

## Out of the Box

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*This article gives a brief and condensed account of the main trajectory (having pruned out many side shoots) of my philosophical career. This started with the philosophy of mathematics and the history of science. A major turning point was the encounter with Gaston Bachelard's philosophy of science, with other French philosophers, and with the history of Chinese mathematics.*

**Key words:** mathematics; history of science; Bachelard; epistemology; ancient Chinese mathematics

To be honest, I rather drifted into philosophy and into university. Once in the sixth form, I just wanted to get out of school and pursue my love of athletics and games of all sorts. My highest ambition was to teach PE. Bristol was chosen because it offered a qualification to teach PE after just one extra year at university. So I went there to study a joint degree in philosophy and mathematics. However, ironically, it was athletics that pushed me toward philosophy of science. En route between school and athletics training I started reading about the history of science in the central reference library in Nottingham. I was supposed to be doing my maths homework, which was easy if I didn't show the methods of working requested by the teacher, and books were more interesting. This was undisciplined, unstructured reading. I remember being particularly impressed by Tobias Dantzig's book *Number the Language of Science*, which is quoted at the beginning of Lancelot Hogben's *Mathematics for the Million* (found amongst my father's books).<sup>1</sup> Dantzig was convinced of the role of intuition in the evolution of mathematical concepts and the importance of the cultural context in which mathematics was developed, and that these are anthropomorphic in origin. His conclusion: "... man is the measure of all things; there is no other measure" (Dantzig 1954: 331). This just reinforced the message of a picture book my father had given me, also by Lancelot Hogben, called *Man Must Measure* (Hogben 1955).

When I got to Bristol, I fell out with the PE instructor and received a mixed training in philosophy. There was the standard analytic philosophy of science, ethics, etc., but I also had to attend seminars on Kant delivered by a Yugoslav (Edo Pivcevic); a non-analytic approach to Kant's philosophy (despite my lack of German) has remained a theme for me (I also attended seminars by Gerd Buchdahl and Alan Montefiore). That emphasis on Kant is vital, in spite of his uncompromisingly universalist ethics (a subdiscipline to which I was never attracted), because he never drew a sharp line between the reflective subject's ability to be aware of rules and the extent to which those rules are being followed.

Looking back at my career, I can see several turning points, not the least of which was getting an appointment at the University of Hawai'i at Manoa (1989). A significant aspect of the Philosophy Department at Manoa was its commitment to Asian philosophy, and although I was hired to teach logic, it proved a useful backdrop for my nascent interest in the history of mathematics in China, as well as a context with the freedom to develop my interests in many related areas less clearly philosophical.

The starting point for my publishing career was Alan Montefiore's request that I write a book on Gaston Bachelard's philosophy of science for a series he edited.<sup>2</sup> That request came on the basis of work on, and arguments about, Kant. At that point I had never encountered

Bachelard's work, and worse for me, his philosophy of science had not been translated into English; my French had been a bare pass at O-level and even that was very rusty.

Initially I was completely baffled by Bachelard's whole approach to philosophy of science, which was so unlike anything I, with my strictly analytic training in Oxford, had been used to. I was familiar with efforts to provide a foundation for mathematics, with attempts by Carnap and others to tie the language of science back to sense-data, with the Fregean notion of a concept, and with debates, sparked by Kuhn and Feyerabend, about whether scientific progress could be portrayed as rational. I was not prepared for the idea that concepts from the various sciences might be at different stages of philosophical development or for the discussions of *La philosophie du non* (Bachelard 1940), where the idea of the epistemological profile of a concept is introduced. In general I had trouble thinking beyond the context of theories and their language in order to incorporate their historical background and the various contexts in which they are used. Recently I looked at the book on Bachelard and realized that when writing it I was gradually dismantling the box in which analytic philosophy (particularly as done in Oxford) had held me. Working on Bachelard thus helped me see what had been blocking the work I had been doing in the philosophy of mathematics. I did not have the framework for escape at the time, but Bachelard helped me reveal the assumptions that had put me in the box—not the least of which was that mathematics needs a foundation.

