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A NOTE ON FREGE'S SEMANTICS

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The Fregean theory of syntax says what the meaningful parts of sentences are, and which combinations of those parts are meaningful. The Fregean theory of meaning says how the meaningful parts of a meaningful expression contribute to that expression's sense and reference. The theory of syntax discerns two basic kinds of meaningful parts of sentences: proper names and function-names.¹ Each function-name has a type determined by the number of argument places it has and the type of expression appropriate to each argument place (*BL*, § 23). Every complex proper name is the result of completing a function name with expressions of the appropriate types (and conversely).

The Fregean theory of meaning assigns objects to proper names as senses and references, and functions to function-names. While proper names and objects are 'complete' or 'saturated', function-names and functions are 'incomplete' or 'unsaturated'.² The reference of a complex expression is the value of the function which is the reference of the expression's main function-name when it takes as arguments the references of the expressions which fill the main function-name's argument places (*BL*, p. 34). Analogously, the sense of a complex expression is a function of the senses of its parts.

Sentences, says Frege, stand for (have as references) truth values (*BL*, § 2). Truth functional sentential connectives stand for functions from truth values to truth values (*BL*, §§ 6, 12, 31). And quantifiers stand for functions from first-level functions of one argument to truth values (*BL*, §§ 8, 31). But Frege overlooks the fact that quantifiers and connectives play a dual role which is ignored by his theories.³ '&' is apparently truth functional when standing between sentences, but is not when contributing to the references of larger quantificational contexts such as ' $\forall x(Fx \& Gx)$ ': here there simply are no truth values to be taken as arguments, no sentences to fill argument places. Similarly, quantifiers are well described as

they attach to unary predicates, yet in ‘ $\forall x \exists y Rxy$ ’ Frege must be at a loss to explain the referential contributions of the existential quantifier since it is not followed by a first-level function-name of *one argument place*.⁴

What one is inclined to say on Frege’s behalf here – indeed the *only* obvious thing that could be said for Frege – is that in these more complex cases connectives stand for functions from first-level functions to first-level functions, and, analogously, quantifiers stand for functions from first-level functions of n arguments to first level functions of $n-1$ arguments. But there are two important difficulties with this.

The first and perhaps least serious problem is that for Frege functions have only objects for values (*BL*, § 31). It is easy to see why this is so. If some functions had functions for values, then the filling of argument places of some function-names with appropriate expressions would produce a function-name, not a proper name as required by the theory of syntax. And attempts to specify the values of such functions for specific arguments will bog down in troubles isomorphic to Frege’s celebrated troubles over ‘the concept *horse*’. For our equation will look something like:

$$(1) \quad F(f(\xi)) = g(\xi).$$

This, though, could not be a true sentence – indeed it is not even a proper name – since it contains gaps, is ‘incomplete’.⁵ These troubles, I think, can be solved by seeing identity here as a full-blooded variable binding second-level function.⁶ Thus instead of (1) our picture should be more like

$$F(f(x)) =_x g(x);$$

and, given hints from Frege, this may be equated with

$$\forall x [F(f(x)) = g(x)].$$

Still, of course, the theory of syntax must be revised so as not to require the ‘completion’ of every function-name to be a proper name.

The second and graver difficulty is that if Frege is to maintain his distinctions of level he will now require infinitely many replacements for a single connective or quantifier. Thus ‘&’ will have different types as it takes function-names of different types. Since function-names of different numbers of argument places have different types, and since the number of argument places a function-name can have is in principle unlimited, ‘&’

will have to assume an infinite number of types. '&'s type differs, for instance, in each of the following: ' $p&q$ ', ' $p&Fx$ ', ' $Fx&Gy$ ', ' $Fx \& Hyz$ ', ' $Hxy \& Rzw$ ',... . Since this is not possible for Frege – each expression for him has a unique type – there must be an endless number of surrogates for '&'. This is a serious problem because Frege is now forced to explain the structure of language by appeal to an infinite number of semantic primitives. But it seems very doubtful that any human could master such a language, and thus doubtful that the theory provides an acceptable explanation of the structure of human languages.⁷ The value of Frege's theories as an account of natural language thus seems thoroughly dissipated. This is apparently a major problem for any theory of language which both maintains some sort of hierarchy of levels and construes connectives and quantifiers as categorematic, as contributing to meaning and reference, that is, by virtue of meant or denoted entities. It is consequently appears that it is a major virtue of Tarski's account of truth that it does not attribute denoted entities to logical vocabulary.

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¹ The theory of syntax is sketched in the early portion of *The Basic Laws of Arithmetic* (University of California Press, Berkeley, 1964 (ed. and transl. by Montgomery Furth)) (hereafter cited as *BL*), and is elaborated in my 'Frege's Problem's With 'The Concept Horse'', *Critica*, forthcoming, §I (hereafter cited as 'Frege's Problems').

² Cf. *BL*, §1. Compare *The Collected Papers of Charles Sanders Peirce* (Harvard University Press, Cambridge, 1931–1958 (ed. by Charles Hartshorne, Paul Weiss, and Arthur Burks)), Vol. 3, paragraphs 421 and 469.

³ As apparently does Michael Dummett, 'Frege, Gottlob', *Encyclopedia of Philosophy* Macmillan, New York, 1967 (ed. by Paul Edwards), Vol. 2, p. 228.

⁴ Similar remarks apply to the horizontal (*BL*, §§5f), and identity (*BL*, §7); and there is a parallel difficulty in the theory of contribution of senses. It goes without saying that Frege countenanced these larger contexts; cf. *BL*, pp. 39, 55, 72, 77ff. In a Tarskian theory of truth the two roles get double treatment, once in terms of satisfaction and once in terms of truth; cf. Definitions 22 and 23 of 'The Concept of Truth in Formalized Languages', *Logic, Semantics, Metamathematics* Oxford University Press, Oxford, 1956 (transl. by J. H. Woodger)).

⁵ For a fuller account of these troubles see 'Frege's Problems', §III.

⁶ Cf. 'Frege's Problems', §IV.

⁷ Cf. Donald Davidson, 'Theories of Meaning and Learnable Languages', *Logic, Methodology, Philosophy of Science* North-Holland, Amsterdam, 1964 (ed. by Y. Bar-Hillel).