a source of information about what children understand about mathematical cognition. Reynolds (2001) described methods used to record speech patterns and gestures during a mathematical problem solving task in a collaborative learning setting. The methodology used in these studies provided structure for the current study.

Chapter 3 Method

In this qualitative descriptive analysis, I investigated how students in a tenth grade geometry class experienced TPTS as they tried to make sense of and then report findings on mathematical tasks. The focus of attention was on how students experienced TPTS during each phase and how they used mathematics vocabulary, references to cultural connections, and gestures during whole class presentation of solutions. Study data included pre- and post-observation interviews with six students, audio- and videotapes of six classroom sessions over an eight-week period, six video-recall interviews of selected students and one focus group interview of the entire class. Additionally, field notes documented vocabulary and student/teacher interactions over a five-month period.

Setting

The study took place in a conveniently chosen Midwestern private Christian school that services all twelve grades. Classrooms were located in an annex to the church building and all areas were accessible without going outside. Every class began with prayer lead by the teacher and the minister served as an interim principal at the beginning of the study. A full-time principal took over in January but the minister was still readily available to students and teachers throughout the day. Figure 3.1shows the number of students and teachers by grade level. The same administrative structure served all grades.

Grade	**PK 3-5**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Students	38	14	10	13	5	9	9	10	5	11	10	8	5
Teachers	3	1	1	1	1	0.5	0.5	5 subject teachers plus Athletic Director, Music, Bible, Physical education, Librarian					
2 Administrators and 2 Staff													

Figure 3.1 School population by grade level.

Pre-observation interviews took place during the first semester of the academic year. The video and audio recordings took place two months later during the second semester. The sample consisted of one 10th-grade geometry class with ten students at the beginning of the study but one student left the school before the study was completed. The teacher did not have a license to teach in Indiana but had teacher training through a Christian college in a nearby state. The state had recently accredited this school so the teachers were seeking state licensure to keep that accreditation. The accreditation also meant that the school would administer state assessments each spring. The entire faculty received TPTS training as part of their professional development. The curriculum director supported the research and all classroom teachers cooperated throughout the study by providing quiet spaces for interviews with study participants in the library or their classrooms.

Participants

All teachers, administrators, staff, and most of the students at this school attended the church and most of the students had known each other since early childhood. However, some students without church affiliation came because of the state's voucher system changes. State laws had recently changed, allowing parents to use state funds, in the form of vouchers, to help pay tuition to private schools. Students wore uniforms and the church provided confidential assistance for those that had difficulty buying them. Demographic information shared by the administration indicated that nearly one-fourth of the students were either foster or adopted children, including many with biological parents who were abusive or in jail. Classes were inclusive; autistic students and students with severe limitations were included in classrooms with their same age peers. The student population was 20% African American, 13% Hispanic, 12% multiracial and 55% Caucasian. Because the school housed Pre-K through 12, entire families were involved in the school and most lived within a thirty-minute drive. School wide, 57% of the students were on free or reduced lunch programs. Students were encouraged to be active in band, sports, and extra-curricular clubs.

As part of this study, I recorded comments from the classroom teacher but the focus of this study was on the experience of the students, not the teacher. The teacher provided instruction using the normal curriculum and used the TPTS strategy with six lessons over an eight-week period. Student participation in the study was strictly voluntary. School administration sent a letter requesting parental permission for the videotaping of the students prior to the opening of school; this letter stated that the student might be included in a series of interviews. A second letter, distributed in class to students just prior to the beginning of the study, requested participation in the interviewing process. All ten students in the geometry class and their parents or guardians indicated permission for videotaping but two asked not to be interviewed, leaving eight students willing to participate in both the videotape and interview phases of the study.

The school administration shared data from the previous year's mathematics and English assessment scores and provided details about learning disabilities to identify students who struggled or had strengths in academic language and mathematics. Although not asked about these facts during the interview, many students mentioned their own academic strengths and weaknesses, diagnosed and supposed learning disabilities, and family structures. This confirmed information shared by the administration. The study included six students, purposefully selected to provide a variety of strengths and weaknesses in language and mathematics, for interviews and audio and videotapes. I chose six students based on the spectrum of abilities and experiences. The remaining four students provided comments in whole class videotapes, including the TPTS sessions and the final focus group interview. One student left the school before the fifth session. Table 3 shows the student profiles and the purpose for selection of the six students chosen for the main part of the study. Anonymity and confidentiality of individual student data and results based on those data were assured. Figure 3.2 shows the six chosen participants' gender, pseudonyms, reported academic strengths and weaknesses, and the reasons for selecting them for the study.

Pseudonym/ Gender	Reason chosen	Reported characteristics
Alberto/ Male	low math low English bilingual	low scores in math (C-) and English (repeating English credit) in coursework and standardized tests but had A+ in biology Mom and Dad were bilingual (Spanish and English), grandparents spoke only Spanish self-reported problems with math vocabulary he frequently used self-talk to understand problems
Bradley/ Male	high math high English monolingual	A's in all coursework. Self-reported that he did not study, did use self-talk to solve problems and would rather work alone.
Chad/ Male	low math low English monolingual	behavioral struggles in academic settings, had D- in geometry and two other courses
Diana/ Female	high math high English monolingual	exceled in both math and English coursework, perfect scores in all classes and high standardized test scores, self-reported that she did not self-talk to problem solve
Ester/ Female	low math Low English bilingual	low scores (Ds) in math and English coursework and low standardized test scores Mother spoke only Spanish and Father was bilingual. Father tried to help with math homework but his definitions of math terms seemed different to her. This was sometimes confusing.
Finn/ Male	average math low English monolingual	reading comprehension difficulty, must read things three times struggled to get Cs and Ds in English coursework Mother suspected he was dyslexic because dad and uncle were diagnosed and symptoms were the same.

Figure 3.2 Background information for the six profiled students.

Curriculum

Students were required to purchase and were encouraged to write notes in the Seymour (2008) geometry textbook, a traditional lecture-based curriculum. The textbook is not on the state textbook list. First copyrighted in 1925, this edition was revised and illustrated in 2006. The progression of topics throughout the text were similar to those recently printed but some terminology was unfamiliar. For instance, Chapter Two deals with *rectilinear* plane figures such as triangles, rectangles, and other polygons. These terms were well defined. I was concerned that *line* was used rather than *line segment* throughout the text. The text defined line as the intersection of two solid surfaces, anchoring it in geometric figures rather than an abstraction in space. The copious text provided consistent structure with definitions, axioms, proofs, Think It Through questions and exercises in each section. Most questions asked students to prove or show how it was possible to do something. There were examples of proofs for most corollaries and theorems. The author expected students to develop two-column proofs but gave them leading questions, stating that propositions would be proved using an "analytic-synthetic method" which allowed the student to work from the end result back to the beginning.

The teacher closely followed a daily schedule provided by the publisher and used book tests for assessing student understanding. Homework assignments involving exercises from the textbook were posted weekly and referenced daily. Students graded their own homework and scores on those assignments were reported to the teacher publically unless the student requested a private report. The teacher never saw the homework. One corner of the blackboard held names of those who were missing quizzes or tests and the Bible verse of the day.

Role of the Researcher

In July 2013, I met with school administrators to discuss the purposes and procedures of the study and to receive assurance of cooperation and then provided a letter of explanation and consent (Appendix C) for distribution to the parents of the class studied. I provided one hour of professional development in August (see Appendix D) to teachers from all grade levels and subject areas to introduce them to the TPTS strategy for cooperative learning. The cooperating teacher received further training during two prep periods prior to beginning use of TPTS and continued to meet briefly with me prior to and after each session. His skills for incorporating TPTS into the normal curriculum improved throughout the study. All teachers at the school had the option to request training in TPTS at no cost to the school throughout the academic year. One teacher requested mathematics tutoring for the PRAXIS exam to meet state licensing requirements. Prior to our first meeting, however, her university provided her with a tutor.

Data Collection

This section describes the three interviews, recorded transcriptions, and field notes used in this study.

Pre- and Post-observation interviews. A pre-observation interview (see Appendices E and F) with the six primary subjects gathered information about attitudes concerning group or individual work, word usage in class, use of written text, and the availability of quiet time for thinking. Many of the questions referenced feelings about group work, think time, and whole group sharing. The interview at the beginning of the study provided clarity about perceived needs for support in the mathematics classroom. The video-stimulated response, post-observation interviews, and focus group interview provided data about the kinds of support the students received through the different phases of TPTS.

Audio/Video Tape Transcriptions. Audio and video recordings were taken for each session. When student voices were weak on video recordings, supplemental audio recordings were synchronized with the video to match words and gestures during each of the phases of TPTS. Transcriptions included gestures and long pauses in parenthesis. Events such as "teacher spent 5 minutes reading announcements" were referenced in field notes but actual interactions during these events were not transcribed.

Video-stimulated recall interviews. Interviews based on selected video clips of TPTS in the classroom gathered information about how students used each of the phases of TPTS. Questions referenced language, mathematics, and cultural connections. Instances where students mentioned connections between mathematical tasks and applications outside of school were considered a cultural connection.

Field notes. I gathered field notes on each visit to record mathematics vocabulary usage and student interactions. I noted student reactions to the teacher's use of mathematics vocabulary, and student gestures, facial expressions, writing and behavior during work times.

Focus group interviews. The entire class participated in a final focus group interview four weeks following the taped sessions. The teacher indicated that he used TPTS in the class on three occasions following the taped sessions and prior to this focus group interview. This interview gathered end-of-the-year reactions. A TPTS format was used to conduct the interview. I asked them for their honest reactions to the use of TPTS and gave them one minute to think alone. They then took another minute to discuss thoughts with a friend sitting nearby, fifteen seconds to think alone, and then reported to the class.

Procedures

This section describes the process used by the teacher for use of TPTS and the procedures for interviewing and transcribing recordings.

Teaching with TPTS. The classroom teacher participated in the training on TPTS provided for all teachers in the school and received additional coaching from me on the use of TPTS in geometry. He incorporated its use in the regularly scheduled curriculum on ten occasions, six for this study and four more by his own choice. He chose pairs and had students move to specific desks around the room to accommodate the video cameras. He chose to use a timer with a buzzer to signal the end of each phase. Students first worked individually for one to two minutes on the task. He then instructed them to work within the groups for two to five minutes, dependent upon the given task, to talk about their different solutions and try to solve the problem. On each occasion, the teacher told the students to think alone for one minute to develop a plan for presenting to the class. They could write in their books, refine their work, quietly rehearse words in sub vocalizations, or simply sit quietly. There was an expectation for all students to be prepared to share with the class.

Interviews. I conducted and videotaped all interviews to assure consistency of procedure. Interviews took place in a quiet area of the school to assure confidentiality of responses and audio clarity. Pre- and post- study interviews were in the empty math classroom during the activity period. Video stimulated recall interviews were in the library.

Audio/videotape transcriptions. Clips of transcriptions documented language and gesture usage, variety of responses, and student use of each segment of TPTS. Video-stimulated recall interviews used these clips. Audio recorders, placed at the desks of three groups of students, captured student reactions to assigned tasks. One stationary video camera was positioned by the teacher’s desk in the front of the room and one roaming video camera that I carried recorded all students but focused mainly on a selected student of the day. Full transcriptions of audible recordings of the six students taken during the TPTS provided data for five of the six weeks. Technology difficulties restricted data collection for one audio recording of one TPTS session.

Video-stimulated recall interviews. Following each of the six videotaping sessions, I interviewed at least one of the purposefully chosen students using video-stimulated recall. Each student sat for the interview at least once based on clarity of video and audio recordings, availability during activity period, and needed clarification of events on the clip. Clips from the most recent videotaping stimulated recall of student responses. Additionally, clips from earlier days were included when they could potentially add clarity to the student's perspectives.

Data Analysis

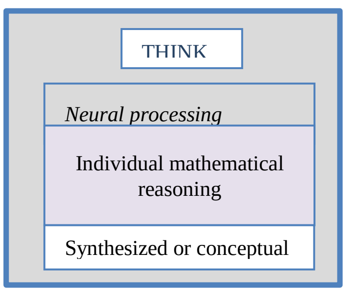
Data were identified according to the four phases of TPTS. I determined whether the action was individual or collective and whether it was manifested publicly or privately according to Ernest’s model of sign appropriation and use. These allowed me to build student profiles that I compared to perceived student experiences.

Interviews and video-stimulated recall (VSR). During initial analysis of the classroom videotapes, I looked for key points where the students had potential disengagement, or excitement, discomfort, shifts in body movement, elation, cognitive shifts registered through gestures and verbal cues. During the VSR, I played the short segments of the video (no more than five) for the student and asked for input into what was happening in an attempt to gain deeper understanding of the student's experience and motives. Following is an explanation of how the interview questions were used in answering the research questions. (See chart with full rationale in Appendix I.)

Question 1. How are the phases of TPTS experienced by students with language differences in a high school geometry classroom?

Audio/video transcriptions provided visual descriptions of the setting and activities for each class; the focus was on one or two students per class period. Field notes included facial expressions, body gestures, word usage, modes of representations for solving tasks, and social interactions.

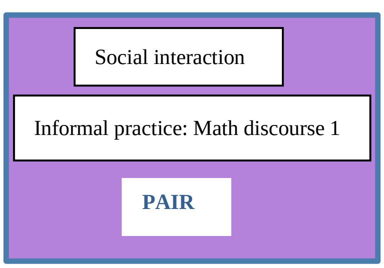
Question 1a. How do students use the first Think time?



Pre-observation interview responses to question #5 (student's perception of the use of quiet time) were used to build student profiles and then compared to post-observation interviews to identify changes in attitudes.

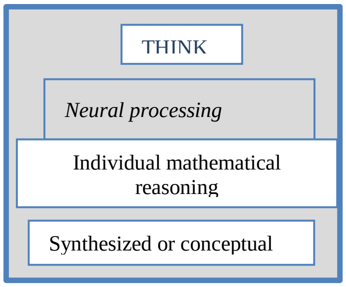
Pre-observation interview responses to question #6 (how would quiet time look to an observer) were used to build student profiles and then compared to post-observation interviews to identify changes in attitudes.

Question 1b. How do students use mathematics vocabulary during Pair time?



Audio/video transcriptions and field notes documented the usage of academic language, specifically mathematics vocabulary, during small group or PAIR time.

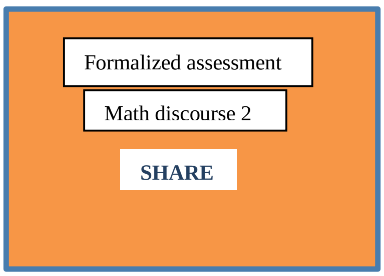
Question 1c. How do students use the second Think time?



Video-stimulated recall prompted student responses about what they were thinking during the second think time. This illuminated other artifacts such as written work, audio/video of gestures and subvocalizations and field notes.

Question 1d. How do students use supportive cultural connections and when do they appear?

Field notes and recordings were inspected for explanations that included cultural references during the Pair or Share phases.



Question 2. How do students report their answers to mathematical tasks in a classroom that uses the Think-Pair-Think-Share cooperative learning strategy?

I inspected field notes and recordings for patterns of responses for the students either individually or by group.

Question 2a. How do students use mathematics vocabulary during the Share phase? Audio/video transcriptions and field notes during whole group sharing documented the usage of academic language, specifically mathematics vocabulary, within the recorded sessions.

Question 2b. How do explanations vary during the Share phase? The number and type of explanations (such as when one task was solved multiple ways or through multiple representations such as diagrams, pictures, equations, or examples) were recorded in field notes and recordings.

Question 2c. How do students use gesturing throughout the TPTS process?

Facial expressions and body movements were recorded in field notes and video tapes. Of particular interest was the shift between quick inward motions and slow outward motions that signal internalization.

Building student profiles. Many of the pre-observation interview questions helped to form student profiles for each of the six selected students. Comparisons to responses in the post-observation interview identified changes in attitudes about group work, quiet time, academic language, and whole class sharing of mathematical understanding. A chart describing the purpose of each task was included in Appendix I. Questions 1 and 2 of the pre-/post-observation interviews (see Appendices E and F) prompted students to discuss their feelings about working alone or in groups (THINK and PAIR) to determine if language concerns might surface in group situations and interfere with mathematical reasoning. All interviews were transcribed and analyzed using the following guidelines to determine student perceptions of individual and group work and the kinds of supports offered in each of these classroom configurations.

- What reasons do students give for their grouping preferences? Do they mention aspects of 1) language, 2) mathematical understanding, 3) classroom or home culture?
- What do students feel are enjoyable and/or difficult about group work? Do they mention aspects of 1) language, 2) mathematical understanding, 3) classroom or home culture?

Interview Questions 3 and 4 asked about words not understood and how students found supports for learning. This information was used in construction of the student profiles.

- When students have difficulty understanding the academic language in the classroom, what methods do they use to make sense of the conversations? Do they mention 1) peers, 2) text, 3) cultural connections, or 4) use of quiet time?
- How do students perceive their own use of quiet time? Do they find value in the first Think time? Do they value in the second Think time?
- What do students report about sharing results for mathematics tasks to the whole class? Do they mention aspects of 1) language, 2) mathematical understanding, 3) classroom or home culture?

Audio and Videotapes. TPTS class sessions were taped on six occasions over a seven-week period. I transcribed segments that included the use of TPTS and wrote summaries for individual students by session number. I then looked across all students for each session, separated by TPTS phase, and analyzed them through an iterative coding process. I first color-coded the following categories: mathematical vocabulary, attentiveness to book references, writing, self-talk, problem solving strategies, gestures, cultural connections, and willingness to share. Comparisons of these categories across the six sessions and an analytic memo Appendix H aided final between- and within-student analysis of data.

Focus Group Interviews. Because all students had agreed to participate, the focus group formed at the end of the study included all students in the geometry class. I asked students as a group about their impressions of learning using TPTS during a relaxed conversation without structured questions. I tried to get information on student experiences during each stage of TPTS.

Pairings

I assigned students to pairs in the first task according to the stated preference in the pre-observation interview for working alone or in groups. If the student listed *distractions* as a reason for working alone, the student was paired with a student who tended to be quiet. The teacher assigned the groups in all of the following sessions.

Tasks

Five of the six tasks for the TPTS sessions came directly from the textbook. Either the teacher used problems he would normally assign or he chose difficult problems that would normally be skipped. He originally chose tasks from the textbook that he felt could be discussed in pairs. Near the end of the study, however, he chose tasks that he felt would help him assess student understanding. A chart with the tasks and pairing of students is included in Appendix J.

Geometry Curriculum and Sessions

The section describes each TPTS session with the date, the assigned task and its placement in the curriculum. Notes about the text's usage of vocabulary are noted.

February 4: session 1. The students were in the middle of the unit on proportions and similar polygons. The prior night's homework dealt with internal and external division of a line where "a point was said to divide a given line into segments internally or externally according as the point is on the line or on the extension of the line"(Seymour, 2008, p. 199). I wondered how a line, infinite in length by definition, could have an extension so I checked the index for references to segment. The book used in the classroom used the word *line* to indicate *line segment* "where there is no ambiguity"(Seymour, 2008, p. 5). The first question in the section asked, "Could the ratio of the two segments be unity (a) when the division is internal? (b) when the division is external?" I sought help from the index to check the use of *unity*. It was not there. I looked in the glossary for *unity* and still found nothing. The teacher explained in class that *unity* meant *being one* or *undivided*. He also explained other terms (ratio, antecedent, consequent, proportion, extremes, means, fourth proportional, mean proportional and third proportional) that were listed in the front of the chapter(Seymour, 2008, pp. 190-191) and indicated that they might see these terms on a test.

This was the first time the teacher had used any teaching method other than direct instruction and he asked for my assistance in choosing the task. I chose a student exercise provided by the book that the teacher normally used as direct instruction. The task asked students to write the converse of the following theorem.

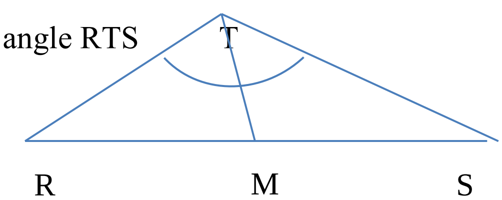
Theorem 55

The bisector of an interior angle of a triangle divides the opposite side internally into segments which have the same ratio as the other two sides.

Given: ΔRST and TM the bisector of interior angle RTS

Prove: RM : MS = RT : TS

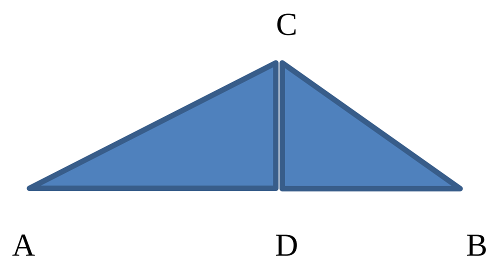
(Seymour, 2008, p. 201).



The teacher provided no explanation for the conditional statement and accepted an informal student definition that *converse* meant *flipping* the order. He gave them the page of the theorem and put them into pairs that I had chosen based on their pre-interview preferences for working in groups.

February 10: session 2. The unit addressed similar polygons, proving triangles similar, and proving lines proportional. In the section on proportional segments of parallel lines, the teacher chose a three-part theorem and its corollary for TPTS. He asked each small group to explain to the class one corollary. (For a list of pairing assignments, see Appendix J session 2).

Theorem 61. If in a right triangle the perpendicular is drawn from the vertex of the right angle to the hypotenuse:



I. The two triangles thus formed are similar to the given triangle and to each other.

II. The perpendicular is the mean proportional between the segments of the hypotenuse.

III. Each leg of the given triangle is the mean proportional between the hypotenuse and the adjacent segment.

Corollary 61-1. If a perpendicular is dropped from any point on a circle upon a diameter, then the perpendicular is the mean proportional between the segments of the diameter.

(Seymour, 2008, p. 210)

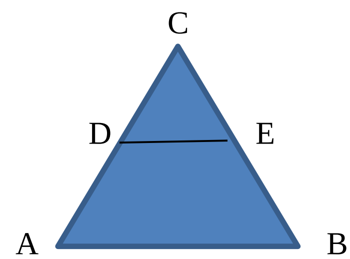
February 13: session 3. The teacher decided to try using TPTS as a tool for reviewing for the mid-chapter test. The chapter included the Pythagorean Theorem, so he chose a word problem from online that used the theorem.

Honey and Money are twin brothers. Mother wants to divide a rectangular shaped cake of 5 cm wide and 6 cm long. Find the length of the third side of the cake if it were cut diagonally? [sic]

February 24: session 4. One full week passed before the next session due to video stimulated recall interviews and weather-related school closings. During that week, the students worked with similar polygons and the *ratio of similitude*. The teacher planned to give a quiz but, instead, decided to use TPTS because he said he could listen to their thinking.

Given the following triangle, find BC.

