

A Review of Allen Forte's *The Harmonic Organization of the Rite of Spring*

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Allen Forte's The Harmonic Organization of the Rite of Spring¹ is a study of the pitch structure of Stravinsky's ballet. It is intended to consider the structure of linear as well as vertical configurations — the harmonic structure of melody as well as chords. The book begins with an introduction to the concepts and vocabulary of set theory, closely following the organization of Forte's The Structure of Atonal Music, although in less detail. Included is discussion of pitch class set and its notation, normal order, cardinal and ordinal numbering and prime form, interval class and interval content. The concepts of transpositional and inversive equivalence, and the inclusion relation are explained, followed by a discussion of invariance and complementation. Finally, the inclusion relationships involved in set complexes K and Kh, and the indexes of similarity relations (Rp, RO, R1, R2) are covered.

In the next chapter, Forte presents the reader with tables showing the distribution of sets throughout the ballet. Although a great variety of sets is used in the work, certain sets can be seen to be more prominent, some appearing in every movement. As an example, Forte discusses the use of set 4-18 (and its complement 8-18) as a formal delineator throughout the work. (Note here that I will follow Forte's procedure in identifying measures in the score: R58+5 indicates the sixth measure after rehearsal number 58.) Referring to the harmonies used at R58+5, a comparison is made among versions of the passage found in the score, Stravinsky's piano duet version, and various earlier sketches. These sources, all somewhat different from one another, reveal Stravinsky's intent to cadence on 4-18, al-

¹Allen Forte, The Harmonic Organization of the Rite of Spring (New Haven: Yale University Press, 1978).

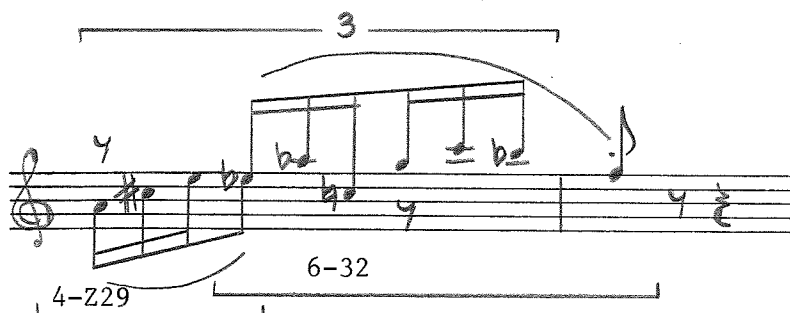
though not necessarily in the same transposition as in the final version. In one sketch, the final 4-18, although containing the same pitches as the final version, has an E in the bass instead of B-flat. Evidently, the sonority of 4-18 was of more importance than the actual pitch content or "root" of the final chord. (See Figure 1.) Forte's other

Figure 1. Three versions of R58+5

orchestra	4-18	5-6	4-Z15	4-18
score	[10,11,2,5]			[3,4,7,10]
piano	4-18	6-Z6	5-14	4-18
duet	[10,11,2,5]			[3,4,7,10]
sketch	4-18	5-6	4-Z15	4-18
	[10,11,2,5]			[5,6,9,0]

examples of 4-18/8-18 as formal delineator are somewhat problematic. Example 1 (p. 23) brings up the question of segmentation. Forte reads the passage as 8-18, leaving out the cadence pitch F. It seems more logical to read the passage as 4-Z29 (0137), the second flute line, followed by 6-32 (the major hexachord) in the first flute. Stravinsky's scoring of the passage for two flutes supports this interpretation. He could as easily have written it as a single line, for a single instrument. Also, the cadential F is certainly part of the line, and should be included in a linear analysis. (See Figure 2.)

Figure 2. Flutes 1 and 2 at R6+10



Example 4 (p. 27) illustrates the use of 4-18 at R84 (Introduction to Part 2). However, the set 4-27 is the accented vertical in the passage; 4-18 appears on weak beats, functioning as a neighbor to 4-27. The example would be clearer if the following measure were included, showing the return to 4-27. (See Figure 3.)

