"Harmonic Dualism" may be defined as a school of musical theoretical thought which holds that the minor triad has a natural origin different from that of the major triad, but of equal validity. Specifically, the term is associated with a group of nineteenth and early twentieth century theorists, nearly all Germans, who believed that the minor harmony is constructed in a downward ("negative") fashion, while the major is an upward ("positive") construction. Although some of these writers extended the dualistic approach to great lengths, applying it even to functional harmony, this article will be concerned primarily with the problem of the minor sonority, examining not only the premises and conclusions of the principal dualists, but also those of their followers and their critics.

Prior to the rise of Romanticism, the minor triad had already been the subject of some theoretical speculation. Zarlino first considered the major and minor triads as entities, finding them to be the result of harmonic and arithmetical division of the fifth, respectively. Later, Giuseppe Tartini advanced his own novel explanation: he found that if one located a series of fundamentals such that a given pitch would appear successively as first, second, third ... and sixth partial, the fundamentals of these harmonic series would outline a minor triad. Tartini's discussion of combination tones is interesting in that, to put a root under his minor triad outline, he proceeds as far as the 7:6 ratio, for the musical notation of which he had to invent a new accidental, the three-quarter-tone flat:

1 G. Tartini, Tratato di musica (1745), p. 66. The combination tones are given one octave too high, an error corrected in Di principe dell' armonica (1767).
It must be pointed out here that Tartini is scrambling mathematics, acoustics, and subjective phenomena; his exposition is less an explanation of the origin of the minor triad than a collection of interesting coincidences surrounding it.

Jean Phillipe Rameau (1684-1763) took his departure from the foregoing. In his Traité de l'harmonie (1722), he advocated an "altered-third" approach: the minor triad is a major triad, but the middle tone has been lowered. Soon after, Enlightenment science acquainted him with the phenomenon of resonance. Convinced that a lower string resonates at its fundamental pitch when activated by the sound of a sympathetic higher string, he proposed a genuinely dualistic theory in his Génération harmonique (1737): the major triad is formed by the upper twelfth and seventeenth, the minor by the lower twelfth and seventeenth, both being reduced to one octave. Thus "C" will generate both the C major and F minor triads. But by 1750, he had been forced by acousticians to admit that longer strings do not vibrate at fundamental pitch when excited by sympathetic tones.²

In 1803, the great German thinker and poet Goethe exchanged views with the composer Zelter on the subject of the minor triad. Zelter presented the minor triad as 6:5:4 (i.e., in string lengths, as harmonic division of the fifth) in opposition to the major triad (4:5:6). Goethe found this unsatisfactory, since only the major triad could be found in the overtone series. He reduced the problem to a syllogism:³

1. Musical practice recognizes the equality of major and minor;
2. The overtone series places this equality in question; therefore:
3. The overtone series does not suffice as an explanatory measure for both modes.

Later, in 1815, Goethe expressed the following opinion:

Thus, the fundamental C yields in an upward direction the C major harmony, downwards the F minor harmony. Major and minor are the polarity of harmony, the first principle of both. The major arises through ascending, through upward tendency, through an extension of all intervals upwards. The minor arises through descending, through downward tendency, through the extension of all intervals downwards....The carrying out of such opposition is the basis for all music.

The German Romantic Weltanschauung is much in evidence here: Goethe's pronouncements proceed from the notion that music (and all art) is somehow an extension of Nature, and that the origins of musical (and artistic) practices and phenomena are to be found in Nature.

Harmonic dualism as a complete musical theory was first seriously proposed by Moritz Hauptmann (1792-1868) in his book, *Die Natur der Harmonik und der Metrik*, published in 1853. The book remained popular enough to warrant a second edition twenty years later. Hauptmann, following a procedure then in vogue, organizes everything according to Hegelian dialectical principles — a method which has been termed both "entirely unnecessary" and "unsuitable." Hauptmann begins by asserting that there are (only) three directly intelligible intervals: the octave, the (perfect) fifth, and the (major) third, which are unalterable. He defines the octave as the interval in which half of a sounding quantity is heard against the whole; in acoustics, he says, it is the expression for the concept of unity. He equates the octave with "unity" because the half which sounds (against the whole) is equal to the remaining half. The
octave/unison becomes the "thesis" of a Hegelian dialectic. Hauptmann proceeds by defining the fifth as the interval in which two-thirds of a quantity is heard against the whole. This interval becomes "duality" in the dialectic, opposing the "unity" of the octave/unison, on the grounds that the two-thirds which sounds (against the whole) is twice the one-third left over, creating conflict within itself. The major third is defined as the interval in which four-fifths of a sounding quantity is heard against the whole. The sounding part is here quadruple the remaining part, that is, "two-times-two"; Hauptmann finds in the major third the "synthesis" of the dialectic: 9

In the quantitative determination of twice two, since the double is here taken together as unity in the multiplicand, and at the same time held apart as duality in the multiplier, is contained the notion of identification of opposites: of duality as unity.

Shirlaw points out that in this system, it is the third which renders the fifth consonant. 10

Having defined his materials, Hauptmann proceeds to the construction of triads. The three "directly intelligible" and "unalterable" intervals are designated by Roman numerals: the octave (unison) as I, the fifth as II, and the major third as III. The major triad is analyzed as a fifth and a major third belonging to a given root:

\[
\begin{align*}
I & \quad - \quad II \\
C & \quad e \\
I & \quad - \quad III \\
\end{align*}
\]

In Hauptmann's terms, this triad is said to be "referred to unity C," 11 that is, all of the pitches involved can be generated from C by means of one of the "directly intelligible" intervals. The minor triad, however, has no unity if referred to its nominal root:

\[
\begin{align*}
I & \quad - \quad II \\
F & \quad a, b \quad C \\
I & \quad - \quad III \\
\end{align*}
\]

8Hauptmann, loc. cit. Shirlaw, predictably, criticizes this also, as the ratio 2/3:1/3 is that of the octave.

9Hauptmann, op. cit., p. 20; quoted from the Heathcote translation, p. 6. The boldface type is from the translation.

10Shirlaw, loc. cit.

