THE NEUTRINO AS A TACHYON

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We investigate the hypothesis that at least one of the known neutrinos travels faster than light. The current experimental situation is examined within this purview.

The idea that faster-than-light particles, or "tachyons", might exist has attracted some attention in the literature over the past two decades [1], following the realization [2] that such particles are not necessarily incompatible with the tenets of special relativity. Impetus for research has been lacking, however, in view of the apparent experimental fact that no tachyon has ever been detected. In this paper, we wish to raise the possibility not only that tachyons exist, but that they have been known for several decades. The hypothesis we shall examine is that at least one of the presently known species of neutrino is in fact a tachyon.

In the standard model of the strong and electroweak interactions, the neutrinos are massless. Each neutrino has a kinetic energy term in the lagrangian density, which may be written in the form

$$L_{\mathrm{KE}} = \mathrm{i}\bar{\psi}\partial\!\!\!/\psi\ , \qquad (1)$$

subject to the Weyl constraint

$$\gamma_5\psi = -\psi\ . \qquad (2)$$

Due to eq. (2), however, the kinetic energy term could also have been written as

$$L'_{\mathrm{KE}} = \mathrm{i}\bar{\psi}\gamma_5\partial\!\!\!/\psi\ . \qquad (3)$$

At the level of the standard model with massless neutrinos, eqs. (1) and (3) are indistinguishable.

Let us suppose that at least one neutrino gains a small mass, due to some short distance interaction such as is present in a GUT. One then removes the constraint (2), and one adds a mass term either to eq. (1),

$$L = \mathrm{i}\bar{\psi}\partial\!\!\!/\psi - m_\nu\bar{\psi}\psi\ , \qquad (4)$$

or to eq. (3),

$$L' = \mathrm{i}\bar{\psi}\gamma_5\partial\!\!\!/\psi - m_\nu\bar{\psi}\psi\ . \qquad (5)$$

At this point, however, there is an important difference between L and L': eq. (4) describes a normal Dirac neutrino, while eq. (5) describes a tachyon [‡1].

We are not claiming that L' serves as a satisfactory starting point for the construction of a consistent quantum field theory of tachyonic neutrinos. Indeed, as yet there does not exist a completely satisfactory quantum field theory of any type of tachyon [3]. Also, most work to date [4] has been concerned with scalar tachyons, because the little group for spacelike momenta is noncompact and, hence, its unitary representations are either one-dimensional (the scalar case)

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or infinite-dimensional. Therefore, the Fock space of the theory given by eq. (5) involves a nonunitary representation of the Lorentz transformations. A related problem is that straightforward quantization of the field theory can be shown to imply negative norm states in the Hilbert space, resulting in serious difficulties with the interpretation of the theory. Essentially, this occurs because γ_5 has both positive and negative eigenvalues, so that the kinetic energy term of the right-handed component of ψ will appear with the wrong sign in the hamiltonian. The problem is foreshadowed even at the level of quantum mechanics: the conserved current is

$$j_\mu = \bar{\psi}\gamma_5\gamma_\mu\psi\ , \tag{6}$$

so that the probability density j_0 is not positive definite.

In our view, such difficulties cannot be used to exclude a priori the existence of tachyons. Rather, they suggest that more theoretical work is required to determine physically acceptable modifications of the usual non-tachyonic quantum field theory. Such efforts would surely be forthcoming, if convincing experimental evidence were presented that at least one of the neutrinos is a tachyon. In fact, despite the catalogue of disasters listed in the previous paragraph, it is interesting that γ_5 makes a natural appearance in the theory of the tachyonic neutrino given by L'. To zeroth order in the neutrino mass, this means that the $(1 - \gamma_5)$ projection operator occurring in the standard model neutrino couplings is just what is required to prevent the interaction of the negative-norm component of the neutrino field. We shall elaborate on this point below.

Let us consider possible experimental tests of the nature of the neutrino. Perhaps the simplest conceptually is a direct time-of-flight measurement. Experiments comparing the neutrino, antineutrino, and muon velocities have in fact been performed [5], but no firm conclusions can be drawn at the current level of precision. The most precise tests, for the electron neutrino, involve the study of nuclear beta decay [6]. Although there are published data [7] bounding the mass of the electron neutrino between 14 and 46 eV, the result is not generally viewed as conclusive [8]. We wish to point out that the data, as usually analyzed, are much less sensitive to the presence of a tachyonic mass parameter than to that of an ordinary mass. This is because the energy of a non-tachyonic neutrino