The bulk of the rest of the text ("Chronological Survey of the Work") goes through the ballet movement by movement, citing sets used prominently in each section. Generally,

Figure 3. R84-R84+2

4-27

4-18

Forte's Ex. 4

Forte refers to the 1921 orchestral version of The Rite of Spring. Other sources consulted are the 1913 piano duet score, prepared by the composer for rehearsals of the first performance, and the autograph collection The Rite of Spring: Sketches 1911-1913. The duet score, parts of which were written before the orchestral score was completed, and the sketches provide insights into Stravinsky's choice of sonorities (as in the above discussion of 4-18 at R58+5).

The chronological survey concludes with a "Summary of Harmonic Relations," in which are discussed the set complex and similarity relations of prominent sets in the work. Since such a great variety of sets is used in the work, some means of selecting the main harmonies must be found. The following criteria are considered in this selection process:

- (1) the number of times a given set appears,
- (2) its mode of occurrence (although this is never really defined in the text), and
- (3) the K and Kh relations of the given set.

Forte selects as the main harmonies three 7-note sets and four 8-note sets: 7-16, 7-31, 7-32, 8-16, 8-18, 8-23, 8-28.

The inclusion relationships among sets of the same cardinality are then illustrated. Each of the three 7-note sets contain as subsets all three of the complementary 5-note sets. (See Figure 4a.) Among the 8-note sets the pattern of inclusions is not as complete. Set 8-18 contains all four 4-note complements; 8-16 and 8-23 contain all but 4-28; and 8-28 contains only 4-18 and 4-28. (See Figure 4b.) In the text the inclusion relations are presented in a rather disorderly way, making the relationships difficult to

comprehend. Not specifically pointed out are the inclusion relations between the selected 7- and 8-note sets. Although they can be seen in examining subsequent tables in the text illustrating the K/Kh complexes about each of the sets, it is useful here to point out the relationships between these main sets. (See Figure 4c.)

Figure 4. Inclusion relations among
main harmonies of the work

(a) among the three 7-note sets

	5-16	5-31	5-21
7-16	X	X	X
7-31	X	X	X
7-32	X	X	X

(b) among the four 8-note sets

	4-16	4-18	4-23	4-28
8-16	X	X	X	-
8-18	X	X	X	X
8-23	X	X	X	-
8-28	-	X	-	X

(c) between 7- and 8-note sets

	8-16	8-18	8-23	8-28
7-16	X	X	-	X
7-31	-	X	-	X
7-32	X	X	X	X

Noting that all the 8-note supersets and 6-note subsets of the three main 7-note sets are used in the ballet, Forte then provides tables and some discussion of the K/Kh complexes about these sets, and their relationships to one another. Included are listings of the intersections of the set complexes, showing which sets the set complexes have in common. The intersection of the set complexes of all three 7-note sets, a relatively sparse set, is of particular interest (Ex. 115, p. 135 in the text).

A group of tables which "show the result of the operation symmetric difference (or exclusive union) on the set com-

plexes about the three 7-note sets" (p. 136) is then presented. These, which are the complement of the tables of intersection, simply show which sets do not form a link between two 7-note sets — an illustration of non-intersection of set complexes.

Next, the reader is given tables showing K/Kh complexes of 8-note sets, and the intersections of these set complexes, as with the 7-note sets. However, the discussion includes two additional 8-note sets (8-Z15, 8-12) which were not indicated as main harmonies at the beginning of the summary. Why were these sets added? What are the inclusion relationships between these and the other 8-note sets? The reader is never told. It is of interest to note that all of the entries in K(8-Z15) and K(8-12) appear in at least one of the K complexes surrounding the four other 8-note sets. Thus the inconsistency shown in introducing these new sets is not only most confusing, but totally unnecessary.

Two items of interest are brought up concerning set 8-28 in this discussion. All of the 5- and 6-note subsets of 8-28 occur in the work, and all of the 7-note subsets of 8-28 are 7-31.