AD=8, CD=6, CE=3



March 12: session 5. Over two weeks passed before I returned for session 5. The teacher chose to use TPTS twice on his own during that time. He had worked extensively with proportions in circles and in polygons so he chose a proof that involved a trapezoid.

Prove that the line joining the midpoints of the bases of a trapezoid divides the trapezoid into two equal parts.

March 17: session 6. One week later, he conducted the final TPTS session for the study. The class studied areas of polygons and was nearing another mid-chapter test. The teacher was also using TPTS with his algebra class and I had suggested a modification in that class of one of the algebra exercises that used the term *perimeter*. He liked the challenge that it gave the algebra students and suggested that we modify it further for the geometry class so it would include area.

The perimeter of a right triangle is 30 cm. One side is 7 cm. longer than the other. What is the area of the triangle?

Focus Group Interview

I finished individual interviews over the next week and returned one month later for the focus group interview.

Chapter 4 Results

I reported individual findings for each of the purposefully selected students by presenting a profile based on information shared during pre-observation, a description of reactions to tasks during each phase of TPTS in a series of sessions, and a comparison of the post-interview to the pre- and classroom reactions. I identified the key kinds of support for each student and described reactions in each of the phases of TPTS.

I analyzed data and arranged the results primarily by individual rather than by session because the research questions asked how individuals experienced TPTS. Each section begins with a profile of the student. Then, that student is followed through a series of TPTS sessions, including notes about the partner. Because a different student was the focus each session, not all sessions were recorded for each of the six chosen students. For instance, there is no mention of session 4 for Bradley. The final section in this chapter reports findings from the focus group interview and includes comments from all ten of the students.

Alberto, Hispanic Male with Low Academic Scores in English and Geometry

I began by analyzing Alberto, picked at random from six focus students. Alberto was a male of Hispanic background who spoke fluent English and seemed to require talking aloud to process mathematics. He used proper grammar with clear enunciation and a broad vocabulary in conversational speech but he mentioned in the pre-observation interview that he had some trouble with mathematics vocabulary. The administration remarked that math grades from his last school were low and he was repeating his English credit. His standardized scores were also low but, in that same year, he had an A+ in biology.

It was Alberto's first year at this school where most of the students had known each other since early childhood. He was the shortest male in the class and appearance before his peers was vital to him. I, therefore, noted height throughout session results. His uncle traveled internationally as a speaker at conferences so public appearance was also important to his family. His parents were bilingual and his grandparents spoke only Spanish.

I compared the transcribed pre- and post-interviews to initially identify Alberto's key kinds of support. The pre-interview indicated that the teacher's step-by-step explanations and willingness to answer questions after class were the main supports. In both interviews, Alberto stated that he sought examples from the presentations and book, frequently memorizing information. There was repeated mention of talking aloud in both interviews. He stated that he talked to himself and often asked the teacher questions and then would repeat the answers to himself so he could better understand. In the post-interview, he stated that the most helpful part of TPTS was explaining the answer aloud to others. He spoke of the Pair phase as teamwork, raising his hand to respond to the teacher, anxiously presenting solutions to the whole group, and using self-talk to help make decisions. Asked if he talked to himself, he responded,

> Yes, I did. I talked to myself and (looks to the table and subvocalizes, "How should I explain?" then looks back at the interviewer). I would ponder, sometimes I would come to two ways where I thought (long pause) I knew there was only one way but I couldn't remember, umm (pause) which one it was. There were two ways so I would ponder against

myself which one I should try first and which one I think would be the right one. So, I kind of talked to myself so kind of debated with myself.

Alberto was emphatic in the post-interview that quiet time allowed him to think of how he would explain the problem properly without using *slurred* language. He added, “I try practicing it in my lang…in my head to make sure that when I was explaining it, it was explained clearly and I wouldn’t have to like repeat it multiple times because I said it off or anything.” I wondered what his reference to language might be but did not elaborate further.

The quiet time in T1 gave Alberto a feeling of peace, according to his post-interview, because he had time to think about a problem on his own without distraction about how to solve the problem and how he thought he should do the problem. After quiet time, he felt determined to describe accurately his solution to his partner during PAIR. Without T1, he feared the group might be more confused and there might be a few arguments over what was right or not right. I noted that his only fear of group work in the pre-interview was the potential for arguments with others. Teamwork and social acceptance were very important to him and he wanted to present himself well in front of others. Although he did not believe he had problems with mathematics vocabulary, he required quiet time to process language. He reported, “I knew what it meant. It was just that the question was worded funny.” This implies that individual vocabulary words were accessible but mathematically stated sentence structure was difficult.

Alberto perceived T2 differently for the first and second half of the study. Prior to his first presentation to the class, he felt *worried* – stage fright – and later, he felt *confident*. Working with a partner and then having time to practice his words reportedly gave him confidence.

The feeling Alberto experienced during the Share phase reportedly also changed midway through the study. In the beginning, he claimed he felt *awkward* and wondered what would happen if his answer was wrong and he looked foolish in front of his peers. In the end, he said he felt *relaxed*. According to the interviews, his ability to speak about mathematics in class changed dramatically over the course of six weeks and it was his perception that he understood math a little more. He believed that everyone made mistakes and he sometimes learned from their mistakes. He claimed that he was at least more willing to ask questions than prior to using TPTS.

I compared these findings to the video clip from the TPTS geometry sessions to establish patterns and identify discrepancies. The first session was during the time when he later described himself as having stage fright. This seemed evident on the video of the first session when he was unable to find words to begin speaking in front of the class.

Session 1 with Alberto and Bradley. Alberto was paired with Bradley, a tall blond-haired male known as one of the top students in the school both academically and in athletics. Both had indicated that they liked to work in groups. During T1, Alberto studied his textbook, wrote in the margins and focused on the task. Immediately upon beginning the Pair phase, his partner spoke rapidly for 26 seconds of the allotted four-minute time. In that time, Bradley read the theorem then said, “In a sense it is saying that if you draw a line here (points to side of the triangle) that the bisector will ultimately create two proportions. So if you invert that, when you draw the line first instead of the bisector the triangle is divided into a ratio where you drew the line. This line will bisect wherever it falls." I had to slow down the tape to run at 58% speed so Bradley's words could be understood.

Alberto rubbed his fingers together, listened intently with mouth slightly ajar, head cocked to one side and his eyes focused occasionally on his partner but mainly on his book. He told me in the later video stimulated recall interview (VSRI) that he was comparing his own work to what Bradley was saying. At the end of the 26 seconds, Alberto pursed his lips, shook his head in light acknowledgement, and gave a faint smile. He turned his head back to his book, erased, rewrote, and studied. Bradley asked if he had any ideas and Alberto shook his head and kept writing. Bradley said, "I guess we will just use this then." They then sat in silence for three and a half minutes while other groups negotiated their final responses. In slow motion, I was able to see Alberto mouthing something while pointing to the book. He scratched his head, rocked his body, played with his pencil between writing and mouthing, checked the index for the definition of converse, and kept his eyes in his own space. He continued to write and study his own words during T2. He carefully wrote the original theorem, placing a checkmark at the end of the If statement. Alberto reported in the VSRI that he had stage fright and tried to form a three-step presentation and add notes so he could have eye contact with his peers during the presentation.

Because this was the first time students shared, students were nervous and told the teacher that they were reluctant about being on tape. We predicted this and told Bradley on the prior day that he would be a presenter so he could prepare himself mentally. This may have been a mistake because, as soon as the buzzer signaled Share, he nearly leapt from his chair to give the presentation with no apparent thought of his partner. Alberto's face turned from tense concentration to playfulness. He played with the recorder, made faces at the video camera, and turned to look at others in the classroom as Bradley wrote,

> If you draw a triangle with a line that divides the four parts into an equal ratio then the line will bisect the interior angle." He gave a 20-second speed-reading of the theorem, his converse, and the following explanation, "So you are just doing the same thing. You are flipping it and so you do the last thing first, so you get the first thing last.

Alberto stated during the VSRI that he understood the explanation given by his partner. It was his belief that the partner's converse was more succinct and therefore better so he decided to adopt it. He stated,

> Whenever I wrote, mine kind of, mine was a little longer because I kind of added in a few more, well, quite a few more words than necessary. So when I saw his, it was… it was kind of like the same and they were similar but mine was longer so I figured his was better because it was faster to say and not quite as complicated as mine. I wasn't going to present mine because I saw the flaw there.

The most important thing on Alberto's mind during the first session was classroom presentation rather than the validity of the mathematics converse. Listening to Alberto, one would never suspect that he came from a home where Spanish was the first language or that Alberto had low scores in English class and troubles with mathematics vocabulary. His words were precise and his structure was exact. He was very conscious of using proper grammar when speaking to adults and during formal presentations. His fluency with spoken language prevented him from being labeled ELL but struggled to decode academic language.

Session 2 with Alberto, Ester, and Hugh. Alberto gave his first presentation in the second session, which was nearly one week later. Students were required to prove that a line drawn from the right angle (labeled C) perpendicular to the hypotenuse of a right triangle (labeled AB) was the mean between the two formed segments (see Appendix J). Two students, Chad and Diana, presented prior to him but had faulty logic in their proofs. Alberto stood at the board speechless while the teacher quietly spoke with Chad about his proof. Alberto had not participated in the Pair discussion with his two partners and he appeared frustrated, unsure how to proceed as he stood before his peers, so I intervened.

R: Were you corollary 1 or 2?

A: two

R: Okay, so point to the perpendicular line.

A: (pause)

R: What line is perpendicular?

A: (He pointed correctly.)

R: CD is the perpendicular line?

A: (He nodded affirmatively)

R: It says it is the mean proportional between the segments.

R: What are the segments of the hypotenuse?

A: AD and DB (correct response)

R: Okay. Were you able to write a proportional for that?

A: (pause)

R: Two fractions equal to each other?

A: Yes. AD and DC are to DA and DB (incorrect)

R: Close.

A: AD and DC to DC and BD (correct)

R: Okay. Write it up there.

A: $\frac{AD}{DC} = \frac{DC}{DB}$

Although it was my intent to speak only when needed to clarify instructions for the students, I felt it was in their best interests to ease the tension for Alberto and continue the discussion of the problem for the other students. This was the beginning of the study and participants were still not comfortable with the Share phase. My decision to join the conversation influenced the result of the day's presentation for Alberto and it is impossible to know what would have happened if I had not intervened. All students watched Alberto's presentation except the three in the group that were engaged in conversation with the teacher. After the presentation, Alberto sat and smiled in relief. The teacher returned to discuss the problem then Diana returned to the board to attempt a new, but still incorrect, solution. The teacher discussed the difficulties he had witnessed in Diana and Chad's problems then asked if the lesson was helpful for anyone. Alberto, Diana, and one student that was failing the class (not one of the six) raised their hands and shook their heads affirmatively.

Session 3 with Alberto and Chad. In the first session, Alberto accepted Bradley's explanation without question. He had a much different reaction in the third session when paired with Chad who was failing most of his classes, reportedly from off-task behavior and lack of effort rather than cognitive ability.

The task required using the Pythagorean Theorem to find the diagonal in a rectangular cake. During T1, Chad stared straight ahead with mouth slightly open. He did not look in his book or write on paper. Alberto once again flipped pages in his book and wrote in the margins. When the teacher indicated that they would have about four minutes to talk, Chad raised five fingers and quietly asked for more time. Chad began explaining the problem to Alberto before the teacher stopped giving directions. “Oh! It’s a rectangle.” Alberto watched closely and responded, “I just can’t get it. Did you get it?” When Chad said that he did, Alberto responded that he had been kidding and actually had it. They both sat staring at the problem on the overhead with hands cupped inward toward the body. They leaned toward each other and spoke interchangeably, Chad bounced his hand in the air and Alberto manipulated his pencil in both hands. Both subjects sometimes looked and wrote in their books and Alberto frequently subvocalized his thoughts. Alberto moved his eyes, pencil toward Chad’s book, and said, “Okay, so, uh, (talking rapidly) you are supposed to multiply 5 times 5 and 6 times 6 and then shhh shhh shhh shhh shhh (Chad trying to speak). He underlined something dramatically on his notepad as he said they had to add the two squares. Chad asked, “What page is that? In the book?” and began flipping pages. Alberto told him it was just a squared plus b squared equals c squared as he looked into space, hand flattened on the book, pencil held closely to his chest and mouth slightly parted. Chad said, “It’s all on my page!” then locked his eyes on Alberto who said, “Shhh, shhh shhh. Let me think.” A member of a different group who wanted a calculator briefly interrupted but they seemed to ignore him, intent on solving their own problem. They worked in tandem, discussing the problem and discovering how to compute using the calculator.

C: This is 6 cm here and then we are cutting it here (looks at the overhead) so we are trying to find this. (Pushes tape recorder away.)

A: Is that a leg? Or is it a hypotenuse?

C: Yeah, this is also 6 cm and this is 5 cm.

A: Shhhh. We have to be quiet.

C: So is this what you do? So you push the square button. And you add 36 and 25 and whatever is added up, you square that? (underlines something)

A: 61? Squared?

C: Let me think. 9 times 9 is 81.

A: What's 8 times 8?

C: 64. Seven times 7 is 49. OK, 8 times 8 is 64. So it would be 7 point something.

A: No, I thought it was point 4. Wait! Shhh.

C: Square, square, square. (They quarrel over the use of the calculator.)

Chad stopped abruptly, looked at the buttons with his tongue partially out and said, "Where's equals? Oh! There it is! 7.8!" and handed the calculator to another group. Alberto smiled and wrote the result in his book. Chad asked, "Do you think that's right?" to which Alberto responded, "I have no idea." Chad read aloud as Alberto read the same passage from the book. Chad asked, "Are you sure that's how you do it?" to which Alberto responded, "I don't know but I think so." Chad told him to "be serious" and Alberto nodded his head as if in understanding and released the pencil from one hand while Chad extended his arm across the desk in a relaxed manner and his eyes moved from his own book to that of his partner. Alberto talked to himself and wrote in his book.

This appeared to be where they came to agreement on the solution to and answer for the problem.

During T2, Alberto looked at the screen, flipped book pages, and tapped his chin with his pencil. Chad wrapped one arm on his chest or rubbed his head and wrote with the other hand. When the buzzer rang for the Share phase, the two partners debated about whose turn it was to go to the board. They both watched intently as three groups presented solutions but they did not have to present. When the teacher questioned the presenters, Chad answered correctly in whispers or used his hands to describe an answer. Alberto looked in the index for the definition of a rectangle. This confirmed his continued need for time and resources to investigate mathematics vocabulary.

Session 4 with Alberto, Isaiah, and James. Alberto was paired with two students who were not the focus of this study so tape recorders were placed elsewhere and most of his comments were inaudible for this session. Field notes showed that his partners were off-task most of the time, discussing capitalism and survival but James suggested that they just, "go with 4." James later said that he saw that 3 was half of 6 so he figured 4 was half of 8 and took that as the answer.

Alberto subvocalized, "c=bc squared" and it appeared that he tried to use the equation somehow in the problem. Still subvocalizing, he wrote in his book and frequently erased. When other groups presented, he listened but continued to subvocalize and write until he said, "Great. BC=7. If he calls on us, we have the same thing." He later indicated that he used the same multiplication process with means equals extremes as one of the presenters.

Session 5 with Alberto and Gabe. The teacher paired Alberto with Gabe in session 5. Gabe agreed to be taped but did not want to be interviewed so there is little background information on him. Field notes indicate that Gabe was unsure about his ability with math and struggled with social skills. Talking on tape continued to be problematic for Alberto; he spoke very softly and it was difficult to discern his words. Much of what was caught on tape during this session was in whispers with both males frequently saying, "I have nothing." Alberto began the pair phrase by saying, "I think. I'm not very sure. Um. So, #4 is prove that the lines joining the midpoints of the bases of a trapezoid divides the trapezoid into two equal parts. Uhhhh." They both expressed a preference for algebra over geometry then Gabe suggested that they "be just like a cheap guy and find *midpoint* in the back." They looked up *midpoint* and *trapezoid*. Alberto noted that the definition of a trapezoid talked about parallel lines and said, "Could that probably, maybe, help?" They used all of their time finding definitions. Alberto expressed regret when the buzzer went off and whispered, "Where to start?" then asked Gabe for a piece of paper. Once again, Alberto used T2 to write and look up definitions. During Share, Bradley presented his ideas then the teacher asked for more ideas. Alberto prodded Gabe to, "just give them definitions" but neither volunteered to present.

Session 6 with Alberto, Isaiah, and Hugh. The task for session 6 involved the Pythagorean Theorem and was modified from a similar problem for the algebra class earlier that day. The teacher decided to keep it as review for the Pythagorean Theorem but modify it further to ask for area. He said that students had trouble using the area formula for triangles and this would help prepare them for a test they would soon be taking. The task asked for the area of a right triangle of perimeter 30 with one side 7 cm longer than the other. It proved to be challenging for most students. Alberto used the last page in his book to record his work. Half way down the open space, he wrote an equation that accurately portrayed the problem.

x + (x+7) = 30 – y

2x + 7 = 30 – y

(Alberto correctly subtracted 30 from each side giving him 2x – 23 = - y then scratched it out.)

~~2x – 23 = - y~~

He correctly simplified the equation but crossed it out when he discovered that he could not get a single number because he still had two variables. He was at the bottom of the page so he began again at the top.

x+7 = 15 – y

y = 8 - y

The new equation, nested on top of the other, still had two variables but he incorrectly changed an x to a y. During the share phase, one student presented an equation like Alberto's first equation and the teacher discussed the need for a second equation.

Post-interview with Alberto. In the post-interview, students used one word to describe their feelings for each of the phases. The responses of Alberto were telling about his experience with TPTS.

T1	Pair	T2	Share
peace	determined	worried (sessions 1-2) then confident (sessions 3-6)	awkward (sessions 1-2) then relaxed (sessions 3-6)

Figure 4.1 Alberto's feelings about TPTS phases

Alberto said that during T1 he felt *peace*. It was a quiet time to think about the problem with no pressure. He knew that he would be able to discuss his ideas with a peer and that the result would be shared. He felt *determined*, during the Pair phase, to convey his understanding to his partner. This was not evident in the first session with Bradley, but it appeared each session after that as he worked with Chad and Gabe. Alberto reported a distinct change in his feelings mid-way through the study. After he presented to the class the first time (when I intervened with factual questions), he said he no longer worried about the presentations. For the first two sessions, he felt *worried* during T2 about what he was going to say and then felt *awkward* sharing. For the last four sessions he felt *confident* during T2 because he knew he had time to develop his thought and then he was *relaxed* during the Share phase as he listened to others, shared ideas, and presented his thoughts.

Summary of Alberto. The key supports for learning for Alberto were the mathematics teacher, examples, periods of quiet time for self-talk and for individual thought, talking aloud to peer groups, asking the teacher and then repeating his response aloud, and listening to whole class sharing. He believed that TPTS supported classroom discourse. During the post-observation interview, Alberto quickly responded that TPTS helped him gain the confidence to speak before his classmates and go to his teacher more frequently when he had questions but he paused at length as he thought about the usefulness in learning mathematics.

> Well, it helped me understand math a little more because he [the teacher]…some of the things I think we already learned. So he had already taught it and, well some of the stuff, if I still had trouble learning and I

> was having trouble doing the problems or even going to ask – I would have to go ask every now and again. And with this, it kind of helped me better knowing how to do the problem.

Although the teacher presented the material previously through lecture, Alberto found he repeatedly went to the teacher to ask how to do the problems. Sometimes he did not even know what to ask. He felt that TPTS helped him understand and gave him confidence to go to the teacher for more help. In the beginning of the study, he told me that he only asked the teacher for help when there was time and that he repeated the teacher's response. At the end of the study, field notes show that he stayed on task longer and asked questions in more ways. He valued his quiet time and wanted even longer Pair and T2 phases.

Alberto used T1 to debate with himself about possible ways to solve the task. He used Pair to check his answers and to work as a team to, at least, attempt the problem. T2 was used to develop a formal three-part presentation and to select the right words. He wanted to have an introduction of the problem, an explanation of the solution, and a summary with the answer. The Share phase was the most important part for him because he got to explain the answer and, in his words, "conquer" his fear of talking to others.

Bradley, Male Caucasian, straight-A student, Athlete

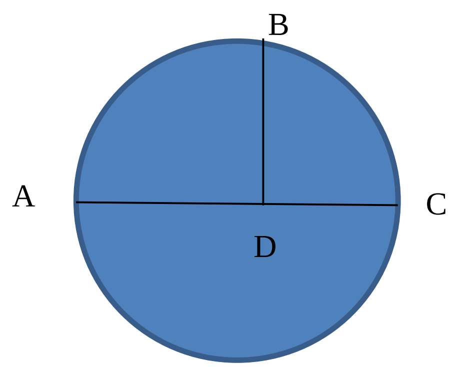
Bradley was a tall, blond male who was active in sports, carried A's in all of his classes, and scored well on standardized tests. Considered one of the two strongest students in the school by both administration and fellow students, he was highly self-confident. He, like most other students in the class, had never used cooperative learning and, according to the pre-interview, preferred direct instruction by a teacher. However, he wanted to try new things and looked forward to working with others with the hope that he could learn things from them. Vocabulary was not a problem for Bradley and he was confident in his ability to learn math. He did not report using drawings or the book to assist problem solving, was comfortable talking in front of peers and felt that quiet time was probably a waste of time.

Bradley stated in his post-interview that he thought others probably believed he was right and did not contribute to the conversations. I found this to be true of both students and the teacher in the beginning of the study. My notes began with the teacher asking for the definition of ratio. Without hesitation, Bradley said, “Things that are equal.” Teacher and fellow students, without question, accepted the incorrect definition.

Session 1 with Bradley and Alberto. Bradley answered questions rapid fire; there were neither probing questions nor challenges. As seen in the description of session one in the prior profile, Bradley gave a 20-second analysis of the problem and solution to Alberto and then asked, “Do you have any ideas?” Alberto sat silent.

Session 2 with Bradley, Chad, and Isaiah. Session 2 provided Bradley with a scenario much different from session 1. Rather than pairing with a student that was new to the school, he was placed in a triad with his longtime friends, Chad and Isaiah, who liked to banter and challenge his ideas. (I noted Chad in an earlier section with Alberto; this section complements that description.) The administrator described Chad as an unmotivated male who enjoyed the social aspects of school rather than the academic. He spent the last two years in a different school where he claimed he learned nothing. The other partner in the triad, Isaiah, was of average ability but was not one of the six selected individuals. The administration said that Isaiah earned B's and C's with seemingly little effort. He enjoyed socializing and courting Ester, a female in the class.

The task for session 2 required students to explain a corollary that involved a perpendicular line as a mean proportional between two segments on the diameter of a circle. The book provided no diagram. I include a diagram and explanation to aid understanding of their problem.