11Hauptmann, op. cit., p. 32.
(That is, all the pitches cannot be generated from one point by means of the "directly intelligible" intervals, in an upward direction.) Hauptmann therefore rejects this approach (the so-called "double-generator" theory) in favor of one in which the elements of the F minor triad are "referred to unity C": 12

\[
\begin{align*}
\text{II} & - I \\
F & - ab - C \\
\text{III} & - I
\end{align*}
\]

Hauptmann explains this radical procedure as follows: 13

The determination of the intervals of the triad have been hitherto taken as starting from a positive unity, a Root, to which the Fifth and the Third are referred. They may also be thought of in an opposite sense. If the first may be expressed by saying, that a note has a Fifth and a Third, then the opposite meaning will lie in a note being Fifth and Third. Having is an active state, being a passive one. The unity, to which the two determinations are referred in the second meaning, is passive: in opposition to the having of the first idea we find the second, being had. The first is expressed in the major triad, the second in the minor.

He also locates the major and minor triads in the harmonic series, as notes 4-5-6 and 10-12-15, respectively, and observes that multiples of these numbers will locate other like triads in the series. He finds opposition in the expression of these proportions as positive and negative powers: 14

\[
\begin{align*}
4:5:6 &= \frac{4:5:6}{1} = (4:5:6)^1 \\
10:12:15 &= \frac{1}{4:5:6} = (6:5:4)^{-1}
\end{align*}
\]

Hauptmann summarizes as follows: 15

In the major triad the unity is the positive

12 Ibid.
14 Ibid.
15 Ibid., pp. 32-33. Quoted in the Heathcote translation.
which determines; in the minor it is the positive which is determined.

The minor triad thus being of a passive nature, and having its starting-point above (not its most real starting-point, yet that which is determined as unity), and forming from it downwards, there is expressed in it, not upward driving force, but downward drawing weight, dependence in the literal, as well as in the figurative sense of the word. We therefore find in the minor chord the expression for mourning, the hanging boughs of the weeping willow as contrasted with the aspiting arbor vitae.

Pursuing the dialectical method still further, Hauptmann explains tonality as a "triad of triads"; he represents the structure of major tonality as follows: 16

\[
\begin{align*}
\text{I} & - \text{III} - \text{II} \\
\text{fa} & - \text{Ce} - \text{Gb} - \text{D}
\end{align*}
\]

Minor tonality begins as:

\[
\begin{align*}
\text{II} & - \text{III} - \text{I} \\
\text{d} & - \text{fa} - \text{Ce} - \text{Gb}
\end{align*}
\]

Hauptmann insists, however, that this does not yet form a complete tonality, because it lacks the positive unity of a major triad; this defect is remedied by the harmonic minor system, which has a major dominant triad: 17

\[
\begin{align*}
\text{II} & - \text{III} - \text{I} \\
\text{fa} & - \text{Gb} - \text{D}
\end{align*}
\]

The above is presented as the structure of C minor, although it has two triads of "unity G." The major tonality includes no two triads of the same unity. Hauptmann therefore press-\[\text{es the dialectic yet further, introducing a synthesis of 16Ibid., pp. 32-33.} \]
\[\text{17Ibid., p. 34. It must be pointed out that, if major and minor (positive and negative unity) were really consid-} \]
\[\text{ered to be of equal validity, the lack of a major triad in the fundamental structure of minor tonality would not be a greater (or lesser) defect than the absence of a minor triad from the basic outline of major tonality.} \]
HARMONIC DUALISM

In this system, there are two triads of "unity C." Hauptmann approaches this tonality from a logical (dialectical) and intellectual viewpoint, offering no musical examples to support his creation, even though mid-nineteenth century harmonic practice did include frequent use of the minor subdominant in major tonalities.

Besides the criticisms already mentioned in footnotes, Shirlaw finds fault with other aspects of Hauptmann's theory. One flaw lies in Hauptmann's idea of "negatively determined" intervals; Shirlaw points out that the interval remains the same regardless of the direction from which it is considered: the fifth c-g is the same fifth whether it is part of a C major or a C minor chord. Furthermore, Shirlaw feels that the "negative unity" of minor triads amounts to the same thing as "positive determination" from "double roots." He also criticizes Hauptmann for his inability to explain minor tonality, as practiced, with only a "trivial of triads"; this is especially true with respect to the melodic minor system. Lastly, and perhaps most significantly, Shirlaw faults Hauptmann's system of minor tonality because of the fact that two triads are "referred" to the unity of the fifth scale degree; this seems to leave the system without a tonic function. A change of harmony from, for instance, G major to C minor would not involve a change of unity, and would not be a harmonic progression; the fifth scale degree would have to serve both tonic and dominant functions. This argument would apply also to the "major-minor" tonality, which apparently has no subdominant function.

The next major writer to pursue harmonic dualism was Arthur von Oettingen (1836-1920), professor of physics at the University of Dorpat. His principal work, *Harmoniesystem in dualer Entwicklung*, appeared in 1866; a revised version was issued in 1913 as *Das duale Harmoniesystem*. The book is in essence an attack on Helmholz' description of the minor triad as a major triad with

---

18 Ibid., p. 35.
19Shirlaw, *op. cit.*, pp. 369-370. Shirlaw does not attack Hauptmann's unsupported assertion that there are only three directly intelligible intervals; on the contrary, he later builds his own theory on the same premise.
an altered third, Von Oettingen begins by criticizing Helmholtz' view that consonance depends upon the coincidence of upper partial tones; he demonstrates that some partials of the (consonant) major triad clash as severely as those of the tones of the (supposedly less consonant) minor triad:

He proposes that an analogy between major and minor may be found in the relationships of the two triads to their two strongest secondary sounds: the "tonic ground-tone" (chief combination-tone) and the "phonic overtone" (the first common partial).

Each construction produces a major seventh chord. Von Oettingen maintains that major has its unity or central point (Einheitbeziehung) in its fundamental, or ground-tone, and that minor has its union in the first partial common to all three tones.

He stops short of saying that 'g' is the root of the C minor
chord, but his "phonic overtone" comes very close to assuming that role.