The summary concludes with tables illustrating similarity relations among some of the more prominent sets of the work, as follows:

- Ex. 142 (p. 146) — main tetrachords (R0, R1, R2, Rp)
- Ex. 143 (p. 147) — main hexachords with their principal subsets and supersets
- Ex. 144 (p. 148) — "The Transitive Hexachordal Quintuple for the Relations R1 and Rp"

In the final example, five hexachords are shown, all of which are Rp, with the common subset 5-31 (which means, of course, that they are all members of the set complex K about 7-31), and all of which are R1, having maximally similar interval vectors.

At the conclusion of his discussion of the set-complex relationships of the main sets, Forte says, "Perhaps this provides some measure of the complexity of the work. I hope that at least it elucidates the complicated connection of components to some extent." (p. 142) I believe that these "complicated connections" can be drawn together into a more logical whole. Let us consider set 6-27 as a candidate for nexus set of the entire work. Forte himself cites it several times as an important hexachord. In the subcomplex Kh(6-27) appear sets 7-16, 7-31, 7-32, 8-28 and 8-18. As we can recall, these five sets also bear a strong relationship among themselves. (See Figure 4.) Each of the 7-note sets contain all three 5-note complements as subsets; both of the 8-note sets contain as subsets both 4-note complements; and 8-18 and 8-28 are both supersets of each of the 7-note sets.

Examples 143 and 144 in the text list the important hexachords in The Rite of Spring. All of the hexachords cited appear in either K(8-18) or K(8-28); all but one Z-related pair (6-217/43) appear in at least one of the set complexes surrounding the three 7-note sets. The set complexes about 8-16 and 8-23, the other two 8-note sets cited by Forte as main harmonies, contain less than half of these important hexachords. While 8-16 and 8-23 are not members of the set complex about 6-27, the set complexes of each of these 8-note sets have many points of intersection with K(8-18) and K(8-28), and both sets are members of the set complex about 7-32. Thus a secondary relationship to K(6-27) can be drawn. (See Figure 5.) At the center of all these interacting relationships, 6-27 can be seen as the unifying set in the complex harmonic structures of The Rite of Spring.

Figure 5. Primary and Secondary inclusion relationships of the main harmonies of The Rite of Spring

6-27		Kh	Kh	-	-
		8-18	8-28	8-23	8-16
Kh	7-16	Kh	K	-	-
Kh	7-31	Kh	Kh	-	-
Kh	7-32	Kh	K	K	K

In reading Forte's text, several questions concerning the style of writing, the analysis of the work itself, and the principles of set theory were raised in my mind. Set theory is a relatively new, and somewhat complicated analytical approach, especially when applied to such a large work as The Rite of Spring. Forte's writing style sometimes makes understanding these complex concepts even more complex. His organization is sometimes confusing, as with the presentation of the inclusion relationships among the main 7- and 8-note sets. At other times, his presentation is even misleading. One example of this difficulty is his discussion of interrelated sets at R67-R70+8 (text, pp. 63-65). Forte summarizes the interlocking inclusion relations of the sets important in the passage in two graphs (p. 64). The section contains two 8-note sets — 8-18 and 8-28. Each graph illustrates the inclusion relations among the sets of lower cardinality, and between those sets and one of the 8-note sets. Forte concludes his discussion by stating that "the union of the two diagrams would of course reveal an even more elaborate interaction of the main components" (p. 65).

This is not so. The union of the two diagrams would add no new information, nor reveal any additional complexities; it would simply point out which sets are not included in $K(8-18)$, which the second graph does, in any case. The inclusion relations shown among the sets of lesser cardinality are the same in both diagrams. The result of Forte's statement is that the reader exerts effort looking for additional relationships which are not there.

I would like to point out one outstanding exception to the "communication problem" discussed above. The introduction to the book, in which the basics of set theory are put forth, is a well-written, remarkably understandable presentation. In seventeen pages Forte covers most of the information contained in his The Structure of Atonal Music, although, of course, in much less detail. I highly recommend this introduction as a clear, concise explanation of set-theoretical concepts.