Given BD perpendicular to AC,

AD : BD = BD : CD

During T1, all three males read quietly. As soon as the Pair phase began, Bradley laughed, looked at the teacher, and raised his hands. He explained during the VSRI that he believed he had the solution at this point. The other two males said they did not understand so Bradley read aloud to them and drew a picture in his book. He explained by saying, "So this is going to be the same as this." Bradley believed that it was a very simple problem and that his answer was correct. The other two males continued to question him, pointing to the book, repeatedly using the terms *this* and *that* and pointing at lines around the picture. Bradley looked at the teacher, laughing, seeking help to quiet his partners. The teacher moved away. Bradley asked why they did not understand and Chad responded, "Because you circled everything."

When the teacher returned, Bradley asked, "Is the perpendicular dividing right there or is it everything? This is the diameter." I coached the teacher that responses should be limited during Pair work and he did not respond. Chad continued pointing and tapping, saying, “So this and this” (moved hand sideways) “are to this” (moved hand up and down the segment perpendicular to the diameter). Suddenly, Bradley stared straight ahead with a blank look and mouth open. He exclaimed, “Oh! I get it now!” His head rotated upward and his eyes moved around the space in front of him. Chad said, “Wait! So am I actually right?” This time, Bradley looked at the book and said slowly, “This is to this as this is to this” as he pointed distinctly at segments. Chad responded, “That… is what I said.” He looked at the teacher, not laughing, mouth slightly open, as if in amazement. After a moment of pause, the three males began arguing over who had the solution first.

Still in the Pair phase, Bradley and Chad began thinking about the presentation. Once again, Bradley used the *this is to this* verbiage. Chad lowered then raised his head, placed his hands in a vertical position, and slowly enunciated, “the vertical, perpendicular line…is the… What is the bottom part of a fraction?” His partners answered. Chad continued, “Denominator, denominator. And the horizontal part of the line is the nominator [*sic*].” Bradley looked at the teacher and said, “I’m not saying I’m not having a good time. But from a learning point of view…” and he shook his head to imply that it was not helping him to learn.

Bradley and Chad both tried to talk during T2 until I reminded them to be quiet. Bradley wrote something in his book and Chad looked around the room, appearing disengaged. When Chad learned he would have to present, he panicked and asked Bradley for help but Bradley said he could not talk. Bradley played with his cell phone.

Chad ran his finger up the perpendicular line segment and tried, unsuccessfully, to get Bradley's attention.

During the Share phase, Finn and Diana presented prior to Chad. Diana's presentation actually involved the right triangle inscribed in the circle for Bradley's task but the book did not point that out and no one mentioned it. All students except Bradley watched her; he looked at his iPad until the teacher began to speak. As soon as the teacher finished, Bradley returned to the iPad. Diana had an error. The teacher asked her to reconsider and called upon Chad for the next presentation. The teacher saw Chad's error and spoke with him and the others in the group while Alberto presented. After the teacher addressed the triad, Bradley shook his head affirmatively, pointed to the book, then picked up his iPad again. He was convinced that he had the correct solution but Chad had not presented it correctly.

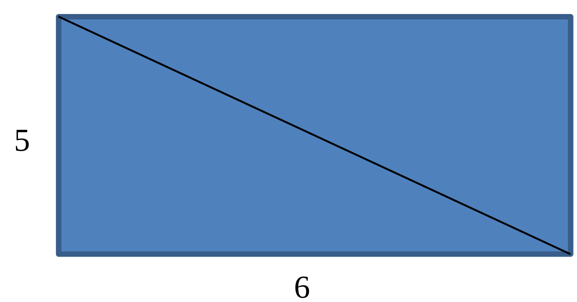
This session was the focus of the VSRI for Bradley and Chad where they revealed differing insights. I will share Bradley's perceptions here and will include Chad's in his own section. Bradley believed that he nearly always solved problems within a few seconds during T1. He claimed to need no writing aids since it was all in his head. He would then sit there and wait. He said that the bantering with his partners in session 2 was merely their effort to confuse him because he already had the solution. He thought they were just pointing to random lines. When shown the clip where he expressed new insight, he said, "I'm sure something probably clicked in my head that was…I was trying to figure out. I might not have been 100% sure about it." We looked at the clip again and I probed further. He stated that the other two were just trying to, "drive him nuts" and that any new insights came from looking over what he already had. He explained that during the Pair phase others usually looked to him, asked what he thought about it, and then agreed - except with these two males. He said that these two always tried to come up with something different. Bradley stated that when he asked other people for their thoughts, they gave no input. “I usually think maybe they assume that I am right so they don’t say anything.”

Asked if he thought others could explain his reasoning after the Pair phase, he said, “They probably could after they heard me explain it.” This was not the case in session 3 when he paired with Ester.

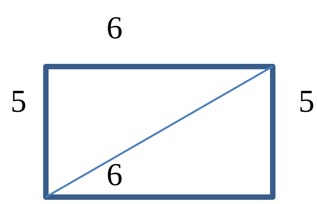
Session 3 with Bradley and Ester. The teacher selected the rectangular cake task because it was a word problem using the Pythagorean Theorem. Students had previously experienced troubles with word problems and the upcoming test over right triangles required knowledge of the Pythagorean Theorem.

Honey and Money are twin brothers. Mother wants to divide a rectangular shaped cake of 5 cm wide and 6 cm long. Find the length of the third side of the cake if it were cut diagonally? [sic]

(Diagram not provided in task.)



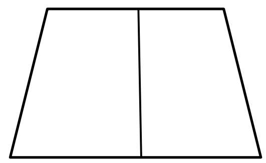
The teacher paired Bradley with Ester, a female of Hispanic background who had weak skills in mathematics and hated speaking in public. Few of her words were audible on the tape so there is little evidence beyond observation and the video recording to use as evidence for her participation. Bradley began speaking immediately and rapidly to explain his solution. She gave him a calculator and wrote on paper for about two minutes while she subvocalized. At the signal for T2, Bradley briefly looked at his IPad and Ester stared into space. She talked little during the Pair phase so there was no opportunity for Bradley to evaluate what she understood. Bradley did not consult with her. According to his VSRI, he assumed that she understood what he told her and he quickly volunteered her to present the problem during the Share phase. She gave him a disgusted sideways glance and stood to go to the board. She asked him what she should do and he said, “Just tell them what I did.” He kept talking to her as she moved toward the board and I reminded him to, “Let her use her own words.” Ester asked if she could just write what was on her paper. She displayed the following:



$$\begin{array}{r} 25 \\ \underline{36} \\ \sqrt{61} \end{array} \quad 7.8$$

As she started to step away, I asked her to explain it. She replied, “All I did was, multiply five by five.” The teacher asked her why. There was no response. He asked, “What was the reason you took five times five?” She replied, “To get … the … length of the squared side.” She then put down the chalk and returned to her chair. After Bradley viewed this clip during VSRI, I asked if he felt she understood him or the concept. Bradley said, “I think so. I think she was a little bit indecisive though. So, not sure.”

Session 5 Share with Bradley. Bradley was the first to share in session 5. The task asked students to prove that the line joining the midpoints of the bases of a trapezoid divides the trapezoid into two equal parts. Bradley began with a statement that used mathematics incorrectly but drew a picture so his peers and teacher did not question him. “The presumption is that you have a trapezoid *with the lines meeting at the midpoints*.” He drew a trapezoid with a line connecting the midpoints of the bases.



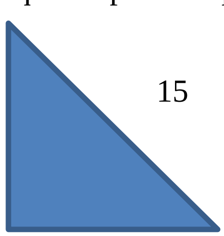
“So if you have the same, *when you divide by the same bases*, and *I assume they are the equal*, then you get that they are equal when you multiply them together.”

The teacher questioned him, not about his improper use of vocabulary, but about other types of trapezoids. “What happens if you extend this further? What happens if that is perpendicular on one side?” Bradley said, “Wait! Wait! Oh, this line would be longer so it would be slanted. So it would have to connect the midpoints, so it that was shorter, then the line would just be longer on one side and shorter on the top so it would just be slanted.” Bradley drew a new trapezoid, still without a perpendicular side, and explained the midpoint of the line splits into two equal parts. There was no evidence that Bradley understood the teacher’s question.

Session 6 with Bradley and Gabe. The teacher paired Bradley with Gabe for session 6. Gabe was socially awkward and unsure of his own academic strengths but he was usually the first to respond with correct calculations. The task required students to find the area of a right triangle whose perimeter was 30 cm. given that one side was 7 cm longer than the other side. The video captured the two Caucasian males at the end of T1 staring at the problem with nothing written on paper. Once again, Bradley began talking as soon as that teacher released them for the Pair phase. As Bradley incorrectly said that the two sides would be the same, Gabe squinted his eyes as if not understanding and asked about the hypotenuse. Bradley said, “Oh, the hypotenuse is seven inches longer.” He, mistakenly, believed the problem involved an isosceles right triangle whose hypotenuse was seven inches longer than the sides. Gabe questioned this notion, bounced his hands, and looked at the space between them. The males debated about the legs and hypotenuse. Bradley seemed to ignore the suggestions made by his partner and determined that the hypotenuse would be fifteen. He wrote that down; the first time I witnessed him writing a solution to a problem. Gabe asked, “One and seven?” and gave a questioning look. The males then voiced different combinations of numbers and settled on eleven and four for the sides of the triangle. They now believed that there was a difference of seven between the two sides but the hypotenuse was half of the perimeter.

During T2, Bradley looked into space while Gabe used his pencils to form a triangle and made small bounding motions. Bradley played with his pen and looked at the work of his partner. As soon as the buzzer rang for Share to begin, Bradley looked at his partner to see if they should volunteer. Gabe nodded affirmatively and the teacher allowed them to choose the presenter. This is the first time I saw Bradley ask a partner before volunteering. Bradley made the following presentation:

B: (Drew a right triangle.) If the whole perimeter is 30 cm, then you have your right triangle, you have your hypotenuse. It just the Pythagorean Theorem that states it is A squared plus B squared equals C squared. Ahhh, these two sides have to equal this one.



So, it’s stating this is always going to have to be 15. (marks the hypotenuse as 15) And once you know that, the other two…one of them is 7 cm longer than the other. So, that would be 11 and 4. So, then you could take this because it is 11 and basically 4. So you have ½ base times height. So ½ of 4 times 11 which gives you 22.

T: Did you test out the theory that both sides equal each other? I mean, you are saying that the two sides have to equal each other but how do we tell that they are right triangles?

B: What do you mean right triangles?

T: Well, it said that it was the perimeter of a right triangle. Correct? So how do we know that that is 11 and 4? How do we test that?

B: With the Pythagorean Theorem.

T: Okay, so did you do that?

B: Ah, yes.

T: Okay, and it came out correctly?

B: Um, I'm not sure. It was just basic understanding so didn't actually solve. (Looks at partner, smiles and shakes head No.) It doesn't work? (Laughs.)

T: All right! So calculations do not compute. All right. Maybe, let's think about this. What were some things that you tried? How did you know some things?

B: Because of some formulas.

T: Okay, because of the formula. So you figured you needed to know your hypotenuse. You needed to know your two legs to deal with this problem. What was, perhaps, your error?

B: Not actually solving the equation.

T: Okay, not solving the equation. I mean if you just really look at it.

Is 11 seven more than 4?

B: Yes.

T: Okay, so you look at it and say, that looks great. However, why is it not 11 and 4?

B: I'm not sure yet.

The teacher asked others for ideas. While the next group presented findings, Gabe continued working, found the correct solution, and presented to the class. Bradley indicated that he was still unsure why his solution had not worked.

Post Interview with Bradley. .Bradley believed, according to his post-interview, that TPTS was only fun because he was able to work with others but he did not believe it helped him understand mathematical concepts. For him, TPTS was an enjoyable time for relaxation.

T1	Pair	T2	Share
Relaxed, just thinking about it	Enjoyable, talk with other people about it	Boring, not much to do Just waiting for it to be over.	Relaxed, not too worried.

Figure 4.2 Bradley's feelings about TPTS phases

Summary of Bradley. Teacher input seemed to the key kind of support for Bradley because he trusted that the teacher was the expert. He was overly confident of his own abilities and was seldom challenged, even by the teacher. The contextual meanings of his responses were basically correct for the normal textbook tasks chosen by the teacher but his use of mathematical language was often vague or incorrect. He was challenged only once during the months that I observed, in TPTS Session 6, but there were many instances where a challenge to his response could have increased understanding for the entire class. In general, he used a few seconds of the first Think time in TPTS to complete the problem to his own satisfaction, seldom listened to others during the Pair phase, read on a hand-held device during the second Think time, quickly volunteered to present during the Share phase and then remained off task while others presented their findings.

Chad, Male Caucasian, Received Ds in Mathematics and English

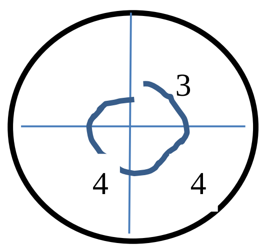
Chad, who was mentioned in sessions with Alberto and Bradley, attended grades 3 through 7 at this Christian school, transferred to a public school in grades 8 and 9, and returned to the Christian school at the beginning of the school year. Field notes show that he frequently reported that he did not believe he learned anything while at the other school and lacked foundational knowledge in math. Bald since the 7^{th} grade due to illness, he struggled with self-esteem in the public school but seemed confident and accepted now. As previously noted, school administrators reported that he had never excelled academically but they felt it was due to a lack of motivation rather than intelligence. Abused by his father as a young child, he now had difficulty staying on task and out of trouble.

According to the pre-interview, Chad's key support for learning mathematics was his teacher because the teacher showed how to do the problem and then how to progress. The material was presented step-by-step with extra hints not provided by the book. The teacher first gave them an example and then let them try one like it before proceeding. Chad liked the idea of working with groups because he would have another person to consult with so he had more answers that might give more understanding. He also wanted to help others with their corrections. Chad feared that he might get mad if someone did not correct him the right way and felt he might get more done if he worked alone because there would be, "no one yelling at you or talking to you telling you it's wrong when it's actually right." He reported that he had no problem with math vocabulary, did not think it made a difference for understanding concepts, and never used his book for anything other than homework problems. Chad reported that he only dealt with numbers, never pictures, and did not talk to himself. When asked what I would see if I watched him working during quiet time, he said I would see him getting frustrated if it was a hard problem; he would write on a piece of paper, throw it away, then write on a different paper. He thought he would be comfortable presenting his work to his classmates and would use quiet time prior to presentation to think of what he would say. I noted that there were frequent long pauses in his responses, indicating that he needed time to formulate answers.

Session 1 with Chad and James. Chad quietly looked at his text during T1, rubbing his head and chewing on his pen. He and James talked off task during the Pair phase and Chad staring blankly at his book during T2. He believed that math was too hard to understand and appeared to give little effort to try during the first three phases. He listened to his peers during the Share phase and laughed at James who had no solution to write on the board.

Session 2 with Chad, Bradley and Isaiah. The next two sessions seemed to create a turning point for Chad. His participation in session 2 was much different, in part, because of his pairing. He was friends with Bradley and he admitted that, as a prank, he was intentionally and routinely "trying to confuse him" in all subjects. In the VSRI he stated that he tried to think of how to solve the problem during T1. During the Pair phase, Bradley drew a picture and tried to explain the problem but he did not use any numbers or letters so they just said "this" and "that." Chad started to understand and began repeating the “this and that” explanation aloud. He explained in the VSRI that it was an attempt to confuse Bradley. However, as described in the results section for Bradley, both males expressed surprise at new understanding during the bantering. Chad then practiced words aloud, preceded by hand movements, to plan for the sharing. He was afraid that continued thinking would confuse him, so he stated that he stopped thinking after they all agreed on the solution and did not use T2 for thinking about math.

During Share, Chad stared at his book rather than looking at Diana’s presentation. Asked to present, he stretched, yawned and went to the board. The teacher explained that the corollary was not displayed for the class so Chad asked if the teacher wanted it drawn on the board. He drew a circle with two diameters then placed another circle inside and labeled three segments.



> So really it is proportions. So like, say that this is 3 and this is 4 and this is 4. Well, you add all these up and well, right here is 11 and 11. $\frac{}{11} = \frac{11}{}$ And these two right here (points to the fours), would be here and here (places 4s in the extreme positions in the proportion).

He then drew a smile at the bottom of the circle to make a smiley face. The teacher explained that the problem mentioned a diameter but did not imply that there were two diameters. The teacher was new to TPTS and did not know how to direct the students so I quietly suggested that he have the next presentation and then explain the corollaries to the class himself so he did not run out of time. The teacher called on

Alberto for the next presentation and talked with Chad and his group while I worked with Alberto.

Session 3 with Chad and Alberto. As noted in the descriptions of Alberto and Bradley, the task for session 3 involved finding the diagonal in a rectangular cake. Chad stared at the problem, mouth open, during T1 in session 3. During Pair phase, the partners talked at the same time, trying simultaneously to explain beginning solutions. Chad leaned toward his partner, bounced his hands in animation, and talked quickly. Alberto leaned back in his chair, bounced his pencil and pronounced authoritatively, “You are supposed to multiply 5 times 5 and 6 times 6 and, shhh, shhh, shhh, shhh (Chad tried to speak) you add them together.” Chad asked for page numbers of definitions and theorems, flipped his book pages, and proudly exclaimed, “It’s all on my page!”

Neither of the subjects was comfortable with the mathematics vocabulary at this point. Chad leaned forward, explained where the cake’s dimensions would be on the drawing, and pointed, without use of the word hypotenuse, to show what they needed. Alberto asked if the side was a leg or the hypotenuse. Chad reiterated what numbers were needed, gave a series of calculations such as “eight times eight is sixty-four” and “seven times seven is forty-nine” and estimated the solution to be, “seven point something.” They found a calculator that they took from each other repeatedly, sometimes with both trying to push keys simultaneously. When it was time to prepare for the Share phase, Chad told Alberto to, “Be serious.” Chad watched intently as the other groups shared their solutions.

Session 6 with Chad and Diana. The teacher chose to pair Chad with Diana to find the length of one side of a right triangle with perimeter 30cm if one side was 7 cm longer than the other side. Diana was a white female who got straight As in her classes and was not shy about asking questions. When she looked for the formula of perimeter in her book, Chad told her it was “two length plus two width.” She replied that his formula was for a rectangle and they needed a triangle. When she stopped looking, he turned to the glossary and read the definition of a perimeter of a polygon: the sum of the sides. When she did not immediately take his response, Chad motioned to the teacher to resolve the issue but the teacher just told them to work together. Diana insisted that she still did not know how to find the perimeter and Chad repeated, “Sum of the sides of the polygon. Sum of the sides of the polygon. So the sum… of the sides… of the things.” Diana held her head and said, “Oh my gosh!” in disgust, blocking her eyes from Chad but he persisted, “The perimeter is 30 cm.” He bounced his hand and enunciated and read the entire problem aloud. He moved his hand in a triangular motion and said, “they are all together 30 cm.” Diana, still exasperated that she did not know how to find the perimeter of a triangle, pinched her face between two fingers. Chad layered his hands on top of each other and said, “A plus B plus C, A plus B plus C, is the perimeter of a triangle.”

During T2, Chad took Diana’s pencil, wrote a note in his book to her, and nudged her to read. She ignored him. They nudged her again and finally said her name so she looked. She wrote her own note, he made a quiet comment, she shook her head in disagreement and continued writing. Eventually, he nodded his head affirmatively, said, “POW!” and leaned back smiling to indicate they were in agreement.

Bradley was the first to present. When his result was shown to be incorrect, the teacher gave Chad the opportunity to present but he stated that he was thinking. The teacher prodded him to describe his process and he said that he used the equation x+x+x+7=30. Prompted to go further, Chad stated that because there was only one variable (the word provided by the teacher), they represented three numbers that were all equal. He showed a triangle with sides x and x+7 and hypotenuse x and noted that this was not right because the hypotenuse would have to be longer. The teacher noted that he knew Chad was ready to sit down so he thanked him for sharing and allowed him to return to his seat while he called on the next group.

Post Interview with Chad. In the post-interview, Chad said that his classmates would listen so his fears about disagreements did not develop. He liked having a second opinion and no longer preferred working alone. His beliefs about vocabulary changed also because he reported that there were math words that he did not understand. He laughed at the thought of using a book during the interview yet during session 3 he asked for page numbers from Alberto so he could look at theorems and definitions. He reported using more techniques for solving problems such as looking at similar problems and drawing pictures.

Chad believed that talking with others, the teacher and peers with second opinions were a key supports for learning. Figure 4.3 shows Chad's feelings about the phases of TPTS. He reported that he normally felt confused at the beginning of T1. However, the fear of arguments that he voiced in the pre-observation interview was replaced in the post-observation interview by his perceived feeling of happiness in the Pair phase, the part he felt was the most valuable component of TPTS. This feeling of happiness continued into T2. He said that, by the end of the study, he felt confident while sharing.

T1	Pair	T2	Share
Confused	Happy	Happy	Confident

Figure 4.3 Chad's feelings about TPTS phases

Summary of Chad. Chad's participation with the class was nearly non-existent in the beginning of the study but he was fully engaged by the end. During session 1, neither he and nor his partner attempted to solve the problem and engaged only in off-task behavior. In the session 2, he began trying to confuse Bradley and ended practicing vocabulary and giving a presentation. His depiction of the problem was faulty but at least he participated. By the third session, he became fully engaged in the task, talking simultaneously with Alberto about possible solutions for the task. Paired with Diana in the final task, he prodded her with vocabulary words, trying to convince her that the perimeter of a triangle was the total distance around rather than thinking of it as a formula.

Chad's beliefs about mathematics vocabulary changed between the pre- and post-interviews. At first, he felt he had no trouble with words but he later acknowledged that some math words gave him trouble. Videos showed him using hands in motion while saying math words to assist his understanding. Although he claimed that he never referenced the book, video recordings and field notes show multiple times when he looked in the glossary, index, or chapter pages for assistance while engaged in TPTS.

Chad used T1 to think about the problem but sometimes wished he could talk with someone else. I was reminded of the articulate but confrontational student in Chazan's (2000) study that often had deep mathematical insights yet turned in blank test papers. That student said that he did not understand what the instructions said. The conversation in the Pair phase was a key support for Chad and seemed to mediate the understanding of instructions. It was there that he practiced using vocabulary, prodded his partners with the parts of the task that he understood, and talked aloud to develop his own thoughts. Chad felt T2 helped him to prepare for the presentation but said he sometimes “stopped thinking” so he would not confuse himself. The math used in the tasks did not seem to have any connections to life at this time but he thought they might sometime later. Although he did not want to be embarrassed and “get stuff wrong” in front of his peers, he felt, in general, confident about sharing his answers to the whole group. He used the words he practiced during Pair to convey his thoughts during Share.

Chad’s gestures helped to analyze his shifts in thought. Hand movements were large and rhythmic for Chad while trying to understand a concept. He often stared into space while trying to process the problem then leaned back to emphasize points to his partner.