Shirlaw criticizes von Oettingen on the grounds that the fifth is the same interval in both major and minor (a criticism also leveled at Hauptmann). He also finds fault with von Oettingen's criticism of Helmholz, maintaining that major is consonant with its upper partials and with its combination tones, while minor is consonant with neither. It must be pointed out, however, that von Oettingen had demonstrated that there is dissonance between the overtones of the members of the major triad—and that Shirlaw had quoted this demonstration. Von Oettingen extended his dualistic approach to functional tonality, which is beyond the scope of this article; he is probably more important for having catalyzed certain of Riemann's ideas. His "phonic overtone" idea is difficult to criticize in itself, because it can be heard (it is the pitch at which "beats" will be most strongly heard if the minor triad is out of tune); the notion that it is the source of unity in the same way as a real root is perhaps rather naive. This theory is important, however, for the influence that it had on others.

Hugo Riemann (1849-1919) was the central figure in the dualistic movement. A prolific writer on music theory, music history, and performance, and a compiler of dictionaries, he was not content merely to present his ideas to the public (many times!), but actively sought to justify harmonic dualism in quasi-acoustical researches and in what he saw as historical precedents. We will first deal with the main features of Riemann's theories regarding the minor triad, then examine his search for historical bases for his thoughts, and conclude by discussing his famous pursuit of undertones. The principal works involved are Musikalische Logik (1874), Musikalische Syntaxis (1877), Natur der Harmonik (1882), Musik-Lexikon (eight editions, 1882-1919), Handbuch der Harmonie (1887), Katachismus der Musik, and Vereinfachte Harmonie (1890). Fortunately, Riemann seems to have formed his opinions on this subject early and to have repeated himself often; there is relatively little development in his thought concerning the minor triad, except for an increase in conviction in the face of accumulating contrary evidence. Indeed, the tenacity with which Riemann held to his views against all criticism is one of the most

---

23 Ibid. This argument appears also in the works of other theorists, notably Hindemith.

24 For a treatment of von Oettingen's theory of functional tonality, and a somewhat different view of the central German dualists, see Mark Hoffman's A Study of German Theoretical Treatises of the Nineteenth Century (Ph.D. Dissertation, Eastman School of Music, 1953).
noteworthy features of his career as a theorist; his attitude on this matter colored even his historical researches.

Following Hauptmann, Riemann allows only three directly intelligible intervals. He accepts Rameau's explanation of the major triad as a result of the "natural" phenomenon of the overtone series, but finds that an equally "natural" explanation of the (to him) equally consonant minor triad is wanting. He seizes upon von Oettingen's "phonic overtone" and describes it as a "Prime." If a series corresponding to the intervals of the overtone series should be constructed downward from such a "Prime," the minor triad would appear where the major had been found in the overtone series - notes four, five, and six:

![Diagram of musical notes](image)

This "Prime" can be a real Prime only if the supposed downward-generated series actually exists. A great deal of Riemann's theoretical writing concerns this problem. He pointed out that the overtone series is produced by dividing a string by successive integers, and reasons that an "undertone series" should be the result of multiplying a given string length by the same succession of integers. In this he was mathematically correct. Riemann believed completely in the existence of this undertone series; he thought that he could hear them at the piano; in Musikalische Syntaxis he advanced the theory that the fibers of the inner ear co-vibrate at the lower 12th, 17th, etc.; and he felt that combination tones reinforced undertones and are proof of their existence. The character of Riemann's beliefs may best be expressed in his own words:

---

26 W. Mickelson, Hugo Riemann's History of Harmonic Theory and a Translation of Harmonielehre, p. 4.
28 Riemann, op. cit., p. 3.
29 Riemann, Musikalische Syntaxis, pp. i-vii, on the authority of Mickelson, op. cit., p. 5, the original being unavailable.
As the consonance of the major chord is explained not only by means of combination tones, but has its real foundation in the phenomenon of the overtones, so likewise for the completely adequate explanation of the minor consonance there is only necessary the opposite phenomenon of the undertones. Even if the existence of such a phenomenon did not admit of positive proof, nevertheless it must be remembered that the minor correlatives have a subjective existence, in that the major proportions may be measured downwards as well as upwards. I have already pointed out that the co-vibration of tones points the way towards the existence of a series of undertones; and the same may be affirmed of such acoustical phenomena as are furnished by the sounds produced by striking rods, metal discs, etc. [Klirrtöne]. If one takes a vibrating tuning-fork and allows the prongs to touch quite lightly a resonance box, or if one sets in violent vibration a loosely held metal plate or disc, there may be heard, instead of the proper tones of the tuning-fork or plate, the lower octave or twelfth, even the lower 15th or 17th, as well as lower undertones. It is even probable that every tone has not merely a series of overtones, but also a series of undertones, of the same proportions, but gradually becoming more feeble as they recede from the prime tone, and being more difficult to distinguish, that is, to separate from the klang of the prime tone, than the overtones.

Riemann also went to great lengths to find the seeds of harmonic dualism in the works of older theorists. He considered Zarlino the first theorist to have discussed the triad, and thus the father of modern harmonic thought. Zarlino explained the major and minor triads as harmonic and arithmetic divisions of the fifth (see above), resulting in the same intervals in reverse order. "Riemann blows this all out of proportion in his attempts to prove that Zarlino found the principle of harmonic dualism in the antithesis of major and minor."31 Zarlino considered the harmonic division of the fifth to be natural and the arithmetic division to be contrary to natural order; he found the major "cheerful" and the minor "plaintive"; consequently, he cannot be in substantial agreement with the harmonic dualists, who insisted upon the equal value of major and minor. 32

31 Mickelson, op. cit., p. 9.
32 Jorgenson, op. cit., p. 43. Zarlino (and Tartini),
Riemann considered Tartini to have been the first truly great thinker since Zarlino (to the pointed exclusion of Rameau, for reasons to be discussed presently). Riemann saw in Tartini's work a "revival" of dualistic thinking based on string lengths harmonically and arithmetically divided, producing the overtone series and major triad, and the "undertone series" and minor triad. Although the principle involved is related to von Oettingen's "phonic overtone," Tartini did not treat the common overtone as a prime. Tartini's explanation of the minor triad, however, is open to some of the same criticisms which may be leveled at the dualists.

Riemann ignored or discounted Rameau's explanations of the minor triad (see above). Regarding Rameau's volte face of 1750, Riemann wrote:

He had discovered the true principle but was talked out of it by the physicist D'Alembert, who told him that the lower strings did not vibrate sympathetically in their whole length, so as to give their fundamentals, but only in such fractions as correspond to the tone of the 'generator'.

