

observed for  $\theta_{lab} > 30^\circ$ . The  $^{16}\text{O}$  analyzing power is very similar to that measured for  $^{12}\text{C}$  at this energy,<sup>1)</sup> while the  $^9\text{Be}$  analyzing power resembles the polarization measured in 180 MeV p + Li scattering.<sup>2)</sup>

The continuation of these measurements to larger angles is expected to be scheduled in early 1981. Optical-model analyses of the full angular distributions will be undertaken following acquisition of the of the large-angle data.

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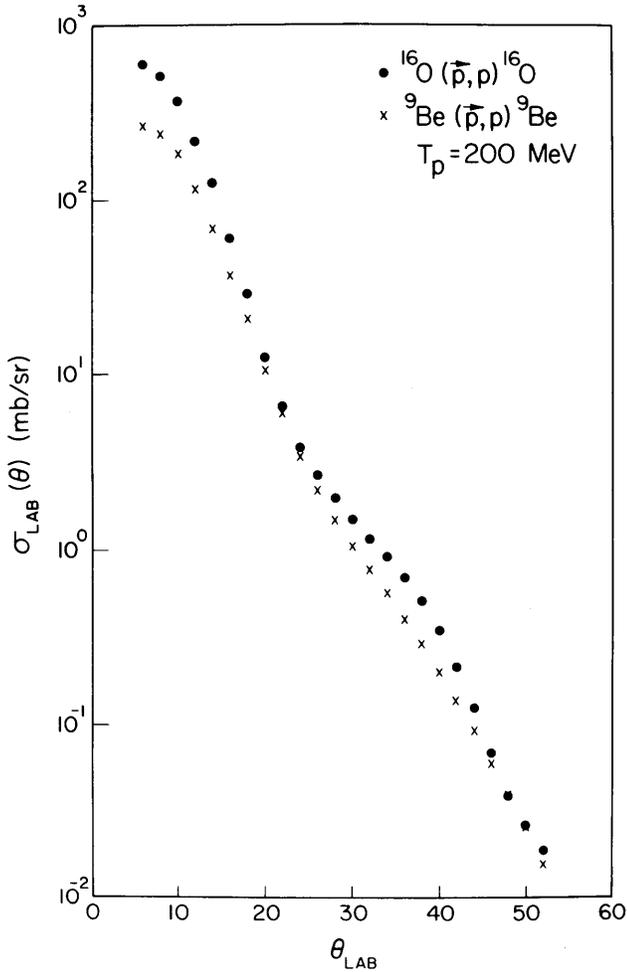


Figure 1. Laboratory differential cross sections for 200 MeV proton elastic scattering from  $^9\text{Be}$  and  $^{16}\text{O}$ .

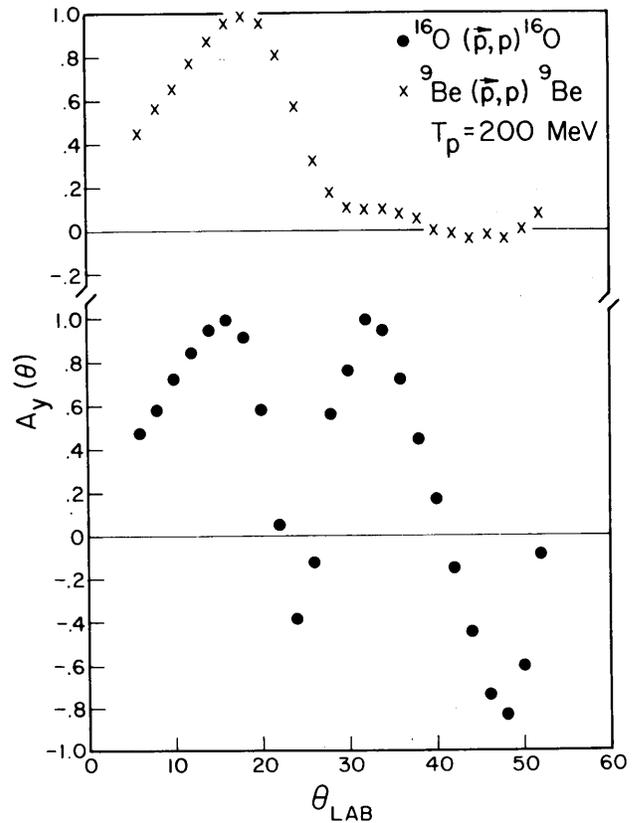


Figure 2. Analyzing powers for elastic scattering of 200 MeV polarized protons from  $^9\text{Be}$  and  $^{16}\text{O}$ .