Diana, Female Caucasian, Straight-A Student

Diana was a tall, dark blonde female who held herself with self-confident posture. The school administrator described her as a straight A student with a “superiority complex” who always turned in work at the last minute without “the need to apply herself to get it.” My field notes show that she was continually asking the teacher questions even though other students sometimes sighed heavily or snickered. Her questions were usually about the meaning of mathematical terms or formulas but sometimes were about deeper conceptual meanings.

Knowing *why* something worked was extremely important to Diana according to her pre-observation interview. The teacher was a named as a key support but I also believe that persistence and speaking thoughts aloud supported her learning. This was evident throughout the study as she relentlessly asked questions about why things were true and what words meant. Field notes indicate that prior to the first TPTS session, she asked, “What does transformation mean?”, “How do you know which are means or extremes?”, and later during the trigonometry session she admitted,” I do not know how to find the angles other than by the chart.” Once she understood how to set up tanA = $\frac{18}{24}$, she did not know how to solve it so she persistently kept asking until she was satisfied that she knew why she should do the steps. During TPTS sessions Diana asked, “What does mean mean?” and fretted when unable to find the perimeter of a triangle because she only had the formula for the perimeter of a rectangle. She wanted a deep understanding so she could find shortcuts on her own and know what was going on instead of just doing steps. She believed that she had no trouble with vocabulary and that she could still understand math even if she did not know the words because she could always ask the teacher for help. She wanted to work quietly alone so she could think for herself. She feared that too many people in a group would cause too many overlapping ideas and conflicts about who should lead.

Diana reported that in prior group experiences, she had to help the group understand then had to help another group and never had anyone that had helped her. She wanted her group members to show her something new or find one of her mistakes. She recognized that she used multiple forms of representation for problems and that she talked to herself inside her head as she worked. When TPTS allowed her to speak aloud and listen to the words of others, she questioned herself. I saw her inner argument in session 1.

Session 1 with Diana and Ester. The first TPTS task involved writing the converse for Theorem 55 which they had considered the prior day.

Theorem 55: The bisector of an interior angle of a triangle divides the opposite side internally into segments which have the same ratio as the other two sides (Seymour, 2008, p. 201).

In pre-session discussion, the teacher gave an example theorem and asked for identification of the hypothesis. Diana said, “The then part?” When the teacher shook his head, she said, “The then part?” and the teacher repeated, “The then part.” While the teacher used the example to show how to reverse the hypothesis and conclusion, Diana picked fingernail polish from her nails, dividing her attention to the teacher. He asked if they had any questions and she said, “No.” Diana described *converse* as “the opposite” which it seemed was the accepted definition in the classroom. Although the teacher added some formal language, he accepted this definition from the students. Diana also called it switching or swapping around so you start with a different statement and find the original statement.

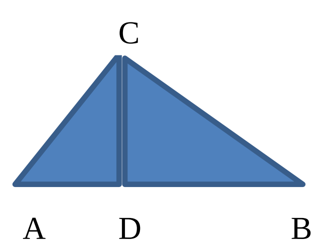
According to the VSRI, Diana used T1 to try to rearrange the statement and then used her partner as a sounding board during the Pair phase. She interrupted Ester at the onset of the Pair phase to say, “We are doing like Theorem 55 so it means if there is a bisector of this angle then it divides this and this so this is D. So A is to B as C is to D. … So the bisector of the angle divides the sides.” She then tried to *flip* the statement by saying, “If one of the sides of the triangle bisects the angles then the other two sides are to each other as the segments of the bisected side are to each other.” Stating this aloud, she realized that it made no sense.

Diana was totally quiet during T2 as she focused on her presentation. She thought that she understood it; but as she looked back at the clip in the VSRI, she was sure she did not understand at that time. She added that she did not think that her partner could understand her reasoning since she could not understand it herself. The second presenter in session 1, she began by explaining the original theorem and using her hands to portray the parts of the triangle referenced in the theorem. Her solution restated the original theorem but reversed parts of the conclusion. She wrote, "If one side of a triangle is divided because of the bisector of the opposite angle, then the sides not divided are proportional to the segments of the divided side." The teacher stated that all of the answers were similar but different and said that we do not all think the same. The teacher did not correct any of the solutions himself but reviewed the meaning of converse and explained converse as the *opposite*, putting the hypothesis and conclusion in the opposite positions.

Session 2 with Diana and Finn. Diana was paired with Finn, a tall white male with what seemed to be a mild learning disability. Though untested, his symptoms emulated those of his father who was dyslexic. He had to read stories three times before understanding them and struggled to get Bs and Cs in English. Diana and Finn's task in session 2 involved finding a mean proportional in a right triangle.

Theorem. 61. If in a right triangle the perpendicular is drawn from the vertex of the right angle to the hypotenuse, each leg of the given triangle is the mean proportional between the hypotenuse and the adjacent segment.

(A correct explanation would include $\frac{AB}{AC} = \frac{AC}{AD}$ and $\frac{AB}{BC} = \frac{BC}{BD}$.)



Prior homework focused on proportional parts of similar triangles. The teacher gave explanations during homework grading but it appeared that Diana (and most other students) were not paying attention. Diana reported the incorrect number of problems correct to the teacher; she said that she had 8 points (full credit) but motioned to her friend that she really had one point.

The words in the task for session 2 were confusing to Diana and she persistently asked about meanings prior to T1.

D: What is a mean proportion?

T: What have we been working on and what happens in a proportion?

D: Yeah, I know, but what does the mean mean?

T: The second and the third in a proportion.

D: Oh, exchanging the means?

T: (explains further)

Diana began the Pair phase asking Finn if he had any opinions. He told her to go to a theorem about proving lines to be proportional in similar triangles in her book and read the theorem to her. She continued doggedly asking about what words meant.

D: What does that mean by the given triangle?

F: Here’s the triangle. The whole thing, ABC. The whole triangle.

D: It’s like the mean proportional. What does that mean? The mean proportional? (The teacher had just spent several minutes trying to explain this concept to her in the whole class setting.)

F: That means… It's like this leg is proportional with this leg and this leg is proportional to this leg and this leg is proportional to this leg …. So they are all equally (inaudible).

Diana prodded Finn to use mathematics terminology to explain mean proportional so she could understand which lines to use.

D: Which ones are proportional?

F: They are all proportional to each other.

D: Tell which lines are proportional.

F: This one and this one.

D: No, say the letters.

F: A to C and A to B. (Finn either gives labels of vertices for a proportion or he is trying to name two lines: AC and AB.)

D: A to C and A to B are proportional … to what? I still don't understand what mean proportional means.

F: The are all proportional to each other.

D: Between the hypotenuse and the adjacent segment to the hypotenuse. What does it mean to have the adjacent segment to the hypotenuse?

F: It's (pause) the segment is connected to, is touching the hypotenuse.

D: So this?

F: You have your hypotenuse area. (incorrect usage of area)

D: So, this is the hypotenuse. This is what I want. What is the adjacent segment?

F: It would be this one.

D: BC?

F: Yeah. Because it's adjacent.

D: So, like … this is to this as this is to this? So AC is to BC as AB is to CB?

F: You are setting it up into a ratio.

D: Yes, so I'm saying AC is to

F: Yeah

D: Is to CB as AB is to CB?

F: Yeah.

Diana correctly used symbols to identify parts of the triangle and she formed a proportional equation but she used the wrong parts of the triangle to form the proportion and Finn agreed with her. In his next response, however, Finn used a different set of segments for the proportion.

D: I don't even understand what we should do.

F: AC is to AD as CB is to CD.

D: Is that it? Or is it another one?

F: No

D: Is that the only answer: So would you say…AC

F: AC is to AB as CB is to CD.

D: OK so each leg of the triangle would be proportional. I don't understand. Is there a definition in the book?

F: Proportional means

Diana looked up the definition of mean proportional in the glossary and became very excited when she found information that finally made sense to her.

D: The ratios ….Mean proportional…In any proportion in which the two means are equal, either (long pause) What is the mean?

F: Oh wait, it is right there! In a proportion, the second and third terms.

D: OK, so (pause)

F: OH Wow!

D: So and then the extremes are the outside. (long pause) The mean proportional – any proportion in which the two means are equal, they are said to ….(fades off as she reads).

F: OK the second and third terms should be equal. AB and CD are definitely not equal.

D: So it is saying that each leg of a given triangle is the mean proportional between,

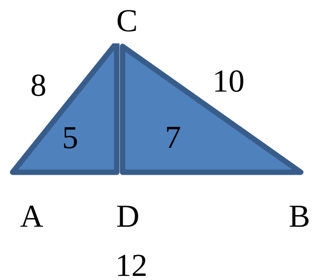
F: OK so the hypotenuse and the adjacent segment of the hypotenuse are going to be going in 1 and 4 and 2 and 3 are going to be equal.

D: So each leg …

F: but that doesn't make sense since there are 3 legs. (Buzzer goes off to end Pair segment)

Because Diana did not know that difference between a leg of a right triangle and the side of a triangle, she was still unable to solve the problem. She continued talking to herself during her partner's presentation, trying to decipher her own work. The teacher noted Finn's frustration at the inability to get a correct solution and allowed Diana to join him. She was unsure of her answer (it was incorrect) so the teacher tried giving hints.

Finn had assigned 8 and 10 to the legs of the given triangle and 12 to the hypotenuse. Diana assigned 7 and 5 to the segments on the hypotenuse.



Diana asked if 7:10 as 10:5, oblivious to the numerical implications ($\frac{7}{10}$ does not equal $\frac{10}{5}$) and thinking only about using the numerals as labels. The teacher hesitated, not knowing how to respond during TPTS. I intervened and began asking guiding questions based on the words in the problem. The dialogue follows.

R: What are the three sides of the given triangle?

D: A to B, B to C, and C to A.

R: Choose one of the legs, not the hypotenuse.

D: B to C.

R: B to C is a leg. Hypotenuse is what?

D: A to B

R: Okay. What is the segment on the hypotenuse that is adjacent to the leg you chose?

D: Ummmm. B to D

R: B to D. So the leg of the given triangle…What was the leg you chose?

D: B to C

R: (reading the theorem) …is the mean proportional between the hypotenuse. (pause) What is the hypotenuse?

D: Ummm. A to B

R: And what was the adjacent segment then?

D: D to B

R: Okay. See if you can write that as a proportional and then we will come back.

Diana said that she wanted to correct her written proportion and she and Finn worked together to get a correct response in front of the class. Interestingly, it was a mathematics term, *adjacent,* that caused Diana to misinterpret the problem. After more presentations, Diana returned to the board with a new, but still incorrect, equation. The teacher told her it was not quite right and began reading it aloud. She exclaimed, "Oh! Sorry! That is supposed to be AB" and she quickly corrected her work. The teacher then asked why a different segment could not be used in that place and she correctly replied that it was not *adjacent*. Several students, including Diana, indicated at the close of the session that it had been beneficial for them. The recognition of the word adjacent was important for several students according to the teacher who reported that scores on the quiz the next day were better than anticipated. They seemed eager to begin the third session three days later.

Session 3 with Diana and Gabe. The cake problem was a simple application of the Pythagorean Theorem. The teacher used it because he said that students tended to miss information in word problems. Diana interrupted T1 and asked if the cake was a triangle or a rectangle. The teacher tried to quiet the conversation because he wanted to see how individuals would interpret the story problem – misinterpretation was frequently a complication in story problems. The entire class kept talking until they jointly decided it was a rectangular cake cut into two triangles. During the Pair phase, Diana began by stating that they needed to find the hypotenuse, C, gave the formula, and then got her calculator. Her partner agreed and the teacher rang the buzzer for the transition. For the Share phase, there were three presentations and all had a correct solution. It was interesting that each presentation built on the prior one and included even more depth of information and use of mathematical terminology. Diana presented last and gave a complete description of the problem and solution.

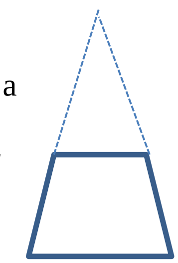
> You have a rectangle and you cut it in half. You get two triangles. And this is a right angle and this is a right angle. You know that this is 6 and these are both 5. So if this is a right angle and this is a right triangle, then this can be the hypotenuse. So according to the Pythagorean Theorem, this is going to be C and this is going to be B so we square A which gives us 25 and then we square B which gives us 36 and then we add those together and we get 61 which is going to be the square of the hypotenuse. So then we find the square root of that which is 7.8. So then the hypotenuse if 7.8.

The teacher continued the conversation, asking Diana how she knew it was a *right* triangle and to define a rectangle. After thought, she correctly responded that a rectangle had four sides, the ones across from each other were parallel to each other and equal to each other and that it had four right angles. Her partner then read the definition from the book, confirming her statement.

Sessions 4 and 5 with Diana and Bradley. Diana and Bradley were considered the top two students in their class. They correctly completed the task in session 4 very quickly as individuals, verified their answers in Pair using about 5 seconds, and talked casually throughout the rest of the time. There was no explanation of how they found the result.

In session 5, they approached the trapezoid problem in their own ways. Bradley presented a solution based on the definition of midpoint and intuitive assumptions about one base. Diana considered an extended case where the sides of trapezoid extend to make a triangle. She related this to the triangle in session 3 where a line parallel to the base divided the sides proportionately.

> Mine is basically the same thing but I was thinking that if you had a trapezoid like that and you continue those two *legs*, then it would make a triangle. So I went off of how, you know, if there is a parallel line and *a triangle that connects the two sides* or whatever, then it divides the two sides proportionally. So that would make like he said, 3 and 4 and would make them like proportional. So that's basically all I have.



I watched Diana's vocabulary grow and witnessed her making more connections as the year progressed. It was, therefore, surprising to me that she struggled again in session 6 with a very basic concept – perimeter of a triangle.

Session 6 with Diana and Chad. As illustrated in the profile for Chad, Diana believed she could not find the area of a triangle because she did not know the formula for finding the perimeter of a triangle. She knew the formula for a rectangle but was disturbed that they had not learned a formula for a triangle. Stuck in this frustration, she did not respond when her partner tried to use his words and hands to illustrate the distance around a triangle. At one point, he wrote the formula x+x+x+7 in his book and she copied that to try, unsuccessfully, to find a solution. She needed *base* and *height* to get area and believed she had no way to find either. Frustrated, it appeared that she stopped trying to solve the problem. She responded again only after the teacher began posing questions that referenced the formula for the area of a triangle.

Post Interview and with Diana. Diana was a young woman of action. She actively pursued her learning goals regardless of the reactions of others. That spirit of action came through in the words that she used to describe TPTS in the post-observation interview. Although I encouraged Diana to use feeling words to describe each of the TPTS phases and gave her examples of *feeling* words, she chose to use explanatory action words. T1 was a time for organizing thoughts and plans. Pair was a time for sharing with a partner and saying things aloud. T2 was a time of planning and Share was a time for explaining.

T1	Pair	T2	Share
Organizing	Sharing Saying what you got out loud	Planning	Explaining

Figure 4.4 Diana's feelings about TPTS phases.

According to her post-observation interview, Diana was irritated when she felt the partner was not trying. She stated that she sometimes used T2 to help the partner prepare, sometimes to better understand the problem and sometimes to just wile away time. The Share phase was the most important one for Diana. She compared it to exercise – she did not want to do it but knew it was good for her.

Summary of Diana. Diana was persistent in her quest to understand. She wanted to understand the meanings of words and the reasons why things worked and she doggedly asked her teacher and peers for more information. Short of that complete understanding, she felt reduced to using formulas and rote manipulation of prescribed steps. She compartmentalized information such as the perimeter of rectangle and triangle and, although she questioned peers, wanted direct guidance from the teacher-expert or textbook to give information credibility.

Diana liked to present ideas to others when she felt they were genuinely trying because they could sometimes see mistakes. She sometimes did not understand the problems because she did not fully understand the words so she would ask the teacher. She claimed that she never used her book other than for homework yet evidence showed her using the glossary, index, and chapter information.

Ester, Hispanic Female, Struggled in Mathematics and English

Ester was a vivacious Hispanic female who lit up the classroom with her smile. Although she enjoyed socializing with individuals, she was exceedingly shy about talking to a large group. Her grades in both English and mathematics were near failing and she seldom asked the teachers in either class for help. Before class, she would usually sit with other girls and talk about makeup. At first, she declined participation in the study. Her homeroom teacher and minister spoke with her to allay her fears and asked her to speak personally with me about the study. She and I talked informally, laughed, and shared background stories. I asked again about participation in the study and assured her of privacy and the ability to withdraw if she felt uncomfortable. She agreed to participate in both classroom sessions and interviews.

The teacher and a few of her classmates were key supports for Ester’s learning of mathematics according to her pre-interview. She sometimes asked her parents for help but language differences made it complicated.

> Just, the words that we use, um, I just get confused a little because the words in Spanish are different. And, just, I feel like in Spanish it's all a little different as in (inaudible), the definitions and what they mean, and in Spanish those same definitions can mean something else.

Ester's mother spoke only Spanish but her father was bilingual. He mainly talked with her in Spanish about math but would sometimes use English to try to help her understand. I asked if she could remember any English words in particular that he used but she said she could not.

> It's just once my dad told me a few words, I was like, I'm okay if I just put it behind my head and just cancel out that stuff because I don't want to get any more confused.

I asked her to describe how she thought about mathematics in her head, whether the words were in Spanish or English.

> I do find myself sometimes I'll just, in Spanish, I'm repeating the same thing he's talking about sometimes in Spanish it's this and that so I'm thinking in my head how would I make this in English and think about the translation to see if I will understand it better. There are times when I will understand it better and there's other times when I just, I won't, and I will just have to try to figure out a way to get it in English. So it's a little twist there.

Ester said in the pre-interview that her preference for individual or group work depended upon the task and the day. She spoke often about being uncomfortable and unsure of her thinking. She did not want to be the only one that was unsure of something. Her hardest task, she said, was sharing her answers with others.

> Because the fact that I'll not be so confident about my answers. I won't want to share them because I won't be sure whether I'm wrong or right. And I will sometimes turn out to have the right answer and I just won't tell anyone.

The teacher sometimes used math words that Ester did not understand and she said that she either "went out of her way" to talk with him after class or she asked her parents. She rarely used her book but would sometimes look up definitions. She did not use pictures or numbers when trying to solve problems but did talk to herself and tried to recall what the teacher had written on the board. If she could not solve a problem on the first try, she stopped and waited for help.

Session 1 with Ester and Diana. Ester was friends with her first partner, Diana; they frequently socialized before class. During the Pair phase of the task that involved finding a converse of a theorem, Ester asked Diana repeatedly for an explanation of the theorem. Ester used her pencil to point to parts of the triangle in Diana's book while she responded in short phrases and sentence stems to prompt her partner's completion such as, "Like how?", "What is the (did not finish the question)?", "I don't have a clue,", "So you would say that (long pause) there (did not complete)" or "These two?" Each time, Diana would explain in a little more depth and try to move into thinking about the converse. There was no evidence that Ester understood anything about the theorem or its converse but her questioning helped Diana to verbalize her understandings.

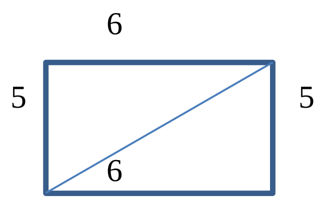
Session 2 with Ester, Alberto and Hugh. The teacher placed Ester in a triad with Alberto and Hugh. Hugh was a tall, blonde male that received Bs in mathematics. He took the center seat and the role of leader throughout the discourse. The group was asked to show the following to be true:

> If in a right triangle the perpendicular is drawn from the vertex of the right angle to the hypotenuse, the two triangles thus formed are similar to the given triangle and to each other.

During T1, Ester and Alberto drew the picture of the problem on their papers while Hugh read silently. When the bell rang for transition to Pair, Ester rolled her eyes upward toward Hugh. Alberto joked and tried to distract Hugh. Ester smiled at him, smiled at the camera and then used the book to locate a likely theorem for a proof and offered it to the group. She and Hugh flipped pages, looking at different theorems and then agreeing that “CSSTP” was the solution to the problem but there was no explanation of what it was or why it worked. I searched the book for a CSSTP theorem and, when finding none, asked the teacher what they meant. He said that they were referring to Corresponding Segments of Similar Triangles are Proportional. This theorem relies upon the assumption that there are similar triangles, a fact not given in the theorem. The teacher had just used that theorem during homework grading prior to the TPTS session. In the video, it appeared that only Hugh seemed to be listening during his explanation. As their discussion continued, Ester began saying CSTP or CCSTP and Hugh would try to correct her. Alberto fussed about the recorder on the table and tried to make jokes. Hugh told him to be quiet because he was bothering their concentration. When Ester asked what Alberto was saying, Hugh replied, “He’s being stupid.” Ester’s voice was too soft to be picked up by the recorder but she could be seen pointing at Hugh's book and asking him questions or staring into space.

During the Share phase, Hugh continued trying to explain his proof to Ester and Alberto. He did not know what *vertex* was referenced in the theorem and he said that the triangles were equal rather than similar. Alberto was required to give the presentation. When he was unable to report, I intervened and asked leading questions because the teacher was helping another group. Alberto was allowed to Phone-A-Friend (ask someone in his group for help) so he asked what mean proportion meant. Ester replied, "What is the question she is asking?" Alberto told her; she repeated the question but gave no answer. Because she merely repeated what someone else said, it is impossible to know whether she lacked understanding of the question, the mathematics behind the query, or an appropriate response.

Session 3 with Ester and Bradley. Ester paired with Bradley to work on the cake problem involving use of the Pythagorean Theorem. As explained in Session 3 with Bradley, he dominated the conversation during Pair. Ester merely gave him a calculator then she wrote on paper for about two minutes and subvocalized. She stared into space during T2. Bradley quickly raised his hand for their group to present but the teacher requested volunteers who had not yet presented. Bradley affirmed that Ester should present. She looked at Bradley with displeasure and stood while she asked him what she should do. He responded, "Just tell them what I did." He tried to keep feeding her information as she headed to the board but I stopped him because I wanted to record her reaction to the use TPTS in mathematics at that point. She asked if she could just write what was on her paper and, without comment, displayed the following:



25
36
$\sqrt{61}$ 7.8

Asked to explain, Ester replied, “All I did was, multiply five by five.” The teacher asked her why. There was no response. He asked, “What was the reason you took five times five?” She replied, “To get … the … length of the squared side” then sat down. She provided no further evidence of her understanding of the problem.

Session 4 with Ester and Hugh. Ester paired with Hugh. She asked at the beginning of the Pair phase if he had a solution. He shook his head no and they remained quiet for the next minute. Ester wrote on her paper, chewed on her finger, rubbed her head, and asked his opinion of her written thoughts. He looked at her work but was unresponsive until about two and a half minutes into the phase. Hugh’s motions then quickened and he made a short response to her notes that I could not hear or see. There was no evidence that either of them even understood what the problem was asking even after the Share phase.

