

DAVID RANDALL LUCE. *A calculus of 'before.'* *Theoria* (Lund), vol. 32 (1966), pp. 25–44.

Luce's motivation for his calculus \mathcal{B} is to provide "a purely relational account of temporal phenomena without events and without momentary particulars." Exactly what a "relational account" amounts to is not specified but apparently it is rooted in Leibniz's classical version that instants apart from things are nothing. In this, the recently resurrected tense logicians would certainly agree, except that their basic formula operators, e.g. P and F , correspond to such phrases as 'it was the case that' and 'it will be the case that,' whereas Luce's basic connective, $\#$, corresponds to 'before' (or better perhaps 'and then'). The usual treatment of temporal reference through the introduction of a time parameter in which things are not said simply to possess a property but rather to possess it *at a time t* is metaphysically misleading unless the phrase 'at a time t ' is given a purely derivative status in the overall conceptual scheme. It is ontologically innocuous to refer to moments of time in the semantical meta-language for tense logic or for Luce's calculus of 'before' so long as such reference is understood in some such derived sense. An alternative semantics could equally well be provided, e.g. in higher-order tense logic, in which such reference need not occur. In his semantics, Luce allows himself to refer—as we shall in this review—to moments of time (cf. his semantical clause 5.12) although he attempts to rephrase such reference (cf. clause 5.13) by means of tensed sentences of a natural language. His rephrasing is inappropriate in that he speaks of such tensed sentences only as being (tenselessly?) true (false) and not of having been true (false), being (in the present tense) true (false) and going to be true (false). More of this anon.

The atomic wffs for Luce are of the form $(X \# Y)$ and $q = r$ and all other wffs, also called *full formulas*, are generated from these by means of sentential connectives and quantifiers. Atomic full formulas have (simple) *components* which are either *partial formulas* (and these are exactly the *wffs* of standard first-order theories with identity) or are generated from partial formulas by means of a formula operator E and the sentential connectives (re-applied after E). The function of E is to lock up "the contradiction inherent in saying that a certain object in the domain of discourse does not exist." Wherein lies the contradiction, the reviewer fails to see. For in the context of tenses, we can, without contradiction, say of a past object that it *does* (present tense) not exist as well as that it *did* exist. Luce could easily have avoided introducing E by utilizing any of a variety of well-known modified versions of quantification theory in which no such "contradiction" occurs. His own quantifier, incidentally, plays an ambiguous role: When affixed to a full formula it ranges over objects existing at some time or other, and when affixed to a simple component occurring within a full formula it ranges only over those objects existing at some time appropriate to the semantics of the full formula.

In Luce's semantics, a partial formula can be said to be true (false) *at a time*, unless the objects denoted by the individual constants occurring therein fail to exist at that time, in which case the partial formula is neither true nor false. Rejecting bivalence for this reason precludes the formalizability in \mathcal{B} of a variety of statements regarding relations between objects whose "life spans" do not overlap. Prefixing a partial formula with E results in a component which at no time is without a truth value. Thus $EA(p)$ is true at t if and only if what p denotes (and apparently for Luce names cannot change their denotation over time) both exists at t and has at t the property denoted by A . No indication is made as to what properties are or how they are assigned to predicates, so that the semantics remains incomplete in this regard. In tense logic an adequate *counterpart* can be constructed for each property, e.g. a function which assigns to each moment that property's extension at that moment. Such counterparts serve equally well for the system \mathcal{B} .

Where $\phi_0, \dots, \phi_{n+1}$ are simple components, the full formula $(\phi_0 \# \phi_1 \# \dots \# \phi_{n+1})$ is said to be (tenselessly?) true (false) iff there are moments t_0, \dots, t_{n+1} such that $t_0 < t_1 < \dots < t_{n+1}$ and for each $i \leq n + 1$, ϕ_i is true at t_i . We note that in tense logic, where $\Diamond\phi =_{df} P\phi \vee \phi \vee F\phi$, Luce's full formula $(\phi_0 \# \dots \# \phi_{n+1})$ is readily definable as

$$\Diamond(\phi_0 \& F(\phi_1 \& \dots F(\phi_n \& F\phi_{n+1}) \dots)).$$

Moreover, as any formula of tense logic of the form $\Diamond\phi$ is true at one time iff it is true at all times (although this can hardly be the meaning of 'tenseless truth'), we can understand how Luce was led to say of his full formulas that they are simply true or false. From this it is clear

that much of what can be said in tense logic simply is not expressible in \mathcal{B} , though \mathcal{B} itself is but a fragment of a definitional extension of tense logic.

