Symmetries and conserved quantities in integrated historical-philosophical perspective

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Notions of symmetry have special prominence in 20th century physics and philosophers have often discusses issues such as their empirical status, their role as heuristic tools, their possible interpretation as indicators of the structure of reality and the significance of symmetry breaking. While not a few authors are content with equating symmetries with mathematical forms and using these as basis for further study, others have pointed out that one should look beyond the purely mathematical aspects. Integrated historical and philosophical analyses are the most promising approach to bring to light the complexity of the subject, as has been shown by Giora Hon and Bernard Goldstein (2008) in their study of "symmetry" from Antiquity to the turning point in Adrien-Marie Legendre's work (1794). Focusing on more recent developments, the present paper addresses an issue closely linked to the prominence acquired by symmetries in the 20th century: their connection to “conserved quantities”. This connection is presented by physicists (and some philosophers) as one overarching principle valid both in classical and quantum theory. However, a closer look at the various theoretical and experimental practices allegedly implementing it shows that such operationalizations take very different and often not mutually equivalent forms.

First of all, the mathematical apparatus necessary to define and manipulate symmetries and conserved quantities radically differs not only between classical and quantum physics, but also between nonrelativistic quantum mechanics and relativistic quantum field theory. Moreover, in the quantum framework invariance over time concerns quantum averages, which in general are not directly measurable, so that the observable signature of conservation takes the form of selection rules establishing which processes have a nonzero probability of taking place. Selection rules have usually been left out of philosophical discussions about symmetry and conservation, but a integrated historical-philosophical study highlights their significance. Another often-neglected aspect that such an analysis brings into the picture is how verbal expressions of a connection between symmetries and conserved quantities play an essential role in bridging the gap between different physical theories - classical mechanics, general relativity, quantum mechanics, quantum field theory - allowing to regard them all as referring to the same physical entity, for example by defining "momentum" as "the conserved quantity associated to translation invariance".

In my contribution I shall address these issues by means of a compact, chronologically ordered overview. First of all, classical mechanics around 1800 already contained the mathematical apparatus linking invariance to constants of motion, but in general "conserved quantities" received attention only after the emergence of energy conservation around 1850. Even then, scientists showed little or no interest in investigating their connection to mathematical invariances, a question explicitly addressed only in discussing the conservation of energy-momentum in general relativity. The result were two theorems by Emmy Noether (1918), which went almost unnoticed at the time of their publications. In fact, it was only with quantum theory that symmetries - and with them conservations - gained centre stage. A premise for the further development of the connection symmetry-conservation was the construction of a new, typically quantum observable: selection rules.

The concept of selection rules emerged in the period 1915-1920 through the close interplay of theory and experiment, but by the early 1920s it was regarded as a purely descriptive tool of spectroscopy. It was thanks to the work of Eugene Wigner (1926-27) that selection rules came to be seen as observable consequences of the transformation properties of quantum systems. One year later Wigner explicitly posed the question, whether and how the classical notion of "conserved quantity" could be reinvented to fit quantum systems, and answered it by drawing from his own work linking invariance to selection rules. Wigner's answer was a verbal definition of a new kind of conservation, but mathematically it contained no novelty. It was immediately and tacitly embedded in the physicists' worldview and is today to be found in textbooks of quantum theory - more often then not under the name of "Noether's theorem." The "obviousness" which was (and still is) attributed to Wigner's result stands in contrast to its physical and philosophical significance: a quantum reinvention of conservation through mathematical invariance and selection rules which established a continuity defying radical changes in mathematical, experimental and epistemic framework.

The significance of the new approach was already evident from Wigner's paper, where the invariance of atomic Hamiltonians under mirror transformations was used to interpret a selection rule previously observed by Otto Laporte (1925) as signature of a nonclassical conserved quantity later known as "parity". Mirror transformations could not be implemented on atomic systems as easily as on their Hamiltonians, and parity was not in itself measurable, but could be empirically grasped only in the form of selection rules - nonetheless, Wigner set the status of their connection equal to that of other, well-established links, such as that between translation symmetry and momentum. It was only in the 1950s that the tacit assumption of universal mirror symmetry of physical systems was empirically tested - and refuted. Meanwhile, in the 1930s, symmetries and selection rules were heavily used in atomic, molecular and nuclear spectroscopy, and further innovations came when the methods developed in those contexts were applied to the study of newly discovered elementary particles. There, the triangle conserved quantities/selection rules/symmetries allowed to glide effortlessly from theory to experiment to physical interpretation and back, proving an invaluable heuristic tool working in all directions. For example, the selection rules observed in the process pion --> two photons could be explained as following from the conservation of total angular momentum (1948-49), while the difficulties in producing the newly discovered antiprotons and the ever-known stability of the proton could both be (re)interpreted as selection rules and linked to a new conserved quantity, later known as “baryon number” (1949- 50). The connection could also serve to drive more formal theoretical developments: it was while trying to formulate a symmetry associated to the conservation of isospin, that Cheng Ning Yang and Robert Mills (1954) introduced non-Abelian local gauge transformations into quantum field theory. The situation became even more complex when symmetry and conservations became entangled with the procedure of renormalization, prompting the rediscovery of Noether's work and giving rise to new facets of their relationship, like the problem of the invariance of the vacuum.

Thus, rather than having to do with one principle of symmetry and conservation expressed with different means, one is confronted with diverse notions of “symmetry” and “conservation” arising from situated practices and variously connected with each other. In epistemic practice, the connection between symmetries and conservation appears not as a static universal principle, but rather as a dynamical network of paths which may (or not) be crossed in all directions.