Session 5 with Ester, Chad and Hugh. The task in session 5 required students to prove that a line joining the midpoints of the bases of a trapezoid divides the trapezoid into two equal parts. During T1, Chad quietly asked the others what they were supposed to do and they showed him in the book. Ester interrupted Hugh to ask if her drawing was a trapezoid. Hugh shook his head affirmatively and drew a modified figure in his own book. Chad drew something in his book, Hugh shook his head that it was incorrect and Ester made an inaudible comment to Chad. Chad changed his drawing and Ester nodded her head in approval. Chad finished his drawing and Hugh and Ester nodded yes while whispering, “That is correct.” When the bell rang for the Pair phase, Hugh said, “This is so simple.” Ester responded, “Yeah.” Hugh quickly read aloud the task and Ester said, “OK, how” and Hugh kept talking. Hugh said that it just automatically cut it in half because it was a midpoint. He used his hands in a chopping motion and divided the two imaginary sections apart. Ester told Chad, “So, we just do what he said.” Chad looked like he did not understand so Ester drew a picture in Chad’s book and said, “I like pictures.” She asked again what to say and Hugh repeated that the midpoints cut it in half. Ester started studying her work during T2 but the three students just took turns turning the recorder around and around and snickering off task.

In the Share phase, Bradley assigned numbers to the bases of the trapezoid. Ester whispered that his solution was like their group’s though there was no mention of numbers in their drawings or discussion. The teacher asked Hugh to explain how they solved it and Ester whispered, “the definition of a midpoint.” Hugh said nothing. Ester and Hugh incorrectly believed that the definition of the midpoint of a line segment proved that the area of the trapezoid given in the task was equally split into two parts.

The teacher noted to the class that several students had commented on the simplicity of the task. He said that if he had assigned it as homework, they would have complained about how hard it was. Ester said, “Because we don’t want to do it.” The teacher said that by doing it in TPTS, they focused on the task and talked with each other. They listened to what others were saying and it “clicked in.”

Session 6 with Ester and Finn. Ester paired with Finn in session 6. The task asked for the area of a right triangle whose perimeter was 30. They were told that one side was 7 cm longer than the other side. Ester subvocalized while reading the problem during T1.

Finn did not originally recognize the right triangular shape and Ester corrected him at the beginning of the Pair phase. Dialogue during this session illustrated her use of repeating her partner's words and completing his sentences to participate in the discussion. She stated in the mid-way interview that she would like the Pair phase to be longer because there was a lot more to discussion needed for her to understand.

F: Ummm. It's half it's (inaudible) so

E: Ummmm. Half base times height

F: (inaudible)

E: What?

F: Of a right triangle

E: Right triangle (she reads problem again)

F: So if it is a right triangle

E: (inaudible)

F: This thing is 30 cm.

E: (looks at Finn's drawing) It is a triangle. It's not a square.

F: Oh wait!

E: Yeah, it's a triangle, not a right triangle.

F: Yes

E: So it it's

F: A triangle

E: So it's 30 cm

F: The whole thing is 30 cm?

E: Yeah and then one side is 7 cm longer than the other. Yeah, I'm guessing that's 7 and then

F: That makes this one… and the other

E: Is 30, but

The task had two numbers, 7 and 30. Ester knew that something had to be done with the two numbers but ignored the "longer than the other" phrase. Her words and thoughts did not always coincide. Though she told Finn that "the whole thing" was 30, she concluded that one side was 7 and the other was 30; there was no mention of the third side. Finn knew that 30 was a total so he subtracted the two numbers and got 23, subvocalizing as he calculated. Ester was confused about the 30.

E: So then what do we do with the 30?

F: That's the total combined.

That's the total. So (paused then subvocalized again 23 divided by …. Is what? Is…. 11.5 So we'll do ___ plus ___ 25 ….equals 22, 23 plus 7 equals… twentyyyyyyyy, thirty. For some reason…..)

Aloud to Ester: What's the area of the triangle? Yeah, that's not right because that's stuff I figured out about the triangle.

E: Do you have a calculator?

F: Yes. Is this yours? (pointed to the recorder)

E: No, that's (indicated it was part of the study)

F: Is this thing on?

E: Yeah

F: Can we turn this thing off? What is it? One-half (buzzer rang to end Pair phase)

Ester stated that she used T2 to “think over the discussion and make sure I have it in my head I guess… (in a whisper) sometimes.” I probed about the “sometimes” and she stated that when she just did not understand, she would just sit there and try to figure out what she was missing. In the video for session 6, she talked to herself during T2, shook her head negatively and put her calculator down with force. She made it clear that she hated talking in front of people so talking about the problem in front of the class was very uncomfortable for her. However, it helped her “a lot” to hear other students talk about their struggles and solutions. She thought more about how to correct the problem when others made mistakes. She was glad that the class took time to discuss the problems during that phase.

Mid-study Interview and Summary of Ester. Ester’s voice was very low during classroom sessions and it was difficult to transcribe her words. Rather than using a VSRI with barely audible sound, I decided to use a mid-study interview.

Early in the study, Ester stated that if she did not understand something, she would just wait to ask someone else or “go out of her way” to ask the teacher but she found it difficult to speak with others when she was not sure of herself. The explanations given by her father were difficult to understand when he spoke a language, used words, different from that spoken in class.

She stated in the mid-way interview that if she did not understand the problem, she did not really think on it during T1 but, instead, waited until she could talk with her partner. She used Pair to listen and she only talked when she believed she actually knew what she was doing. She sometimes felt that she was “too far off” so she did not speak.

She believed that she could sometimes understand what they were trying to say to her but only questioned them when she was sure about what her task was asking. Figure 4.5 shows her responses in the post-observation interview. She stated that she liked working with others, sharing her answers and seeing what her classmates thought. She was *anxious*, implying excited, to see what others had and to compare answers during Pair.

T1	Pair	T2	Share
Anxious	Worried	Frustrated and nervous	Mad if it's me, relieved if it's not.

Figure 4.5 Ester's feelings about TPTS phases

Finn, Male Caucasian, Struggled with Mathematics and English

Finn's mother suspected he was dyslexic because his symptoms mimicked his dyslexic father and uncle. He had difficulty with reading comprehension and, according to his English teacher, required reading things three times and struggled to get D's. In mathematics, he rapidly responded to teacher questions during whole group instruction and could readily compute using whole number facts yet he struggled to get C's in coursework. He indicated in his pre-observation interview that he felt he learned math best when the teacher did the work on the board and then give him a similar problem to "fill out." This allowed him to first see how it was done and then prove to himself that he could do it before leaving to work on his own. He often asked the teacher and his parents for help. His parents explained what they knew and then had him expound upon it and continue on. I do not know how well the parents understood mathematics.

In the pre-observation interview, Finn stated that he preferred to work alone because he got more done but was confident that he could talk freely in classroom groups to further their conversations. Completion of the task and getting a correct answer was important to him; he mentioned it repeatedly in the interview. He feared that the group might use his incorrect ideas and make a mistake. Quiet time was important to Finn for reading his book which he often used to work ahead or to look back to find errors. He needed quiet time for thinking about the problem and getting his thoughts together. He claimed that he did not use pictures but he sometimes talked to himself and liked to ask other students for help.

Session 1 with Finn and Hugh. I paired Finn with Hugh for this session because they both expressed a desire to work alone in their pre-interviews. Hugh reported that he was easily distracted and often got confused. His sister was his key support but he sometimes asked the teacher for help. He stated that if he did not understand a problem, he just stopped working but in this task, paired with Finn, he persevered and came up with a solution that satisfied both partners. The task asked students to find the converse of the following theorem.

Theorem 55. The bisector of an interior angle of a triangle divides the opposite side internally into segments which have **the same ratio** as the other two sides.

During T1, Finn used his book to look at the theorem and at prior examples. He read the theorem aloud, pointing to and isolating the math words. He spoke aloud during T1 while thinking about how to reverse the *if* and *then* statements in theorem. Finn took the lead during the Pair phase but Hugh developed the converse.

> F: Finding the converse of Theorem 55 which means we are going to have to try to take the ummm if then statement. Make and if/then statement

H: If this side, what’s the opposite side, if the opposite segments are equal? But the bisector of the interior angle divides those two opposite sides. Internally. Do we have to write it down?

F: I’m going to.

H: The bisector of an interior angle. My hands smell like cheese right now. The bisector of an interior angle divides the opposite side. (whispers) I’m fine.

Finn tried to write down the converse but had problems replicating what Hugh said.

H: The opposite segments are in the same ratio as the other two sides then the bisector of the interior angle divides the two sides internally. Basically just change these two things around. That’s all you have to do.

F: The bisector of a triangle. The opposite.

H: If the triangle is divided by…no. If the triangle’s two segments are in the same ratio as the other two sides then the bisector of the interior angle of the triangle divides the opposite two sides internally. Errr, not internally, but equally.

H: Yeah, there you go. Okay I get you.

F: Okay, we’re done.

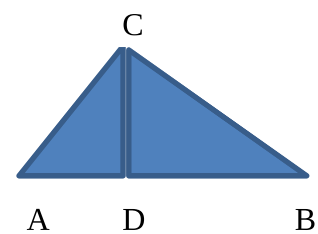
H: We’re so smart it only takes two minutes.

F: Yeah.

Hugh concluded with a solution that Finn found satisfactory. Hugh repeated the converse slowly upon Finn’s request and they decided it made sense. Finn wrote notes in his book during T2 and compared his writing to the theorem. Both males offered to write their solutions on the board during the Share phase, but Hugh persuaded Finn to allow him to write. The teacher called two students to the board to explain but did not indicate whether any of the solutions were correct; rather, his emphasis was on the possibility that multiple solutions might work.

Session 2 with Finn and Diana. Finn paired with Diana to understand mean proportion.

Theorem 61. If in a right triangle the perpendicular is drawn from the vertex of the right angle to the hypotenuse, each leg of the given triangle is the mean proportional between the hypotenuse and the adjacent segment.



Diana began the pair session by asking Finn for opinions. He guided her to a section in the book on similar triangles used as a means for proving lines were proportional. Diana questioned what “the given triangle” meant and Finn outlined the largest triangle with his finger and stated, “The whole thing. ABC, the whole triangle.” For each question she posed, he gave an answer that included pointing and mathematical terminology. Words were often used incorrectly or imprecisely. For instance, when asked which lines were proportional, Finn said, “A to C and A to B.” This would imply that vertex A was to vertex C as vertex A was to vertex B. Diana pressed him to explain further without success. She then read, “Between the hypotenuse and the adjacent segment to the hypotenuse. What does it mean to have the adjacent segment to the hypotenuse?” Finn correctly explained that the adjacent segment *connected to* or *touched* the hypotenuse. However, he then referred to the *hypotenuse area*, confusing language for area with a line segment. Diana then pointed to sides of the triangle and said, “So AC is to BC as AB is to CB? Right?” The pair did not identify the hypotenuse but I assume that Diana saw BC (CB) as the hypotenuse rather than AB. Finn responded that she was setting it up in a ratio. Although Diana had used the language for proportion (is to/ as/ is to), she claimed that she did not understand what to do because she did not know what proportional meant. Finn started to explain but Diana looked it up in the book. She expressed a moment of insight as she read that the second and third terms were the means in a proportion and the first and fourth were the extremes and recognized that her statement was incorrect because the second and third terms were not equal (AB and CD). She apparently did not recognize the difference between legs and sides and it was unclear whether Finn recognized the difference. He incorrectly told her that she had a correct proportional statement then restated it using different segments (AB rather than AD).

Finn correctly shared with the whole class that AB was the hypotenuse of the given triangle, and BC was adjacent to the hypotenuse. He then assigned 12 inches to the hypotenuse and said "let's say BC is the adjacent angle [sic] to the hypotenuse. Then *the ratio is, let's say 8 inches* then" and he was unable to go further. He wrote 12 on side AB and 8 on BC.

Finn's explanations comingled correct assumptions with misinformation, and though he clearly stated definitions, he misused terms such as area, angle, and ratio. The incomplete understanding of terminology and mathematical concepts left him unable to complete the solution. His partner did not accept his responses even when he was correct.

Session 3 with Finn and James. For the cake problem, Finn paired with James, a student who left the school prior to the end of the study. Finn struggled to keep James on task and worked on the problem on his own. His presentation followed Ester's cursory attempt. Finn added to her information by saying that the problem required use of the Pythagorean Theorem.

F: Um. Okay! Um You are going to use the Pythagorean theorem so that's going to be, umm…

J: Don't use your head.

F: five squared plus 6 squared equals and we don't know the answer so we've got um. So then after we do five times five equals 25 then 6 squared is 36. I add together the two equals 61. Then you take the square root of 61. The square root ends up being 7.81 so on and so on. So just 7.8 would be your answer. I guess you could just round it up and it would be just 8.

T: Why did you take the square root?

F: Because I don't know the diagonal so you have to take the square root of the number. I think.

T: From the equation, where does it tell you to take the square root.

F: I don't know.

T: Okay. What are you missing?

F: Ummm. The square up there.

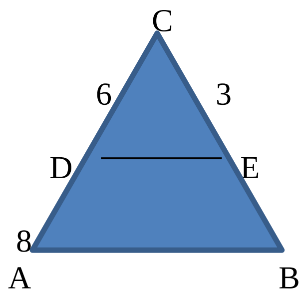
T: Okay. Next group?

Finn knew that the Pythagorean Theorem required squaring and adding but did not know why the square root was required in the final step. The teacher called his attention to the equation, he noticed the squared term, but there was still no evidence that he understood why to take the square root.

Session 4 with Finn and Gabe. The teacher forgot to include the second think time so this session was taught in a Think-Pair-Share format. Finn paired with Gabe to determine the length of BC in session 4. (The diagram below was labeled but was not drawn to proportion.)

Given the following triangle, then BC = __?__ .

AD=8, CD=6, CE=3



Finn read the problem on the board, looked in to his book for 9 seconds without pausing on any page, held his pencil in both hands, then looked again at the board. He began the Pair phase with an explanation that compared the right side of the triangle to the left, noting that it was half of the other.

F: If C to D is 6. Half of CD is 3 (uses fingers to emphasize “3”). Half of AD is 4. So the whole line is 7.

G: Yeah, that sounds about right. Because the lines would continue at the same rate. (pause) Because if you ….we know that if you cut the triangle in half would be well, (pause) the drawing… so (pause) B to E is going to be 4.

This exchange took one minute. They then checked their solution by using proportions. The teacher left the students in the Pair phase for three minutes and forgot to split into the additional quiet time. The pair used the remaining time to discuss how the proportion would simplify.

F: Do you have something else?

G: Nah…I’m just checking it. (pause) Yeah, that works. I just checked it.

F: The problem would be…8 over 6.

G: Now wait a minute. That would be 6 over 4.

F: (Wrote it on his paper while speaking.) It would be 8 over 6 and 4 over 3 or it could be 6 over 8 and 3 over 4.

G: Yes, that is correct.

F: Not too bad. It's just a confusing question.

G: This one was actually much easier.

F: Now we need to figure out how to explain.

G: Ummm, I'll plead the 5^{th}. (Buzzer signaled end of phase.)

Finn volunteered to give the first presentation. Although his original explanation to his partner involved halving the left side, his presentation used proportions which he called a ratio.

> Because we have A to D, we are going to start off with a ratio. 6/8 = 3/x. And to solve we cross multiply 8 times 3 = 24. So we have 24 = 6x. Divide by 6 would give us 4 and so x would be equal to 4. And since they are looking for the full B to C and not just B to E, we would have to go back and add 4+3=7. So B to C is 7.

Confident, he sat down and smiled at his answer. The teacher asked for alternative solutions and the next presenter discussed looking at the right side as half of the left side; this had been Finn's first method for finding the solution.

Session 6 with Finn and Ester. Finn paired with Ester for session 6. The task asked for the area of a right triangle whose perimeter was 30 and had one side 7 cm longer than the other. Both Finn and Ester struggle with language and mathematics and this was a task chosen because it was more challenging than the last two problems. It became obvious in this task that the struggle with the language of mathematics created a need for more time to solve the problem for these two students. They supplemented each other's voices as they worked to solve the task.

Finn and Ester began the Pair phase with the formula for the area of a triangle. Finn began voicing the equation, struggled, and Ester finished.

F: Ummmm. It's half, it's…so

E: Ummm. Half base times height.

Although they voiced "right triangle" and "perimeter", the connection with the geometric figure was not immediate. Finn said, "so… a right triangle and perimeter" then drew a square on his paper. They both repeated "right triangle" three times before Finn said, "So if it is a right triangle, this thing is 30 cm." Ester corrected him emphatically, "It is a right triangle. It's not a square." Finn hit his pencil, said, "Oh, wait!" and drew an equilateral triangle below the rectangle. As soon as he saw the triangle, he said, "Ohhh," sat up straight, scratched his elbow and focused on the drawing. Ester said, "It's a triangle, not a right triangle." They continued to finish each other's sentences while they corrected the drawing of the right triangle and both wrote on the diagram.

E: So it it's

F: A triangle

E: So it's 30 cm.

F: The whole thing is 30 cm?

E: Yeah and then one side is 7 cm longer than the other. Yeah I'm guessing that's 7 and then

F: That makes this one…and the other

E: Is 30, but

F: No, the other is equal to 23 (subvocalized as he calculated)

E: So then what do we do with 30?

Finn and Ester postulated that one side was 7 and began finding the other two sides by dividing the remaining 23 cm in half. They struggled with the calculation and looked for a calculator.

F: That's the total combined. (pause) That's the total. So… (Subvocalizes again 23 divided by… Is what? Is… 11.5. So we'll do ___ plus ___ 25 … equals 22, 23 plus 7 equals… twentyyyy thirty. For some reason…(pause) What's the area of the triangle? Yeah that's not right because that's stuff I figured out about the triangle.

E: Do you have a calculator?

The bell rang to end the Pair phase without further mention of one side being 7 more than another. During T2, Finn scratched his neck and wrote diligently on his paper. Finn continued writing on his paper even after the bell rang to begin the Share phase but listened to other presentations.

Post Interview and Summary with Finn. Finn's placement in the first session was based on his preference to work alone. However, in the post-observation interview, Finn changed his preference to working with others because they helped him to figure out the steps if he had problems on his own. This preference, he said, depended upon how much he felt he could contribute to the group. "Sometimes I felt like I wouldn't need their achievements with their grades being higher than me. I felt like I would be the underman going into the group. But then other times I would feel like I knew what I was talking about and could solve it." He preferred to work alone when he felt he understood how to solve it and could get it done. Quiet time was still important to him for thinking through what he was saying, but he claimed that he now only used his book for assigned problems. Finn's responses about how he used quiet time to work on assigned problems also changed from the pre-interview. In the post-interview, he stated that he used pictures for geometric shapes and he definitely liked being able to ask someone else for help.

Finn was quick to report that TPTS helped him to understand math. The additional quiet time was helpful when he tried to gather thoughts together but was a waste when he completely understood. It helped him to see the steps that other students took to get solutions; it gave him more ways to look at math. He stated that the last part of TPTS, Share, was probably the most important for him because he saw how others used their thoughts. He reiterated in the focus group that TPTS helped him see the ways other students thought and integrate those methods into his own thinking (Figure 4.6).

Think 1	Pair	Think 2	Share
Excited to try to come up with the answer. To try to solve it, like a puzzle.	Nervous. Having to share my ideas.	(Long thought and then sigh). Don't know, it's a lot. I'm trying to think of a word that would fit. (Long pause) hmm, probably a little confused. Trying to put together their ideas with my ideas to come up with the conclusion of what the answer would be.	Nervous. Having to give my answer so everyone else could understand.

Figure 4.6 Finn's feelings about TPTS phases

Ancillary Profiles

Incomplete data were collected from the last (non-selected) four students to provide a more complete picture of the classroom. For instance, Gabe was included in the results section even though he was not one of the six students originally chosen for the study because his insights often differed from other students. These students were videotaped in classroom settings including during the focal group interview.

Gabe, male Caucasian, struggled with mathematics and English

Gabe allowed videotaping but not interviewing so data were limited to classroom sessions and profile information from the administration prior to the study. He told Alberto that geometry was more difficult than algebra. Field notes showed he responded quickly to calculation questions and frequently utilized the back of his book for definitions and theorems. I suspected that he was at Van Hiele's visualization level because he was unable to recognize a right angle in session two when the hypotenuse faced the bottom of the page. He quietly studied tasks during T1 in each session. Script from session 3 showed him inquiring about alternative solutions when he paired with Diana, an inquisitive female who considered herself a top student in the school.

Session 3 with Gabe and Diana. The pair went to the front of the class to get a better view of the problem. Diana asked if the cake was rectangular or a triangle and the teacher told her to interpret it how she read it. She then asked Gabe rather than relying on her own reading and figured it out when someone in the back of the class said, "Oh! He's going to split it in half!" The teacher reminded the class that it was quiet thinking time because some were already talking.

Diana quickly determined that they wanted to find the hypotenuse of a right triangle so they needed to use the Pythagorean Theorem. They calculated the result and rounded it off to their mutual satisfaction. Although Diana did much of the calculating, Gabe experimented, interchanging labels for the legs to see if they got the same result. Gabe said, “Well, that was pretty simplistic. I wonder…if 6 changes (puts 6 on a different leg than they had pictured). I’m not sure it will change it. It’s still the same, isn’t it? See if you can figure out how to work it.”

Gabe did not want to present their findings but he tried to help Ester when she struggled at the board. When she stated that she took 5 times 5, the teacher asked her why she did that. During her silence, Gabe whispered to her, “because A squared plus B squared equals C squared” but she did not use his response.

The next presenter used the term Pythagorean Theorem and Gabe looked up the theorem in the book. This presentation needed correction so the teacher invited one more presentation. Diana volunteered. Her explanation was precise but the teacher asked more probing questions that required definitions. Gabe aided her by reading the definitions from the book.

Session 4 with Gabe and Finn. (See <u>Session 4 with Finn</u> for dialogue.) Students had to find the length of a segment on the following triangle.

AD=8, CD=6, CE=3

Find BE

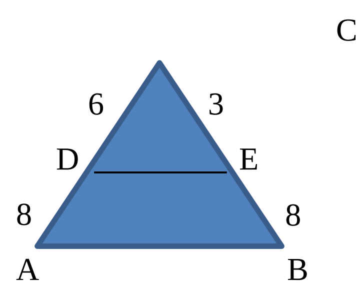


Figure 4.7 Session 4 task

Gabe looked at the sides of the triangle as objects in motion when he stated, “the lines would continue at the same rate.” He agreed with Finn’s logic and checked their work but he miscalculated the simplification of 8 over 6 when he said it would be 6 over 4. I wondered if his proportional thinking might to be additive rather than multiplicative but there was no occasion to test this premise.