#### STUDY OF RELATIVISTIC EFFECTS IN OPTICAL MODEL CALCULATIONS

H.O. Meyer

Indiana University Cyclotron Facility, Bloomington, Indiana 47405

Recently, unconventional (non-Woods-Saxon) shapes have been encountered in the analysis<sup>1)</sup> of 200 MeV proton scattering from  $^{12}\text{C}$ . One of the open questions has been whether relativistic effects in a conventional phenomenological optical-model analysis can be

responsible for part of this phenomenon. This is investigated for a model case, representative of proton scattering from  $^{12}\text{C}$ . The model case was defined by the choice of a standard potential  $\{P_0\}$  consisting of a complex central part (simple WS form) and a real spin-

orbit part (Thomas form) with the following parameters:  
 $V = 7.69$  MeV,  $r_v = 1.392$  fm,  $a_v = 0.489$  fm,  $W = 13.6$  MeV,  $r_w = 1.068$  fm,  $a_w = 0.674$  fm,  $V_{s0} = 4$  MeV,  $r_{s0} = 0.912$  fm,  $a_{s0} = 0.512$  fm.

The investigation was carried out using the code HAVOC, a spin 1/2-spin 0 optical-model code with a Marquardt fitting routine, written for diagnostic purposes and independently of existing software. Correctness and accuracy of HAVOC calculations were established by a comparison with the codes SNOOPY<sup>2)</sup> and A-THREE<sup>3)</sup>. For bombarding energies up to 500 MeV, the accuracy and stability of the calculation at backward angles (i.e., at the nanobarn level of cross section where measurements are now available) crucially depend on appropriate choices for the step size in the numerical integration of the Schrödinger equation and for the radius of matching to Coulomb wavefunctions. A step size of about 0.03 fm (0.015 fm) and the inclusion of partial waves up to  $l=35$  (50) have been found necessary for (p,p) calculations at 200 MeV (500 MeV).

Relativistic effects, strictly, cannot be incorporated in the framework of a Schrödinger equation. Basically, two approximate approaches can be found in the literature:

(i) RKV: The kinematical variables in the wave equation (i.e., the wave number  $k$ , the Sommerfeld parameter  $\eta$  and the reduced mass  $\mu$ ) are replaced by their relativistic values. We calculate  $k$  from the relativistic momentum in the center-of-mass system and choose  $\eta$  and  $\mu$  in accordance with ref. 4, chapter 2-9. This choice is, of course, not unique (c.f. refs. 5,6 for alternative choices).

ii) MWE: It has been shown<sup>7,8)</sup> that the Schrödinger equation can be modified such that it retains its mathematical form but represents an appropriate 2nd-order reduction of the Dirac equation

for the upper (large) components of the Dirac spinor. The modification consists of a real mass- and energy-dependent factor applied to the total nuclear potential (which is still treated purely phenomenologically). Difficulties arise in this case with the proper choice of Coulomb functions (relativistic vs. non-relativistic) for the external wave functions. Clearly, it is necessary to guarantee that the solution of the wave equation in the absence of a nuclear potential is equal to the Coulomb functions specified in the exterior region. This can be achieved by a suitable redefinition of the Coulomb potential. Again, unambiguous modification of the scattering problem to include relativity is not possible.

