APPLICATION OF A Δ-ISOBAR RESCATTING MODEL TO THE $^{12,13,14}$C(p, $\pi^-$)$^{13,14,15}$O REACTIONS

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The achievement of a quantitative understanding of pion production and absorption in nuclei at the microscopic level is a central problem in intermediate-energy nuclear physics. Recent theoretical work has focused on the two-nucleon model, which relates the production (absorption) process in complex nuclei to the elementary NN + NNn reaction.

The philosophy and structure of our two-nucleon model and tests of elements of the model using the $^3$He(p, $\pi^+$)$^4$He reaction are described in the preceding two contributions to this annual report.\cite{1,2} We present here applications of the model to the A(p, $\pi^-$)A+1 reaction.

We choose the (p, $\pi^-$) reaction for early testing of our model because of the simplifying features of this reaction, which have been demonstrated by recent experiments carried out at the Indiana University Cyclotron Facility (IUCF). They result from the fact that only target neutrons contribute to $\pi^-$ production whereas both protons and neutrons contribute to $\pi^+$ production. This simplifies the nuclear structure input to the calculations and reduces uncertainties due to poorly known nuclear wave functions, making tests of the reaction dynamics more transparent. The primary aim of the calculations presented here is to test the reaction dynamics of our model; the sensitivity of the model to details of nuclear structure and other complications will be investigated in future work.

In view of the exploratory nature of this (p, $\pi^-$) study, we keep the nuclear structure input relatively simple. Explicitly, we neglect ground correlations in the various nuclei and keep, except for $^{12}$C, only $\frac{1}{2}$-shell active nucleons. The ground state of $^{12}$C contains substantial admixtures of $lplh$ configurations. For the other cases, our assumed pure wave functions should exhaust 80-90% of the "exact" wave functions. Studies employing more sophisticated nuclear structure input are in progress.\cite{3}

In the two-nucleon mechanism, the bulk of the momentum sharing is incorporated explicitly at the microscopic level. Corrections to account for higher-order multiple-scattering effects in the incident and outgoing channels are included via standard proton and pion optical potentials.

The proton distorted waves were generated using the optical model code SNOOPY\cite{4} with the "standard geometry" potential parameters of Meyer et al.,\cite{5} which account reasonably well for the forward angle ($\theta < 80^\circ$) differential cross section and analyzing power data. Investigation of the sensitivity of (p, $\pi^+$) calculations to non-standard radial shapes of the central and spin-orbit potentials we defer to the future.

The pion distorted waves were generated using the pion optical model code DWPIES\cite{6} with a modified Kisslinger potential, which has been used extensively in theoretical analyses\cite{7} of pion-nucleus elastic and inelastic scattering data over a wide range of energies. The potential parameters were calculated internally by the code directly from free pion-nucleon phase shifts. Only the lowest-order optical potential
was used. Studies of the sensitivity of the 
\((p,n)\) reaction to higher order contributions introduced 
by Johnson and Siciliano\(^6\) to account for pion double 
charge exchange data will be the subject of future 
investigations. The optical potential employed 
reproduces reasonably well elastic scattering data at 
the appropriate energy.

For both protons and pions, we use the technique 
of Charlton\(^8\) to make a Bessel function expansion of the 
radial part of the distorted waves:

\[ \chi_L(k,r) = \sum_n c_{2n}(k,n,j)j_L(k,n,r) \]

where

\[ c_{2n}(k,n) = \int_0^{\infty} \chi_L(k,r)j_L(k,n,r) r^2 dr \frac{2k_n^2}{\pi} \]

The normalization is chosen to yield for the 
coefficients

\[ c_{2n}(k,n) = \delta_n,1 \delta(k,n) \]

in the plane wave limit (no distortions). The \(k_n\)'s are 
chosen to minimize the number of states needed, and the 
same set of \(k_n\)'s is used for all L-values. Care must 
be taken in choosing the \(k_n\)'s: the values of \(k_n\) and 
\(k_{n+1}\) must not be too close together, and for large L's 
no more than one state may be included with a value 
\(< (L/R_L)\). The latter condition means that for the 
larger L-values, a few of the states with small values 
of \(k_n\) must not be used. The efficiency of this 
technique depends strongly on how many states are 
needed in the Bessel function expansions. This was 
determined by making a variety of calculations with 
different choices for the spacing and range of the \(k_n\) 
values and comparing the original distorted waves with 
their Bessel function expansions. It was found that 
good reproduction of the original distorted waves could 
be achieved by using 13 \(k_n\) values for protons and 17 \(k_n\) 
values for pions. The spacing and range of the \(k_n\) 
values used are shown in Fig. 1.

