

***CPT*, strings, and meson factories**

V. Alan Kostelecký

Physics Department, Indiana University, Bloomington, Indiana 47405

Robertus Potting

U.C.E.H., Universidade do Algarve, Campus de Gambelas, 8000 Faro, Portugal

(Received 17 May 1994)

Spontaneous breaking of *CPT* is possible in string theory. We show that it can arise at a level within reach of experiments at meson factories currently being built or designed. For ϕ , B , and τ -charm factories, we discuss the likely experimental string signatures and provide estimates of the bounds that might be attained in these machines.

PACS number(s): 11.30.Er, 11.25.-w, 13.20.Gd

I. INTRODUCTION

To be generally accepted, any new theory purporting to describe successfully a physical system must ultimately satisfy two criteria. First, it must be viable, in the sense that it must provide a physically accurate model for observables while maintaining internal consistency. Second, it must be falsifiable; i.e., it must predict new effects that can be tested experimentally.

String theory provides an ambitious framework within which to seek a consistent quantum theory of gravity incorporating all fundamental interactions. The issue of the viability of string theories has been the subject of much research and appears promising. However, progress on the issue of falsifiability has been slower, since it entails addressing the difficult problem of identifying a potentially observable effect in string theory that cannot occur in a particle theory.

Although such stringy effects are likely to exist because strings are qualitatively different from particles, the search for an experimentally testable stringy effect meets several obstacles. One problem arises from the absence of a satisfactory and completely realistic string theory. This can be partially avoided by examining features typical of string theories and by looking for ways that stringy effects *could* appear in present experiments, without requiring that they *must* occur. However, even this weakened notion of falsifiability faces difficulties because any stringy effects are likely to be highly suppressed at present energies. The Planck scale M_{Pl} is the natural string scale, while present experiments allow access only to energies of order of the electroweak scale m_{ew} . Stringy effects are therefore likely to be suppressed by some power of the ratio $m_{\text{ew}}/M_{\text{Pl}} \simeq 10^{-17}$, making them difficult to observe. For most purposes, at present energies the string appears pointlike and so presumably is well described in terms of a four-dimensional renormalizable gauge theory.

One method of minimizing the impact of the suppression is to seek string properties that violate an exact

symmetry of particle theory, preferably one that can be measured to a high degree of precision. One candidate symmetry of this type is the discrete symmetry *CPT*. Although *CPT* is known to be a symmetry of local relativistic point-particle field theory under a few apparently mild assumptions [1–7], the extended nature of strings means that *CPT* invariance is not apparent *a priori* in the string context. In addition, *CPT* is indeed accessible to high-precision tests. Presently, the best experimental bound on *CPT* violation is obtained from observations of the $K\bar{K}$ system [8,9], where one figure of merit is

$$\frac{m_K - m_{\bar{K}}}{m_K} \leq 5 \times 10^{-18}. \quad (1)$$

Together, these features of *CPT* suggest it could provide a signature for string theory.

A mechanism by which certain string theories could spontaneously break *CPT* invariance is known [10–12]. A summary of the mechanism is presented in Sec. II. The main goal of this paper is to address the issue of how stringy *CPT* breaking might manifest itself at present energies and how best to attempt to observe it experimentally. In Sec. III, we consider possible effects of the *CPT* breaking in a generic neutral meson-antimeson system, denoted $P\bar{P}$. The possibility of using various meson factories to measure *CPT*-violation parameters is discussed in Sec. IV. We provide some estimates on the limits that could be attained in machines currently being built or considered, including ϕ , B , and τ -charm factories. Section IV A presents features generic to any $P\bar{P}$ system, while Secs. IV B through IV D discuss aspects specific to the various mesons. We summarize in Sec. V. Some details of our error analysis are relegated to an Appendix.

Throughout, we use the symbol P to denote any of the relevant neutral mesons. Our notation is based on that of the classic work on the $K\bar{K}$ system by Lee and Wu, Ref. [13]. We have, however, adopted slightly different phase conventions for the *CP* properties of the states $|P\rangle$; see Sec. III.

II. BACKGROUND: CPT AND STRINGS

The possibility that certain string theories could spontaneously break CPT invariance, at a level that might be observable in the $K\bar{K}$ system, was suggested in Ref. [10] on the basis of an investigation of CPT in string field theory. To make this paper more self-contained, this section summarizes some of the earlier analysis; the reader desiring to skip the preliminaries can proceed directly to Sec. III. In what follows, we work at the level of field theory so that off-shell properties are correctly incorporated.

The issue of the CPT properties of any given particle or string theory can be addressed via several approaches, at least in principle. A highly sophisticated approach for particle field theories is provided by the axiomatic method. At present, however, this method is unavailable for strings because appropriate axioms have not yet been formulated. Another general approach is the constructive model, in which particle field theories are viewed as built from products of anticommuting irreducible spinors, with overall CPT properties inferred from those of the underlying spinors. This method, too, is not presently practical for strings. Both these approaches remain open areas for future investigation.

An approach that can be applied to individual particle and string theories exists [14]. The method begins by establishing suitable C , P , and T transformations for the free quantum theory and proceeds by applying these to the interacting fields by expanding them in a Dyson series, thereby establishing the CPT properties of the full theory. Application of this method to string theory requires some care; for example, string interactions modify canonical quantization so the compatibility of the initial free-field C , P , and T transformations must be verified.

In Ref. [10], this method was applied to the field theories for the open bosonic string [15] and the open superstring [16,17]. Some standard assumptions were used including, for example, the connection between spin and statistics. It was shown that the bosonic-string action is C , P , T , and CPT invariant, and that previously proposed superstring actions violate C , P , and T but preserve CPT . Whereas the superstring P and T violation results from the appearance of massless chiral fermions in ten dimensions, the C violation was unexpected. However, Ref. [10] presents a modified superstring action maintaining both C and CPT . The analyses suggest that, despite the extended nature of the string and the corresponding infinite number of particle fields, string and particle theories are sufficiently alike that CPT is preserved at the level of the action.

In addition to the CPT properties of the action, the CPT behavior of the ground state must also be considered. Noninvariance of the vacuum then corresponds to spontaneous CPT violation. While this is unnatural in conventional particle models, the situation is less obvious in string theory, which has an off-shell description naturally lying in higher dimensions. If strings are to describe the real world, the corresponding higher-dimensional Lorentz invariance *must* be broken, to at least an approximate four-dimensional one. Moreover, the mechanism for breaking Lorentz invariance should be

implicit in the action for the higher-dimensional string.

A natural mechanism of this type does exist in string field theory [18]. The mechanism also breaks the higher-dimensional CPT invariance. It then becomes an open question as to whether four-dimensional CPT is preserved. Consider, for example, the interaction Lagrangian of the field theory for the open bosonic string. This contains a term $\mathcal{L}_1 = \phi A_\mu A^\mu$, which controls the coupling between the tachyon ϕ and the massless vector A_μ . However, the tachyon vacuum expectation value $\langle\phi\rangle$ is driven away from the origin since the effective potential has a local maximum there. For the appropriate sign of $\langle\phi\rangle$ a similar instability is generated in the effective potential for A_μ , and any resulting $\langle A_\mu\rangle \neq 0$ spontaneously breaks the 26-dimensional Lorentz symmetry. Since A_μ changes sign under CPT , CPT -violating terms appear in the action. The mechanism is stringy in the sense that particle gauge invariance excludes terms of the form \mathcal{L}_1 in a standard four-dimensional gauge field theory, whereas they are compatible with string gauge invariance. Moreover, the mechanism can involve other Lorentz scalars S and tensors T because string field theories typically have cubic interactions of the general form $ST \cdot T$.

Since no zeroth-order CPT violation appears in laboratory experiments, any observable effects must be suppressed, perhaps because higher-level fields of Planck-scale mass are involved. The ratio $m_l/M_{\text{Pl}} \lesssim 10^{-17}$ of the low-energy scale m_l to the Planck scale M_{Pl} provides the natural dimensionless quantity governing the suppression. Observable effects could arise in a variety of $P\bar{P}$ systems, as we discuss below. For instance, in the $K\bar{K}$ system one might expect [10,19,20]

$$\frac{m_K - m_{\bar{K}}}{m_K} \sim \frac{m_l}{M_{\text{Pl}}}, \quad (2)$$

which includes a range just below the present bound, Eq. (1).

We remark here that the ratio m_l/M_{Pl} might be expected to arise in the context of any gravitational mechanism that violates CPT . Even in canonical general relativity the precepts of the usual CPT theorem may not hold [21–23] so CPT violation at observable levels might occur [24,25]. However, the stringy spontaneous CPT violation we discuss here arises from a specific mechanism and has a particular characteristic signature (see below). A recent discussion of spontaneous CPT violation for strings in the gravitational context is given in Ref. [26].

III. STRINGY CPT VIOLATION IN A $P\bar{P}$ SYSTEM

To make contact with experiment, we must first connect the potential spontaneous CPT violation in a string theory to terms in a four-dimensional low-energy effective action and then determine the effect on measurable quantities [19,20]. In the absence of a specific, satisfactory, and completely realistic string field theory, it is necessary instead to work within a generic theoretical framework. In this spirit, the present section considers a class of possible terms and investigates their implications.