Some axioms for \mathcal{B} are listed, but no claim is made in regard to completeness. Indeed, no notion of semantical validity is defined for \mathcal{B} which even at least minimally encompasses its theorems, still less coincides with them. A notion of "full logical truth" is given which is characterized in terms of the logical truth of certain standard formulas associated with full formulas and called their defined correlates. However, there are theorems of \mathcal{B} , e.g. $(Ap \# Bq \# Cr) \rightarrow (Ap \# Cr)$, whose defined correlates are not standard logical truths and which therefore are not themselves "full logical truths." The relevance for \mathcal{B} of the latter notion is lost on this reviewer.

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JAN SALAMUCHA. *The proof 'ex motu' for the existence of God: Logical analysis of St. Thomas' arguments.* English translation of 4212 by Tadeusz Gieryski and Marian Heitzman. *The new Scholasticism*, vol. 32 (1958), pp. 334-372.

This intriguing piece, by a Polish Catholic priest who studied under Łukasiewicz, was written to convince Catholic philosophers that first-order predicate logic is better than syllogisms. The bulk of the work, which is dated 1934, is a painstaking translation into first-order logic of the first proof of God's existence in Aquinas's *Summa Contra Gentiles*. Father Salamucha is quite right; Aquinas's argument is so tangled that one almost has to use the modern symbols to keep track of the threads. And as footnote 30 (p. 363) remarks: "In the arguments formulated in colloquial language, . . . assumptions easily slip in unnoticed."

Mathematically the piece is near-trivial, though there is enough sheer detail here to keep a Thomist happy for several days. We find some derivations proving that this follows from that, and some neat model-theoretic examples showing that that doesn't follow from this. There are eight pages (337-345) to explain what first-order logic is. There are also some errors, none disastrous for the argument. On page 345 Salamucha maintains that "It makes sense to speak of a first or last element only in connection with [a linearly] ordered set"; this mistake leads him to plant on Aquinas (p. 347) the assumption that the relation 'x moves y' is connected and transitive. On pages 353 and 362 (twice) Salamucha assumes that in a valid inference no predicate symbol can occur in the conclusion which did not occur in the premisses. In the footnote on page 352 he seems to be saying that in a discretely ordered set every bounded interval is finite; but this may be clumsy translation. Formula 3.3 on page 361 is badly misprinted and should be corrected from page 369.

A biographical note by Sobociński prefaces the essay. It tells us that Father Salamucha fell victim to Hitler's troops in Warsaw in 1944, and his notes and manuscripts on the history of logic were destroyed at the same time.

WILFRID HODGES

CZESŁAW LEJEWSKI. *On Leśniewski's Ontology.* *Ratio* (Oxford), vol. 1 no 2 (1958), pp. 150-176.

CZESŁAW LEJEWSKI. *Zu Leśniewskis Ontologie.* *Ratio* (Frankfurt a.M.), vol. 2 (1957-58), pp. 50-78.

Most of Leśniewski's published works present his systems in the process of formalization. Reading Leśniewski's papers is not an easy matter, chiefly because their symbolism is entirely different from the standard one.

Lejewski's essay contains a comparatively simple exposition of one of Leśniewski's most important systems, namely, his Ontology. Lejewski starts his considerations by constructing the so-called Ontological Tables, meant as an extension of the Euler diagrams. Using the tables, Lejewski defines some of the proposition-forming functors for one and for two nominal arguments (among them the functor of singular inclusion, the only primitive term of the system). Those functors and their mutual relationships are further characterized by a number of theorems of the form of biconditionals. These theorems, which are provable with the help of the axioms of Leśniewski's Ontology, are accompanied by their counterparts in everyday language.

The author is chiefly concerned with the successive simplifications of the well-known axiom of the Ontology: $(\alpha) [ab] :: a \in b \equiv :: [\exists c] \cdot c \in a :: [c] : c \in a \supset \cdot c \in b :: [cd] : c \in a \cdot d \in a \supset \cdot c \in d$. He also presents his own results in this connection; e.g. he proves that it is possible to found a system of Ontology on an axiom which contains the functor of partial inclusion as the