Using the standard potential  $\{P_0\}$ , calculations

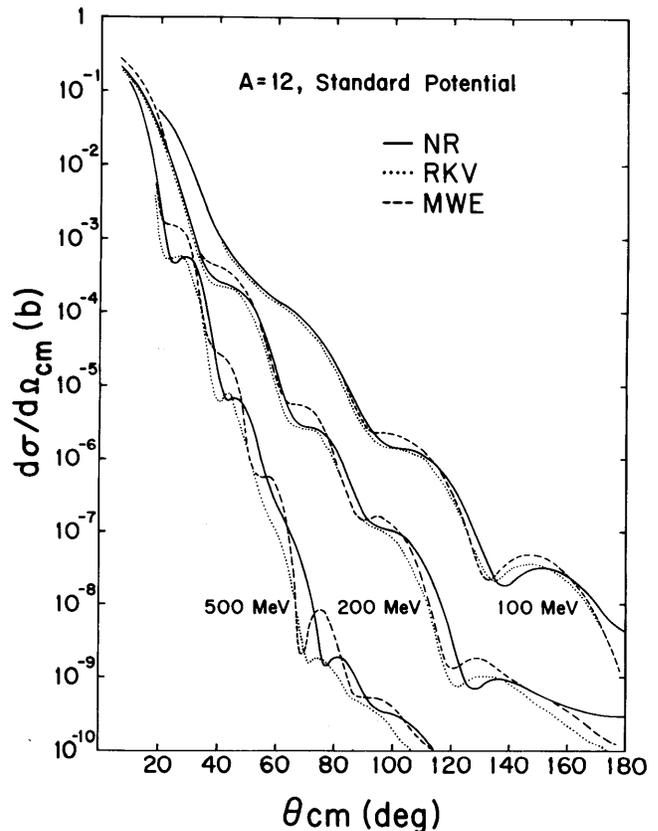


Figure 1. Calculated proton elastic scattering differential cross sections at three bombarding energies, using the same potential, but different treatment of relativistic effects.

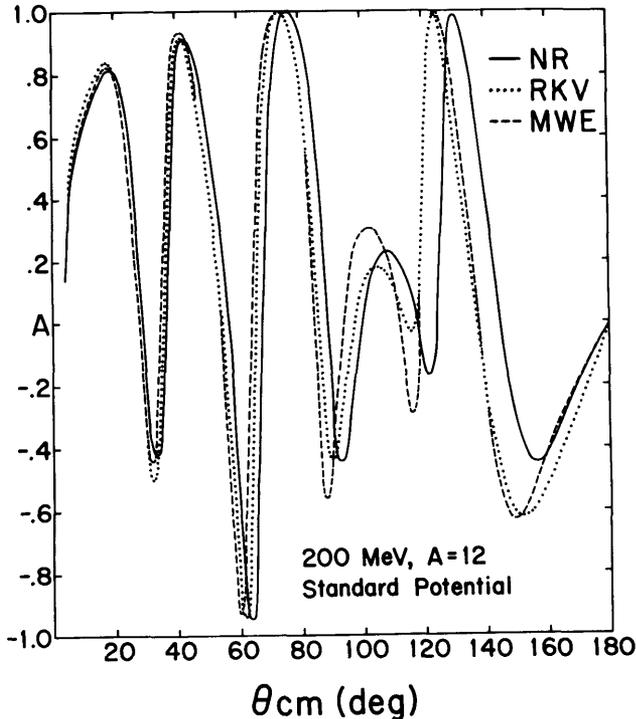


Figure 2. Different treatment of relativistic effects in the calculation of the analyzing power in proton elastic scattering at 200 MeV.

have been carried out for bombarding energies between 50 and 500 MeV, either in the conventional non-relativistic framework (NR) or using one of the relativistic options (RKV or MWE). Some results are shown in Figs. 1 and 2.

We draw the following conclusions from this investigation:

Conclusion 1. Even at energies as low as 100 MeV sizeable relativistic effects are present for  $\theta > 100^\circ$ . As the bombarding energy is increased, they do not become larger, but the angle of onset is shifted forward.

Conclusion 2. Two a priori equally acceptable methods to take into account relativistic effects yield quantitatively very different results. Relativistic corrections to the optical model as commonly used must therefore be viewed with reservation.

To study the impact of relativistic corrections on the potential itself, a search was made for an equivalent potential  $\{P_{equiv}\}$  which - if used in a non-relativistic (NR) calculation - would yield the same observables as the  $\{P_0\}$  potential using the MWE option.