We assume that the four-dimensional effective theory arises from a string theory compactified at the Planck scale,¹ leading among other terms to four-dimensional effective interactions of the schematic form

$$\mathcal{L}_I \supset \frac{\lambda}{M_{\text{Pl}}^k} T \cdot \bar{\psi} \Gamma (i\partial)^k \chi + \text{H.c.} \quad (3)$$

For notational simplicity, the Lorentz indices in this equation are suppressed. The field T is a four-dimensional Lorentz tensor, while ψ and χ are four-dimensional fermions, possibly identical. The symbol Γ is used to denote a gamma-matrix structure, while $(i\partial)^k$ represents a k th-order derivative coupling with the four-dimensional derivative ∂_μ . By definition, $k \geq 0$. Each term in the interaction Lagrangian of the string field theory is a Lorentz scalar, so $T \cdot \Gamma (i\partial)^k$ represents a spinor matrix with derivative entries. The coupling λ is dimensionless. The inverse of the k th power of the Planck scale arises from derivative couplings in the string field theory and the compactification process and ensures the correct overall dimensionality.

Following the mechanism described in Sec. II, we suppose that the tensor T acquires an expectation value $\langle T \rangle$. The interaction terms in Eq. (3) then generate terms in the Lagrangian quadratic in the fermion fields and hence produce a tree-level contribution $\Delta K(p)$ to the fermion inverse propagator $K(p)$. Suppressing the Lorentz indices on the momentum factors, $\Delta K(p)$ is given by

$$\Delta K(p) = \frac{\lambda}{M_{\text{Pl}}^k} \langle T \rangle \cdot \Gamma p^k. \quad (4)$$

To incorporate a measure of the possible suppression, we write the expectation $\langle T \rangle$ as

$$\langle T \rangle = t \left(\frac{m_l}{M_{\text{Pl}}} \right)^l M_{\text{Pl}}. \quad (5)$$

Here, t is a numerical factor incorporating the Lorentz structure of T . It is tempting theoretically to assume numerical values of order 1 for the nonzero components of the product λt , for all P . However, this assumption presumably would hold at best only at the scale at which Eq. (3) becomes applicable, below which running would occur. Moreover, it would appear that any assumption of this type should be treated with caution in view of the observed differences in the scales of the Yukawa couplings in the standard model. These are also trilinear boson-fermion-fermion couplings, which range from about 10^{-5} for the up quark to about 10^{-1} for the bottom quark.

The ratio $(m_l/M_{\text{Pl}})^l$ in Eq. (5) allows for possible suppression by a power of the low-energy scale to the Planck scale. By assumption, $l \geq 0$. Since we are in-

terested in the situation where T has nontrivial Lorentz structure and incorporates CPT breaking, it appears on experimental grounds that a realistic model must have $\langle T \rangle \ll m_l$, i.e., $l \geq 2$. A theoretical argument deducing that this *must* occur would require the solution to a hierarchy problem that evidently lies outside the scope of the present work. Its resolution may be related to that of the usual hierarchy problem since the same scales appear. Indeed, we are using the experimental fact of its existence to motivate the introduction of the small scale m_l in Eq. (5). In any case, an understanding of the mechanism responsible for the appearance of disparate scales in nature presumably awaits the formulation and study of a satisfactory, realistic string (field) theory.

For a fermion of mass m_f , the order of magnitude of the change in the fermion inverse propagator relative to its size is therefore

$$\frac{\Delta K}{K} \sim \left(\frac{p}{M_{\text{Pl}}} \right)^k \left(\frac{m_l}{M_{\text{Pl}}} \right)^{l-1} \left(\frac{m_l}{m_f} \right). \quad (6)$$

For purposes of comparison with experiment, we assume the fermions to be observable so that $m_f \sim m_l$ and $p \ll M_{\text{Pl}}$. Since no zeroth-order CPT violation has been detected in laboratory experiments, it follows that $k+l > 1$. Any associated Lorentz breaking is then small and has no effects other than the CPT -related ones discussed below. With the above assumptions, the question of the experimental detection of lowest-order stringy spontaneous CPT violation reduces to determining the observable consequences of terms with $k=0$ and $l=2$.

It can be seen from Eq. (5) that terms of this type produce effects suppressed by about 17 orders of magnitude, so their direct experimental detection is difficult. Instead, we consider interferometric experiments, which are the ones of choice for high-precision purposes. In this paper, we concentrate on the interferometric experiments that can be developed for the $P\bar{P}$ systems, where we use P generically to denote one of the neutral mesons. The fermions ψ and χ of Eq. (3) can be taken as one or more component quark fields of the meson P . Since we are interested in the dominant contributions from the effective interactions in Eq. (3), where needed in what follows we work in lowest-order perturbation theory. For example, this implies the replacement of $\Delta K(p)$ with its amplitude in the quark wave functions. It also means that terms involving both flavor changes and CPT violation can be neglected.

To make further progress, we must consider effects on the $P\bar{P}$ system of the changes ΔK in the matrices of the quark inverse propagators. In principle, there are many different linearly independent ways for CPT to be violated by each term of the form (3), one for each possible CPT -violating expectation of the tensor T . However, the $P\bar{P}$ system acts as an *energy* (mass) interferometer (as can be seen from the effective-Hamiltonian formalism below), so the only experimentally detectable effect involves an energy shift. For example, an interaction of the form $\lambda_q T_{\lambda\mu\nu} \bar{q} \gamma^\lambda \gamma^\mu \gamma^\nu q$ together with an expectation value $\langle T_{000} \rangle = t_{000} (m_l/M_{\text{Pl}})^2 M_{\text{Pl}}$ provides a direct energy shift $E \rightarrow E + \lambda_q m_l^2 t_{000}/M_{\text{Pl}}$ in the q -quark inverse propagator $E\gamma^0 - \vec{p} \cdot \vec{\gamma} + \dots$. Indirect energy shifts are

¹String theories partially compactified at larger length scales can produce other string signatures; see Refs. [27,28] and references therein.

also possible. For example, any term leading to a three-momentum shift also changes the energy through the dispersion relation. The combination of such shifts may lead to (partial) directional dependence of the energy effects. For simplicity, we disregard any directional effects in the present work, and we focus on the experimental detection of the possible energy shifts.

The energy shifts in the quark propagators induce corresponding shifts in the mass and decay matrices for the $P\bar{P}$ system. Before proceeding further, we first introduce the notation and conventions we use for these matrices, allowing for CPT violation. Define the eigenstates $|P^0\rangle$ and $|\bar{P}^0\rangle$ of the T - and CPT -invariant part of the full effective four-dimensional action by

$$CP|P^0\rangle = |\bar{P}^0\rangle, \quad CP|\bar{P}^0\rangle = |P^0\rangle. \quad (7)$$

In the $|P^0\rangle$ - $|\bar{P}^0\rangle$ space, the time evolution of a linear superposition of $|P^0\rangle$ and $|\bar{P}^0\rangle$ is governed by 2×2 effective Hamiltonian Λ , which can be written in terms of two Hermitian operators M and Γ called the mass and decay matrices, respectively [13]:

$$\Lambda = M - \frac{1}{2}i\Gamma \equiv \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{pmatrix}. \quad (8)$$

The matrix elements can be determined in a degenerate perturbation series using the T - and CPT -violating terms in the full action as the perturbation. If CPT invariance holds, $\Lambda_{11} = \Lambda_{22}$. In what follows, it is useful to introduce the parametrization

$$i\Lambda = \frac{1}{2}\Gamma + iM \equiv \begin{pmatrix} D + iE_3 & iE_1 + E_2 \\ iE_1 - E_2 & D - iE_3 \end{pmatrix}, \quad (9)$$

and, for convenience, the quantity

$$E^2 = E_1^2 + E_2^2 + E_3^2. \quad (10)$$

In the above expressions, a subscript P distinguishing analogous quantities in distinct $P\bar{P}$ systems is understood but omitted for notational simplicity.

The physical particles² P_S and P_L are represented by the two eigenvectors $|P_S\rangle$ and $|P_L\rangle$ of the matrix Λ . The corresponding eigenvalues λ_S and λ_L are combinations of the masses and lifetimes of the physical particles:

$$\lambda_S = m_S - \frac{1}{2}i\gamma_S, \quad \lambda_L = m_L - \frac{1}{2}i\gamma_L. \quad (11)$$

Note that a subscript P on each of these quantities is again understood.

In terms of the parametrization of Eq. (9), we find

$$i\lambda_S = D + iE, \quad |P_S\rangle = \frac{(1 + \epsilon_P + \delta_P)|P^0\rangle + (1 - \epsilon_P - \delta_P)|\bar{P}^0\rangle}{\sqrt{2(1 + |\epsilon_P + \delta_P|^2)}}, \quad (12)$$

$$i\lambda_L = D - iE, \quad |P_L\rangle = \frac{(1 + \epsilon_P - \delta_P)|P^0\rangle - (1 - \epsilon_P + \delta_P)|\bar{P}^0\rangle}{\sqrt{2(1 + |\epsilon_P - \delta_P|^2)}}.$$

Here, the parameter

$$\epsilon_P = \frac{-iE_2}{E_1 + E} \quad (13)$$

is a measure of T violation and

$$\delta_P = \frac{E_3}{E_1 + E} \quad (14)$$

is a measure of CPT violation. We have explicitly included the subscript P on the parameters ϵ_P and δ_P to avoid confusion in what follows. Also, it follows that, for each P ,

$$E = -\frac{1}{2}\Delta m - \frac{1}{4}i\Delta\gamma \\ = -\frac{1}{2}ia \exp(-i\hat{\phi}), \quad (15)$$

with³

$$\Delta m = m_L - m_S, \quad \Delta\gamma = \gamma_S - \gamma_L, \quad (16)$$

$$a = (\Delta m^2 + \frac{1}{4}\Delta\gamma^2)^{1/2}, \quad \hat{\phi} = \arctan \frac{2\Delta m}{\Delta\gamma}.$$

For the purposes of Sec. IV, it is also convenient to define

$$m = m_S + m_L, \quad \gamma = \gamma_S + \gamma_L, \quad (17)$$

$$b = (\Delta m^2 + \frac{1}{4}\gamma^2)^{1/2}, \quad \bar{\phi} = \arctan \frac{2\Delta m}{\gamma}.$$

The parameters of interest for CPT studies are the δ_P , introduced in Eq. (14) above. Combining this equation with Eqs. (4) and (5) and the discussion in the paragraphs following Eq. (6) permits the various δ_P to be expressed in terms of quantities appearing in the effective low-energy action (3) and the experimentally accessible parameters Δm , $\Delta\gamma$, and $\hat{\phi}$. We next proceed to show this.