The formulation of our model\(^1\) includes the 
one-nucleon term and the two-nucleon terms 
corresponding to s-wave rescattering and both resonant 
and non-resonant p-wave rescattering (see Fig. 2); 
however, only the resonant p-wave (intermediate \(\Delta\) 
rescattering term is included in the present version of 
the computer code. The low-energy tail of the strong 
\(\Delta\)-resonance is expected to dominate the other channels 
even at energies well below the peak of the resonance.

To test this simple mechanism, we have applied 
the model to recent IUCF data\(^9\) on the 
\(^{12,13,14}\text{C}(p,n)^{13,14,15}_0\) reactions. The restrictive 
nature of the ground state transitions allows 
semi-classical, model independent predictions for the 
analyzing powers to be made.\(^9\) Assuming the dominant 
configurations shown in Fig. 3, the final two protons

\[ \begin{align*} 
\text{PROTON } K_n &- \text{VALVES} \\
K_1 &\rightarrow K_{13} \\
K_{PW} &\rightarrow K_n \\
K_1 &\rightarrow K_{17} \\
K_{PW} &\rightarrow K_n \\
\end{align*} \]

\[ \text{PION } K_n &- \text{VALVES} \]

\[ \begin{align*} 
\text{ONM} &\rightarrow \Theta \rightarrow \Delta \rightarrow K_n \\
\text{TNM} &\rightarrow \Theta \rightarrow \Delta \rightarrow K_n \\
\end{align*} \]

Figure 1. Choice of \(k_n\)-values for the Bessel 
function expansion of the proton and pion 
distorted waves.

Figure 2. Diagrams included in the 
formulation of the microscopic, meson-exchange 
model of pion production. Only the resonant 
p-wave (intermediate \(\Delta\)) rescattering term is 
included in the present version of the code.
are required by the Pauli principle to be in a spin singlet state. Conservation of angular momentum and parity then requires the initial proton and neutron to be in a spin triplet state. Since 200 MeV bombarding energy is subthreshold for pion production in free nucleon-nucleon collisions, the Fermi momentum of the active target neutron must be directed toward the incident proton. Then, for a polarized spin-up incident proton, active neutrons in the $\lambda+1/2\ (p_3/2)$ shell-model orbit are on the left side of the nucleus, and $\lambda-1/2\ (p_1/2)$ target neutrons are on the right side (see Fig. 3). This leads to the prediction that absorption effects should cause the sign of the analyzing power for the $^{12}\text{C}(p,\pi^-)^{13}\text{O}_{g.s.}$ reaction to be opposite to those of the $^{13}_1^{14}\text{C}(p,\pi^-)^{14}_1^{15}\text{O}_{g.s.}$ reactions. Our TNM calculations of these analyzing powers, including pion distortions only, are shown in Fig. 4a. They agree in sign with the semi-classical predictions. This agreement is a check of sorts on the complicated $(p,n)$ code, in so far that it produces qualitatively the results expected on physical grounds.

Although the model appears to behave as expected, nature doesn't! The signs of the measured analyzing powers shown in Fig. 4b are opposite to those of the TNM calculations.

A possible explanation for this apparent discrepancy between the measured and calculated analyzing powers for the $^{12,13,14}_1^{15}\text{C}(p,\pi^-)^{13,14,15}\text{O}_{g.s.}$ ground state transitions is the following. We have calculated only the resonant $p$-wave (intermediate $\Delta$) part of the two-nucleon mechanism (see Fig. 2). Since the isospin of the $\Delta$ and the nucleon are $3/2$ and $1/2$, respectively, the intermediate $\Delta N$ state can have $T=1$ or 2. This requires that the initial nucleons must be in a $T=1$ state. From the generalized Pauli principle, the allowed initial $\text{NN}$ states are then $^{15}_1\text{O}_{g.s.}$, $^{3}_1\text{P}_{0-},^{1}_1\text{P}_{-2-},^{1}_1\text{D}_{2+},^{3}_1\text{F}_{2-},^{3}_1\text{S}_{-4-} \cdots$. At low energies, the $\Delta N$ system will be predominantly in a relative $S$-state with total spin and parity equal to $1^+$ or $2^+$ (the intrinsic spins and parities of the $\Delta$ and $N$ are $3/2^+$ and $1/2^+$, respectively). Hence, only the singlet $D$ initial state can contribute to a process involving formation of an $S$-wave intermediate $\Delta N$ state. The