Session 5 with Gabe and Diana. Gabe and Diana went to the front of the class to get a better view of the problem. Diana asked if the cake was rectangular or a triangle and the teacher told her to interpret it how she read it. She then asked Gabe rather than relying on her own reading and figured it out when someone in the back of the class said, “Oh! He’s going to split it in half!” The teacher reminded the class that it was quiet thinking time because some were already talking.

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Session 6 with Gabe and Bradley. Gabe was paired with Bradley who, as previously noted, was considered the top student in the class in all subjects. Gabe struggled with communication yet, in this task, he was the only student that found a correct solution. Gabe wore thick glasses and walked to the front of the room to inspect the problem more closely. During T1, he leaned forward, rolled his eyes upward, and moved his pen back and forth. During the Pair phase, Bradley talked rapidly, explaining his thinking for the problem. Gabe sometimes questioned him and Bradley appeared to modify some of his numbers while he continued to talk. Gabe frequently furrowed his eyebrows as if not following completely Bradley's logic. He sometimes nodded his head in approval and sometimes challenged Bradley's ideas. Gabe questioned when Bradley used ½ hb rather than ½ bh for the area of a triangle so Bradley showed him that it worked both ways.

Gabe presented the final solution. He erased Bradley's numbers and replaced them with his own.

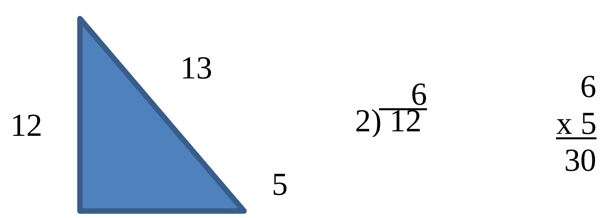


Figure 4.8 Gabe's solution to the task in session 6.

Gabe took one-half the height of 12 rather than one-half of 5 (the base on his drawing) because it was easily divisible. He used long division rather than the fractional form as the other students had done. He then wrote 6 X 5 = 30 vertically, looked at the original problem again, and added *sq. cm.* He sat down without saying a word.

> T: Where did you come up with 5, 12, and 13?
>
> (Diana whispered, “That’s good!”)
>
> G: Well, I just used the original process and went from there. (He looked at Bradley, indicating that it was his original idea.)
>
> T: What was your original process?
>
> G: Ask him. (indicated Bradley who looked blankly and shrugged his shoulders) Uhhhh. I just knew that we had to find the two that equaled whatever it was and so we just kind of guessed.
>
> T: So just the guessing method?
>
> G: Well, more like a logical guessing.
>
> T: Okay. So you knew that one was going to be 7 cm longer and so you knew they had to be 7 apart. Why?
>
> G: We used the Pie-tha-gorian Theorem (another student quietly chuckled) after using these two figured out. **So I just subtracted two and added one to each and that way you can logically do it.** So then you have got to go from there. Til they equal by the way. Because if not, one side is going to be too big on the Pythagorian equation. Like the 11 and 4, that was too big on the hypotenuse. This one equals.

Gabe returned to his seat without further comment.

Summary of Gabe. In this session, Gabe's strategy of taking away two from one side and adding one from each of the other two sides and completion of the task revealed much about his mathematical understanding. Rather than randomly assigning numbers, he devised a plan that would conserve length while varying the sides of the triangle. He then tested these combinations, correctly using the Pythagorean Theorem, until he discovered a triplet that worked. Perhaps because his partner had called to his attention the ability to switch base and height in the area formula, he chose to find one-half of the even side so he could work with whole numbers.

No individual interviews were held with Gabe; he agreed to taping but was uncomfortable with being interviewed. However, his insight during the focus interview, described in the next section, added greatly to the conversation for future research.

Focus Group Interview

This study sought to describe student experience during TPTS. In the focus group interview, students were directly asked, "How did you experience Think Pair Think Share?" Most students described it as "interesting" and "different" but were not sure if it was beneficial. Gabe claimed that it definitely helped him with proofs and Diana said, "At least I paid attention." It was suggested that it was helpful for those who were "too proud" to go to the teacher for help. Ester and Finn said that TPTS allowed them to work with those who gave them ideas that they could implement and Ester said they helped with ways to fix her work. When asked to describe some particular difficulties that they experienced, Bradley stated that working with those who thought they were right when they were actually wrong could increase confusion and Gabe said that it was easy for him to get distracted.

Although not a component of this study, Gabe noted in the focus group interview that TPTS appeared to be a useful tool for assessing student understanding; the teacher confirmed this. At the end of the focus group interview, Gabe said, “I think it could help the teacher also because if he needs a break or to assess the class understanding, he could just break them up into groups because if they don’t understand the lesson at all, then you can fix it. This is a good assessment.” Mr. T concurred and said, “If I am teaching there is not as much interaction. Whereas, if the class is interacting with each other, I have the opportunity to hear, listen, observe and see, OK, what part do I need to go over again? What part is maybe, perhaps, clear?”

In general, students who received high English class grades and performed well on standardized tests were least impacted by the use of TPTS in this study but found it different and kept them more on task. Those who had language differences (Hispanic descent and dyslexic in this study) reported that the quiet time helped them process the tasks and that working with others helped when they were unable to get help from the teacher. The strategy was deemed most beneficial for development of proofs and as an assessment tool for the teacher. The teacher found it was a good way to increase student participation and interaction.

Chapter 5 Discussion

This was the first formal study on Think Pair Think Share (TPTS), a teaching strategy for cooperative learning. Designed to aid learners unaccustomed to the language of use in the classroom, TPTS differed from the traditional Think-Pair-Share learning strategy in that it allowed two quiet times for cognitive processing prior to moments of speech. The classroom teacher taught six TPTS sessions in a high school geometry classroom. I videotaped each of the sessions, interviewed six selected students on three occasions, and took field notes for three months. In this chapter, I report patterns of behaviors between and among participants in response to research questions.

I found that students with language differences valued the phases of TPTS. Specifically, the two whose mother tongue was Spanish valued T2 as a time to prepare for public speaking. The student suspicioned to be dyslexic valued both T1 and T2 for quiet processing. Student experience with the phases of TPTS varied not only with language differences but, more generally, according to what I will call *use of external supports* to solve problems. Students who read their books, used pictures, or solved problems using equations and procedures (external supports) valued the use of quiet time. When students solved intuitively with little use of books or pictures, they reported little value in the extra quiet time (See Figure 5.1).

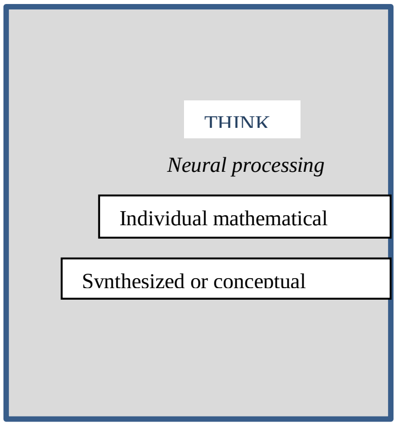
	Does quiet time help you to understand things?	Do you read your math book for more than assignments? Pre-	Post-	Used book/pictures in quiet times (out of 12 sessions)
Alberto	Yes	Yes	Sometimes	12
Ester	Yes	Yes	No	12
Finn	Yes	Yes	No	8
Diana	Yes	No	No	6
Bradley	No	No	No	1
Chad	No	No	No	4

Figure 5.1 Pre- post-observation preferences for quiet time and book usage

Students who stated that quiet time was useful for gaining understanding also claimed to read their math books in the pre-observation interview but not in the post-interview. These same students, Alberto (Hispanic background), Ester (Hispanic background), and Finn (dyslexic) used their texts in the majority of sessions. My observations indicated that Diana (straight A student in all classes) used her book extensively except when she felt she had already mastered the concept yet she claimed that she never used it. On several occasions, she told her partner to, "Look it up." Gabe (who struggles with English) tended to use the book and pictures during T1 and then think internally during T2. Bradley (straight A student in all classes) and Hugh (easily distracted) solved their mathematics problems without external aids in both T1 and T2. Chad (failing grades in all classes) worked intuitively during T1 but, as the sessions progressed, used T2 for external aids in development of presentations.

From the research of Rowe (1974, 1975, 1986), Tobin (1987), Stahl (1994), and Geiger (2010), I expected that T1 would become a comfortable spot where all students, regardless of external supports, knew they would have time to quietly ponder a question which would prompt more participation during the Pair phase. I expected students to have also increased usage of mathematics vocabulary during the Share phase because of the added T2. I used the research questions to directly guide the final analysis.

Research Questions



Research Question 1: How are the phases of TPTS experienced by students with language differences in a high school geometry classroom? This question subdivides into experiences during the first three phases of TPTS (T1, Pair, and T2) and the use of cultural connections.

Research question 1a: How do students use the first Think time? Rowe reported that quiet time following questioning increased the length and correctness of students' responses and that more students volunteered. Tobin (1987) reported improved classroom management with extended wait time. From these studies, I expected that students might experience a level of comfort during the first quiet time as they read and considered the mathematics task. The post-observation interview asked students to give one word describing how they felt for each of the four phases. These were open responses and 4 of the 6 students relayed a word that implied comfort for that person (peace, anxious, excited, relaxed) while working on the problem during T1 (Figure 5.2).

THINK1			
Alberto	peace	**Bradley**	relaxed
Ester	anxious	**Chad**	confused
Finn	excited	**Diana**	organizing

Figure 5.2 Post-observation feelings about T1

Ester's choice of the word *anxious* must be placed in context to see understand her level of comfort. She found it hard to find words to explain her feelings but reported that she was anxious to see what others were doing and to make comparisons to her own work. She implied that she was not fearful because she knew she could get help. Her anxiousness was a positive feeling of anticipation. Finn's word, excited, also needs contextual clues. He said he was excited to try to imagine what the answer could be and to try to solve it like a puzzle. Comments by Alberto, Ester and Finn indicated that T1 gave them time to process problems before speaking.

For Bradley, T1 made no difference. He said that he just relaxed because he nearly always found an answer within a few seconds. Diana expressed no feelings but said that T1 gave her time to organize thoughts. Chad used a word normally associated with a negative feeling, *confused*. When asked how he felt about T1, he had no immediate response. After thinking about how much he enjoyed the Pair phase, he returned to say that he guessed he could call T1 his time of confusion because it was the time when he was trying to figure out the problem. Although Chad did not express a feeling of comfort, his confusion was with the task and was normal.

The TPTS adaptation of Ernest's model places T1 in the quadrant where social location is private and the manifestation is individual. This indicates that new learning is attaching to prior knowledge and experience within the individual. The learner uses his own individual reasoning abilities to understand the problem. The collected data from observations of T1 indicated that some learners reason internally, without physical aids such as books, drawings, or labels while others needed these external supports. Figure 5.3 classifies student use of quiet time as either internal or external based on video observations. I identified internal processing as problem solving absent of the use of external resources such as books or pictures or equations.

TPTS SESSION	First Think Time EXTERNAL 1	2	3	4	5	6	INTERNAL 1	2	3	4	5	6	Second Think Time EXTERNAL 1	2	3	4	5	6	INTERNAL 1	2	3	4	5	6
Alberto	x	x	x	x	x	x							x	x	x		x	x						
Bradley							x	x	x	x	x	x							x	w	w		w	w
Chad	w				x		w	x	x			x	x		x			x		w			w	
Diana	x	x	x			x				x	x		x	x	x		x	x						
Ester	x	x	x	x	x	x							x	x				x					w	
Finn	x		x	x		x		x			x		x	x			x	x						
Gabe	x	x	x	x							x	x	x	w	x									x
Hugh	x							x	x	x	x	x	x	x	x								w	
Isiah					x		x	x	x	x		x	x	x	x		x							
James	w		x	x			w	x	x				w	w					w					

Not at school. No T2 in session 4.

External support: books, writing Internal: intuitive processing, no external markers

x = Student used external/internal support w = Off-task

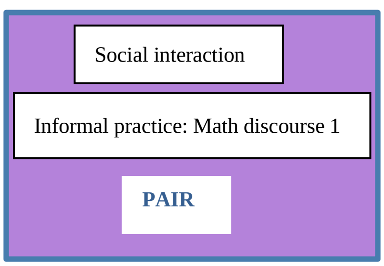
Figure 5.3 Video observations of student use of quiet time

This coding indicated that Alberto, Diana, Ester, Finn and Gabe used external supports while Bradley and Chad relied on more intuitive reasoning. Hugh and Isiah worked intuitively during T1 and sometimes used external supports during T2 to plan their presentations.

The six selected students were accustomed to working alone on a task so their pre- and post-observation interview responses about T1 were the same. The only change from their normal classroom routine to T1 was that they were not allowed to ask the teacher questions but were expected to try to figure out the problem on their own. This was not a problem for Diana, for whom language and math was a strength, because she knew there would be time later for discussion. However, for Ester who struggled with language, T1 helped only when she understood the problem. Otherwise, she said, it was a waste of time. I observed her drawing many pictures throughout the sessions, sometimes interrupting her partner to check the accuracy of the picture. I classified this use of pictures and textbook as external resources.in Figure 5.2. Ester subvocalized while reading the problem and stated in the mid-way interview that if she did not understand the problem, she did not really think on it during T1 but, instead, waited until she could talk with her partner.

Three students, Alberto, Diana and Finn reported that T1 was helpful. It gave Finn, who had trouble with reading comprehension, time to think through the problem without distraction. He read aloud, pointing to and isolating the math words and thinking about how to solve the problems. Alberto, an Hispanic student, said he could think about how to do the problem before talking with others. He indicated that pondering and debating with himself first resulted in less confusion and fewer arguments during the Pair work. He felt peaceful because he could talk about the problem later with his peers. He spent a lot of time flipping through pages, drawing pictures and writing equations in the margins of this book. Diana used T1 to read problems, organize thoughts, rearrange if/then sentences, and draw pictures to aid her understanding.

Two students, Bradley and Chad, felt that T1 was a waste of time. Bradley, considered the top student in the class, felt he could get an answer within a few seconds and did not need a full minute to sit quietly. He never used writing aids because he performed all work in his head. Chad, failing both math and language despite what seemed to be good intuition, reported that he got bored with the quiet and said that he found it a waste because he could not discuss his ideas with others. Although Chad claimed to find the time wasteful, videos documented changes in the way Chad used T1. In the beginning, he spent no time working on the problem throughout the entire session. By session 3, he began staring at the problem with his mouth open and began talking about his ideas and questions on the task as soon as allowed during Pair. He stated that he sometimes wished he could talk with someone else so he could understand what the problem was asking. This appeared similar to Ester's feelings.



Research question 1b: How do students use mathematics vocabulary during Pair time? Student discourse is assumed to have a major impact on mathematical understanding (*Professional standards for teaching mathematics*, 1991) but communicating ideas is difficult when language skills are weak. Geiger (2010) reported that allowing students to choose their own partners increased the rate of participation. I expected to find more discussion when students were seated with friends and this study provided support for that expectation. Bradley spoke very rapidly and went unchallenged when partnered with each of the two Hispanic students. It was his friend, Chad, who challenged his thinking and demanded to understand the mathematics vocabulary. Ester had trouble speaking until she was paired with her friend, Diana.

Roth (2000) found that gesture preceded speech when students begin to consider possible words that might be used to convey their thoughts. Chad continually used his hands to describe shapes and then asked for help finding appropriate words so he could use them during presentations. In general, students clarified meaning, analyzed definitions, and practiced using mathematics vocabulary during the Pair phase. Talking with a peer, knowing that they would report to the class later, allowed them to hone vocabulary skills through practice.

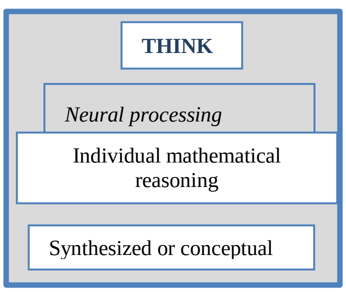
In session 1, pairs read the theorems to each other, emphasizing words such as bisector, sides, and opposites. They defined these words, both informally and by use of the book, to each other and had lengthy discussions about what they thought were If and Then portions of the theorem.

Session 2 required students to explain theorems that included mathematics vocabulary presented in the prior lesson and used in homework. They struggled to understand the meaning of mean proportional and given triangle, found they needed to standardize and formalize their naming of the sides of triangles (using two letters rather than just one) so they could communicate with each other, and practiced saying words together with hand movements for vertical, perpendicular and denominator.

The third task involved finding the diagonal of a rectangle. This was a straightforward use of the Pythagorean Theorem and students used the words diagonal, hypotenuse and Pythagorean Theorem with ease. Little formal mathematical discussion took place during the Pair phase of the fourth task and in the fifth task, students clarified the possible ways to draw a trapezoid and they used informal definitions of midpoints to make intuitive claims about equality.

The final session involved finding the area of a triangle with given perimeter. Students discussed the meaning of perimeter of a polygon, how to find the area of a triangle, and debated about the length of the sides and the hypotenuse of the triangle.

Field notes showed only informal usage of mathematics vocabulary prior to the beginning of the sessions. By the end, students were more regularly injecting proper math terms into questions and answers during classroom discussions with their peers and with the teacher. The teacher reported that he sometimes included vocabulary on his quizzes and that scores improved over the course of the study for that vocabulary and on problems using that vocabulary.



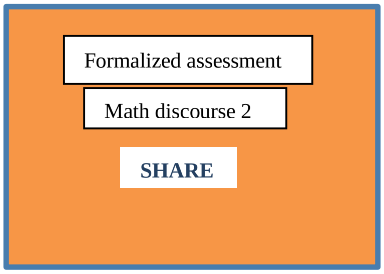
Research question 1c: How do students use the second Think time? Zhu (2005) found that students had trouble changing registers and speaking with special terminology. I expected T2 to help alleviate some of the stress that develops before presentations – especially for those who had low grades in both mathematics and English. Students reported in their interviews that the second Think time was used to finish unresolved solutions, practice for the Share phase, or just rest. Students were told to work quietly but some struggling students tried to get further feedback from their partners. The students classified as external in Figure 5.2 drew pictures, subvocalized solutions, clarified definitions and theorems, wrote equations, and wrote in their books to prepare for the classroom report. The difficulty of the task and the student's comfort with speaking in front of peers affected whether TPTS was deemed helpful or a waste of time by the students. Students who struggled with language and mathematics, such as the two Hispanic students and the one with supposed dyslexia, found it to be very helpful.

THINK2			
Alberto	worried/confident	**Bradley**	bored
Ester	frustrated	**Chad**	happy
Finn	confused	**Diana**	planning

Figure 5.4 Student feelings about T2.

As indicated in Figure 5.4, Alberto expressed worry about the presentation in the first half of the study but became confident, knowing that he had time to prepare during T2. Ester was frustrated because she needed more time for translation and she was nervous that she might have to speak. Finn needed time to process, even during the interview when trying to find a suitable feeling word. He said T2 was a time of confusion because there was a lot to do as he tried to put together his ideas with his partner's ideas and come up with a final solution. This made the T2 time very valuable to him. Diana found it helpful for planning tasks where she was unsure of concepts or the meanings of math words. However, she found it to be a waste of time when the task was not challenging to her because she was able to formulate her presentation during the Pair phase. Bradley was bored because he did not feel he needed T2. He processed what he believed were the solutions long before the buzzer rang for phase change. Bradley stated that he never needed to think in advance of speaking; he just said what came to his mind.

Research question 1d: How do students use supportive cultural connections and when do they appear? Neither the textbook nor the students made an effort to make cultural connections to the tasks used during TPTS in this geometry classroom. The students made the only connection to outside experience during the cake problem. In their experiences, students cut rectangular cakes into pieces that were rectangles and squares so they questioned how a right triangle could form. The teacher was unable to quiet them for T1 until they resolved the issue as a class. It was not possible to draw any conclusions with respect to this subquestion.



Research Question 2: How do students with language differences report their answers to mathematical tasks in a classroom that uses the Think-Pair-Think-Share cooperative learning strategy?

To answer this question, I examined mathematics vocabulary usage, variance in explanations, and gestured signals during the Share phase. Vocabulary, explanations, and gestures were examined in each of the six sessions and then combined with others to identify patterns.

Research question 2a: How do students use mathematics vocabulary during the Share phase? Baleghizadeh (2010) found that peer talk helped to decode the academic language of the classroom. I expected that the Share phase would encourage students to move into a more formal register, using math terminology more frequently and fluently. From the work of Roth (2000) I expected gestures to co-occur with speech, indicating that they were gaining familiarity. The first task asked all students to write a converse for the same theorem. Bradley gave the first presentation. He informally, and incorrectly, explained the theorem (draw the line that divides it into 4 parts) and said to *flip* the words to create the converse. Diana's presentation followed. She said there was a line *dividing* then corrected herself to say *bisecting an interior angle*. Hand motions  slanting apart from each other emphasized her description as she spoke of the *undivided sides*.

Each group in session 2 had the same theorem but a different corollary. In order to explain the corollaries, students needed to know nuances about the differences between the legs, sides and hypotenuse of a right triangle. Four other terms, *adjacent given segment, vertex, perpendicular line and mean proportional* became key obstacles for interpretations. Diana's initial explanation commented on the *hypotenuse and its adjacent leg* (there would be two of these) rather than *a leg and the segment of the hypotenuse adjacent to that leg*. Hugh could not identify a given vertex. Sides were referred to as *A and B* rather than *AB* so a proportional statement became A and B is to B and D as A and C is to C and D. I suspect that the additional "and" confused the naming of the lines and the identification of a vertex. Alberto hesitated when asked to identify two perpendicular lines and Diana labored over the meaning of mean proportional.

In session 4, Finn's use of mathematics vocabulary was informal but it supported learning. He used the word *ratio* to indicate a *proportion*. From my experience, this is a common mistake that does not confuse learning in the initial stages but that will need correction over time. He used the informal cross-multiply terminology rather than the equality of the product of the means and extremes. His name for a segment was A to D rather than AD.

By session 5, students were routinely using *proportional* in correct contexts. They discussed midpoint both formally and informally. Ester requested assurance from her partner that she was drawing the trapezoid correctly with bisected bases and Diana spoke of continuing legs of a trapezoid to form a triangle. She referred to a (singular) parallel line in a triangle but used her hands to demonstrate a line parallel to the base of a triangle.

Students used minimal mathematical terminology in their Session 6 presentations showed students using minimal terminology but the terminology they did use was used with ease. Bradley drew a right triangle, explained his assumption that the hypotenuse had to be half of the perimeter (incorrect assumption) and gave the area of a triangle and formula for the Pythagorean Theorem. Chad explained that he knew his solution was incorrect because something was wrong with the *variables* in the equation. Gabe mispronounced Pythagorean but correctly and succinctly explained all aspects of the problem. This session sounded much more like a mathematical discussion of possibilities rather than the informal and often incorrect usages found in the beginning sessions.