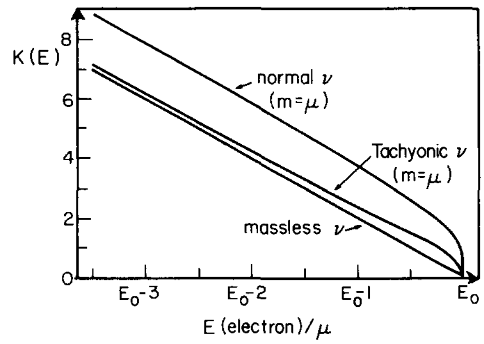

Fig. 1. Kurie plot of the electron energy spectrum near the upper end point, showing the expected behavior for massless neutrinos, for non-tachyonic neutrinos, and for tachyonic neutrinos. Note that the end point values have been chosen equal.

cannot be less than its mass m_ν, so that the location of the end point of the electron energy spectrum changes with m_ν. For a tachyonic neutrino this is no longer true, since it can have zero energy for any m_ν. The effect of this may be seen in the Kurie plot [9] of fig. 1, in which the quantity

$$K(E_e) = \{[F(Z, E_e)p_eE_e]^{-1}\, \mathrm{d}N/\mathrm{d}E_e\}^{1/2}\ , \tag{7}$$

is plotted versus E_e near the upper end point. Here, p_e and E_e denote the momentum and energy of the emitted electron, $\mathrm{d}N/\mathrm{d}E_e$ denotes the density of final states per unit range of total energy, and $F(Z, E_e)$ denotes the Coulomb correction factor. In terms of the deviation D from the upper end point energy, the spectrum near the upper end point energy is as follows: for massless neutrinos

$$K(E_e) \sim D\ ; \tag{8}$$

for non-tachyonic neutrinos,

$$K(E_e) \sim (D + m_\nu)^{1/2}(D^2 + 2m_\nu D)^{1/4}\ ; \tag{9}$$

and, for tachyonic neutrinos,

$$K(E_e) \sim D^{1/2}(D^2 + m_\nu^2)^{1/4}\ . \tag{10}$$

From fig. 1, it is evident that the electron energy spectrum data [7] bounding the electron neutrino mass also provide a weaker bound on the tachyon mass parameter.

Since the magnitude of the momentum of a tachyonic neutrino is bounded from below by m_ν, a

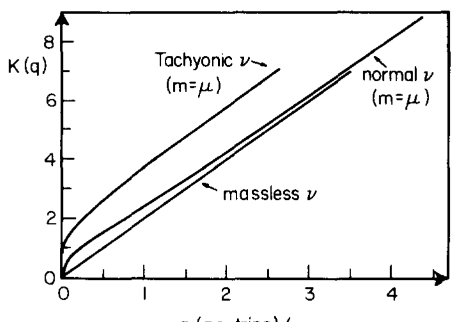

Fig. 2. Kurie plot of the neutrino momentum spectrum near the upper end point, showing the expected behavior for massless neutrinos, for non-tachyonic neutrinos, and for tachyonic neutrinos. Note that the end point values have been chosen equal.

more sensitive test for the tachyonic case is obtained by plotting the number of events as a function of the neutrino momentum. We illustrate this in fig. 2. We have conservatively chosen to plot the curves as though all of the end points were at zero neutrino momentum, $q = 0$. If an experiment is done that can measure the location of the end point, then the tachyonic curve will be displaced relative to the others and the difference will be even more pronounced.

Unfortunately, the neutrino energy is much easier to determine experimentally than its momentum because, in the limit of infinite nuclear mass, the recoiling nucleus carries an unknown amount of momentum but absorbs no kinetic energy. Thus, increased sensitivity to the existence of a tachyonic neutrino requires an experiment capable of determining the momentum of the recoiling nucleus. Also, since the tachyonic and non-tachyonic cases have the same qualitative shape, both the electron energy spectrum and the neutrino momentum spectrum are required to distinguish conclusively between the two types of neutrino.

The experiments that yield the most precise results on the muon neutrino mass involve the study of the reaction $\pi^+ \to \mu^+ \nu$. Essentially, the method involves determining the momentum $\boldsymbol{p}_\mu$ of the emitted muon in the rest frame of the pion and using known values of the pion mass, m_π, and muon mass, m_μ, to obtain the neutrino mass m_ν:

$$m_\nu^2 = m_\pi^2 + m_\mu^2 - 2m_\pi(\boldsymbol{p}_\mu^2 + m_\mu^2)^{1/2} \,. \tag{11}$$

The square of m_ν is calculated, so this method is less sensitive to small neutrino masses than the method involving the end point spectrum. We find the weighted average $\overline{m_\nu^2}$ of four independent experimental results [10] to be

$$\overline{m_\nu^2} = 0.02 \pm 0.18 \text{ MeV}^2 \,, \tag{12}$$

weakly favoring a non-tachyonic muon neutrino. However, it is amusing that three of these four experiments provide negative values for m_ν^2.