Equally important was a visit to China in 1987, where I was (unwittingly) it seems instrumental in fostering the development of analytic philosophy in China. The visit to China was sponsored by the British Council. I was to negotiate an academic agreement between the Royal Institute of Philosophy (where I had a part-time secretarial job), the Chinese Academy of Social Sciences in Beijing, and the University of Oxford. My Chinese hosts seemed to assume that “secretary” was a prestigious position. The British Council was very keen to ensure that the Chinese not use whatever was agreed upon as a way of rewarding political favorites with trips abroad. What was proposed, and eventually agreed upon, was that philosophers from the UK would teach a Summer School in Philosophy in China. Those instructors would then examine the pupils on the basis of what had been taught. The top scorers in the exam would be sponsored to spend time at a UK university (see Yi Jiang and Tongdong Bai 2010).<sup>3</sup> It is little known that China had one of the original members of the Vienna Circle, Professor Hong Qian (T'scha Hung), who told me that he continued to hold his position throughout the Cultural Revolution by insisting that his work had no political significance. But even if not required to do physical labor, he was forced to remain silent since logical empiricism was condemned as capitalist and imperialist. Later he was for many years Director of the Institute for Foreign Philosophy at Beijing University, and his prestige was apparently crucial to the success of my trip (for more details see the prefaces to Cohen 1996 by Cohen and Fan Dainian).<sup>4</sup> It is worth noting that in China, Russia, and the McCarthy era in the US, philosophers of science had, as a matter of self-preservation, to claim their work to be of no political significance. This makes understandable their insistence on a sharp distinction between fact and value, between ethics and philosophy of science, between theory and practice. This was true in the UK but for different reasons; it was home to a strong empiricist tradition sharpened by opposition to German idealism (whether in its Marxist or fascist guises).

In the middle of the trip I left my Chinese hosts with the proposal and went to Wuhan University to deliver lectures (in English) on Bachelard's philosophy of science at the invitation of Professor Jiang Tian-ji, who had been teaching his graduate students analytic philosophy of science, in English; the dominant philosophy of science in China at that time was Engel's *Dialectics of Nature* (in Chinese translation). Jiang Tian-ji had been one of Hong Qian's students, but came to appreciate that the analytic framework had its limitations, hence his interests in historicism and in the American pragmatists. After the Cultural Revolution his reputation in China was enhanced by the widely read *Logical Empiricism Epistemology: Contemporary Western Philosophy of Science*, published in Chinese by Wuhan University Press (1991). He and his students became interested in the work

of Karl Popper, partly for its attention to political issues as well as those of the philosophy of science (see Newton Smith 1992).<sup>5</sup>

The visit to China, particularly the trip to Wuhan, impressed on me the gulf between Chinese and western culture (China was at that time only just opening up—it is quite different now). As a consequence, I became interested in the history of pre-Jesuit (pre-sixteenth–seventeenth-century) Chinese mathematics and its cultural embedding. What I came to understand is that the fundamental (metaphysical) difference between China and the west stems from what is presumed to be natural. For China, change is natural, whereas for the west, the Ideal, behind superficial change, is unchanging. In China, change is natural, whereas order and stability must be imposed. For the west, change needs explanation; left to itself, nature will order itself. In neither case were technical skills valued.

This feeds into the positioning of mathematics as an educational endeavor. In China's history, mathematics was never held up as paradigmatically rational science; it belonged more to practice and had only a minor role in the education of the elite (mandarins) (see, for example, Martzloff, *A History of Chinese Mathematics*, and Kline, *Mathematics in Western Culture*).<sup>6</sup> Thanks to Plato and Aristotle, mathematics was in the west held to be inextricably tied up with its image of reason and (according to neo-Platonists) a route to the mind of God. In China mathematics had a purely practical, administrative function—the collection of taxes, surveying (including map-making), construction, etc. Its role was more something that administrators needed to use, and hence have a knowledge of, rather than being intimately bound up with conceptions of reason.

Primed by Bachelard's work and the emphasis it placed on the importance of cultural context, I wanted to know more. The classic *Nine Chapters on Arithmetic* (anonymous but possibly composed 200 BCE—50 CE), in stark contrast to Euclid's *Elements*, does not contain any proofs. Instead it has a series of problems and their method of solution. These range from the kind of problems that would be useful to a tax collector to those which might be useful when constructing fortifications. Those employing mathematics in Ancient China used counting rods which they set out in arrays and then manipulated, so calculations were primarily procedural but had a problem-solving power roughly equivalent to the use of matrices. The placing of the counting rods was important and equivalent to having a positional notation (including, some would argue, a zero—empty place). Calculation beyond a few decimal places was not important.

Education, until Mao intervened, was all about passing the Imperial exams for admission to the class of mandarins, and because these were perceived as open to all who could afford it, they also served as a vehicle for self-cultivation; they thus assured a kind of cultural hegemony and order. Education itself consisted of committing much of the pre-Qin Classics (and commentaries on them) to memory, accompanied by an ability to creatively apply passages from those classics to a given situation. The Classics themselves contain a lot about cultivating excellence (*de*) which may have been emphasized more in some eras than in others. It embedded in the educational system a core of neo-Confucian texts, which provided a vehicle for cultural uniformity, even as it encouraged adaptation to changing circumstances. As Liu Hui (an early commentator on the Nine Chapters) put it, “I told him what had gone before and he understood what followed: showed him a corner and he replied with the other three.”