²The subscripts are defined in analogy with the CP properties of the kaon system, where they refer to the particles with “short” and “long” lifetimes. Note, however, that the particle P_S is not necessarily the shorter-lived state in general.

³Note that in general Δm and $\Delta\gamma$ are not necessarily positive quantities, since the correspondence between CP eigenstates and the relative masses and lifetimes of the physical particles may not be the same as that in the neutral kaon system from which the notation is abstracted. The sign changes that ensue do not affect the results that follow.

Since the CPT -violating term is so small, it is sufficient to work at lowest order in perturbation theory in the CPT -violating coupling λ of Eq. (3). As discussed above, the only observable effects arise as energy shifts in the quark inverse propagators. At this level in perturbation, terms of the form (3) then contribute only energy insertions on quark lines in the P and \bar{P} mesons. Moreover, the CPT violation means that quark and antiquark insertions of given flavor have the same magnitude but opposite sign. For instance, in the example considered above with interaction term $\lambda_q T_{\lambda\mu\nu} \bar{q} \gamma^\lambda \gamma^\mu \gamma^\nu q$ and expectation value $\langle T_{000} \rangle = t_{000} (m_l/M_{P1})^2 M_{P1}$, any q -quark line in the meson would receive a perturbative correction with Feynman weight $\lambda_q m_l^2 t_{000}/M_{P1}$ while a \bar{q} -antiquark line acquires the same weight but with opposite sign.

The P and \bar{P} mesons are comprised of valence quarks lying in a sea of gluons, quarks, antiquarks, and other particles present in the effective action for the low-energy theory. Let us first disregard the presence of the sea; below we show that this is a good approximation within the level of accuracy needed for our purposes. Since the P and \bar{P} are antiparticles, each energy insertion on a valence-quark-antiquark line in P (or \bar{P}) has a corresponding contribution of equal magnitude but opposite sign in \bar{P} (or P). The contributions from the valence-quark energy insertions evidently provide equal-magnitude but opposite-sign contributions to the two diagonal elements of the effective Hamiltonian Λ , given at lowest order in the CPT -violating coupling λ by the expectation of $-\mathcal{L}_I$ in the P -meson wave function:

$$\Delta\Lambda_{11} = -\Delta\Lambda_{22} = -\langle P^0 | \mathcal{L}_I | P^0 \rangle. \quad (18)$$

Note that contributions to $\Delta\Lambda_{12}$ and $\Delta\Lambda_{21}$, present in principle, are suppressed because they involve either two CPT -violating flavor-changing insertions or higher-order weak effects along with a first-order CPT -violating flavor-neutral insertion as above.

In general, the contribution h_{q_j} to $\Delta\Lambda_{11}$ from the j th valence quark, $j = 1, 2$, is given in terms of the corresponding Feynman weight arising from Eqs. (4) and (5):

$$h_{q_j} = r_{q_j} \lambda_{q_j} t \frac{m_l^2}{M_{P1}}, \quad (19)$$

where the factors r_{q_j} , discussed below, have been included to compensate for the use of the valence-quark approximation. It follows that the shifts in the diagonal elements of the effective Hamiltonian are

$$\Delta\Lambda_{11} = -\Delta\Lambda_{22} = h_{q_1} - h_{q_2}. \quad (20)$$

Note that in writing this expression we have implicitly assumed that for each quark flavor only one term of the form in \mathcal{L}_I appears, or at least that one such term dominates. However, additional terms can readily be incorporated by summing over the different h_{q_j} contributions. Their presence would not affect the order-of-magnitude estimates and other results presented below.

One possible contribution to the factors r_{q_j} arises because energy insertions could be made on sea quarks. However, where the sea quarks occur as same-flavor

quark-antiquark pairs, the two possible first-order CPT -violating contributions cancel because they have opposite sign. The only sea-quark contributions therefore arise from flavor-changing interactions, which are suppressed. Whether or not there are CPT -violating terms, the presence of the quark-gluon sea itself accounts, for example, for the difference between the mass of a light meson (kaon) and the sum of the current masses of its quarks. For instance, in the example considered above with interaction term $\lambda_q T_{\lambda\mu\nu} \bar{q} \gamma^\lambda \gamma^\mu \gamma^\nu q$ and $\langle T_{000} \rangle \neq 0$, the expectation $-\langle P^0 | \mathcal{L}_I | P^0 \rangle$ reduces to the Feynman weight multiplying the meson wave function expectation of the q -quark number operator. Disregarding flavor-changing effects, the latter is just 1. In any event, although the factors r_{q_j} might be important for a detailed calculation, they play no significant role for the order-of-magnitude estimates below. In what follows, we therefore take these factors to be 1 for simplicity.

We can now combine the above results. Setting $E_3 = \Delta\Lambda_{11}$ and making for convenience the approximation $E_1 \approx E$, which is valid provided T and CPT violation are small (i.e., E_2 and E_3 small compared with E), we obtain the expression

$$\delta_P = i \frac{h_{q_1} - h_{q_2}}{\sqrt{\Delta m^2 + \Delta\gamma^2/4}} e^{i\hat{\phi}}. \quad (21)$$

This equation applies to any $P\bar{P}$ system for any CPT -violating term of the form \mathcal{L}_I in Eq. (3). We remind the reader that subscripts P are understood on all parameters on the right-hand side.

Establishing the experimental signature of stringy CPT violation is at this point reduced to determining the net contributions to δ_P from different interaction terms (3) via Eq. (21). An important aspect of this process in what follows is that the energy shifts in the effective Hamiltonian are real. This is true because the dominant effects from the CPT -violating terms arise as simple matrix elements of the interaction terms in Eq. (3), which are real since the fundamental string theory is Hermitian. Using Eq. (21), it follows that

$$\text{Im}\delta_P = \pm \cot\hat{\phi} \text{Re}\delta_P, \quad (22)$$

a result that we use in Sec. IV.

A range of plausible values of $|\delta_P|$ can be identified from Eq. (21). The key issues are the sizes of the light mass scale m_l and the factors $r_q \lambda_q t$ appearing in Eq. (19). The natural value of m_l lies between the mass m_P of the P meson and the electroweak scale m_{ew} . As discussed above, the values of λt could be taken of order 1, or, perhaps more plausibly, of order of the Yukawa coupling G_q for the corresponding quark.

To gain intuition, we have considered four scenarios, each leading to a characteristic value of $|\delta_P|$ as follows:

$$\text{Scenario 0: } \lambda t \sim 1, m_l \sim m_{\text{ew}}, |\delta_P| \approx \frac{km_{\text{ew}}^2}{aM_{P1}},$$

$$\text{Scenario 1: } \lambda t \sim G_{q_j}, m_l \sim m_{\text{ew}}, |\delta_P| \approx \frac{G_q m_{\text{ew}}^2}{aM_{P1}},$$

$$\begin{aligned} \text{Scenario 2 : } \lambda t \sim 1, \quad m_l \sim m_P, \quad |\delta_P| &\approx \frac{km_P^2}{aM_{P1}}, \\ \text{Scenario 3 : } \lambda t \sim G_{q_j}, \quad m_l \sim m_P, \quad |\delta_P| &\approx \frac{G_q m_P^2}{aM_{P1}}. \end{aligned} \quad (23)$$

The parameter k appearing in two of the scenarios arises from the combination of the two h_{q_j} factors in Eq. (21). Assuming no fine tuning, k is presumably of order 1. The factor G_q refers to the heavier of the two quarks in the P meson, which dominates the term $(h_{q_1} - h_{q_2})$ in Eq. (21). Scenarios 0 and 3 are of lesser interest because scenario 0 is probably experimentally excluded already while scenario 3 results in CPT violation too small to be detected in the foreseeable future. The feature of interest is that intermediate scenarios generate values of δ_P comparable to the range to be probed in the next generation of experiments. The intuition gained from these four candidate cases suggests that spontaneous CPT violation in a realistic string model might generate experimentally measurable effects.

