Two preliminary beam transfer function experiments were performed this funding period. The most important objective for these experiments was to measure general properties of the Cooler Ring. From these measurements, we will determine the future direction of the beam transfer function experiment. First we measured the bunch length as a function of RF voltage and beam intensity. For a bunched beam at equilibrium in a synchrotron, the impedance of the ring can be measured from the threshold current, $I_{\text{thr}}$, the RF voltage, $V_{\text{rf}}$, and the bunch length at threshold, $\sigma_{\text{thr}}$:

$$\frac{|Z_{\|}|}{n} = \frac{F h V_{\text{rf}} \cos \phi_s}{\sqrt{2\pi I_{\text{thr}}}} (\omega_0 \sigma_{\text{thr}})^3,$$

(1)

where $F = 1$ for a capacitive impedance and about 1.4 for a resistive impedance, $h$ is the harmonic number, $\omega_0$ is the angular revolution frequency and $\phi_s$ is the RF stable phase angle. For our experiment, we have $h = 1, \phi_s = 0$ and $\omega_0 = 2\pi f_0$ with $f_0 = 1.03168$ MHz for a proton energy of 45 MeV. This simple well-known formula corresponds to the threshold of the turbulent microwave instability limit. Figure 1 shows the measured $V_{\text{rf}}^{1/3} T_{\text{FWHM}}$, where $T_{\text{FWHM}}$ is the full-width-half-maximum of the bunch length. Our experimental data show that the result can be fitted with $I_{\text{thr}}[\mu\text{A}] = (\frac{1}{83})^3 V_{\text{rf}}[\text{V}] T_{\text{FWHM}}^3[\text{ns}]$. From Eq. (1), we obtain the magnitude of the impedance to be $|Z_{\|}| = 4.8 \text{ k}\Omega$. It is well known that the low energy accelerators are dominated by the capacitive space charge impedance given by,

$$Z_{\text{sc}} = \frac{Z_0 g_0}{2\beta \gamma^2},$$

where $Z_0 = 377 \text{ } \Omega$ is the vacuum impedance, $g_0 = 1 + 2 \ln \frac{b}{a}$ is the well-known geometry factor with $b$ and $a$ the radii of the vacuum chamber and the beam respectively, and $\beta$ and $\gamma$ are relativistic Lorentz factors. A similar measurement at the IUCF Cooler Ring was performed and published which, however, omitted the factor 1 in the geometric factor $g_0$ and
led them to prematurely conclude that the beam distribution must be parabolic with zero momentum spread. A nonzero momentum spread plays a critical role in Landau damping, an important mechanism in producing stability against coherent oscillations. From such a measurement, the best one can conclude is that the magnitude of the impedance is near that of the space charge impedance alone.

Using the longitudinal beam transfer function technique, we measured the response of a coasting beam due to the excitation of the $h = 1$ RF cavity. Figure 2 shows a typical data of the inverse beam transfer function (BTF) in polar coordinates. The small loop in Fig. 2 indicates that the beam was unstable at the specific frequency. In our longitudinal BTF measurements we have observed narrow dips in the beam’s response to external perturbation at frequencies which are correlated to the electron energy in the electron cooling system. In Fig. 3 the location of these dips is plotted as a function of the electron energy. We are currently working to understand the physics of these sharp dips in the BTF. Also, effects of the linear coupling on the BTF and on the transverse feedback system have recently been studied. Data analysis is currently underway.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Plot of measured $V_{\text{rf}}^{1/3} T_{\text{FWHM}}$ vs. $I_{\text{thr}}$. The points marked with crosses are from Ref. 2.}
\end{figure}
Figure 2. Photograph of network analyzer measurement of the inverse of the BTF in polar coordinates.

Figure 3. Plot of electron energy in the electron cooling system vs. frequency location of the sharp dips in the longitudinal BTF.