Recent experiments at the IUCF Cooler Ring have shown that Siberian snakes can overcome depolarizing resonances and thus preserve the proton spin polarization during acceleration. However, we found that the $G\gamma = 2$ imperfection depolarizing resonance appeared to occur about 1.9 MeV below the expected beam energy of 108.4 MeV. Since the energy calibration of the Cooler Ring was believed to be accurate to better than $10^{-3}$, this apparent 1.9 MeV miscalibration caused some concern.

An alternate explanation for the observed discrepancy was that the spin tune, $\nu_s$, was somehow shifted upward by about 0.0036. This shift would make $\nu_s = 2.0036$ at 108.4 MeV and thus move the $G\gamma = 2$ resonance to about 106.5 MeV. Recently, Pollock suggested that the magnets in the electron cooling system of the IUCF Cooler Ring could act as a partial type-3 Siberian snake which might cause such a spin tune shift. A type-3 snake is a device which rotates the spin around the direction perpendicular to the ring plane. In the electron cooling region of the Cooler Ring, each proton’s spin is rotated by several magnets including the main cooling solenoid, two toroids, two compensating solenoids, four vertical steerers, and four weaker horizontal steerers. Since spin rotations do not commute, these magnets can rotate a proton’s spin and thus shift the spin tune, $\nu_s$. At a fixed energy, a shift in $\nu_s$ would change the proximity to any nearby depolarizing resonance. This proximity can be measured by studying how sharply the radial and vertical polarization components depend on the net longitudinal $\int \mathbf{B} \cdot \mathbf{dl}$ of the cooling magnets.

We recently stored and cooled vertically polarized proton beams at nominal energies of 104.6, 105.9, 106.9, 107.8, and 110.5 MeV. The stored beam intensity was 15 to 50 nA and the cycle period ranged from 4 to 20 seconds. The vertical and radial components of the
polarization were measured simultaneously using a 4.5 mm thick Carbon skimmer target and the cylindrically symmetric CEO1 detector. The beam polarization was measured before injection into the Cooler Ring using a beam line polarimeter; it was found to be 77 ± 2% at all incident beam energies.

At each beam energy we measured both the vertical and radial components of the beam polarization in the Ring, while varying the net longitudinal field, ∫B·dl by changing the currents in the main cooling solenoid and the two compensating solenoids and holding fixed the steerer and toroid magnet currents. At each energy, the width of the resulting vertical polarization profile was a measure of the actual proximity to the Gγ = 2 imperfection resonance, which should occur at 108.4 MeV. The data are shown in Fig. 1.

One can better parameterize this proximity by plotting the ratio Pradial/Pvertical against ∫B·dl. At each energy Pradial/Pvertical is proportional to the tangent of the angle φ between the vertical and the stable spin direction. Since ∫B·dl is small one can also show that Pradial/Pvertical is proportional to ∫B·dl by using equations 2, 3, and 5 of ref. 6,

\[
\frac{P_{\text{radial}}}{P_{\text{vertical}}} = -\tan \phi \cdot \sin \left(\frac{2\pi G\gamma}{3}\right) = S \int B\cdot dl. \tag{1}
\]

The \(\sin \left(\frac{2\pi G\gamma}{3}\right)\) term occurs because the polarimeter was 120° downstream of the Snake. The quantity, S, is given by

\[
S = \frac{e(1+G)\sin \left(\frac{2\pi G\gamma}{3}\right)}{2\sin(\pi \nu_s)}, \tag{2}
\]

where e and p are the proton's electric charge and momentum, while c is the speed of light.

The new experimental data are plotted against ∫B·dl in Fig. 1. The measured values of the vertical polarization are plotted on the left, while the measured radial polarization values are plotted in the center. The ratio of P_{\text{radial}} to P_{\text{vertical}} is plotted on the right. The solid lines in Fig. 1 are straight line fits to the ratio P_{\text{radial}}/P_{\text{vertical}}; the slope of this fit is the quantity, S, defined above. The measured slopes at each energy are plotted in Fig. 2 along with previously published 104.5 and 120.0 MeV data and a less detailed new data set at 110.5 MeV. The curve in Fig. 2 is obtained from equation 2 with the best fit value of \(\nu_s = G\gamma + 0.0036 \pm 0.0003\). This corresponds to a shift in the resonance energy of \(-1.9 \pm 0.2\) MeV.

We directly tested for the existence of a partial type-3 Siberian snake by measuring the beam polarization after turning off all the magnetic fields in the cooling system. Turning off these magnets should eliminate any possibility of a type-3 snake and thus give an absolute energy calibration. The ratio P_{\text{radial}}/P_{\text{vertical}} is plotted against ∫B·dl in Fig. 3 at 105.9 MeV. The dependence of the ratio on ∫B·dl was clearly much flatter with the cooling magnets off. This change clearly demonstrates that the spin tune, \(\nu_s\), and thus the resonance energy were significantly shifted by the cooling magnets. The best fit to the data in Fig. 3 with the cooling magnets off was \(\nu_s = 1.9949 \pm 0.0005\); this agrees well with the calculated value of Gγ = 1.9951 at 105.9 MeV. This agreement directly supports the hypothesis that the apparent 1.9 MeV energy miscalibration was caused by a partial type-3 Siberian snake due to the cooling magnets in the Cooler Ring.
Figure 1. The measured vertical (left) and radial (center) polarization at 104.6, 105.9, 
106.9, 107.8, and 110.5 MeV are plotted against the net longitudinal magnetic field integral 
in the Cooler Ring solenoids. Vertically polarized protons were injected into the ring. The 
narrow dips in the radial and vertical polarizations at 104.6 and 107.8 MeV are probably 
due to synchrotron depolarizing resonances. The ratio $P_{\text{radial}}/P_{\text{vertical}}$ is plotted on the 
right; the curves are straight line fits to the data at each energy.
Figure 2. The experimental slopes of the $P_{\text{radial}}/P_{\text{vertical}}$ curves are plotted against the energy. The curve is given by equation 2.

We plan to make more precise calibrations of the energy and the spin tune by using an rf driven solenoid to induce depolarizing resonances at various rf frequencies.\textsuperscript{8,9} The ability to induce a depolarizing resonance will also allow us to study overlapping depolarizing resonances, which are expected to present serious difficulties at TeV energies. In a preliminary test we showed that it is indeed possible to induce a depolarizing resonance. The measurement was made by connecting a transverse injection kicker dipole to the output of an rf knockout amplifier, which had been used in betatron tune measurements. The resultant oscillating vertical field of about 100 $\mu$T-m produced a few hundred $\mu$rad of spin precession per orbit in the ring. With the Siberian snake turned on, we then injected horizontally polarized beam at 105.9 MeV and varied the kicker driving frequency. The measured radial polarization is shown as a function of the frequency in Fig. 4. The plot shows an extremely narrow dip in the measured polarization. From the expanded view in the inset, the center of this dip occurred near a frequency of $771.0 \pm 0.2$ kHz.

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Figure 3. The measured ratio $P_{\text{radial}}/P_{\text{vertical}}$ is plotted against the net longitudinal $\int B \cdot dl$. Vertically polarized 105.9 MeV protons were injected into the ring. The data shown as squares at the top were taken with the cooling magnets turned on. The data shown as circles at the bottom were taken with the cooling magnets set near zero.
Figure 4. The radial polarization is plotted as a function of the rf kicker driving frequency. Horizontally polarized 105.9 MeV protons were injected into the ring.