So far, we have discussed only the effect of string-inspired spontaneous CPT violation in the P - \bar{P} effective Hamiltonian. *A priori*, one might expect additional CPT -violating effects to appear in ratios of decay amplitudes along with those appearing directly in the P mass matrix. However, in the string-inspired scenario this is not the case because there is no direct effect of the energy shifts on the matrix elements of the decays. Although contributions do appear from the interaction terms of Eq. (3) when ψ is q_1 and χ is q_2 or vice versa, these effects are unobservable. They can cause decays that are forbidden in the standard model, but with a suppression of at least two powers of m_l/M_{P1} . Alternatively, they can produce unobservable additional effects in electroweak-type flavor-changing processes. In principle, an indirect effect could also appear via energy-dependent normalizations of the eigenstates, but it too is unobservable because it is suppressed by at least one power of the ratio m_l/M_{P1} .

IV. STRINGY CPT VIOLATION IN MESON FACTORIES

We have seen in the previous section that stringy CPT violation can generate nonzero values of the parameters δ_P . In the present section, we consider the detection of such effects. There are two issues involved for each P : the expected signature of δ_P , and the procedure for observing it. Relatively good bounds on δ_P can be obtained in appropriate meson factories, designed to generate high fluxes of correlated P_S - P_L pairs. In Sec. IV A, we discuss the general framework of P production in meson factories and present some formulas of use in later sections. Section IV B, IV C, and IV D consider in turn the cases of $P \equiv K$, B , and D mesons, corresponding to ϕ , B , and τ -charm factories, respectively. For each case, specifics of the signature of stringy CPT violation are discussed

and an analytical estimate is provided relating the number of mesons produced and the precision with which δ_P is measured. For these estimates, we have exploited the elegant methods used by Dunietz, Hauser, and Rosner [29] for their analysis of the K system in the context of a ϕ factory.

A. General framework

The general idea behind a meson factory designed to study properties of a P meson is to generate large numbers of a quarkonium state lying just above the threshold for P^0 - \bar{P}^0 production. A state of this type strongly decays with a relatively large branching ratio into correlated P^0 - \bar{P}^0 pairs. Since e^+e^- machines generate virtual photons, it is relatively easy to produce quarkonia with $J^{PC} = 1^{--}$. For example, for the case $P \equiv K$ the appropriate quarkonium is the ϕ meson, while for $P \equiv B_d$ it is $\Upsilon(4S)$, for $P \equiv B_s$ it is $\Upsilon(5S)$, and for $P \equiv D$ it is $\psi(3770)$.

Since C and P are conserved in strong interactions, immediately after the strong decay of the quarkonium the initial PP state $|i\rangle$ has $J^{PC} = 1^{--}$. For simplicity in what follows, we work in the rest frame of the decay and we choose the z axis as the direction of the momenta of the two P mesons. Denoting the mass eigenstates by $|P_S(\pm\hat{z})\rangle$ and $|P_L(\pm\hat{z})\rangle$, where $(+\hat{z})$ means the particle is moving in the positive z direction and $(-\hat{z})$ means it is moving in the negative z direction, the normalized initial state $|i\rangle$ is given by [30]

$$|i\rangle = N[|P_S(\hat{z})P_L(-\hat{z})\rangle - |P_L(\hat{z})P_S(-\hat{z})\rangle], \quad (24)$$

where the normalization constant is $N^{-1} = \sqrt{2}|1 - \epsilon_P^2 + \delta_P^2|$. The subsequent time evolution of the two P mesons is governed by the effective Hamiltonian Λ of Sec. III:

$$|P_S(t)\rangle = e^{-im_S t - \gamma_S t/2} |P_S\rangle, \quad (25)$$

$$|P_L(t)\rangle = e^{-im_L t - \gamma_L t/2} |P_L\rangle,$$

which follows from Eq. (11).

Eventually, the two P mesons decay. Let the meson moving in the positive z direction decay into the final state $|f_1\rangle$ at time t_1 while the other decays into $|f_2\rangle$ at t_2 . We measure the times t_α , $\alpha = 1, 2$, in the rest frame of the quarkonium decay, so they are given by the proper time of the P meson divided by the appropriate Lorentz γ factor. In what follows, it is convenient to define for each α the complex parameter

$$\eta_\alpha \equiv |\eta_\alpha| e^{i\phi_\alpha} = a_{\alpha L}/a_{\alpha S}, \quad (26)$$

as the ratio of the amplitude $a_{\alpha L} = \langle f_\alpha | T | P_L \rangle$ for the transition between f_α and P_L to the amplitude $a_{\alpha S} = \langle f_\alpha | T | P_S \rangle$ for the transition between f_α and P_S . Then, the amplitude $\mathcal{A}_{12}(t_1, t_2)$ for the decay is

$$\mathcal{A}_{12}(t_1, t_2) = N a_{1S} a_{2S} \left\{ \eta_2 \exp[-i(m_S t_1 + m_L t_2) - \frac{1}{2}(\gamma_S t_1 + \gamma_L t_2)] - \eta_1 \exp[-i(m_L t_1 + m_S t_2) - \frac{1}{2}(\gamma_L t_1 + \gamma_S t_2)] \right\}. \quad (27)$$

The interference between the two amplitudes on the right-hand side is what makes possible the extraction of information about *CPT* (and *T*) violation. Note that $\mathcal{A}_{11}(t, t) = 0$.

In Sec. III, Eqs. (16) and (17), we defined Δm , m , $\Delta\gamma$, and γ . It is useful to introduce in addition the quantities

$$\Delta\phi = \phi_1 - \phi_2 \quad (28)$$

and

$$t = t_1 + t_2, \quad \Delta t = t_2 - t_1. \quad (29)$$

The decay rate $R_{12}(t, \Delta t)$ as a function of t and Δt can then be found:

$$\begin{aligned} R_{12}(t, \Delta t) = & |Na_{1S}a_{2S}|^2 e^{-\gamma t/2} \\ & \times [|\eta_1|^2 e^{-\Delta\gamma\Delta t/2} + |\eta_2|^2 e^{\Delta\gamma\Delta t/2} \\ & - 2|\eta_1\eta_2| \cos(\Delta m\Delta t + \Delta\phi)]. \end{aligned} \quad (30)$$

Experiments at meson factories will measure integrated decay rates. Following Ref. [29], we consider first the once-integrated rates for complete t acceptance:

$$I_{12}(\pm|\Delta t|) = \frac{1}{2} \int_{|\Delta t|}^{\infty} dt R_{12}(t, \pm|\Delta t|), \quad (31)$$

where the time difference Δt is kept constant. The definition keeps separate track of the cases where the decay

into f_1 occurs before and after that into f_2 . Writing $v = |\Delta t|$, a short calculation gives

$$\begin{aligned} I_{12}(\pm v) = & \frac{|Na_{1S}a_{2S}\eta_1|^2}{\gamma} e^{-\gamma v/2} \\ & \times [e^{\mp\Delta\gamma v/2} + |r_{12}|^2 e^{\pm\Delta\gamma v/2} \\ & - 2|r_{12}| \cos(\Delta m v \pm \Delta\phi)], \end{aligned} \quad (32)$$

where we have defined $r_{21} \equiv \eta_2/\eta_1 = |r_{21}| \exp(-i\Delta\phi)$. Nonzero values of δ_P (and ϵ_P) induce asymmetries between the two rates $I_{12}(\pm v)$, entering through contributions to the η_α .

Typically, a detector has limited Δt acceptance, lying in the range $\tau_1 \leq |\Delta t| \leq \tau_2$, say. Define the twice-integrated rates $\Gamma_{12}^\pm(\tau_1, \tau_2)$ by

$$\Gamma_{12}^\pm(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} dv I_{12}(\pm v). \quad (33)$$

For most geometries $\tau_1 = 0$. We assume this in what follow, and write $\tau_2 = \tau$.

The asymmetries induced by nonzero values of δ_P (and ϵ_P) can be probed in terms of the rate asymmetry $A_{12}(\tau)$, defined as

$$A_{12}(\tau) = \frac{\Gamma_{12}^+(0, \tau) - \Gamma_{12}^-(0, \tau)}{\Gamma_{12}^+(0, \tau) + \Gamma_{12}^-(0, \tau)}. \quad (34)$$

A calculation gives

$$A_{12}(\tau) = \frac{(1 - |r_{21}|^2)[h_S(\tau) - h_L(\tau)] - \frac{4}{b^2} \text{Im}r_{21}[\Delta m - be^{-\gamma\tau/2} \sin(\Delta m\tau + \tilde{\phi})]}{(1 + |r_{21}|^2)[h_S(\tau) + h_L(\tau)] - \frac{4}{b^2} \text{Re}r_{21}[\gamma/2 - be^{-\gamma\tau/2} \cos(\Delta m\tau + \tilde{\phi})]}, \quad (35)$$

where $h_{L(S)}(\tau) = [1 - \exp(-\gamma_{L(S)}\tau)]/\gamma_{L(S)}$, and b and $\tilde{\phi}$ are defined in Eq. (17). In the subsequent sections, we make particular use of the idealized asymmetry $A_{12}(\infty)$, defined as the limit of $A_{12}(\tau)$ as $\tau \rightarrow \infty$:

$$A_{12}(\infty) = \frac{(1 - |r_{21}|^2)(\gamma_S^{-1} - \gamma_L^{-1}) - 4 \text{Im}r_{21} \Delta m/b^2}{(1 + |r_{21}|^2)(\gamma_S^{-1} + \gamma_L^{-1}) - 2 \text{Re}r_{21} \gamma/b^2}. \quad (36)$$