Conclusion 3: Given the standard potential  $\{P_0\}$  in a MWE framework, there exists an equivalent potential  $\{P_{equiv}\}$  which in the NR mode reproduces the observables for  $5^\circ < \theta < 175^\circ$  within numerical accuracy (tested for  $T_p < 500$  MeV). Thus, the omission of relativistic effects cannot be the reason for failure to fit a given set of data.

Conclusion 4: Since both  $\{P_0\}$  and  $\{P_{equiv}\}$  have simple Woods-Saxon (WS) forms, the presence of relativistic effects does not fake non-WS form factors for the potentials.

The search for  $\{P_{equiv}\}$  has been carried out for  $p+^{12}C$  at a number of energies between 50 and 500 MeV. Generally, all of the 9 potential parameters had to be varied. Their final values were found to depend smoothly on bombarding energy. As an example, the fractional change in the rms radius  $R$  of the real central potential,  $\Delta R/R$ , depends linearly on  $\beta^{3.28}$  ( $\beta = v/c$ ). Thus, at 500 MeV, the omission of relativistic effects would lead to a real central potential whose rms radius is too large by  $\sim 0.4$  fm (to be compared with  $R=3.0$  fm of the standard potential).

Conclusion 5: Aside from the validity of any microscopic optical potential model and other caveats, the deduction of believable matter rms radii from intermediate-energy proton scattering depends critically on the treatment of relativistic aspects of the wave equation.

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ENERGY-DEPENDENT FORM FACTORS OF THE OPTICAL POTENTIAL FOR INTERMEDIATE-ENERGY PROTONS FROM  
A RELATIVISTIC MODEL

P. Schwandt  
Indiana University Cyclotron Facility, Bloomington, Indiana 47405

L.G. Arnold and B.C. Clark  
The Ohio State University, Columbus, Ohio 43210

R.L. Mercer  
Watson Research Center, IBM Corp., Yorktown Heights, New York 10598

Realistic Brueckner-Hartree-Fock (BHF) calculations<sup>1,2,3</sup> of the optical potential in nuclear matter lead to very different energy-dependences of the real potential strength for different values of the matter density. When applied to finite nuclei in a local density approximation (LDA), this results in potential radial shapes which in the 150-400 MeV "transition" region of nucleon kinetic energy differ drastically from the radial shape of the nuclear matter density distribution. Specifically, in the (normal-density) interior of the nucleus the potential changes from attractive to repulsive at a much lower energy (200-250 MeV) than in the (low-density) nuclear surface where the potential remains weakly attractive even at several hundred MeV. A characteristic "wine-bottle-bottom" shape for the real central potential in the transition energy region is thus a natural result of the BHF approximation.

Strikingly similar energy-dependent potential shapes are obtained in an entirely different approach to the microscopic formulation of the optical potential, namely the relativistic Dirac-equation model (or Dirac-Hartree model) based on essentially static,

heavy-meson exchange nucleon-nucleon (NN) interactions<sup>4,5</sup>). The relativistic treatment of the many-body system in its simplest form leads to an average, local one-body potential which is the sum of a Lorentz scalar field,  $U_S$ , and the 4th (time-like) component,  $U_0$ , of a Lorentz vector field. The Dirac equation for the one-body wavefunction with these potentials is written as ( $\hbar = c = 1$ ):

$$\{\alpha \cdot \vec{p} + \beta [m + U_S(r)] + [U_0(r) + V_C(r)]\} \psi_D(\vec{r}) = E \psi_D(\vec{r})$$

where  $V_C(r)$  is the Coulomb field,  $m$  the nucleon mass and  $E$  the nucleon total energy in the c.m. frame. For positive-energy (scattering) states the Dirac equation potentials  $U_S$ ,  $U_0$  are taken to be complex:

$$U_S(r) = V_S f_S(r) + iW_S g_S(r),$$

$$U_0(r) = V_0 f_0(r) + iW_0 g_0(r).$$

One expects  $V_S$  to be attractive since its origin is in neutral scalar meson exchange, and  $V_0$  to be repulsive since it is associated with the exchange of neutral vector mesons. The form factors  $f_S(r)$  and  $f_0(r)$  for the real parts of  $U_S$  and  $U_0$  are identified with effective scalar and vector target density distribu-