Riemann also failed to mention that Rameau had first proposed the "double generator" theory of minor harmony. His prejudice is due, of course, to the fact that he considered D'Alembert wrong, and believed that lower strings would vibrate sympathetically in their entire lengths.

Riemann wished to supplant all other theories with his undertones, but could not provide an objective, scientific proof of their existence. In his article "Mollakkord" in his Musik-Lexikon, he defines the minor triad as a prime sounding with its (perfect) under-fifth and (major) under-third; he then concedes the usual definition, beginning with the lowest tone. He proceeds to express doubt as to how the minor triad can be heard in an upward direction, on account of the clashing of overtones. He erroneously credits Helmholtz with the "double generator" theory (see above), and however, have often been described as dualists, due to Riemann's biased accounting of their work. See Carl Dahlhaus' "War Zarlino Dualist?," Musikforschung X (1957), 286ff.

33 Riemann, op. cit., p. 12; quoted in Mickelson, op. cit., p. 73.
34 D'Alembert had developed a theory of his own from Rameau's passing interest in the "double generator" idea: see J. LeR. D'Alembert, Elements de musique theorétique et pratique suivant les principes de M. Rameau. See also Mickelson, op. cit., p. 75.
one Ottokar Hotinsky (Die Lehre von den Musikalischen Klängen, 1879) with a "triple generator" theory: c-g as a "c-klang," e\textsubscript{b}-g as an "e\textsubscript{b}-klang," and a\textsubscript{b}-c as an "a\textsubscript{b}-klang." These theories are dismissed in favor of the undertone-series explanation, which seems simpler and more "natural" than any other.

In his article, "Untertöne," in the same dictionary, Riemann admits difficulty in proving the existence of these undertones; yet he still insists on their reality:

The compiler of this dictionary has made repeated attempts to demonstrate the existence of the undertones, corresponding to the overtone series; in his Musikalische Logik he has demonstrated their objective existence in the ear, and from various signs, he thinks himself justified in believing in their objective existence. In his Katachismus der Musik (p. 79) he has shown finally by proof of a scientific character why, in spite of the commensurability of the vibration forms, a tone by summation of its vibrations cannot produce the undertone series, and that the question may thus be considered to be finally closed.

Shirlaw quotes this much, allowing the matter to seem indeed settled. But Riemann continues:

Each tone, of necessity, produces the whole series of undertones, but each, according to its ordinal number, so many times; the second, twice, the third, three times, etc., proceeding exactly thus, so that by interference, they neutralise one another.

This last paragraph is of particular importance in under-

\textsuperscript{36}Ibid., p. 934. I have been unable to locate Hotinsky's work for inspection; it seems not to have been very influential. Cf. Ortmann's theory, to be discussed presently.

\textsuperscript{37}Riemann, op. cit., p. 1460. Quoted from the Shedlock translation of an earlier edition (Riemann had not changed his mind!), p. 816. I have not been able to determine if these passages remained in the last edition prepared by Riemann. They were not in the twelfth edition (1967), although the editors still treated them seriously. The new Brockhaus Riemann Musiklexikon (1978) does not treat undertones at all. As recently as 1975, however, Riemann's theory of undertones was taught in all seriousness at the University of Bonn. (I am indebted to Susan McLean for this information.)
standing Riemann's actual position on the subject: far from conceding defeat, he has found a way to use the fact that undertones cannot be heard (or otherwise detected) to "prove" that they exist! It is certainly not clear how a tone could produce a given undertone (or overtone) more than once, or how such multiple undertones (or overtones) could neutralize themselves or each other by interference. It is clear that overtones do not behave this way, and there is no reason to believe that a (hypothetical) mirror image of the overtone series should produce such radically different acoustical phenomena. It is unfortunate that Riemann allowed himself to become so obsessed with this notion, as it colored and distorted his otherwise outstanding and scholarly researches. 38

The last German theorist of the dualistic school was Sigfried Karg-Elert (1877-1933), whose Polaristische Klang- und Tonalitätslehre (Harmonologik) appeared in 1931. 39 Karg-Elert takes dualism as his starting-point, a sine qua non, and proceeds through extended tuning cycles upwards and downwards, arriving at a musically impossible situation involving "enharmonic" intervals, many diéses, etc. He is very much concerned with thirds in all of this, going so far as to include the small minor third (7:6). Regarding the perception of various intervals, he is an enthusiastic subscriber to the dualistic position, maintaining that harmonic intervals can be perceived only 'upwards' or 'downwards', and that symmetrical harmonies are therefore ambiguous to the ear. (Symmetrical chords, such as the diminished seventh chord or the augmented triad, are certainly ambiguous functionally; we will return to the perceptual aspect later.) This aspect of Karg-Elert's theoretical framework is expressed in a chart (Figure 1). Karg-Elert's "Polaristische" theory is often rather polemical: 40

38 It is perhaps significant that no mention of undertones or dualism is made in the articles on Riemann in both the 5th edition of Grove's and the New Grove; in both cases, emphasis has been placed on his historical work and on his theory of functional harmony. In the New Grove, the (unsigned) article on "Dualism" omits Riemann altogether, making von Oettingen the central figure in the movement. While von Oettingen's role is certainly important, the complete omission of Riemann's efforts is, in this writer's opinion, a serious shortcoming.

39 This rich and complex work remains in need of thorough investigation; particularly interesting would be a comparison with Hindemith's ideas.

40 Ibid, p. ii; p. 18; Quoted in the translation given by Jorgenson, op. cit., pp. 31-32.
HARMONIC DUALISM

Karg-Elert's dualistic account of intervals

- Perceived as C major or c minor
  [Third lacking!]
- Perceived as F major or d minor
  [Fifth lacking!]
- Perceived as E♭ major or g minor
  [Prime lacking!]
- Perceived as F♯ major or a minor
  [Third and Fifth lacking!]
- Perceived as B♭ major or f minor
  [Prime lacking!]

In use, as an inversion-form (a) perceived as D major or f minor; in normal form (b), as C major and g minor

The theory which disregards effective logic — which regards the minor triad as no natural phenomenon but only a poor contrivance, derived from an a priori framework of nature — brings into question whether one should degrade the minor triad to be a major dissonance or a deformed formation of nature (such as the theory of the lowered third).