Introducing the conventional variables

$$x = \frac{2\Delta m}{\gamma}, \quad y = \frac{\Delta\gamma}{\gamma}, \quad (37)$$

we find

$$A_{12}(\infty) = \frac{2x(1 - y^2) \text{Im}r_{21} + y(1 + x^2)(1 - |r_{21}|^2)}{2(1 - y^2) \text{Re}r_{21} - (1 + x^2)(1 + |r_{21}|^2)}. \quad (38)$$

B. The $K\bar{K}$ system

In this subsection, we consider the particular case $P \equiv K$, for which the existence of *CP* violation has been

known for three decades [31]. For this system, experimental values of $\tau_S = (8.922 \pm 0.020) \times 10^{-11}$ s, $\tau_L = (5.17 \pm 0.04) \times 10^{-8}$ s, and $\Delta m = (3.522 \pm 0.016) \times 10^{-15}$ GeV are available [32], from which we calculate

$$\begin{aligned} \Delta\gamma = & (7.367 \pm 0.016) \times 10^{-15} \text{ GeV}, \\ \hat{\phi} = & (43.71 \pm 0.14)^\circ. \end{aligned} \quad (39)$$

It follows that

$$x \approx y \approx 1. \quad (40)$$

A high intensity of correlated K_S - K_L pairs can be produced in a ϕ factory [30,29]. Several are now in the planning stages or under construction, including symmetric factories (in which the ϕ are produced at rest) at Frascati [33], Novosibirsk [34], and KEK [35], and asymmetric ones (in which the ϕ are produced with a boost) at UCLA [36] and CEBAF (photoproduction) [37]. For electron-positron colliders, typical peak luminosities being discussed are of order 10^{32} - 10^{33} cm⁻² s⁻¹. The relevant cross section is of order 4 μb corresponding to a peak rate of order 10^2 - 10^3 ϕ per second, and one running

year's worth of data could produce up to about 10^{11} ϕ . For photoproduction off protons, the cross section is of order $0.3 \mu\text{b}$ and the peak rate of tagged photons might be as high as 10^8 or even 10^9 s^{-1} , with corresponding ϕ production rates again of order 10^2 – 10^3 per second.

For this system, the discussion of possible stringy spontaneous CPT violation in Sec. III specializes as follows. Experimentally, $\Delta\gamma \approx 2\Delta m$, and $\hat{\phi} \simeq 45^\circ$. Equation (21) therefore becomes

$$\delta \approx \frac{\pm(h_s - h_d)}{\sqrt{2}\Delta m} e^{+3\pi i/4}. \quad (41)$$

Together with Eq. (22), this shows that in the kaon system stringy spontaneous CPT violation gives $|\text{Im}\delta_K| \approx |\text{Re}\delta_K|$. Plausible values of $|\delta_K|$ encompass a range including ones within reach of the next generation of experiments.

In the remainder of this subsection, we combine the methods of Refs. [38,39,29] with the formulas of Sec. IV A and some additional results to estimate analytically the

level at which stringy spontaneous CPT violation could be probed in a ϕ factory. Since the branching ratios of the kaons are dominated by the semileptonic and double-pion decays, we focus on these cases. Our analytical method provides a relatively quick means of estimating the relevant limits. A Monte Carlo study of T - and CPT -violation tests at a ϕ factory is presented in Ref. [40]. A discussion of the possibility of measuring T , CPT , and $\Delta S = \Delta Q$ violations in the kaon system has recently been given in Ref. [41]. A useful collection of papers on CP violation is given in Ref. [42].

Consider first the situation where the two final states from the double-kaon decay are semileptonic: $f_1 = l^+\nu\pi^-$, $f_2 = l^-\nu\pi^+$. Assuming $\Delta S = \Delta Q$ and substituting Eq. (12) into the definition (26) gives

$$\eta_{l\pm} = \pm 1 - 2\delta_K + O(\epsilon_K^2, \delta_K^2). \quad (42)$$

Using $\gamma_L \ll \gamma_S, \gamma$, Δm and taking $\text{Re}\delta_K \ll \text{Im}\delta_K$ on the grounds of the string-inspired scenario, it follows from Eq. (35) that

$$A_{l+l-}(\tau) = \frac{-4 \text{Re}\delta_K [h_S(\tau) - h_L(\tau)] + \frac{8}{b^2} \text{Im}\delta_K [\Delta m - b e^{-\gamma\tau/2} \sin(\Delta m\tau + \tilde{\phi})]}{h_L(\tau) + h_S(\tau) + \frac{2}{b^2} [\gamma/2 - b e^{-\gamma\tau/2} \cos(\Delta m\tau + \tilde{\phi})]} \quad (43)$$

and so, using Eq. (36) or (38),

$$A_{l+l-}(\infty) \approx 4 \text{Re}\delta_K. \quad (44)$$

Note that in the semileptonic case the value of $\text{Im}\delta_K$ cannot be determined from the limiting asymmetry with $\tau \rightarrow \infty$. Instead, one must fit directly to Eq. (43) for finite $\tau = O(\gamma_S^{-1})$.

Using the above information together with experimental data on the branching ratios of the semileptonic decays, we can obtain the number of ϕ events needed to detect a nonzero asymmetry at the $\pm N\sigma$ level, where σ is a standard deviation. For an asymmetry $A = (N_+ - N_-)/(N_+ + N_-)$, the binomial distribution implies that the expected number of events $\langle N_+ \rangle$ required to observe a nonzero $\langle A \rangle$ at the $N\sigma$ level is $N^2(1 + \langle A \rangle)(1 - \langle A \rangle^2)/(2\langle A \rangle^2)$. In the present case, this factor must be multiplied by the inverse branching ratio for ϕ decaying via two kaons into two final semileptonic states, which is approximately 7800. Using Eq. (44), we find that the number N_ϕ of ϕ events needed to reduce the error in $\text{Re}\delta_K$ to $\pm 1\sigma$ is

$$N_\phi(\text{Re}\delta_K) \simeq \frac{500}{\sigma^2}. \quad (45)$$

In this calculation, the statistical accuracy is improved by about a factor of 2 by allowing the two charged leptons in the final states to be any combination of electrons and muons. This is possible because the $\eta_{l\pm}$ in Eq. (42) are independent of l^\pm and because the decay-rate dependence on l^\pm cancels in the asymmetry.

We next consider double-pion final states, which can provide information on $\text{Im}\delta_K$. The parameters η_α of Eq.

(26) for the case of $2\pi^0$ and $\pi^+\pi^-$ kaon decays are conventionally denoted as η_{00} and η_{+-} , respectively. Present experimental values are [32]

$$\begin{aligned} |\eta_{+-}| &= (2.268 \pm 0.024) \times 10^{-3}, \\ \phi_{+-} &= (46.6 \pm 1.2)^\circ, \\ |\eta_{00}| &= (2.253 \pm 0.024) \times 10^{-3}, \\ \phi_{00} &= (46.6 \pm 2.0)^\circ. \end{aligned} \quad (46)$$

The standard theoretical calculation of these parameters treats the $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = \frac{3}{2}$ transitions as distinct, and allows for isospin dependence in any T and CPT violation in the decay amplitudes. A relatively tight bound on $|\text{Im}\delta_K|$ can be derived [38,39] from these values provided one neglects contributions to the effective Hamiltonian coming from 3π decays and from violations of the $\Delta S = \Delta Q$ rule in semileptonic decays. Introducing $\phi_\epsilon \approx (2\phi_{+-} + \phi_{00})/3$, we find

$$\begin{aligned} |\text{Im}\delta_K| &\approx |\eta_{+-}| \cos \hat{\phi} \sin(\phi_\epsilon - \hat{\phi}) \\ &= (8.3 \pm 2.9) \times 10^{-5}. \end{aligned} \quad (47)$$

A completely reliable bound taking into account the effects mentioned is less constrained [43] because then $39.5^\circ < \phi_\epsilon < 47.4^\circ$, from which we find $|\text{Im}\delta_K| < 1.3 \times 10^{-4}$. In either case, the real part of δ_K is not as tightly bounded because in 2π decays it always appears in conjunction with CPT -violating pieces of the decay amplitudes. The discussion in Sec. III implies these are zero for stringy spontaneous CPT violation, in which case we can apply the condition $|\text{Re}\delta_K| \approx |\text{Im}\delta_K|$.