In his analysis of The Rite of Spring, Forte disregards two aspects of the music which seem to be of great importance: diatonicism and triadic chord structures. Commenting on the opening bassoon solo, one of the many instances of diatonicism in the work, Forte says, "Clearly, what Stravinsky wished to express with these diatonic formations is the folk mysticism of the ballet; however, the music in toto is far more complex and quickly breaks out of the sphere of naive diatonicism" (p. 300). Instances of diatonicism are mentioned, but not discussed in Forte's analysis. Diatonic formations need not be gone into in great detail; it can be assumed that the reader is familiar with their workings. However, these instances of diatonicism are a prominent feature of the work, and should be considered in any discussion of the ballet's harmonic structure. In regard to 6-32, Forte says, "The famous opening melody presents set 6-32 (the major hexachord), which is, of course, the source of many of the diatonic melodic figures that characterize certain parts of the music — notably themes and motives" (p. 29). Surely a major source of melodic material is an important element in the harmonic structure of any composition. However, Forte does not list 6-32 as a main hexachord of the work. Upon examination we find that 6-32 bears only a weak relationship to the 7- and 8-note nexus sets. 6-32 is, of course, a subset of 7-32, but is not related to either of the other two 7-note sets. 6-32 is a member of $K(8-23)$, but is not related to the other 8-note sets. The prominence of 6-32 as a melodic source does not fit in with the theory of set complexes. Since its function in the music does not support a basic set-theoretical concept, is 6-32 therefore to be disregarded? The author's characterization of diatonicism as "naive" perhaps reveals a prejudice on Forte's part. It appears, in fact, that Stravinsky's use of diatonicism in The

Rite of Spring moves far from naivete.

A possible related problem is Forte's treatment of large sets found in the work. Many of the verticals have the appearance of either decorated triads (triads with added pitches) or of combinations of triads. Robert Craft, in the preface to the sketchbook, states that Stravinsky's "first idea was the focal chord of F-flat major in the bass combined with the dominant-seventh chord of A-flat in the treble" (p. xvii), a reference to the famous "Augurs of Spring" chord. (See Figure 6.) Most probably the listener

Figure 6. "Augurs of Spring" chord, R13



does not hear the sonority as two superimposed tertian chords, but that is nonetheless the source of its structure. The passage following the introduction of the chord is really a more linear presentation of basically the same sonority, but Forte does not point this out. Referring to the passage at R14 (Ex. 19, p. 37), Forte says, "Far from being an arbitrary conflation of major triads in some vague polyharmonic sense, the passage forms sets that are of significance in a number of other parts of the work." Forte assumes here, as he does elsewhere, that the triad formations are incidental, or even irrelevant. While the sets he points out are indeed present, and are certainly significant, the relationship of this passage to the chord at R13, heard immediately before and after, is more pertinent, and more likely to be audible to the listener.

Forte states in the text that he "will make no attempt to cover such features of the music as tonality, large-scale linear connections, register, or orchestration" (p. 29). Yet some of these factors must be taken into consideration when making decisions on segmentation. For instance, in Ex. 1 cited earlier, Forte identifies the melodic line as 8-18 (ignoring the cadence on F). Examining the passage, we can see the decorated triad (A, C[#], E, plus E-flat) present in the second flute part; this aspect is disregarded by Forte. (See Figure 2.)

Example 17a (p. 35) is presented as an illustration of the prominent use of 5-31, which indeed it is. Note, however, the essentially triadic voicing of the chord. It

can be viewed (and heard) as a seventh chord built on E, containing both a major and a minor third.