The epistemological emphasis here is on being able to develop the skill of grounding one's understanding in past examples whilst creatively extending to the future, which was why when westerners encountered these methods they (mis)classified them as reliance on induction. In the case of mathematics, it not about definitions that determine what it is, for example, to be a triangle, and then using these as the basis for a proof about all possible triangles (as in Euclid), but of taking solutions offered to problems whose solution is known and finding ways to adapt the methods used to the situation at hand. This attitude toward generalization makes it unnecessary to determine a class in advance (by a fixed definition); instead, it is something that can be extended in a variety of ways (which is very reminiscent of Wittgenstein's notion of family resemblance).

This, coupled with the fact that practical, technical skills were not highly prized in traditional Chinese culture, meant that repeatedly, mathematical methods, like technological methods more generally, were developed and became part of practice, with their basis subsequently lost because there was no community with the tradition of requiring justification for the methods. Chinese history is littered with amazing technical developments—printing press, paper, methods for raising water, irrigation systems, a water clock, gunpowder, the compass, surveying techniques... But for lack of a community communicating about them and probing the reasoning behind the inventions, they were forgotten as the inventions either dropped out of use, or their use became routine and just a matter of habit.

This was very much the background that Jesuit missionaries encountered when they entered China (in the late Ming dynasty). They had hoped to convert the Chinese to Christianity using their “superior” mathematics and its associations with rationality (leading back to God). Instead they found that the Chinese accepted the methods but rejected all attempts at proof. The first Qing emperor, Kang Xi, took a personal interest in mathematics, employing Jesuits in charge of astronomy as tutors. Mei Wending, one of the most famous post-Jesuit Chinese mathematicians, who had met Kang Xi and was approved by him, founded a program of rediscovering older Chinese methods and extending them to claim new results (for more detail, see Jami 2012.)<sup>7</sup> The particular problem that prompted Jesuits to intervene was (as in the west) calendar reform. The power of their methods was demonstrated by success in predicting a solar eclipse. But practical success did not lead, as they had hoped, to widespread adoption of Christianity.

My work on Bachelard was vital to this story not because he was ever interested in China or its contribution to philosophy, but because of his turn to focus on the dynamics of scientific thought rather than (as in analytic philosophy of science) its theories. He grants a role for the creative subject in his account of scientific objectivity, emphasizes the importance of cultural context, and stresses the need to understand the history of a science in order to get a sense of its dynamics; the focus is on epistemology rather than ontology or metaphysics. His concern is with the dynamics of scientific thought, not with its foundations. However, the development of any science will be discontinuous (ruptured) when what was once taken for granted comes to be questioned (the shift from Euclidean to non-Euclidean geometry; the shift from Newtonian to relativistic or to quantum physics) or when new practices or new measuring instruments are introduced (the shift from analogue to digital devices, the introduction of computer models).

Bachelard thus rejects the assumptions behind all foundationalist approaches to the philosophy of mathematics, which presume that mathematical reasoning is reducible to formal logical deduction and conceive of objectivity as requiring subjective passivity. His approach is Kantian in spirit (he is classified as neo-Kantian), even though he departs very significantly from Kant’s actual views about mathematics. He is thoroughly Kantian in his link between the demands of rigor and voluntary action. His account of objectivity extends from the community of mathematicians demanding proofs to the practices of laboratory life and scientific observation, where instrumentation plays a crucial mediating role. For him, phenomenology has to be replaced by phenomeno-technique while mathematics provides the space within which scientists dream and create possibilities.

This helped me out of the box I was in because it swept away the presumption that analysis required foundations. This enabled me to then convert my thesis into two books, *The Philosophy of Set Theory* and *Mathematics and the Image of Reason*.<sup>8</sup> That thesis had been prompted by a vague, ill-formulated unease at the way in which model theory was at the time being used both to provide a theory within which to study formal systems and their semantics, so providing completeness or incompleteness proofs, while at the same time being used as a tool for studying axiomatizations of set theory itself. This seemed to me to be the kind of reflexivity revealed by Gödel’s theorems and other paradoxes of self-reflection. This is why much of the focus of the thesis was on what produces those paradoxes.