Since today the minor chord is still considered by many theorists to be a major chord with altered third, one had better cry than laugh! A hand-worker knows the distinction between his materials better than these people, who consider themselves to be musically learned.

Karg-Elert constructs a very elaborate theory of tonality, based on dualistic principles, which is beyond the scope of this article. This work represents the furthest extremity of dualism, but came after the heyday of the movement, and had little lasting effect.

Meanwhile, harmonic dualism had spread beyond Germany. The French composer Vincent d'Indy enthusiastically promoted his own brand of dualism in his Cours de Composition Musicales (1909 and later editions). In Livre I, Chapitre VI
Overtone series (d'Indy's résonance supérieure)

"Undertone series" (d'Indy's résonance inférieure)
"L'Harmonie"), he derives the major triad from the division of a string by successive integers or the multiplication of a frequency by successive integers, a process which produces the overtone series. D'Indy calls this "résonnance supérieure." He then determines the minor harmony by the inverse procedure, multiplying string lengths and dividing frequencies; this yields the "undertone series," which he calls Resonnance inférieure.

Figure 3 Major and minor modes, after d'Indy.

Résonnance supérieure (Major mode)  
Résonnance inférieure (Minor mode)

Authentic Cadence:  
Dominant tonic

Plagal Cadence:  
Subdominant tonic

The minor authentic cadence (B) is a simple transposition of the plagal major (C) into the minor mode; likewise, the major authentic (A) is a simple transposition of the minor plagal (D).

And so, these two forms result:
"résonnance inférieure" (Figure 2).\(^{41}\) He goes on to state that the "prime" of the major triad is the lowest tone (of c-e-g, c); the "prime" of the minor harmony is the highest tone (of a-c-e, e).\(^{42}\) Finally, d'Indy constructs two scales: "supérieure" (major) and "inférieure" (Phrygian, i.e., the inversion of major). The chapter concludes with a presentation of the overtone and "undertone" series, each through the sixteenth partial.

In the next chapter ("La Tonalité"), d'Indy applies this inversion principle to functional harmony, reversing the terms "authentic/dominant" and "plagal/subdominant" vis-à-vis the minor mode. This has the effect of equating the minor subdominant occurring in the minor mode with what we would call the major dominant occurring in the minor mode: to d'Indy, they are both altered subdominants (Figure 3).\(^{43}\) This owes much to Hauptmann, but avoids the earlier theorist's heavy-handed dialectical approach in favor of a more complete symmetry. D'Indy also includes a brief chapter (IX, "Histoire des Théories Harmoniques") on the history of harmonic theory, which is dependent on Riemann and amounts to a rather simple (almost naïve) justification of dualism.\(^{44}\)

In an article, "The Minor Harmony" (Musical Quarterly, October, 1931), Matthew Shirlaw (1873-1961) entered the fray. He begins by summing up the state of knowledge regarding the minor triad—as he sees it. First, he discusses a theory (which he credits to the French and Germans, without naming sources)\(^{45}\) in which the minor triad is found in the (dominant) thirteenth chord:

![Minor Triad](image)

This explanation is dismissed on the grounds that G cannot be the root of the a minor triad, if C is the prime of the scale.\(^{46}\) He then attempts to locate the same triad in the C harmonic series, as partials 13, 16 and 20 (see Figure 2). This also proves unsatisfactory, as the ratio of a-c (13:16)

\(^{41}\) V. d'Indy, Cours de Composition Musicale, 5th ed. (Paris: Durand et Fils, 1912), Livre I, 95-100.

\(^{42}\) Ibid., p. 100. Cf. Hauptmann, von Oettingen, Riemann.

\(^{43}\) Ibid., p. 109-111.

\(^{44}\) Ibid., pp. 133-142.

\(^{45}\) The author has been unable to find any other report of this theory, much less an original source.

\(^{46}\) M. Shirlaw, "The Minor Harmony" (MQ, Oct. 1931), 509. Shirlaw ignores the fact that the minor triad can be found in lower-level tertian sonorities; cf. Ortman infra.
is not a minor third, and the fifth a-e (13:20) is not a perfect fifth. If the overtone series on G is used, c and e are both so out of tune that they cannot be used in C major.\textsuperscript{47} He then raises the view that major is natural and minor is artificial, only to dismiss it by saying that the fifth a-e is nature's fifth and the third c-e is nature's third; in short, all the intervals of minor are found in major.

Shirlaw states that if minor harmony is considered a descending formation, it must arise from the same natural order as major, but in the opposite direction. Here he is proposing, essentially, harmonic dualism. He proceeds to compare string length and cycles-per-second for the major and minor triads, and finds that they are the reverse of one another:

<table>
<thead>
<tr>
<th>Major</th>
<th>Minor</th>
</tr>
</thead>
<tbody>
<tr>
<td>string lengths</td>
<td>string lengths</td>
</tr>
<tr>
<td>Hz</td>
<td>Hz</td>
</tr>
</tbody>
</table>

These ratios are expressed in musical notation as follows:

\begin{center}
\includegraphics[width=0.5\textwidth]{harmonic_dualism}\end{center}

Shirlaw claims that the foregoing is the extent of knowledge concerning the minor harmony. He reports that Dr. Riemann "exerted himself, but without success, to find an objective series of 'undertones' corresponding to the series of overtones."\textsuperscript{48} Shirlaw considers that if the minor harmony is to be perceived in the same way as the major (upwards), then it must signify duality (a conclusion necessitated by the fact that Shirlaw, like Hauptmann and Riemann, allows for only three directly intelligible intervals: octave, perfect fifth, major third.) Since Shirlaw wishes to avoid this

\textsuperscript{47}Ibid. Shirlaw gives no reasons for his choice (or omission) of series, or other possibilities in the series used. He seems to be concerned not only with explaining minor harmony, but with explaining it in terms of the relative minor; in short, he is mixing acoustics with functionality, assuming that not only the materials, but also the practice of music, are "natural."