The error in $|\text{Im}\delta_K|$ in Eq. (47) is dominated by that

in ϕ_ϵ . It can therefore be decreased by improving the precision with which ϕ_{+-} and ϕ_{00} are measured. Their difference $\Delta\phi$ can be determined from Eq. (32), for example, by taking as the two final states $\pi^+\pi^-$ and $\pi^0\pi^0$ and fitting for finite values of Δt . The precision in the value for $\Delta\phi$ thus obtained can be estimated analytically using Eq. (A9) from the Appendix, where the two functions $f(t), g(t)$ and the amplitude α are taken as

$$\begin{aligned} f(v) &= e^{-\gamma_S v} + e^{-\gamma_L v} - 2e^{-\gamma v/2} \cos \Delta m v \cos \Delta\phi, \\ g(v) &= e^{-\gamma v/2} \sin \Delta m v, \\ \alpha &= 2 \sin \Delta\phi. \end{aligned} \quad (48)$$

Combining the result of this calculation with the appropriate inverse branching ratio for the double- 2π decay of the ϕ , which is approximately 4800, we obtain the number N_ϕ of ϕ events needed to reduce the error in $\Delta\phi$ in degrees to within $\pm 1\sigma$ as

$$N_\phi(\Delta\phi) \simeq \frac{3.2 \times 10^9}{\sigma^2}. \quad (49)$$

Determining ϕ_{+-} or ϕ_{00} separately is less straightforward. A relatively good approach is to look at final states consisting of $f_1 \equiv 2\pi$ with appropriately charged pions and $f_2 \equiv \pi^\mp l^\pm \nu$, for which $\eta_{l^\pm} = \pm 1 - 2\delta_K \simeq \pm 1$. For example, using Eq. (32) with $\Delta t > 0$ permits ϕ_{+-} or ϕ_{00} to be determined from the interference term, which carries opposite sign for opposite lepton charges. We use the method described in the second part of the Appendix, which provides a means to estimate analytically the errors in asymmetries consisting of linear combinations of known functions. Taking $\alpha = 2|\eta_{\pi\pi}|$ and $\lambda \approx \beta = \Delta m$, we find from Eq. (86) that a measurement in degrees of ϕ_ϵ to an accuracy of $\pm 1\sigma$ requires a number N_ϕ of ϕ given by

$$N_\phi(\phi_\epsilon) \simeq \frac{1.3 \times 10^{12}}{\sigma^2}. \quad (50)$$

In this estimate, we summed over both electron and muon contributions, obtaining a combined inverse branching ratio of approximately 2700. Treating the error in $|\eta_{+-}|$ and $\hat{\phi}$ as negligible, the result (50) implies that the number N_ϕ of ϕ events needed to reduce the error in $\text{Im}\delta_K$ to $\pm 1\sigma$ is

$$N_\phi(\text{Im}\delta_K) \simeq \frac{1100}{\sigma^2}. \quad (51)$$

C. The $B\bar{B}$ system

We next examine the $P\bar{P}$ system with P taken as one of the two neutral B mesons, B_d or B_s . We primarily focus on the case where $P \equiv B_d$. Towards the end of this section we comment on the other choice.

The experimental data for the $B_d\bar{B}_d$ system are [32] $\bar{\tau}_B = (12.9 \pm 0.5) \times 10^{-13}$ s and $|\Delta m| = (3.6 \pm 0.7) \times 10^{-13}$ GeV, which implies $|x| = 0.71 \pm 0.14$. However, these values have been derived under the assumption that CPT is

preserved. If this assumption is relaxed, the experimental value of x becomes a lower bound [44]:

$$|x| \geq 0.71 \pm 0.14. \quad (52)$$

We use this bound in what follows.

At present, an experimental value of y is unavailable. On theoretical grounds, perturbative calculations via the box diagram are expected to provide an accurate estimate of y for this system because the dominant intermediate states are the top and charm quarks and so short-distance effects should dominate over dispersive ones. For details of these calculations and a guide to the large literature, the reader is referred to any of the standard reviews on this subject; see, for example, Refs. [45–48]. The results of these calculations suggest

$$|y| \sim 10^{-2} \ll |x|. \quad (53)$$

A relatively large number of correlated $B_d\bar{B}_d$ pairs can be generated in a B factory, which provides an intense source of $\Upsilon(4S)$. Discussions of some of the many proposals now being considered can be found in Ref. [49]. In particular, Cornell and SLAC in the U.S. and KEK in Japan are proceeding with asymmetric B factories. Peak luminosities anticipated are in the range 10^{33} – 10^{34} $\text{cm}^{-2}\text{s}^{-1}$. The cross section for $\Upsilon(4S)$ production in electron-positron machines is of order 1.2 nb, so peak production could lie in the range 1–10 Hz. A running year therefore could provide about 10^7 – 10^8 correlated $B_d\bar{B}_d$ pairs.

The discussion in Sec. III of potential stringy spontaneous CPT violation specializes for this system as follows. Equation (53) implies $\Delta m \gg \Delta\gamma$, so $\hat{\phi} \simeq \pm\pi/2$ and Eq. (21) becomes

$$\delta_{B_d} \approx \pm \frac{h_b}{\Delta m}. \quad (54)$$

It follows from this or Eq. (22) that stringy spontaneous CPT violation predicts $|\text{Re}\delta_{B_d}| \gg |\text{Im}\delta_{B_d}|$ in the B_d system, with magnitude potentially accessible to experiment.

We next discuss briefly the issue of measuring δ_{B_d} experimentally in a B factory. Consider the case in which the $\Upsilon(4S)$ produces a double-semileptonic final state, $f_1 = l^+\nu\pi^-$, $f_2 = l^-\nu\pi^+$. At present, it is not known whether T and CPT violation are large, small, or zero in this system. If the violation is large, the integrated asymmetry (36) or (38) provides a relation between the real and imaginary parts of the ratio r_{21} .

If, however, the violation is small, the ratio r_{21} can be related to δ_{B_d} using an expression analogous to Eq. (42). Then, Eqs. (36) and (38) reduce to

$$\begin{aligned} A_{l+l-}(\infty) &\approx \frac{-4 \text{Re}\delta_{B_d}(\gamma_S^{-1} - \gamma_L^{-1}) + 8 \text{Im}\delta_{B_d}\Delta m/b^2}{(\gamma_L^{-1} + \gamma_S^{-1}) + \gamma/b^2} \\ &\approx 4 \frac{x \text{Im}\delta_{B_d} + y(1+x^2)\text{Re}\delta_{B_d}}{2+x^2}. \end{aligned} \quad (55)$$

A measurement of this asymmetry therefore provides one probe of δ_{B_d} .

For stringy spontaneous CPT violation Eq. (22) gives

$$y \operatorname{Re} \delta_{B_d} \approx \pm x \operatorname{Im} \delta_{B_d}, \quad (56)$$

and Eq. (55) reduces to

$$A_{l+l-}(\infty) \approx \pm \frac{4x(1+x^2 \pm 1)}{2+x^2} \operatorname{Im} \delta_{B_d}. \quad (57)$$

Taking $|x| \simeq 0.71$ gives

$$|A_{l+l-}(\infty)| \approx k |\operatorname{Im} \delta_{B_d}|, \quad (58)$$

where $k \simeq \frac{1}{2}$ or $k \simeq 3$, depending on the choice of sign in Eq. (57).

To improve statistical accuracy, we can again take advantage of the fact that several decay channels have equal values of $\eta_{l\pm} = \pm 1 - 2\delta_{B_d}$. These include semileptonic decays, summing to a branching ratio of about 10%, along with any other channels in B_d decay forbidden to a good approximation in \bar{B}_d decay, paired with the corresponding CP conjugates. In principle, this includes a total branching ratio of at least about 20% and perhaps much larger. Taking this into account and using Eq. (57) with $k \sim 1$, we find that the number $N_{\Upsilon(4S)}$ of events needed to reduce the error in $\operatorname{Im} \delta_{B_d}$ to within $\pm 1\sigma$ is no more than about

$$N_{\Upsilon(4S)}(\operatorname{Im} \delta_{B_d}) \simeq \frac{5}{\sigma^2}. \quad (59)$$

It would also be desirable to distinguish experimentally the case of stringy spontaneous CPT violation from a generic scenario. If, for example, one allows for values of $\operatorname{Im} \delta_{B_d}$ comparable to those of $\operatorname{Re} \delta_{B_d}$, the asymmetry (55) reduces instead to

$$A_{l+l-}(\infty) \approx \frac{4x}{2+x^2} \operatorname{Im} \delta_{B_d} \simeq \operatorname{Im} \delta_{B_d}. \quad (60)$$

Although we are unaware of any explicit theoretical mechanism that could produce this result, it nonetheless is not experimentally excluded. It would therefore appear desirable to have an alternative independent means of measuring δ_{B_d} . The resolution of this issue depends on successfully disentangling different CP -violation effects and therefore awaits a complete study of T and CPT violation in the B system.⁴

So far, our focus has been on the case $P \equiv B_d$. In the remainder of this subsection, we consider the alternative choice $P \equiv B_s$.

The experimental data for the key parameters in the B_s - \bar{B}_s system are limited at present, so we content ourselves with theoretical predictions. See, for example, [46–48] and references therein. On the basis of current knowledge of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the B_d system, it is expected that

$$\begin{aligned} |x_s| &\gtrsim 10, \\ |y_s| &\simeq 2 \times 10^{-2}. \end{aligned} \quad (61)$$

The inequality again reflects the uncertainty in x arising from the possibility of CPT violation, as in Eq. (52) above. The theoretical uncertainties in these numbers are probably at least a factor of 2.

The system might be explored in a B factory focusing on the production of $\Upsilon(5S)$. However, both the cross section for production and the branching ratio for B_s - \bar{B}_s decay are smaller than in the B_d case, so a luminosity of perhaps two orders of magnitude greater than that in the planned B factories would be required to produce comparable numbers of correlated B_s - \bar{B}_s pairs.

Anticipated effects arising from possible stringy spontaneous CPT violation are analogous to the B_d system. Thus, we obtain

$$\delta_{B_s} \approx \delta_{B_d} \approx \pm \frac{h_b}{\Delta m}, \quad (62)$$

and $\hat{\phi} \simeq \pi/2$ again, so $|\operatorname{Re} \delta_{B_s}| \gg |\operatorname{Im} \delta_{B_s}|$ in the B_s system too.