But the box had already been partially unlocked by encountering Michael Dummett in Oxford. He had a longstanding interest in both philosophy of mathematics and its Intuitionist foundations (advocated initially by Brouwer). To see why his influence was important, I quote:

If we think that mathematical results are in some sense imposed on us from without, we could have instead a picture of a mathematical reality not already in existence but as it were coming into existence as we probe. Our investigations bring into existence what was not there before, but what they bring into existence is not of our own making. (Dummett 1978: 18)<sup>9</sup>

This is why *The Philosophy of Set Theory* sets out from the question, “Did Cantor discover the rich and strange world of transfinite sets, or did he create it?” And its conclusion, after my encounter with Bachelard’s work, was that, in a sense, the answer is neither. Cantor was exploring the infinite possibilities inherent in, and constrained by, the indeterminacy of the “formless” geometrical continuum, but by methods (that included new notation) that had the power to realize or actualize possibilities that could not have been realized previously. By assuming that he was revealing realities rather than exploring possibilities by setting up a framework for their representation, he was led to assume that the continuum has a structure which can be determined once and for all out of points. The subsequent history of set theory suggests that mathematics remains the art of the possible (the potential) even when it has come to terms with the actual infinite.

If Cantor had just created his transfinite numbers but they made no sense to anyone else, he would have achieved nothing and would not figure in histories of mathematics. What he had to do was convince other mathematicians that he had good reason for his innovations even as they went against some of what had been generally accepted. So his work had to be objectively assessable. It had to be the thought of a rational subject, not of an individual, and yet one which called into question one or more accepted norms.

History is vital here because moves to extend the number system and to arithmetize the continuum had been made since before the time of Descartes and were given particular impetus by Newton, given the central role of calculus in Newtonian mechanics. But calculus was itself on a very shaky footing. It was necessary to Newtonian thought but was mired in disputes with Leibniz about the need for infinitesimals, with Berkley who deplored appeals to the infinite, with Barrow and others. Reliance on mathematical intuition was not good enough. This makes understandable the quest for a secure foundation on which Frege (logicism), Hilbert (formalism), and Brouwer (intuitionism) all embarked. The problem lies in the contrast between the smooth, flowing continuum delivered by mathematical intuition and the need to put numbers on the points at which it might be divided. Calculus, with its higher order differential functions (and the increasing reliance on these functions in applications of mathematics), seemed to presume a continuum composed of points. The puzzle was how dimensionless points could add up to a continuous whole something with length. So Cantor, unlike his Chinese predecessors, was not working in isolation but within a community of mathematicians, regularly publishing his proof proposals and so subjecting them to the critical scrutiny of others. This grounds his sense of objectivity; he had to prove his results and communicate them. Cantor’s understanding that his results needed to be proved to the satisfaction of his interlocutors is important. The crisis in the foundations of analysis forms a significant backdrop against which Cantor’s attempts need to be assessed—it is another attempt to bring rational order to a domain which had been acknowledged to be lacking.

Logicism, despite the crucial advances in logic made by Frege, Russell, and Whitehead, had problems when it came to dealing with paradoxes (although this was not acknowledged by many realist philosophers of science who wrote as if it had been successful). Their quest for foundations was not primarily epistemological, not with how *we* come to know the truth of mathematical propositions but with how their truth can be proved. There is a presumed empiricism here; truths

are forced on us by encounters with an external reality, and the mind is as passively receptive in inference as when it perceives sense data. An alternative, formalist or conventionalist approach was suggested by Hilbert. According to this, forms of logic are linguistic forms. Hilbert, in his axiomatization of geometry, acknowledges that there is a shift in meaning for the term “point,” for example. The unit of meaning is now the whole set of axioms for a geometry, and one has to choose whether this is to be Euclidean or not. One cannot ask for the meaning of a term outside the context of the axioms in which it appears. In the light of Kuhn’s work (*Structure of Scientific Revolutions*),<sup>10</sup> this threatens any account of the history of science which supposes that it has been one of rational advance; instead, rational discontinuities have to be acknowledged.

Formalism and conventionalism threaten to turn reasoning into the reasoning of robots, something that can be turned over to machines (AI), or to matters of linguistic convention. How then can any objectivity be claimed for its rules? Bachelard rejects such positions by reverting to epistemology.

For him, reasoning demands the exercise of reason (a human faculty); the subject’s consciousness of their own cognitive activities cannot be dropped out of an account of scientifically objective knowledge. Bachelard too acknowledges discontinuities of many kinds in the history of a science, but they are part of its internal dynamics, when what was previously assumed without question (whether in experimental practice or in methods of proof) becomes called into question.