\textsuperscript{48}Ibid., p. 510.
particular form of dualism, he dismisses the "double generator" theory, and reports that all writers except Helmholz agree that minor harmony can be conceived and appreciated, as a total unity, only as the inversion of the major harmony.49

Shirlaw then switches to a more subjective tack:50

Dismissing for the moment ratios and proportions, we may at first concentrate on a certain distinguishing feature of the minor harmony about which probably the majority of musicians are agreed. It is this, that while in the major harmony the tonal weight seems to gravitate towards and centre in the fundamental note, in the minor harmony, which is allowed to retain some at least of its original purity, and is not approximated to what we may call its tonic major harmony, the sound that impresses the ear as of quite peculiar importance is not the reputed fundamental note but the fifth: i.e., in the minor harmony a–c–e, not a, but e.

Shirlaw recognizes that objection may be made on the grounds that the minor harmony is "often"51 used as if its fundamental were the same as it would be for its parallel major. He counters by saying that such usage is not purely minor, but is approximated to major, and is heard as duality. This comes close to Helmholz' "Getrubte Dur-Klang" theory and, like that theory, contains a contradiction that is removed by the Picardy third. Shirlaw takes the widespread usage of that device as evidence that his theory is correct.52

Shirlaw must now find some model of "pure minor," for which he turns to the inversion of the major scale, which he calls "Palestrina's Phrygian mode":

49Ibid. This last assertion contains a weakness: if minor and major are equally valid, why must he claim that minor can be heard only as the inversion of major? No one has insisted that major is to be heard as the inversion of minor! Cf. Ortmann, infra.

50Ibid., p. 510.

51Shirlaw's word "often" is a gross understatement of the musical practice he purports to explain.

52Ibid., p. 512.
What Shirlaw calls its dominant (c), mediant (a) and final (e) constitute in his view the purest form of minor harmony. If c-e-g represents the complete major harmony c-e-g-c (upwards), then a-c-e represents the complete minor harmony e-c-a-e (downwards). Shirlaw maintains that pure minor harmony does not possess any fundamental, but "floats, as it were, in the air." Examples of this "pure" minor are, to be sure, extremely rare. Shirlaw offers only the "Allegretto" from Beethoven's Seventh Symphony, and of that, only the opening and closing chords:

"For what other reason should Beethoven elect to begin and end with what appears to be a 6/4 chord, the weakest position of the triad a-c-e?" Harmonic dualism will be criticized generally, later in this article, but this particular question will be answered now. Regarding the opening, Beethoven may have used the unusual inversion simply to create instability — he was quite famous for beginning pieces in a jarring manner. As for the ending, a glance at the full score shows that Beethoven considered the fundamental, if not also the bass, to be 'a':

---

53 Ibid.
54 Ibid., p. 513.
55 Ibid.
The octave a's in the 'celli and bassi are both below the low e of the second horn. In piano music, these tones would probably be sustained by the pedal; piano notation often assumes that this technique will be applied. It is not unreasonable to wonder, then, whether these tones were intended as the true bass of the chord. In any case, Beethoven's counterpoint (which Shirlaw conveniently ignores) makes it very clear that a is the tonic and root of the chord in question. Furthermore, although Shirlaw attributes the "pure" minor scale (Phrygian mode) to Palestrina, he certainly cannot cite so free a usage of second-inversion triads in the works of that master.

Shirlaw then pursues the problem of hearing minor harmony in the way in which he has described it. He claims that the third arises through the division of the fifth, and that only the major third is comprehensible in itself. Thus, c and e can be related (although the ear remains "baffled," as a major third "presupposes" a harmonic fifth), but a and c cannot be related in the ear. In the minor harmony, the third and fifth must be generated downward, by arithmetic division of the fifth. The minor third "searches" for a major third with which to be united; combination tones and overtones are both possibilities:

"The minor third, unintelligible in itself, becomes intelligible through its union with the major third."56

56 Ibid., p. 513-514.
Shirlaw finds no difficulty in two-way perception: 57

That the ear should possess the ability to appreciate harmonic intervals in two directions is not so very astonishing. In nature we have both height and depth and we rightly distinguish between them. And yet these are one and the same thing, but considered in different directions.

This would seem to be enough, except for objective proof, or at least a substantial body of examples from musical literature, but Shirlaw at this point introduces the fact that the minor triad may be located at partials 10, 12, and 15 in the harmonic series. He seems to ignore the fact that this would have belonged to the first section of his article more than to the last, and that it weakens most of his arguments. He states that "In such a descending series the multiples actually vibrate in their totality." (Shirlaw is here merely considering these partials from the highest to the lowest, which he considers the normal order for a minor triad.) If this statement were literally true, of course, Riemann would be proven correct and the argument closed. Almost in the next sentence, however, Shirlaw contradicts himself, and tells us that the lower string will vibrate only in sections corresponding to the pitches of the higher partials. 58 Shirlaw's discourse on the minor triad, which had been to this point orderly if unsubstantiated, is now thrown into considerable confusion; Shirlaw therefore resorts to eloquence: "The minor harmony in its true aspect, although it is the reflection of the major harmony, has none of the brightness of the latter but is as moonlight to sunlight." 59

The above-mentioned writers constitute the mainstream of harmonic dualism. Hauptmann and von Oettingen played key roles in formulating the principles of the movement, but Riemann occupies center stage, having been the most industrious, most zealous, and most determined advocate of the idea; consequently, his work, which represents the apex of dualistic thought, must bear the weight of any general criticism of dualistic thought. To begin with an argument of a negative character: suppose that undertones do exist for all pitches, just as overtones do. In that case, the clashing of secondary tones should be as violent for the major triad as for the minor; negative combination tones should exist, consonant with minor and dissonant with major

57 Ibid., pp. 515-516.
58 Ibid., p. 517-18. I have no convincing explanation for these sudden about-faces.
59 Ibid., p. 523.
— but no one has claimed to have heard them. The fact that these dissonances have not been heard, even by those who claim to have heard undertones, leaves the existence of the phenomenon open to doubt. It seems that the origin and consonance of the minor triad are the primary reasons for pursuing the undertone theory (or, for that matter, the "phonic overtone," "double generator," or "downward appreciation" theories); but, in fact, if the undertone theory (or any other of these theories) were substantial, the origin and consonance of the major triad would be equally troublesome.