In brief, students used informal and inexact vocabulary in the beginning sessions, often using long pauses and supplementing words with hand movements. They became more comfortable and exacting over time. By the sixth session, they used mathematical terminology more fluently in a manner that fostered mathematical discourse.

Research question 2b: How do explanations vary during the Share phase? Students seemed to spend more time on task as the sessions progressed throughout the year. They learned to build upon each other's presentations as they progressed through the Share phase in each session. Vocabulary or concepts used in the first presentation were clarified or expounded upon in subsequent presentations. During the first session, most students were nervous, one group had nothing to present because they talked off task throughout the entire session and another group argued because they both wanted to present. Bradley gave the first presentation with informal and confusing explanation about a line that divided a triangle into four parts. The second presenter used the dividing line terminology and then quickly corrected to discuss the bisector of the interior angle.

Diana's uncertainty in the first presentation of the second session brought to light the weak understanding of mathematics vocabulary for the students in the classroom. Students listened intently to each other, trying to understand terms that they were unable to decipher in their own corollaries.

I coached Alberto by asking him factual/leading questions in the third session to help him isolate necessary information. The three students' presentations for this session built upon each other. The first presentation was a simple explanation of the step-by-step computational process used to find the solution. Arithmetic computations used vertical addition. The second described the need for the Pythagorean Theorem and set it up as an algebraic equation. Prompted by the teacher, the student explained what part of the equation showed him the need to take a square root. The last presentation added that a rectangle formed two right triangles when it was cut in half diagonally. Because the teacher prodded further, the class used a book definition and explained why the diagonal formed two right triangles rather than two ambiguous triangles.

Finn used an algebraic equation and cross-multiplication (the product of the means and extremes) to solve his proportion in session 4 then explained that they needed to add the result to another segment to get the full side of the triangle. James used the same initial proportion but solved by dividing the numerator and the denominator by 2 to get the missing segment.

Three explanations were given for the solution in session 5; two were very distinct. The first presenter assigned numbers to segments on the bases of a trapezoid whose midpoints were joined. He said that the divided bases would now form *the same* bases (rather than equal bases) and concluded that you would therefore be multiplying by the same numbers for the areas of each figure. The next student used the definition of a midpoint of a line to show that the first presenter had a correct solution. The third presenter used an entirely different method. She likened the trapezoid to the triangles in an earlier lesson and used a theorem about a line parallel to the base to show proportionality.

Students began looking for alternate solutions by the sixth session and used each other’s work to build on their own. Bradley and Gabe had correct notions about how to solve the problem but Bradley made an incorrect assumption about the relationship of the hypotenuse to the perimeter of a right triangle that lead to a false conclusion. Gabe saw the problem during Bradley’s presentation and continued to work until he made a correct analysis. In the meantime, Chad presented his attempt at using an equation even though he knew it was incorrect; this prompted the teacher to discuss possible errors in reasoning with his group. Students challenged and supported each other and one girl congratulated Gabe when he found the answer.

Students indicated during informal discussions at the beginning of the year that math problems normally had one answer and the teacher’s step-by-step process allowed them to find the answer. Knowing mathematical vocabulary was not recognized as necessary if you understood the concept. By the end of the sixth session, both students and the teacher recognized that different students could use different methods to complete the tasks. Although most students claimed in the post-observation interview that math could be understood without understanding the words, the students used math vocabulary to explain and explore the solutions during mathematical discourse in the final session.

Research question 2c: How do students use gesturing throughout the TPTS process? I hoped to identify transitions in thinking through gestures in the videos. Roth (2000) found that gestures shifted from quick inward to slow outward to demonstrate cognitive shifts so I examined videos for the shift. During the sixth session, Gabe demonstrated a shift through gesture when he found the sides of the right triangle with one leg 7 cm longer than the other. As he worked intently on the problem, his entire body was close to his paperwork. He quickly flipped his index finger upward when considering the addition one to one side and subtraction of two from the other. When he found the solution, he rapidly tapped his pencil on the answer and looked up. He moved his hands to cup upwards as if to release the answer when his partner accepted his calculations. This occurred during the Share phase as another student presented his work. Although in a whole group setting, Gabe was working alone as if in an additional quiet time. Another transition involving a gesture was identified in session 6 during the Share phase when Finn realized that the shape he had drawn was incorrect. His pencil had been moving quickly; he suddenly tapped it hard with his other hand and said, “Oh!” before changing his shape and revising his thinking. Both of these instances occurred after mathematical discourse with peers. The first was in a self-imposed quiet time and the second was correction by a peer.

Other gestures were representative of movements commonly seen in the classroom for clarification of information. Gestures were used to clarify imprecise or unfamiliar math words during explanations to others and to self. Chad practiced vertical, perpendicular line, and denominator using hand motions. Finn pointed to words to aid in comprehension. Students pointed to pictures and moved their fingers as if drawing shapes to direct attention to the referenced parts of the figure. Chad bounced his hand to create emphasis.

While reading the tasks, Alberto and Chad cupped their hands inward toward their bodies. During the first session when Chad did not work on task, his body was folded forward toward the desk with shoulders slumped. When he was sure of an answer in later sessions, he sat back in his chair when arms folded on his chest. Many students leaned toward their partners when actively listening or speaking. To signify deep thinking, Alberto’s fingers rubbed together and both he and Chad had mouths opened with heads cocked and sometimes with tongue extended. Alberto pursed his lips when he was ready to transition from listening to thinking. Several students moved their lips in subvocalizations during quiet times. Alberto and Finn rocked and played with objects between writing and thinking. The recorders made Alberto, Gabe and Hugh nervous so they made faces at the camera and turned the recorders.

The expectation for finding shifts of cognitive thinking through gestural shifts did not appear in these sessions during the two quiet times in TPTS. Instead, it appeared during or after mathematical discourse with a peer.

Ancillary Observations

I describe in this section changes in the classroom atmosphere and teacher beliefs through the course of the study. I compare Chazan’s ideal model for discourse with the discourse that developed in this study’s classroom.

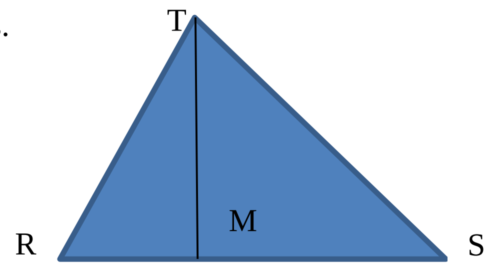
Evolution of instructional practice. Prior to the first TPTS session, I took field notes for one week of classes. Teaching was strictly step-by-step procedural (e.g. FOIL for factoring). The teacher tried to explain why the mathematics worked even though students did not ask but consistently reverted to, “in this method we can get the answer quickly.” The teacher gave 15 minutes of class time each day for homework but only Alberto used the time for study. Some students talked with the teacher, Chad took a water bottle from Ester and she hit him jokingly to get it back. Even though Alberto used his time for study, he reported the next day that he did not have his homework done. Chad stood outside during the homework review to imply (falsely) that he had been absent and could make up the work later. During homework time on the second day, all students were off task except the two Hispanic students, Alberto and Ester, who were taking makeup quizzes. These patterns continued throughout the week.

By the end of the study, the majority of the students used homework time to work on problems and discuss solutions with other students. Teacher questions and his reaction to student response also changed throughout the study. At the beginning of the study, the teacher only asked factual questions. Bradley and Finn made quick responses and the teacher moved forward without offering probing questions or challenging the responses. Only Diana slowed the pace when she asked, “How do you know which are means and extremes?” “What does transform mean?” “Do we actually have to know these transformations?” Her questions were met with Finn holding his head to indicate they were trivial. Hugh always had his calculator ready and was the first to do computations. By the end of the study, some student responses continued to be rapid fire but they were less frequently unquestioned. The teacher probed them further for explanations and students challenged each other.

At the beginning of the study, the teacher used appropriate mathematics vocabulary but accepted imprecise student answers. One of the example problems was as follows:

Given ΔRST and angle bisector TM,

Prove: RM : MS = RT : TS



Isaiah responded that RM would be 6 because TM is a bisector. It was given that TM bisected angle RTS, not line segment RS. The teacher did not correct the response.

Although the teacher used mathematics vocabulary (equivalent, simplify, multiples, ratio, proportion, segments, bisector, unity, the quantity x plus 8), the students usually did not. Field notes show the only instance of student use was with common words such as *divide*. The teacher restated student informal language. For instance, when a student said that in 3 (x + y) you “multiply by both”, the teacher responded, “Yes, distribute.” He did not always catch the imprecisions though. On one occasion, the student asked if answers would be *even* when she meant *a whole number* or *integral*. The teacher responded that the answers would not all be *even*.

There were changes in the use of TPTS time throughout the study. In the first session, all groups gave very brief overviews of their solutions and lasted only a few minutes before the teacher took back his role of instructor rather than mediator. By the sixth session, most of the class period was allotted to the Share phase. Students gave richer explanations of their work, used mathematical terminology, questioned each other’s solutions, and were eager to share additional methods or information. The teacher probed for deeper explanations and alternative ways of viewing the problem.

Evolution of teacher beliefs. The teacher expressed nervousness prior to the professional development on the first day of school. Cooperative learning was different and he was skeptical that he could cover everything necessary in the curriculum while allotting time for TPTS. By the end of the professional development session, though, he was excited and saw possibilities. He thought of a few things in geometry he transformed into TPTS tasks but had no ideas for his algebra class. I assured him that we would work together to develop tasks until he felt comfortable on his own.

Student responses in the first session caught the teacher by surprise. He told me that he did not know that there were so many ways of thinking about solving problems. He liked seeing how students thought and felt they asked deeper questions. Throughout the remaining five sessions, his support of TPTS grew and he told me informally that he did not think the students understood just how much the use of TPTS had improved their thinking. He found himself asking them deeper questions and giving more time for thought.

The teacher verbally gave homework answers then asked if there were questions. Students graded their own papers and called out their scores to the teacher who recorded them at the computer. There was no evidence that students graded their homework accurately; I saw some staring around the room as the teacher read answers yet they reported that they had near perfect scores. The teacher recorded missing quizzes for individuals on the blackboard. When students asked him questions, the teacher responded quickly and called the students by name.

After session 5, the teacher said that he was amazed by the student conversations and by how they actually talked it out and listened to each other. He said that it was interesting to hear them say that the problem was simple because he was sure that is not what he would hear normally in an assignment problem. The teacher told me that he did not think the students realized how much the process helped but he could see evidence of changes in student understanding of math words through the tests he had administered. More importantly to him, he said he had a better understanding of the ways that his students understood mathematics.

Student verbal communications

I noted that Chad told me in the first interview that he had no problems with mathematics vocabulary yet in the post-interview he said there were math words he did not know. Throughout the sessions, students repeatedly asked what a word meant or looked it up in the glossary so they could accomplish a task. Some theorems had multiple words that they questioned (adjacent, vertical, mean) yet most of the students claimed that they had no trouble with vocabulary. Only one student thought that words had to be understood in order to understand the concept.

Bradley felt confident in his speaking and mathematics abilities. This was in stark contrast to Alberto who valued the extra time and desired even more time for thinking about words and the math concepts. Both males indicated in the pre-interview that they valued teacher input but Alberto said he needed to repeat what the teacher said. Although both were open to working with others, Alberto feared there might be arguments. Bradley reported that he never used the book while Alberto used it continuously and tried to memorize words. Bradley neither drew pictures nor used self-talk; Alberto used both. Bradley did not believe he needed help from people, drawings, or speech outside of his own thoughts therefore time was not important and TPTS was just a fun activity. It appeared after analyzing the first two males that belief in one's ability to convey thoughts about mathematics was associated with belief about the supportive nature of TPTS.

Discourse during cooperative learning

Central to this study was the premise that mathematical discourse among peers in a classroom setting builds understanding. It was concern for the language learners in my own classroom that prompted the use of TPTS with the hope that the quiet times would aid their participation. My students that struggled in both English and mathematics were reluctant to talk on task. From the studies of Rowe (1966, 1968, 1969, 1974, 1975, 1986), Stahl (1994), and Tobin (1987), I knew that quiet time that followed questioning in science classes increased length of correctness of student responses and volunteered answers. Tobin confirmed this and called for studies of extended wait times with language learners, Baleghizadeh (2010) reported that peer talk helped college students to decode the academic language of the classroom. In this study, I found that those who struggled with both language and mathematics in a high school geometry classroom deemed peer and whole group discussion, each preceded by quiet time, helpful for understanding mathematical concepts.

I used Chazan's ideas about the kinds of conversations he wanted in his classrooms as a lens to view the discourse in the classroom in this study. *"Everyone must be seen as a participant in the unfolding understanding of the whole class and as a resource for learning"(Chazan, 2000, p. 23).* The teacher recognized and positively reinforced each student's comments. In the beginning of the study, the students viewed Bradley as the major participant. Other than computational answers, textbook definitions or whole class response answers, they allowed Bradley's voice to be the final answer.

Other students and the teacher relied upon him to have a correct answer. Diana was normally the only one that asked for further explanations from the teacher. While they had expectations of Bradley, just the opposite was true for Ester. She was not expected to speak because the teacher did not want to put her into uncomfortable situations.

By the final session, students understood that participation was universally expected and that Bradley did not always have the correct answer; other students were resources for learning. Students began to question each other more during the time to work on homework and the line of students waiting to ask questions at the teacher's desk reduced in size.

"The communication is careful enough to allow members of the class to appreciate ways in which classmates had diverse ideas, understand the same concept or term differently, and have conflicting ideas"(Chazan, 2000, p. 23). Respect was a high expectation in this classroom; a Bible reference to respect was always written on the board and the teacher stressed the importance. Students respectfully listened to the presentations of others but the credence they gave to responses changed across the course of the study. In the first session, both the students and the teacher looked for one exact answer and one particular method to solve the problem. By the final task, they appreciated different ways of thinking and began questioning each other when they thought differently about the problem.

"Everyone needs the opportunity for extended turns"(Chazan, 2000, p. 23). Prior to beginning TPTS, the teacher presented one method and students listened to his solutions. As the study progressed, the time needed for the Share phase extended as the teacher probed students for more information about the meaning of math words and

concepts. He allowed and encouraged student groups to present their solutions and allowed as much time as they needed to complete their responses. If the students wanted to correct an answer and then return to the board, they were allowed to do so.

"Students need to be able to come to the board to draw or to make reference to objects that could be used to clarify their intent"(Chazan, 2000, p. 23). Each student was required to come to the board to present a solution at least one time and they were encouraged to do it as often as possible. Some students naturally drew pictures to reference and others began drawing their own more frequently throughout the study.

"Students need to be able to ask for clarification and elaboration when they do not understand a speaker"(Chazan, 2000, p. 23). From the beginning of the visits, all students expressed a comfort with the teacher about questioning him in person. He was their main support and they had no doubt that he would try to explain the mathematics to them either privately or publically. Their comfort with asking peers varied according to friendships and connections. Bradley always began speaking immediately in the Pair phase and his words came quickly; Ester was hesitant to ask him for clarification. His partners provided little feedback for him.

More students questioned their peers about the meaning of mathematical vocabulary by the end of the study. In session 6, both Diana and Finn struggled with the problem because the illustration for formula for perimeter was a rectangle in their books. Their partners, Chad and Ester, persistently questioned, prodded, and used hands to illustrate to them the meaning of perimeter outside of the formula. Interestingly, Chad and Ester were considered two of the weakest students in the class yet they were the ones that persisted to bring understanding beyond the use of formulas to this task.

“*They should be the audience for one another’s comments—that is, they should speak to each other, aiming to convince or to question their peers*”(Chazan (2000), quoting National Council of Teachers of Mathematics [NCTM], 1991, p. 45). Students used the Pair phase to explain ideas to the peers and sometimes practiced words for the anticipated Share phase. They listened to each other during both Pair and Share.

Summary

We know that academic literacy for adolescent English language learners requires double the work (Short & Fitzsimmons, 2006), and therefore creates a heavier cognitive load for these students. Furthermore, students with reading differences such as the dyslexic frequently struggle with tasks that involve word problems, symbols, and unfamiliar vocabulary. Mathematics is a language on its own and students must learn not only the concepts but also the associated language to communicate with others effectively. TPTS was designed to help these students. While student experiences with TPTS were positive for all students in this study, students that struggled with both language and mathematics found it to be helpful for understanding math. The two Think phases helped them to begin thinking about the problem and to develop a presentation.

Limitations

This was a small study, limited to a geometry classroom of only ten students in one private Christian school in the Midwest. Most of these students and the teacher attended church together and the majority of the students had been classmates since elementary school; trust and respect were highly valued. This is very different from a large public school with high transient populations. In addition, tasks in geometry differ from those in algebra or higher levels of mathematics so the findings might differ in those settings. None of these variables, however, reduces the fact that TPTS appeared to be helpful.

Implications of the Study for Practice and Research

This study illuminated a plethora of possible areas for further study. I would like to see studies designed for larger schools, classrooms using different textbooks and technology, teachers and students with more experience with cooperative learning or TPS, and TPTS sessions using prechosen rich mathematical tasks. This study clearly showed that TPTS helped students who struggled with both language and geometry at a small private school but there is nothing in the data that suggested that it would be any less effective in different school settings. The TPTS strategy easily assimilated into a lecture-based geometry class by a teacher who had no prior experience with cooperative learning and thus content and lack of experience with cooperative learning do not appear to be limitations. Traditional textbook exercises for independent student use transformed quickly into TPTS tasks and it is likely that more progressive textbooks would have tasks that would work as well or better. The teacher was receptive to learning the technique. It required little professional development, gave the teacher another form of student assessment, and provided learning support for struggling students. What aspects of TPTS were beneficial to the teacher is yet to be studied.

Richer, more challenging tasks may provide results that are more rewarding for students of higher mathematical ability than those seen in this study. Bradley finished the first five tasks within the first few seconds of T1 and he was bored during the remaining quiet times. Only the more challenging sixth task caused cognitive dissonance for Bradley when the teacher pointed out that his solution was incorrect and Bradley was unable to say why. While this study allowed the teacher to choose traditional textbook tasks, future studies might examine student experiences and achievement with carefully structured problem solving tasks for cooperative learning.

Although this study showed that most students benefited from TPTS in terms of expressing themselves, experimental studies measuring achievement would be useful in determining the extent to which TPTS results in higher student performance. Schools need demonstrable improvement in student achievement to meet state mandates. They look for research-based practices that will aid that improvement. Future studies of TPTS should include large public schools with certified teachers in a variety of demographic settings.

Ester felt that T1 was a waste of time when she did not understand the problem. I have seen this same frustration in college students who have math anxiety; ways to ease this frustration should be studied. Among those ways, researchers could look at the usefulness of training students how to glean information from available resources such as books and electronic information. Students in the current study used the glossary more frequently as time passed. In recent substitute teaching experiences, I found several middle and high school students who were unfamiliar with the index and Table of Contents and relied solely on iPad search engines.

Factors such as social structures, affect, specific neurological processing, and the changing dynamics between teacher and student were not a part of this study but TPTS provides an excellent vehicle for examining each of them. I know that TPTS made a difference for the students in this study but I do not know what factors within the strategy mattered.

There were noted changes in teacher beliefs about the understanding of mathematics, student assessment, and strategies for teaching but I do not know the longitudinal effects on either teacher or student. This study included one two-hour session of professional development and continued informal support for the teacher for three months. Observations of the teacher were outside the scope of this research but it would be helpful to know what aspects of TPTS he found challenging to implement. I do not know if additional training would improve teacher experience.

Final Reflections

For me, this was merely a starting point for my work with TPTS. I intend to continue my research, speak to teacher groups and publish, as widely as possible, about the potential usefulness of this technique. This study demonstrates potential benefits for teachers as they add TPTS to their teaching tool belts and build mathematical discourse in their classrooms. In TPTS, students have the opportunity to make sense of problems and persevere in solving them. It gives them time to reason abstractly and quantitatively and to look for structure. TPTS allows support for students who are developing understanding through the Common Core Mathematical Practices.

I continue to wonder about the neural correlates of the use of quiet time for language learners in mathematics. The neuroscience of education is an emerging field. Educational neuroscientists are continually looking for studies of the brain that can inform the classroom. Neurological studies of TPTS could reveal biological differences for students who struggle with mathematics and English. I will seek international partners in the neural sciences to conduct this research.

When conducting research, we must consider (a) whether the research is conducted with concern about how the results will be used, and (b) whether the research is conducted with concern for fundamental understanding (Stake, 1997). Research conducted with concern only for use (without understanding) is *pure applied research* (William, 2008). Research conducted with concern only for fundamental understanding (without application) is *pure basic research*. William (2008) dubs research concerned with both *use-inspired basic research*. This study was use-inspired basic research. It is my hope that this study will demonstrate the potential of TPTS, move researchers from the fields of education and the brain sciences into action, and give practitioners a tool to aid language learners in their classrooms. This is the long overdue call to action made by William, Dylan and Lester in 2008, that the focus would be that of actually moving people - teachers, teacher educators, school administrators, policy makers, and others to action.

Appendix

Appendix A   Slope Task

1) Points (3, 5) and (-2, 4) lie on a line.

Cain uses the formula:

$m = \frac{y_1 - y_2}{x_1 - x_2}$

Abel uses the formula:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

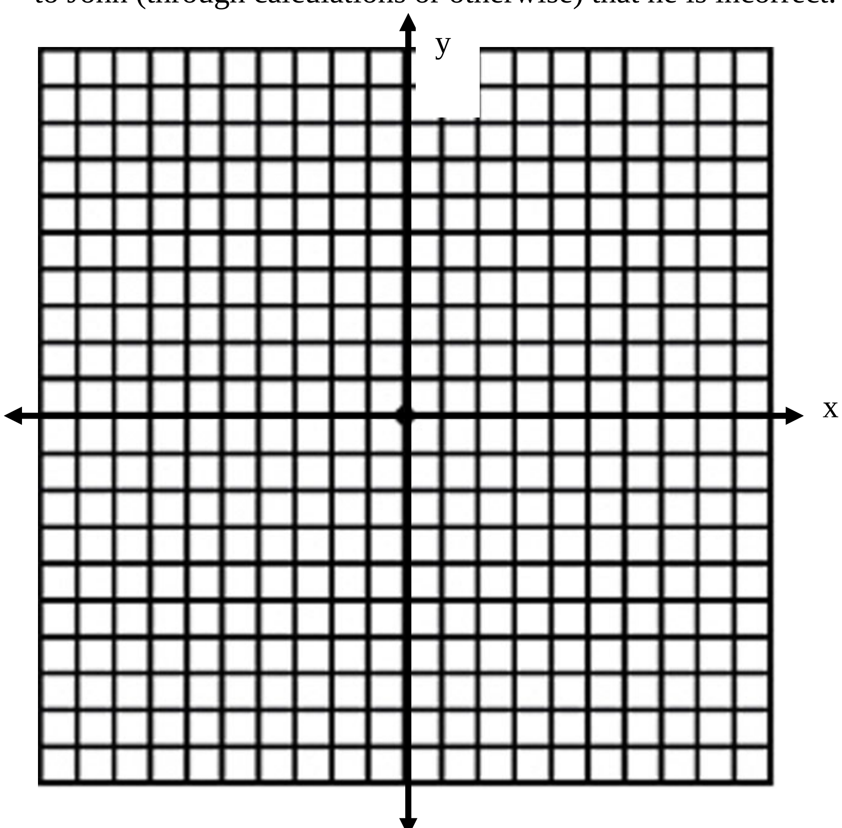
a) Can both Cain and Abel be correct?