The continuum, however, remains the source of chaos theory, of uncertainty, and of potentiality. When teaching a course on the use of computer models in environmental science, I was struck by the fact that every computer model first has to digitize (use numerical methods) to replace the continuous functions that are regularly used in fluid dynamics, the dynamics which underlie many climate and other models. It was after all in the context of climate models that Lorentz first encountered what came to be called chaos theory.

More recently, and in retirement from teaching, my thinking about the acquisition of practical skills has been shaped by learning change ringing (which, like all skills, is much harder than it looks), by the philosophy/sociology of Pierre Bourdieu, who has written extensively on the theory of practice, and by reflection on surrounding events.<sup>11</sup> What sense does it make for a joiner with a lifetime of work experience to be told in his fifties that because he doesn’t, like most of his generation who have working class origins, have the right pieces of paper, he cannot pass those skills on, when we are short of those very skills? What sense did it ever make to shut down polytechnics and convert them to universities or to starve colleges of higher education of funds? The UK has persistently undervalued the skills involved in performing practical tasks (partly because mastery requires performance to seem effortless) while at the same time depending on people who have those skills for the smooth running of a just-in-time economy. This was as much the case during the Industrial Revolution and the expansion of the British Empire as it is today. Bruno Latour made much the same point in *We Have Never Been Modern*, when he pointed out that the whole Enlightenment program depended on, but refused to acknowledge the existence of, what he calls “hybrids”—technological products that humans have created but which do not exist in the natural world.<sup>12</sup>

### Brief Biography

I was born in March 1946, Halesowen, Birmingham (UK), to seriously but unconventionally religious parents who were going to be missionaries until I came along. My mother was socially ambitious (her father was from New Foundland); my father had been brought up in penury by his mother (his father had died at 80 when he was 10). They joined Moral Rearmament (headquartered in Geneva) and in Keswick that dominated their social life. We seemed to move every 3–4 years, so I have lived in Halesowen; New Malden, Surrey; Keswick, Cumbria; Amersham,

Buckinghamshire; and Nottingham, where I did my last four years at school. 1952 was a difficult year; my brother and I both had polio (I was hospitalized and coped by escaping to an inner, fictional life). In addition, both grandmothers died, my father's killed at 80 by the last London Smog, and my mother's in her fifties from heart trouble. By Keswick, I had developed the mental habit of commenting on things around me as if to Shakespeare or someone else; experience was coupled with commentary. In Amersham, we attended Quaker Meetings, with children being taken out part way through for discussion, which gave voice to some of my commentary.

In 1969 I married an American, Jim Tiles, who was a Marshall Scholar at Bristol. The US Congress had just ruled that conscientious objector status could be granted on nonreligious grounds. His family put pressure on him to return home, and he said I might as well get paid while he was away, so we got married. Given the circumstances, we had no long-term plans (although in fact the marriage lasted until he died in 2014). He trained as a medic, and that is why my first teaching job was in the US (1970–1971) at the University of South Carolina (where I taught 11 sections of logic in one year) while my husband did his military service at Fort Jackson. I then followed him to Oxford where I was very firmly put in an analytic box, if I hadn't been before. There were no permanent jobs, especially for a woman, so I did a whole variety of temporary things. The book on Bachelard sealed my fate in England; I became tainted by continental philosophy. But we were offered a temporary job in the US, at the rank of Associate Professor, at Swarthmore College by Hans Oberdiek, whom I had met whilst tutoring at Lady Margaret Hall. From Swarthmore we applied for all available US joint positions. We ended up in Hawai'i, where I was introduced to the department as the token woman; by hiring me they had discharged the federal requirement to hire at least one woman. While in the Department in Hawai'i I made sure that I was on appointment committees and two more women were hired—Tamara Albertini and, as part of a couple, Vrinda Dalmiya. I also started a discussion group specifically for female graduate students whom I sensed needed support at that time. After a while that ceased to be necessary. Since I left, most of the hires have been male.

Education: Bristol University, BA Joint Hons. Mathematics & Philosophy 1967, and PhD 1973; B.Phil. Philosophy, University of Oxford 1974; M.A. University of Cambridge, 1978.

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  - 10 Thomas Kuhn, *The Structure of Scientific Revolutions* (Chicago: University of Chicago Press, 1962).

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- <sup>11</sup> Pierre Bourdieu, *Practical Reason* (Stanford: Stanford University Press, 1998); *Pascalian Meditations*, trans. Richard Nice (Stanford, CA: Stanford University Press 2000).
- <sup>12</sup> Bruno Latour, *We have Never Been Modern*, trans. Catherine Porter (Cambridge Mass.: Harvard University Press 1993).