Some differences between the two systems arise because $|x_s| > |x|$. The implications of this for experiments can be found using the methods discussed above. For example, the string scenario implies that the double-semileptonic asymmetry Eq. (55) reduces to

$$A_{l+l-}(\infty) \approx \pm 4x_s \operatorname{Im} \delta_{B_s} \gtrsim \pm 40 \operatorname{Im} \delta_{B_s}. \quad (63)$$

This theoretical result suggests that a measurement of $\operatorname{Im} \delta_{B_s}$ is about three orders of magnitude more favorable than in the B_d case; cf. Eq. (59). However, as mentioned above, the experiment may be two or more orders of magnitude harder because of the sizes of the associated production cross sections and branching ratios.

D. The $D\bar{D}$ system

In this subsection, we consider the case $P \equiv D$. At present, there is no experimental evidence for mixing in the D - \bar{D} system. This issue could be examined to a relatively high degree of precision in τ -charm factory such as that originally envisaged in Spain [51]. With a design luminosity peaking between 10^{32} and 10^{33} $\text{cm}^{-2} \text{s}^{-1}$, a machine operating at the $\psi(3700)$ could generate yearly between 10^7 and 10^8 correlated D - \bar{D} pairs.

Calculations of short-distance effects via the box diagram suggest only very small effects, giving $|x| \approx |y| \simeq 10^{-6}$. Summaries of the situation and a guide to the relevant literature can be found in [45,47]. In addition to the short-distance effects, long-range or dispersive contributions to x and y must be considered. These are dominated by intermediate lower-mass states and are difficult to estimate with precision. In the kaon system, the short- and long-range contributions are believed to be comparable [52–54]. In the B systems the short-range contributions dominate, as discussed above. In contrast, the D system is believed to be dominated by dispersive

⁴Since the completion of the present work, some progress along these lines has been made; see Ref. [50].

effects [55,56]. These generate values of $|x|$ and $|y|$ larger than the box contributions by several orders of magnitude. In what follows, we take for illustrative purposes $\Delta m \approx \Delta \gamma$ such that [55]

$$|x| \approx 2|y| \simeq 2 \times 10^{-2} . \quad (64)$$

With this assumption, Eq. (21) of Sec. III becomes

$$\delta \approx \frac{\pm i \sqrt{2} h_c}{\sqrt{5} \Delta m} e^{i\hat{\phi}} , \quad (65)$$

with $\hat{\phi} \simeq 63^\circ$. The relationship corresponding to Eq. (22) is $2|\text{Im}\delta_D| \approx |\text{Re}\delta_D|$. Plausible values of $|\delta_D|$ again encompass a range within experimental reach.

Information about δ_D could be obtained from a measurement of the double-semileptonic asymmetry of Eq. (36) or (38). Assuming T and CPT violation are small, as usual, Eq. (55) holds except with δ_B replaced by δ_D . The largest asymmetry here appears when the signs are such that a term linear in x survives, whereupon we find

$$A_{l+l-}(\infty) \approx 4x \text{Im}\delta_D \simeq \pm 10^{-1} \text{Im}\delta_D . \quad (66)$$

In this case, the asymmetry provides a measurement of $\text{Im}\delta_D$ rather than the real part, as occurred in the kaon system. This arises even though the mixing parameters obey $|x| \approx |y|$ in both cases because for K the mixing parameters are of order 1 whereas for D they are much less than 1.

Branching ratios for the D meson are better known than for the B system. We can again use a combination of semileptonic and other decay channels with $\eta_{\pm} = \pm 1 - 2\delta_D$, which here includes a total branching ratio of at least 50%. Assuming Eq. (66), we obtain the number $N_{\psi(3770)}$ of events needed to reduce the error in $\text{Im}\delta_D$ to within $\pm 1\sigma$ as approximately

$$N_{\psi(3770)}(\text{Im}\delta_D) \simeq \frac{200}{\sigma^2} . \quad (67)$$

In principle, further information about δ_D is encoded in asymmetries involving non-semileptonic final states. We do not pursue this issue in the present work.

V. SUMMARY

In this paper, we presented a generic theoretical framework for possible stringy spontaneous CPT violation and examined some experimental consequences. We concentrated on the detection of effects using a sensitive tool: the $P\bar{P}$ interferometers. We found that a range of string-inspired values of the CPT -violating parameter δ_P in these systems can be explored in suitable meson factories. For the case $P \equiv K$, the string scenario gives $|\text{Re}\delta_K| \approx |\text{Im}\delta_K|$. For $P \equiv B_d$ or B_s , $|\text{Re}\delta_B| \gg |\text{Im}\delta_B|$, while for $P \equiv D$, a rough estimate suggests $|\text{Re}\delta_K| \approx 2|\text{Im}\delta_K|$. Estimates were given of the number of events needed to reduce the error in the real or imaginary part of δ_P to within one standard deviation. These are presented in Eqs. (45) and (51) for the

case $P \equiv K$, in Eq. (59) for $P \equiv B_d$, and in Eq. (67) for $P \equiv D$. The case $P \equiv B_s$ was also briefly discussed. The anticipated mesons containing the top quark, T_u and T_c , were not considered since it is likely the top-quark decay proceeds too rapidly to permit hadronization. Effects on other physics, including, for example, the lepton sector, may also exist and are under investigation. Note, however, that although the stringy mechanism might also lead to violation of other discrete symmetries, any effects are likely to be masked by violations appearing from known effects in particle field theories.

One point that emerges from this work is that detecting stringy effects, while certainly difficult, is not necessarily impossible. Although the absence of a satisfactory and realistic string theory precludes definitive predictions for low-energy effects, several possibilities do exist. Among them is the stringy spontaneous CPT violation discussed here. We have seen that, if it does occur, the violation appears in a particular sector and can have magnitude accessible in the next generation of experiments.

ACKNOWLEDGMENTS

We thank Stuart Samuel for useful comments. V.A.K. has benefitted from the hospitality of the Aspen Center for Physics in the summers of 1992 and 1993, when parts of this work were performed. R.P. thanks the high-energy physics group at Indiana University for hospitality during an extended visit. This work was supported in part by the United States Department of Energy under Grant No. DE-FG02-91ER40661.

APPENDIX

Suppose we want to measure an asymmetry in a decay curve, obtained as a function of the elapsed time t since production in the inertial frame of the decay. Let there be two possible decay products, denoted by $+$ and $-$, occurring with probabilities $f_+(t)$ and $f_-(t)$. For the cases of interest in the text, we have

$$f_{\pm}(t) = f(t) \pm \alpha g(t) , \quad (A1)$$

where f and g are known functions and α is a constant that is the focus of the experimental measurement. We take the asymmetry $A(t)$ to be defined by

$$A(t) \equiv \alpha g(t) = \frac{1}{2}[f_+(t) - f_-(t)] . \quad (A2)$$

Let the experiment involve values of t lying in the range $t \in [a, b]$. First, we divide the interval into bins. After N measurements, the number of \pm events in bin n is

$$N_{\pm}(n) \approx C f_{\pm}(t_n) , \quad C = \frac{N}{\sum_n [f_+(t_n) + f_-(t_n)]} , \quad (A3)$$

where C is a normalization constant. We can use the form of Eq. (A1) to find an expression for α in terms of

a function $h(t_n)$ to be determined below:

$$\alpha = \frac{\sum_n A(t_n)h(t_n)}{\sum_n g(t_n)h(t_n)}. \quad (\text{A4})$$

The error in the measured value for the asymmetry $A(t_n) = [N_+(n) - N_-(n)]/2C$ is $\sqrt{N_+(n) + N_-(n)}/2C \approx \sqrt{f(t_n)}/2C$. Thus, using Eq. (A4) to obtain α , the resulting error in α is

$$\begin{aligned} \delta_h \alpha &= (2C)^{-1/2} \frac{\left[\sum_n h(t_n)^2 f(t_n) \right]^{1/2}}{\sum_n g(t_n)h(t_n)} \\ &= (2N)^{-1/2} \frac{\left[\sum_n f(t_n) \right]^{1/2} \left[\sum_n h(t_n)^2 f(t_n) \right]^{1/2}}{\sum_n g(t_n)h(t_n)}, \end{aligned} \quad (\text{A5})$$

or, taking the continuum limit,

$$\delta_h \alpha = (2N)^{-1/2} \frac{[\int f(t)dt]^{1/2} [\int h(t)^2 f(t)dt]^{1/2}}{\int h(t)g(t)dt}. \quad (\text{A6})$$

At this point we can determine $h(t)$ by requiring that the resulting error in α be minimal. Thus

$$\frac{\delta}{\delta h(t)} \left[\frac{\sqrt{\int h^2 f dt}}{\int h g dt} \right] = 0. \quad (\text{A7})$$

This yields

$$h(t) \propto g(t)f(t)^{-1}, \quad (\text{A8})$$

resulting in an error

$$\delta \alpha = (2N)^{-1/2} \left(\frac{\int f dt}{\int g^2 f^{-1} dt} \right)^{1/2}. \quad (\text{A9})$$

This result can be easily extended to a situation where the asymmetry is a linear combination of known functions:

$$A(t_n) = \sum_j \alpha_j g_j(t). \quad (\text{A10})$$

In this case, we introduce functions $h_i(t)$, chosen to satisfy

$$\sum_n g_j(t_n)h_i(t_n) \propto \delta_{ij}, \quad (\text{A11})$$

so that the quantities α_i are determined by

$$\alpha_i = \frac{\sum_n A(t_n)h_i(t_n)}{\sum_n g_i(t_n)h_i(t_n)}. \quad (\text{A12})$$