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Assumed shell model configurations and semi-classical predictions for the signs of the analyzing powers of the $^{12,14}_1^{15}\text{C}(p,\pi^-)^{14,15}\text{O}_{g.s.}$ transitions.}
\end{figure}
$^{12,13,14}_C(p,\pi^-)$ ground state transitions then are forbidden because they require the initial nucleons to be in a spin triplet state. These transitions are allowed for a p-wave intermediate AN state and this presumably is what the code calculates, but this is expected to be a relatively very weak process. Thus, other processes such as s-wave and non-resonant p-wave rescattering may predominate for these ground state transitions. The analyzing powers resulting from the interference of several competing processes may be quite different from those of the single resonant p-wave process which we have calculated. Fig. 5 shows the calculated differential cross sections for the $^{14}_C(p,\pi^-)^{15}O_{g.s.}$ transition, which are indeed several orders of magnitude smaller than the data at large angles. The data points themselves are very low compared to those for the strong $(p,n^-)$ transitions discussed below.

The $(p,\pi^-)$ reaction is known to populate preferentially high-spin states having a simple two-particle one-hole configuration with respect to the target nucleus. This results from the inherently high momentum transfer of the $(p,\pi)$ reaction together with a presumed simple two-nucleon reaction mechanism. Such a case is the $^{14}_C(p,\pi^-)^{15}O$ transition to the $7/2^+$ state at 7.3 MeV. For this case, the initial nucleons are not constrained to be in a spin triplet state and consequently the reaction can go via an intermediate S-wave AN state. Fig. 6 shows our TNM calculation and IUCF data for this transition. Both proton and pion distorted waves are included in these calculations. Note that these cross sections are much larger than those for the ground state transitions (compare with Fig. 5). The agreement between theory and experiment is as good as can be expected at this point, considering that no attempt has been made to fit the data by varying parameters in the model, and small admixtures in the nuclear wave functions have not yet been included.

Figure 5. Calculated and measured differential cross sections for the $^{14}_C(p,\pi^-)^{15}O_{g.s.}$ transition at $T_p = 183$ MeV.

Figure 6. Calculated and measured differential cross sections and analyzing powers for the $^{14}_C(p,\pi^-)^{15}O(7.3\text{ MeV}, 7/2^+)$ transition at $T_p = 183$ MeV.
In conclusion, the present calculations show that at 200 MeV bombarding energy the strong \(^{14}\text{C}(p,\pi^-)^{15}\text{O}\) transition to the \(7/2^+\) state at 7.3 MeV appears to go via a two-nucleon reaction mechanism dominated by intermediate \(\Delta\) formation. This bombarding energy corresponds to an outgoing pion center-of-mass energy of only 30 MeV, which is well down on the low energy tail of the (3,3) resonance.

For the weak \(^{12,13,14}\text{C}(p,\pi^-)^{13,14,15}\text{O}\) ground state transitions, intermediate \(\Delta\) formation is strongly inhibited, and competing processes such as s-wave and non-resonant p-wave rescattering, as well as the one-nucleon mechanism, probably are important. The interference of several competing processes will make realistic calculations for these cases much more difficult.

1) M. Dillig, J.S. Conte and R.D. Bent, this report p. 25.
2) J.S. Conte, M. Dillig and R.D. Bent, this report p. 28.
3) Z.-J. Cao, R.D. Bent, J.S. Conte, M. Dillig and H. Nann (private communication).

NUCLEAR STRUCTURE EFFECTS IN THE \(^{7}\text{Li}(p,\pi^-)^{8}\text{B}\) REACTION

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A microscopic pion exchange model\(^1,2\) for nuclear pion production near the pion production threshold has been under development at IUCF for several years. This model includes both one- and two-nucleon terms and deals with the multiple-scattering effects in the entrance and exit channels via proton-nucleus and pion-nucleus optical potentials. The present version of the computer code, however, contains only pion production through the (3,3) resonance (\(\Delta\)-isobar excitation). The relative technical simplicity of the code makes it possible to analyze systematically a large amount of existing data at IUCF. There is also the flexibility to employ realistic nuclear wave functions in the code, which allows investigation of nuclear structure effects in the \((p,\pi)\) reaction.

In this report, we present preliminary results on