It seems premature to consider published results on the tau neutrino mass [11], given that its existence has only been indirectly inferred from the decay of the tau lepton. Instead, let us consider another popular method of searching for a neutrino mass: neutrino oscillations [12]. These occur when the neutrino mass eigenstates differ from the weak interaction eigenstates. The amplitude of the oscillations between any two flavors ν_a, ν_b of neutrino is determined by the mixing angles, while the oscillation length X_{ab} is proportional to the difference between the energies E_1, E_2 of the mass eigenstates ν_1, ν_2. For neutrino beam momenta $\boldsymbol{p}$ satisfying $|\boldsymbol{p}^2| \gg |m^2|$, the oscillation length is given by

$$X_{ab} = (\pm m_1^2 \mp m_2^2) d/2|\boldsymbol{p}| \,, \tag{13}$$

where d is the distance of the detector from the source of neutrinos of flavor a and where the lower sign in front of any mass eigenvalue is to be taken when the associated neutrino mass eigenstate is tachyonic. Unfortunately, eq. (13) makes it clear that, short of an experiment capable of determining the sign of X_{ab}, neutrino oscillations do not provide a good means of investigating our hypothesis.

One might wonder whether there are any qualitative differences between the tachyonic and non-tachyonic cases that do not disappear as the mass parameter tends to zero. This question is difficult to answer in general, but it seems that no such "zeroth order" effects appear in the model with L' given by eq. (5) and with neutrino couplings involving currents of the form $\bar{\chi}\gamma^\mu(1-\gamma_5)\psi_\nu$. To show this, we adopt the heuristic attitude that the usual field theory rules are to be followed in constructing amplitudes, except that integrals over tachyonic momentum magnitudes $|\boldsymbol{p}_\nu|$ are restricted [4] by the constraint

$$|\boldsymbol{p}_\nu|^2 \geqq m_\nu^2 \,. \tag{14}$$

The neutrino propagator in momentum space is then found to be

$$S^{(\nu)}(p) = (\not{p}\gamma_5 - m_\nu)/(p^2 + m_\nu^2)\,, \qquad (15)$$

subject to the constraint (14). The extra factor γ_5, which one might have expected to lead to zeroth order effects, always occurs in the combination

$$(1-\gamma_5)\not{p}\gamma_5 \equiv (1-\gamma_5)\not{p}\,. \qquad (16)$$

Thus, if terms depending on the mass parameter are ignored, the presence of the extra factor γ_5 is irrelevant. This verifies the assertion made above that the factor $(1-\gamma_5)$ is, at least to zeroth order, just what is needed to eliminate the effects of the negative norm component of the neutrino field.

Eq. (5) is, however, qualitatively different from eq. (4) in at least one respect: it seems to exclude the possibility of neutrinoless double beta decay [13]. This result follows from the properties of the charge conjugation matrix C', obtained by imposing a symmetry of the form

$$\psi \to \psi' = C'\bar{\psi}^{\mathrm{T}}\,, \qquad (17)$$

on the tachyonic Dirac equation. We find

$$C'\gamma_5\gamma_\mu^{\mathrm{T}}C'^{-1} = \gamma_5\gamma_\mu\,. \qquad (18)$$

It is, however, impossible to impose a Majorana condition of the form $\psi' = \lambda\psi$ for any phase λ, as the reader is invited to verify. Thus, although C' defined in this way is a symmetry of the equations of motion, it cannot be used to project out a charge self-conjugate spinor. Although there might be a subtler definition of C' that could circumvent this problem, with our definition eq. (5) cannot be used to describe Majorana tachyons. Therefore, processes that violate lepton number conservation due to the involvement of tachyonic neutrinos are forbidden.

In this note, we have raised the possibility that a neutrino, upon acquiring a small mass, might become a tachyon. We have surveyed the existing neutrino mass searches with this in mind and have suggested some ways to detect a tachyonic neutrino. Unfortunately, there is no "smoking gun", i.e., no piece of experimental evidence that strongly implies the tachyonic nature of the neutrino. There are, however, some interesting aspects of the neutrino that might bear reexamination under the hypothesis that it is a tachyon. For example, at one time it was hoped that the near masslessness of the neutrino might be explained [14] by interpreting it as the Goldstone fermion associated with supersymmetry breaking; perhaps this idea could be revived in conjunction with the tachyonic neutrino. Another possibility is that tachyonic neutrinos might explain the uniformity of the universe over distances that are large relative to the horizon distance. These and other ideas are being investigated.

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Note added: After this paper was written, a new value for the square of the muon-neutrino mass was published by Abela et al. [15]. This new value supersedes the one quoted by Lu et al. in ref. [10]. When we recompute the world average for m_ν^2, we find, instead of eq. (12), the following result:

$$\overline{m_\nu^2} = -0.166 \pm 0.091\,.$$

Thus, the data no longer weakly favor a non-tachyonic neutrino; instead they favor a tachyonic neutrino by almost two standard deviations.

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