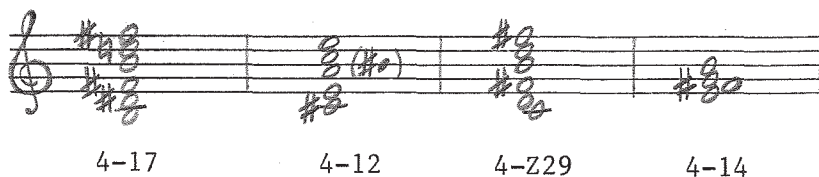
An interesting example of triadic structure occurs at R37 (text, Ex. 28a, p. 45). The chord presented is 6-27, which contains as a subset 5-31. In the following measure, F-sharp is added to the sonority, forming 7-31, complement of 5-31. The complex relationship of subset and embedded complement is notable. Even more notable is that C, the root of the triad formation at R37 and certainly a most important pitch in the sonority, is the one member of this statement of 6-27 which is not part of 5-31. The C, along with the added F-sharp, is the pivot on which the complement relationship hinges, and is important both aurally and set-theoretically.

Example 62 (p. 82) illustrates a careful harmonization of the Khorovod tune, a melody which appears in several variants throughout the ballet. The melody is based on the major pentachord (5-28). The verticals accompanying it are almost all significant sets in the work:

- (a) 5-31, 5-32, 4-28 are all nexus sets;
- (b) 4-229, 4-17, 4-12 are all members of Kh(6-27) and the set complexes about 7-16, 7-31 and 7-32;
- (c) 5-27 is a subset of 6-27;
- (d) 4-14 is a subset of 8-18.

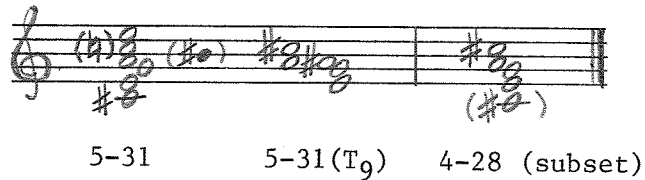
4-14 alone is a set which does not often occur in the work; its use here, however, is confined to weak beats. Parenthetically, since Forte does not recognize 6-27 as nexus set, he cannot justify the prominent use of 5-27 at R92-92+1, which he says "provides an exception to the generalization that metrically accented harmonies are significant" (p. 81). Note the structure of the tetrachords in this passage; each is a triad with a note added. The scoring of the chords reinforces the triadic nature of the

Figure 7. Tetrachords, R91-R92+3



sonorities. (See Figure 7.) The cadence at R91+1 is built on two statements of 5-31, using a transposition which yields the invariant subset 4-28, another triadic structure. (See Figure 8.) This is not at all to say that the harmony in this passage is functional, but to demonstrate that some complex sonorities in the work have their basis in the deco-

Figure 7. Sonorities at R91+1



ration of triadic structures.

The concept that "a set may be represented by its...complement" (p. 23) is central to the application of set theory. But is the complement relationship audible to the listener? I have severe doubts. It is difficult to believe that the absence of a thing can represent that thing. It is true that a relationship can be shown on paper, but that relationship is not necessarily discernable to the ear. A more accessible concept is that of the embedded complement; there the relationship is much more apparent. As an example of Stravinsky's use of this relationship, Forte cites the passage beginning at R47. Starting with 5-16 [5,6,8,9,0], a note is then added, forming 6-15 [4,5,6,8,9,0]. At R47+12 an additional note creates 7-16 [3,4,5,6,8,9,0], the complement of 5-16, "a beautiful example of Stravinsky's idiomatic manner of harmonic development and an instance of embedded complement" (p. 51). It is my contention, however, that the close relationship between 5-16 and 7-16 in this passage is dependent much more on the inclusion relation than on the complement relation.

Finally, Forte's disregard of sets smaller than cardinal 4 seems to be a basic flaw in his analysis. The prominent trichord motive of the "Augurs of Spring" movement, for instance, is ignored by Forte. In a footnote (p. 36) he explains that "in general this study does not deal with trichords, because they are easily identifiable components of larger sets." It seems to me rather more likely that the larger sets are, to the ear, expansions of the smaller. A theoretical approach to pitch which regards a prominent recurring motive merely as part of a larger complex of pitches seems not to be a valid approach to the study of the composition's structure.