Minimizing the error, we find that the h_i are proportional to $g_i f^{-1}$. Then, Eq. (A11) implies that the g_i must satisfy the orthogonality conditions

$$\sum_n \frac{g_i(t_n)g_j(t_n)}{f(t_n)} = 0, \quad i \neq j. \quad (\text{A13})$$

If the chosen functions g_i do not satisfy these conditions, the Gram-Schmidt orthogonalization procedure can be applied. We finally obtain the induced error in the parameters α_i :

$$\delta \alpha_i = (2N)^{-1/2} \left(\frac{\int f dt}{\int g_i^2 f^{-1} dt} \right)^{1/2}. \quad (\text{A14})$$

In the text, we are interested in applying this method to a case with

$$f(t) = e^{-\lambda t}, \quad (\text{A15})$$

$$A(t) = \alpha e^{-\lambda t} \cos(\beta t - \chi), \quad (\text{A16})$$

where λ and β are known parameters while α and χ are to be measured experimentally. We are particularly interested in the induced error in χ . Here, the antisymmetry is *not* in the form of a linear combination of basis functions. However, this can be remedied by setting $\cos(\beta t - \chi) = \cos \beta t \cos \chi + \sin \beta t \sin \chi$ and using Gram-Schmidt orthogonalization on the functions $e^{-\lambda t} \cos \beta t$ and $e^{-\lambda t} \sin \beta t$. We thereby find that the functions

$$g_1(t) = e^{-\lambda t} \cos \beta t, \quad (\text{A17})$$

$$g_2(t) = e^{-\lambda t} \left(\sin \beta t - \frac{w}{2+w^2} \cos \beta t \right), \quad (\text{A18})$$

where $w \equiv \lambda/\beta$, satisfy the condition (A13). We can then directly apply (A14) to obtain the errors in the coefficients α_1, α_2 . For the error in $\chi = \arctan(\alpha_2/\alpha_1)$, we obtain

$$\begin{aligned} \delta_\chi &= (2N)^{-1/2} \alpha^{-1} (4+w^2)^{1/2} \\ &\times \left[\frac{\cos^2 \chi}{2+w^2} + \frac{1}{2} \left(\sin \chi + \frac{w \cos \chi}{2+w^2} \right)^2 \right]^{1/2}. \end{aligned} \quad (\text{A19})$$

- [1] J. Schwinger, Phys. Rev. **82**, 914 (1951).
- [2] G. Lüders, Det. K. Dan. Videnk. Selsk. Mat.-Fys. Medd. **28**(5), (1954); Ann. Phys. (N.Y.) **2**, 1 (1957).
- [3] J. S. Bell, Ph.D. thesis, Birmingham University, 1954; Proc. R. Soc. London **A231**, 479 (1955).
- [4] W. Pauli, *Niels Bohr and the Development of Physics*, edited by W. Pauli (McGraw-Hill, New York, 1955), p. 30.
- [5] G. Lüders and B. Zumino, Phys. Rev. **106**, 385 (1957).
- [6] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That* (Benjamin Cummings, Reading, MA, 1964).
- [7] R. Jost, *The General Theory of Quantized Fields* (American Mathematical Society, Providence, 1965).
- [8] R. Carosi *et al.*, Phys. Lett. B **237**, 303 (1990).
- [9] M. Karlsson *et al.*, Phys. Rev. Lett. **64**, 2976 (1990).
- [10] V. A. Kostecký and R. Potting, Nucl. Phys. **B359**, 545 (1991).
- [11] V. A. Kostecký, R. Potting, and S. Samuel, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics*, Geneva, Switzerland, 1991, edited by S. Hegarty, K. Potter, and E. Quercigh (World Scientific, Singapore, 1992).
- [12] V. A. Kostecký and R. Potting, in *Proceedings of the XX International Conference on Differential Geometric Methods in Theoretical Physics*, New York, New York, 1991, edited by S. Catto and A. Rocha (World Scientific, Singapore, 1992).
- [13] T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. **16**, 511 (1966).
- [14] R. G. Sachs, Prog. Theor. Phys. Suppl. **86**, 336 (1986); *The Physics of Time Reversal* (University of Chicago Press, Chicago, 1987).
- [15] E. Witten, Nucl. Phys. **B268**, 253 (1986).
- [16] C. Preitschopf, C. B. Thorn, and S. A. Yost, Nucl. Phys. **B337**, 363 (1990).
- [17] I. Ya. Aref'eva, P. Medvedev, and A. Zubarev, Nucl. Phys. **B341**, 464 (1990).
- [18] V. A. Kostecký and S. Samuel, Phys. Rev. D **39**, 683 (1989); Phys. Rev. Lett. **63**, 224 (1989); Phys. Rev. D **40**, 1886 (1989).
- [19] V. A. Kostecký and R. Potting, in *Gamma Ray-Neutrino Cosmology and Planck Scale Physics*, edited by D. B. Cline (World Scientific, Singapore, 1993).
- [20] V. A. Kostecký and R. Potting, in *The Fermilab Meeting*, Proceedings of the Meeting of the Division of Particles and Fields of the APS, Batavia, Illinois, 1992, edited by C. H. Albright *et al.* (World Scientific, Singapore, 1993), Vol. I.
- [21] S. W. Hawking, Phys. Rev. D **14**, 2460 (1975); Commun. Math. Phys. **87**, 395 (1982).
- [22] D. N. Page, Phys. Rev. Lett. **44**, 301 (1980); Gen. Relativ. Gravit. **14**, 299 (1982).
- [23] R. M. Wald, Phys. Rev. D **21**, 2742 (1980).
- [24] J. Ellis, J. S. Hagelin, D. V. Nanopoulos, and M. Srednicki, Nucl. Phys. **B241**, 381 (1984).
- [25] P. Huet and M. E. Peskin, SLAC Report No. 6454, 1994 (unpublished).
- [26] J. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, CERN Report No. TH.6755, 1992 (unpublished).
- [27] V. A. Kostecký and S. Samuel, Phys. Lett. B **270**, 21 (1991).
- [28] I. Antoniadis, C. Muñoz, and M. Quirós, Nucl. Phys. **B397**, 515 (1993).
- [29] I. Dunietz, J. Hauser, and J. L. Rosner, Phys. Rev. D **35**, 2166 (1987).
- [30] H. J. Lipkin, Phys. Rev. **176**, 1715 (1968).
- [31] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. **13**, 138 (1964).
- [32] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, SI (1992).
- [33] *DaΦne Physics Handbook*, edited by L. Maiani, G. Pancheri, and N. Paver (INFN, Frascati, Italy, 1992).
- [34] S. Eidelman, E. Solodov, and J. Thompson, in *Proceedings of the Workshop on Testing CPT and Studying CP Violation at a φ Factory*, Los Angeles, California, 1990, edited by D. B. Cline [Nucl. Phys. (Proc. Suppl.) **A24**, 174 (1991)].
- [35] M. Fukawa *et al.*, KEK Report No. 90-12, 1990 (unpublished).
- [36] D. B. Cline, UCLA Report No. CAA0091, 1992 (unpublished); see also A. Boden and D. B. Cline, UCLA Report No. CAA0095, 1992 (unpublished).
- [37] A. Dzierba (private communication).
- [38] V. V. Barmin *et al.*, Nucl. Phys. **B247**, 293 (1984).
- [39] N. W. Tanner and R. H. Dalitz, Ann. Phys. (N.Y.) **171**, 463 (1986).
- [40] C. D. Buchanan *et al.*, Phys. Rev. D **45**, 4088 (1992).
- [41] M. Hayakawa and A. I. Sanda, Phys. Rev. D **48**, 1150 (1993).
- [42] *CP Violation*, edited by C. Jarlskog (World Scientific, Singapore, 1989).
- [43] L. Lavoura, Mod. Phys. Lett. A **7**, 1367 (1992).
- [44] M. Kobayashi and A. I. Sanda, Phys. Rev. Lett. **69**, 3139 (1992).
- [45] L.-L. Chau, Phys. Rep. **95**, 1 (1983).
- [46] P. J. Franzini, Phys. Rep. **173**, 1 (1989).
- [47] E. A. Paschos and U. Türke, Phys. Rep. **178**, 145 (1989).
- [48] I. I. Bigi, in *CP Violation and Beauty Factories*, edited by D. B. Cline and A. Fridman, special issue of Ann. N.Y. Acad. Sci. **619** (1991).
- [49] *Beauty '93*, Proceedings of the International Workshop on B Physics at Hadron Machines, Melnik, Czech Republic, edited by P. Schlein [Nucl. Instrum. Methods Phys. Res. Sect. A **333** (1993)].
- [50] D. Colladay and V. A. Kostecký, Phys. Lett. B **344**, 259 (1995).
- [51] J. Kirkby and J. A. Rubio, Particle World **3**, 77 (1992).
- [52] L. Wolfenstein, Nucl. Phys. **B160**, 501 (1979).
- [53] C. Hill, Phys. Lett. **97B**, 275 (1980).
- [54] J. F. Donoghue, E. Golowich, and B. R. Holstein, Phys. Lett. **135B**, 481 (1984).
- [55] L. Wolfenstein, Phys. Lett. **164B**, 170 (1985).
- [56] J. F. Donoghue, E. Golowich, B. R. Holstein, and J. Trampetić, Phys. Rev. D **33**, 179 (1985).