Use the points given above to support your answer.

b) Will this (your answer for part a) be true for any 2 points?   Why?

2) John calculates the value of the slope of the line and says m=1. Sue take a look at the line and without any calculations says he is incorrect.

a) Explain how Sue knows this without calculating the value of the slope herself. Prove to John (through calculations or otherwise) that he is incorrect.



(Cross, 2009)

Appendix B Dialogue

Excerpt of dialogue from (Cross, 2009)

Teacher: So what does that mean?

Eddie: Oh yeah… I see…

Teacher: So do you know why that is though?

[about 5 seconds later]

Winsome: Because it's just the variables that you switch around so it doesn't matter.

Teacher: Can you explain that again and tell everybody?

Winsome: Well because you could have chosen any number to be y_1 either 4 or 5 or you could have labeled any of the points [pointing to the x-values] x_1 or x_2 so it would work out to be the same.

Deana: Oh… I see

Eddie Yeah

Winsome: So it doesn't matter.

Teacher: So do you really understand [looking at Deana]? Explain it to me.

Deana: Yeah, I get it because you usually ask us which one do we want to be y_1 and y_2 and we usually choose the first one but if we switched it around it would be Abel's formula.

Appendix C Indiana University Informed Consent Statement

Think-Pair-Think-Share and language in the high school mathematics classroom

Dear Parents,

Your child is invited to participate in a research study of the way structured think-time affects the use of language when learning of mathematics. In brief, the new technique adds more time for children to think before responding to verbal questions from the teacher. Your child was selected as a possible participant in the study because it will take place in the Algebra 1 and Geometry classes your child is taking this year. We ask that you read this form and ask any questions you may have before agreeing to be in the study.

The study is being conducted by Nancy Schoolcraft and Dr. Peter Kloosterman in the Mathematics Education program at Indiana University. Your child will be taught at least six regular math lessons using the new strategy by Mr. Cory Tahtinen, the algebra and geometry teacher. These sessions will be video-taped. If you and your child agree, your child may be selected for the three interviews in which she or he will be asked a questions to determine his/her attitudes about working individually or in groups and the use of math vocabulary.

The risks for participation in this study are minimal and involve the fact that some children may be uncomfortable answering questions in interview situations and there is always the possibility of loss of confidentiality. Participants will have the option to decline to answer any question that is asked and any comments made by your child will be reported anonymously. We cannot guarantee absolute confidentiality because personal information may be disclosed if required by law. There is no direct benefit to your child for participating in the study although we expect the new technique to improve the learning of many if not most children in the classes being studied. If you decline allowing your child to participate in the study, (s)he will still be expected to participate in the classroom activities without having the data collected for use by the researchers.

If you have any questions at any time about the study or the procedures, you may contact the principal investigator, Peter Kloosterman, at 812-856-8147 or by email at klooster@indiana.edu. If you feel that your child's rights as a participant in research have been violated during the course of this project, you may contact the office for the Human Subjects Committee, Bryan Hall 110, Indiana University, IN 47405, 812-855-3067, or by email at iub_hsc@indiana.edu.

Your child's participation in this study is voluntary; you may decline to participate without penalty by signing and returning the attached form. If no form is returned, it is understood that your child may participate. You or your child may withdraw from the study at any time without negative consequences. If your child withdraws from the study before data collection is completed and you would like your child's data destroyed, I will do so.

Thank you,

Nancy Schoolcraft, Research Director, Indiana University

Think-Pair-Think-Share: Language learners in high school mathematics

YOU MAY DECLINE PARTICIPATION IN THE STUDY WITHOUT PENALTY BY SIGNING AND RETURNING THIS FORM.

If no form is returned, it is understood that your child may participate.

I have read and understand the above information. I have received a copy of this form.

I do not wish to permit my child to participate in this study.

____

Child's full name

____________________________ _______________________

Parent's name Parent's signature

Appendix D Professional Development Plan

THINK-PAIR-THINK-SHARE

Materials: Handouts of pages 325, 326 from 1998 Glencoe Algebra I Integration, Applications, Connections; graph paper, plain, paper, 4 colors of paper for constructing Think/Pair/Think/Share signs, pencils, copies of Beowulf, timers, colored stones for partner selection

Participants are given a stone from the jar as they enter the room and are greeted. They are instructed to pick up one paper from each stack, fold in hamburger style, and write one stages of TPTS on each sheet. Examples are provided.

3:00 **Introduction** - self, purpose of meeting, description of academic language learner (ALL). If there is an odd number of participants, make one group of 3 and instruct them that two people talk during the share phase and the third one will be the reporter.

3:10 Language and math connections

Turn to page 325 of handout and select words that could pose problems for ALL

How many words are used for slope? (grade, steepness, slope, ratio of rise/run, vertical/horizontal)

Turn to page 326

How many words are used here for slope? (pitch, m, ratio of change, $\frac{y2-y1}{x2-x1}$)

3:15 Demonstration of TPTS

Describe Think, Pair, Think, Share

3:20 Show slopes in graphs that are positive, negative, 0, and undefined.

Teach how to compute using formula $\frac{y2-y1}{x2-x1}$

(Remind teachers that there are other ways to teach this and we will allow discussion of these later. For now, just focus on the technique being demonstrated.)

3:25 Analytical connection Task:

What is the value of *r* if the line through (*r*, 6) and (10, -3) has a slope of $-\frac{3}{2}$?

Think: Allow them to use graph paper or plain paper, pencils, manipulatives. Give them 90 seconds to work alone.

Pair: Let them work with a partner for 2 minutes. (They should either finish the problem or find other ways to solve and then explain how they got the answer to the partners.)

Think: Take about 1 minute of quiet time to prepare for your individual report. Think carefully about the words and pictures you will use to explain. Try to make a cultural connection if possible.

Share: Volunteers first and then selected participants

3:50 If the formula has not yet been used, show -3/2=(-3-6)/(10-r) solution.

3:55 Analytical connection Task:

What is the value of p if the line goes through (-2,4) and (p,5) with slope m=1/5? (May also use (3,4) (-1,p) with m=-3/4)

Think: Allow them to use graph paper or plain paper, pencils, manipulatives. Give them 90 seconds to work alone.

Pair: Let them work with a partner for 2 minutes.

What ways did you try to solve this problem?

How did you try to make sense of the problem?

Think: Take about 1 minute of quiet time to prepare for your individual report.

Think carefully about the words and pictures you will use to explain.

Try to make a cultural connection if possible.

Share: Volunteers first and then selected participants.

What happened during the first THINK?

What happened during PAIR?

What happened during the second THINK?

What happened during SHARE?

4:05 TPTS in literature. Pass out Beowulf

One person reads aloud while the other times and records the number of words mispronounced or faltered upon. Other person reads aloud while the first person times and records. Each person writes (in home language) what the passage meant. (THINK) The passage is discussed as a PAIR. Each participant writes individually, using formal academic language, about what they believe the passage meant.(THINK) Participants report to the entire group. (SHARE)

Did that help you to stay focused on the reading? Write a reflection on the process.

How can you use this in your classroom? Discuss answers.

Appendix E Pre-observation Interview

(This might address comfort level with different groupings and academic language. It should illuminate perceived needs for supports.)

Grade Level: ________________________________

Date: ______________________________________

Introduction

Good afternoon (morning)! I am Mrs. Schoolcraft

I will sometimes be sitting in on your math class for the next few weeks and I wondered if you would be willing to help me understand a few things before I get there. All of your answers will be used only by me for my research. No one else will know who said the things that you tell me. There is no right or wrong answer; I just want to know how you think about a few things. You can stop me at any time.

Is it alright if I ask you a few questions? (If "no", the interview is ended. If "yes", then I say, "Thank you.")

Perceived support systems

What do you think helps you to understand math? Why?

Who do you think helps you to understand math? Why?

Questions about Groups or Pairs

1. On a normal day in math class, would you rather work alone or with others?
    a. Would you tell me why?

2. Over the next few weeks, your teacher will be doing some group work where you are placed with other students. How do you feel about working with others?

    a. Does it matter how many are in your group? (two, three, four?)

        i. Would you please explain?

    b. When you are working with others, what do you like most?

    c. Are there things that are difficult for you when you are in a group?

    d. Are [these things] better when you work alone?

        i. Why (not)?

3. Does your teacher sometimes use words that you don't understand?

    a. (If yes) Are they math words?

    b. Even though you don't understand all of the words, can you still understand what she is trying to teach you?

    c. How do you deal with it when you don't understand?

    d. Does use of quiet time help you understand what was being said?

4. Do you ever read your math book for more than just the assigned problems?

    a. How do you use it?

Questions for Think$_1$

5. If your teacher gives you quiet time after she/he assigns the problem, what do you do?

    Probing questions - Do you try to think of a way you have done this before? Do you draw pictures? Do you try to work with numbers? Do you *talk to yourself? Do you look up similar problems in the book? Do you look at the whiteboard? Do you wish you could ask someone else?*

6. What do you think I would see when if I looked at you during quiet time tomorrow?

Questions for Share and Think$_2$

7. You will be asked to explain your answers in front of the class. How does that make you feel?

    a. If your teacher gives you a little quiet time before you get in front of the class, how do you think you would use that time?

    b. Do you think the quiet time would help you get ready to talk to the class or would it be a waste of time?

Thank you for your help. I look forward to seeing you in class.

Appendix F Post-observation Interview

(This might address comfort level with different groupings and academic language. It should illuminate perceived supports given during TPTS.)

Grade Level: ________________________________

Date: ______________________________________

Introduction

Good afternoon (morning)!

As you know, I watched your classroom over the last few weeks and I wondered if you would be willing to help me understand a few things before I leave. All of your answers will be used only by me for my research. No one else will know who said the things that you tell me. There is no right or wrong answer; I just want to know how you think about a few things. You can stop me at any time.

Is it alright if I ask you a few questions? (If "no", the interview is ended. If "yes", then I say, "Thank you.")

Questions for Pair

1. On a normal day in math class, would you rather work alone or with others?
    a. Would you tell me why?

2. You were sometimes placed with other students for group work. How did you feel about working with others? (Show Task Card 2a)
    a. Did it matter how many were in your group? (two, three, four?)
        i. Would you please explain?

b. When you were working with others, what did you like most?

c. Are there things that were difficult for you when you were in a group?

d. Were [these things] better when you worked alone?

   i. Why (not)?

3. Did your teacher sometimes use words that you didn't understand?
   a. (If yes) Were they math words?
   b. Even though you didn't understand all of the words, could you still understand what she was trying to teach you?
   c. How did you deal with it when you didn't understand?
   d. Did use of quiet time help you understand what was being said?

4. Did you ever read your math book for more than just the assigned problems?
   a. How do you use it?

Questions for Think$_1$

5. If your teacher gave you quiet time after she/he assigned the problem, what did you do?

   Probing questions - Did you try to think of a way you have done this before? Did you draw pictures? Did you try to work with numbers? Did you talk to yourself? Did you look up similar *problems in the book? Did you look at the whiteboard? Did you wish you could ask someone else?*

6. What do you think I saw when I looked at you during quiet time?

Questions for Share and Think$_2$

7. You were asked to explain your answers in front of the class. How did that make you feel?

    a. If your teacher gave you a little quiet time before you got in front of the class, how did you use that time?

    b. Do you think the quiet time helped you get ready to talk to the class or was it a waste of time?

8. Your teacher used the TPTS method with you. (Explain this strategy.) Do you think that strategy helped you understand math?

9. Could you describe for me how you felt about each part of that process?

    a. Did you feel it helped you in any way?

b. Which part - working on the problem by yourself, talking with a partner, having time to talk to the class, or listening to others at the end - was most helpful to you?

Thank you for your help. It has been nice working with you.

Appendix H Analytic/Interpretive Memo

- What seem to be the key kinds of support for this person and why do they matter?
- How are they familiar or different from my first subject?
- Are my findings about key supports for this person connected to particular characteristics of the person (level of understanding of English language)?
    - To what extent do I think these findings might be generalizable or unique to this person?
- What is the big picture of this person's story of using T_1PT_2S (in one paragraph)?
- If I were to answer the research question today, what would I say?
- How does this answer compare to the answer I would have given after just one interview?
- What did I learn about the person or the research question?
- What questions remain?

Appendix I

How are the phases of TPTS experienced by students with language differences in a high school geometry classroom?

How do students use the first THINK time?

How do students use mathematics vocabulary during PAIR time?

How do students use the second THINK time?

How do students with language differences report their answers to mathematical tasks in a classroom that uses the Think-Pair-Think-Share cooperative learning strategy?

How do students mathematics vocabulary during the Share phase?

How do students use gesturing throughout the TPTS process?

Pre-observation interview for student profile	**Addresses Research Question**	
What do you think helps you to understand math? Why?	Does this differ from post-interview?	
Who do you think helps you to understand math? Why?	Does this differ from post-interview?	
1 Prefers working alone or with others? Why?	Does this differ from post-interview?	
2 Feelings about working in groups? Explain	Does this differ from post-interview?	
2a How many in a group?	Does this differ from post-interview?	
2b Like most about group work?	Does this differ from post-interview?	

2c Difficulty in group work	Does this differ from post-interview?	
2d Better than alone? Why?	Does this differ from post-interview?	
3 Teacher's use of academic language	Does the student recognize difficulty/strength with language?	
3a Math words?	Subject specific?	
3b Do you still understand the math even when you don't understand the words?	Context specific?	
3c How do you deal when you don't understand?	Perceived supports – Does this differ from post-interview?	
3d Use of quiet time	Perceived supports – Does this differ from post-interview?	
4 Read math book?	Perceived supports – Usefulness of written text	Q1a
5 Use of quiet time		Q1a
6 How would this look?		
7 Feelings about whole group sharing	Perception of voicing answers around peers	Q1c
7a Thoughts about possible T1		Q1c
7b Thoughts about possible T2		Q1c

Audio/Video Tape Transcriptions

Visual description of setting and activities for class with focus on two students per class

	Q1
Instances of math words	Q2a
Examples of different explanations	Q2b
Instances of hand and body movement to explain understandings	Q2c

Video-stimulated recall interviews

Show clips from each stage of TPTS

What was happening (in your head, on paper, etc.) in this clip?	Q1 a,b
Do you think you understand the math words used today?	Q2a
Can you describe to me more than one way to solve today's problem?	Q2b

Show teacher's presentation of task

Did it seem connected to something you already knew? In 'real life'?	Q1d

Pre-observation interview	**Addresses Research Question**	
1 Prefers working alone or with others? Why?	Does this differ from pre-interview?	
2 Feelings about working in groups? Explain	Does this differ from pre-interview?	
2a How many in a group?	Does this differ from pre-interview?	
2b Like most about group work?	Does this differ from pre-interview?	
2c Difficulty in group work	Does this differ from pre-interview?	
2d Better than alone? Why?	Does this differ from pre-interview?	

3 Teacher's use of academic language	Does the student recognize difficulty/strength with language?	
3a Math words?	Subject specific?	
3b Do you still understand the math even when you don't understand the words?	Context specific?	
3c How do you deal when you don't understand?	Perceived supports – Does this differ from pre-interview?	
3d Use of quiet time	Perceived supports – Does this differ from pre-interview?	
4 Read math book?	Perceived supports – Usefulness of written text	Q1a
5 Use of quiet time		Q1a
6 How would this look?		Q1a
7 Feelings about whole group sharing	Perception of voicing answers around peers Do perceptions of sharing change after using TPTS?	Q1c
7a Thoughts about T1		Q1c
7b Thoughts about T2		Q1c
Did TPTS help understand math?		
Description of feelings about each part of the process		Q1, Q2

Focus Group
Discrepancies and unanswered questions

Appendix J Tasks and Pairings

Session 1

Task: p. 201 #3 Write the converse of Theorem 55. The bisector of an interior angle of a triangle divides the opposite side internally into segments which have the same ratio as the other two sides.

Pairs and their resultant converse statements:

Bradley and Alberto

> "If you draw a triangle with a line that divides the four parts into an equal ratio then the line will bisect the interior angle."

Finn and Hugh

> "If a triangles two segments have the same ratio as the other two segments, then the bisector of an interior angles of that triangle divides the opposite two segments equally."

Chad and James

> No converse developed

Isaiah and Gabe

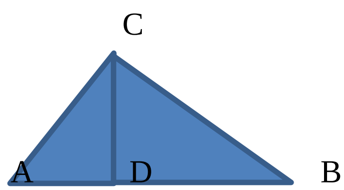
> "The bisect angle of an interior angle combines/divides the alternate side into segments which have same ratios as the other two sides."

Diana and Ester

> "If one side of a triangle is divided because of the bisector of the opposite angle, then the sides not divided are proportional to the segments of the divided side."

Session 2

Task and Pairs: p. 210 Theorem 61. If in a right triangle the perpendicular is drawn from the vertex of the right angle to the hypotenuse:



Gabe and James

I. The two triangles thus formed are similar to the given triangle and to each other.

Alberto, Ester and Hugh

II. The perpendicular is the mean proportional between the segments of the hypotenuse.

Finn and Diana

III. Each leg of the given triangle is the mean proportional between the hypotenuse and the adjacent segment.

Isaiah, Bradley and Chad

Corollary 61-1. If a perpendicular is dropped from any point on a circle upon a diameter, then the perpendicular is the mean proportional between the segments of the diameter.

Session 3

Task: Honey and Money are twin brothers. Mother wants to divide a rectangular shaped cake of 5 cm wide and 6 cm long. Find the length of the third side of the cake if it were cut diagonally?

Pairs:

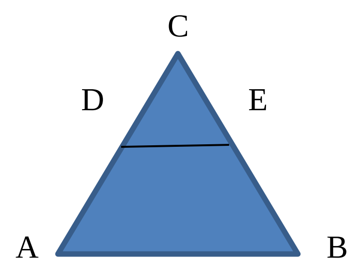
Bradley and Ester Diana and Gabe James and Finn

Alberto and Chad Hugh and Isaiah

Session 4

Task: Given the following triangle, find BC.

AD=8, CD=6, CE=3



Pairs:

Gabe and Finn Bradley and Diana Alberto, Isaiah, and James Ester and Hugh

Session 5

Task: Prove that the line joining the midpoints of the bases of a trapezoid divides the trapezoid into two equal parts.

Pairs:

Bradley and Finn   Diana and Isaiah   Alberto and Gabe   Chad, Ester and Hugh

Session 6

Task: The perimeter of a right triangle is 30 cm. One side is 7 cm longer than the other. What is the area of the triangle?

Pairs:

Finn and Ester   Chad and Diana   Bradley and Gabe

Alberto, Hugh and Isaiah

Task Pairings

	Alberto	Bradley	Chad	Diana	Ester	Finn	Gabe	Hugh	Isaiah	James
Alberto		1	3		2		5	2, 6	4, 6	4
Bradley	1		2	4	3	5	6		2	
Chad	3	2		6	5				2	1
Diana		4	6		1	2	3		5	
Ester	2	3	5	1		6		2, 4, 5		
Finn		5		2	6		4	1		3
Gabe	5	6		3		4			1	2
Hugh	2, 6		5		2, 4, 5	1			3, 6	
Isaiah	4, 6	2	2	5			1	3, 6		4
James	4		1			3	2		4	

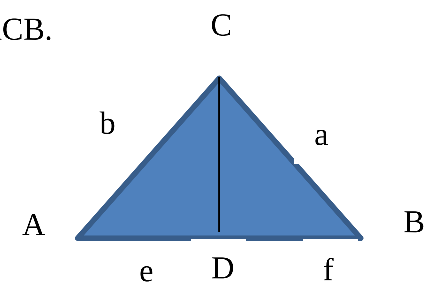
Pairings in red - not video or audio recorded.

Appendix K: Discussion of Task 1

Write the converse of Theorem 55. The bisector of an interior angle of a triangle divides the opposite side internally into segments which have the same ratio as the other two sides.

Given: Triangle ABC with point D on AB and line segment CD. IF CD bisects angle ACB, THEN e:f as b:a.

Converse: IF e:f as b:a, THEN CD bisects angle ACB.



Bradley and Alberto: If you draw a triangle with a line that divides that four parts into an equal ratio then the line will bisect the interior angle.

Correct conclusion. Ambiguous language about four parts and incorrect use of "equal ratio."

Finn and Hugh: If a triangles two segments have the same ratio as the other two segments, then the bisector of an interior angle of that triangle divides the opposite two segments equally.

Incorrect conclusion. Correct use of "same ratio as" but ambiguous use of four segments.

Chad and James: No solution

Isaiah and Gabe: The bisect angle of an interior angle combines/divides the alternate side into segments which have same ratios as the other two sides.

Restatement of original theorem. Incorrect use of *bisect*.

Diana and Ester: If one side of a triangle is divided because of the bisector of the opposite angle, then the sides not divided are proportional to the segments of the divided side.

Restatement of original theorem. Two parts of the original conclusion written in reverse order.

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Curriculum Vita

Nancy K Schoolcraft Fulaytar was born on a farm in Indiana and schooled in a building that housed all twelve grades. She earned a B.S. and M.A. in Mathematics and Journalism with a concentration in Education at Ball State University where she later taught core mathematics and methods classes for pre-service and in-service elementary and secondary teachers.

Nancy worked with different models of teaching as a mathematics teacher at Marion High School and she founded and coordinated the Indiana team for the American Regional Mathematics League (ARML). She received several awards such as the Outstanding Teacher of the Year Award from Rose-Hulman Institute and the Presidential Awards for Excellence in Teaching Mathematics and Science Indiana Semi-finalist in 1991, the same year that her husband died. A single mother of three children, Nancy was an Outstanding Educator Fellow at the Indiana Academy of Science, Mathematics, and the Humanities. After over twenty years of teaching in a high needs public high school, Nancy formed her own company where she trained state certified mentor teachers and conducted workshops that included sessions such as mathematics teaching strategies, learning and the brain, and Women in Boots for remodeling homes while learning math.

In 2008, she earned a MEd. in Mind, Brain, and Education at the Harvard School of Education and a PhD in Mathematics Education at Indiana University in December 2015. From her home in Anderson, IN, Nancy continues to work with the Indiana Department of Education and various educational entities to further the field of mathematics education.