The existence of undertones may be disputed on physical grounds also. Even if strings were to covibrate at a pitch lower than the stimulating tone, the lower pitch would be a property of the second string, not of the original sound source. That is, the lower pitch could not exist in the absence of a properly tuned lower string, and the source could not be said to produce undertones. A string (or any other medium) produces overtones (easily demonstrated as harmonics) because it can vibrate in sections of itself, but undertones could exist as a property of a single sound source only if the medium could vibrate in sections longer
than itself, which is patently impossible for a string: it would require that the string be loose at one end (or both), in which case it could not retain the tension required to make it vibrate at all (see Figure 4).

Von Oettingen's "phonic overtone" is not immune to this attack, nor is Tartini's sequence of combination tones. Both rely on separate and different overtone series, the lowest pitches of which are in effect arbitrarily chosen. The separate series are not all produced at the same time by the common overtone; they are a result of a prearranged minor-triad formation, and are neither the source nor the explanation of it. One could as easily claim that the following chord is consonant because of a common overtone shared by its members:

\[
\begin{array}{c}
\text{C} \\
\text{G} \\
\text{E} \\
\text{B} \\
\text{F} \\
\text{D} \\
\end{array}
\]

The single gravest error made by the writers discussed to this point, however, is the assertion (made by Hauptmann and accepted unchallenged by the others) that only the octave, perfect fifth, and major third are directly intelligible. Riemann points out that those intervals with the simplest ratios are most easily heard, which is probably true, but he accepts Hauptmann's dictum at face value. Perhaps intervals of more complex ratios are more difficult to hear, but it is unreasonable to limit human perception to these three intervals without the slightest offering of evidence, experimental or otherwise. Furthermore, all of these theo-

---

60 The "subharmonics" employed by George Crumb in his quartet Black Angels would seem to provide contrary evidence. It is beyond the scope of the present article to explain this interesting technique in detail, but it is necessary to point out that it involves bowing a string with a great deal of pressure, at a nodal point—thus, the string producing "subharmonics" is not a freely-vibrating medium. The high-pressure bowing effectively stifles a proportion of the string's vibration, causing it to vibrate at a lower frequency than normal. This is not the same thing as harmonics, in that these lower tones are not present in the string's sound under any other circumstances.

61 Riemann, Harmony Simplified, p. 2.
rists exclude the fourth from the list of directly intelligible intervals even though its ratio (4:3) is simpler than that of the major third (5:4). However convincing other arguments for or against harmonic dualism may seem or be, one has only to expose the weakness of this basic and unfounded assumption (that is, one has only to learn to hear the minor third), and the entire house of cards collapses.62

Otto Ortmann (b. 1889), in his article, "The Fallacy of Harmonic Dualism," which appeared in the Musical Quarterly in July, 1924 (and thus antedates Shirlaw's article), takes an altogether different tack with regard to the problem of hearing the minor triad. He deals with the physiological and, to some extent, the psychological aspects of hearing, and of cognition in general. He begins by pointing out that endpoints tend to be more clearly perceived than intermediate points (other factors being equal), thus leaving three possibilities, musically speaking: a tone may be "referred" to the highest tone, to the lowest tone, or to some other tone which, according to the particular method of chord-construction in use, becomes the lowest or highest tone in terms of perception.63 If all tones heard were referred to the highest tone, a system of harmony based on a "figured soprano" would result. This statement becomes the basis for a negative proof of the invalidity of harmonic dualism: if minor were the opposite of major (the difference being determined solely by the direction of generation), then a descending harmonic system (i.e., figured soprano) would be as readily conceived. Since no such system has ever evolved, Ortmann concludes that the premise must be false.64

Ortmann must concede the visual and mathematical symmetry of the dualistic system, citing the Major-Phrygian inversion, the overtone-undertone series symmetry, and the ratios of the major and minor triads:65

\[
\begin{align*}
\text{Major} & \quad \frac{4}{4\cdot5\cdot6} \\
\text{Minor} & \quad \frac{6\cdot5\cdot4}{6}
\end{align*}
\]

In practice, however, he finds that the minor triad is not treated as the equal of the major. He points to the longer acceptance of the major triad, and to the overwhelming

---

62 It is worth noting here that the Kodály method widely used in teaching music to children takes significant advantage of the fact that in many different cultures, the first interval a child learns to sing is the minor third.

63 This last possibility is not very clearly defined; Ortmann may mean here "root."


65 Ibid., p. 371.
HARMONIC DUALISM

preponderance of major tonality in popular music (excluding "associative" pieces, such as pseudo-oriental numbers, etc.) He also cites the Picardy third as an example of the inequality of major and minor: no similar cliche exists in reverse. (Chopin's B Major Nocturne is mentioned, but Ortmann, without naming sources, reports that this piece is sometimes played and has even been printed with a major close — which Ortmann takes as evidence of the non-acceptance of a reversed Picardy third.)

The Tierce de Picardie is a concrete instance of the reaction to the minor triad as a dissonance, and to the major triad as a consonance. If to-day this difference is no longer clearly marked, the primacy of major in both theoretical and practical treatment, which I have attempted to point out, although it may be obscured, still exists. Psychologically, it could not be otherwise.

Further aural evidence against dualism is offered, this time aimed at the undertone series. Ortmann invites the reader to play at the piano, pedal depressed, the overtone and undertone series, each through the seventh partial. He says that the unequal degrees of dissonance should convince the listener of the inequality of the two series. Ortmann also states (with no support) that students commonly know major scales, but not minor scales, etc., and that students generally prefer major to minor. He also reports that he has examined a large number of textbooks and found that, without exception, all treat major first. What Ortmann considers his "most potent proof," however,

---

66 Ibid., p. 374. Ortmann is obviously and ethnocentrically referring to Northwest European popular music.
67 Ibid., p. 373. This writer would be interested in any printings or performances of the piece in question that employ a major close.
68 Ibid., p. 374. Ortmann errs by writing "6th partial"; the notated examples clearly indicate that he means the 7th. He notates this as a minor seventh above or below the fundamental of the series. The term "partial" is inappropriate for the undertone series — "multiples" would be the correct term. Ortmann completely ignores the problems of temperament and intonation, especially concerning the seventh partial.
69 Ibid., p. 372. This assertion is weak in the absence of objective data; Ortmann has again ignored cultural factors.
70 Ibid. Hauptmann and Riemann also treated major before minor.
has to do with the "place theory" of pitch perception. He diagrams the "unrolled" cochlea as follows:

He claims that low tones have a larger area of sensation, and therefore always will be perceived as "bigger" and more important than high tones. 71

Having dispensed with downward derivations of the minor triad, Ortmann attempts to present a viable alternative. In fact, he presents two viewpoints, one harmonic, the other melodic. He states that the primary harmonic relationship is by fifth below, and that secondary relationships are by thirds above and below (the fifth above is "anti-harmonic"). The c minor triad is sounded and various possible relationships are considered; the results are given below (black notes indicate the pitches actually sounded and heard; half-colored notes, the pitches sounding in the ear as "after-images"; and white notes, the pitches manufactured in the ear): 72

<table>
<thead>
<tr>
<th>Primary</th>
<th>Secondary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Relationships which form augmented, diminished, or other minor triads are excluded, leaving relationship number two as the strongest association. He says, "We may summarize, therefore, with the statement that, harmonically considered, the true root of a minor triad is a major third below the

71 Ibid. Ortmann's physiology is exceedingly naïve. He has omitted several important membranes, not to mention the all-important Organ of Corti, which actually contains the sensory cells. The Organ of Corti does not accord more surface area to any particular register; Ortmann's theory is totally groundless.

72 Ibid., p. 379.
A similar process is applied to melodic relationships, the primary relationships being by half-steps and the secondary relationships by whole-steps:

The triads marked with asterisks are excluded because they are also minor; the tonic major remains as the closest relationship. "In conclusion, then, we may say that, considered independently of tonality, a minor triad is melodically referred to its tonic major."  

On the strength of the first of the above conclusions, Ortmann claims that these two progressions are equivalent:

Furthermore, Ortmann claims that his "real" root theory explains why, from a minor key, the key-relationship of a major third below is so satisfying, citing as examples Beethoven's "Pathetique" and "Appasionata" Sonatas, and the same composer's Fifth Symphony.

Ortmann offers no evidence that tones can produce "after-images," or that the ear will manufacture the tones he says it will. He does not even state specifically whether he himself can hear these tones. Composers have not treated the minor triad as if it were an incomplete major seventh chord; the two progressions equated (above) are, perhaps, functionally equivalent to some degree, but that has to do more with the fact that two of the three chords are the same in both progressions (actually, eleven of the twelve pitches

---

73 Ibid. The italics are Ortmann's. Cf. Hotinsky's "triple root theory," supra.  
74 Ibid., p. 380. Italics again Ortmann's. Note the persistence of the idea of "referring" one sound to another for meaning!  
75 Ibid.
are identical) than with the nature of the minor harmony. Furthermore, if the tone located a major third below the nominal root of a minor triad is the real root of that chord, and the key relationship between a minor key and its submediant is especially satisfactory on that account, then it should be most unfulfilling to return to the original tonic after having been in the submediant. This is simply not the case, as Beethoven does return to the original tonic in all of the cases cited by Ortmann. Ortmann's faulty physiology of the ear has already been cited.

We may say, then, that Ortmann, in attempting to debunk harmonic dualism in general, succeeded mainly in substituting another type of dualism in place of the traditional variety, for he has not renounced the notion, fundamental to dualism, that sounds have meaning only when "referred" to some other sound in a particular direction; i.e., that chords are perceived melodically.

Harmonic dualism has faded considerably in recent years, but has never completely disappeared. Heinrich Schenker had to acknowledge the dualistic school in his early writing. Paul Hindemith, although certainly not himself a dualist, nevertheless felt obliged, in his own theoretical writing, to refute the undertone theory of the minor harmony, saying that such a multiplying of wave-lengths is possible only in an electronic studio. 76 In 1952, Elizabeth Godley revived and furthered d'Indy's Major-Phrygian inversion theory; her principal thrust, however, was through the metaphor of the mirror image. 77 And two years later, Edmond Costère published a theory that takes dualism into the world of microtones, basically following d'Indy's principles of inversion and symmetry; unfortunately, he often employed a notation that is practically unreadable. 78

* * *

Goethe's syllogism, stated at the beginning of this article, may be re-examined by way of summation. As to the first point: Ortmann, at least, was unwilling to recognize the equality of major and minor (as evidenced by the Picardy third); at any rate, Goethe's premise has not and should not go unchallenged. The second point is meaningless if the validity of the first point is at all in doubt. The last point, however, states the problem in a nutshell, though not

as Goethe intended: not only does the overtone series not account for both modes, there is no reason to say that it explains in any meaningful way either mode. The overtone series — much to Rameau's consternation — will not produce a subdominant (assuming that the root of the series is the tonic of the key); any attempt to identify this phenomenon of nature with the musical practice of functional tonality is therefore doomed to failure. Musical practice in general may be quite arbitrary; the music of non-Western cultures seems to Westerners anything but "natural," and the reverse is probably also true. Therefore, the real error of harmonic dualism lies in its supposition that either major or minor need be explained apart from practice. As Schenker wrote in dismissing the dualistic theory of the minor triad: 79

Why are the theoreticians so confused when they try to derive from Nature this system [minor tonality] which they have themselves so arbitrarily created? ... The explanation becomes much easier if artistic intention rather than Nature herself is credited with the origin of the minor mode.

We may conclude that composers create and theorists analyze their creations, but any attempt on the part of theorists, physicists, or composers themselves to pre-analyze music is likely to be nothing more than an elaborate but nonmusical intellectual game. The dualists suffered from faulty logic, inaccurate physiology, incomplete psychology, and nonsensical acoustics, but they failed most importantly in their loss of contact with the actual music they purported to explain.

79 H. Schenker, Harmony, ed. and annotated by Oswald Jonas, trans. by Elisabeth Mann Borgese (Cambridge, MIT Press, 1954); 51-52; 50. I have taken the liberty of reversing the order of the two sentences, as Schenker had answered in advance his own rhetorical question.
BIBLIOGRAPHY


Dahlhaus, Carl, "War Zarlino Dualist?" Musikforschung X (1957), 286ff.


Hauptmann, Moritz, Die Natur der Harmonik und der Metrik, 2te Auflage; Leipzig, Breitkopf und Hartel, 1873.


Hoffman, Mark, A Study of German Theoretical Treatises of the Nineteenth Century, Ph.D. dissertation; Rochester: Eastman School of Music; Ann Arbor: University Microfilms, 1953.


