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Calculating Welfare Costs of Inflation in a Search Model with Preference Heterogeneity: A Calibration Exercise

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Calculating Welfare Costs of Inflation in a Search Model with Preference Heterogeneity: A Calibration Exercise

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Abstract

Using U.S. cross-sectional data, this paper calculates the welfare cost of a 10% inflation for different individuals and finds that the difference in cost between the poorest 10%, measured by their expenditure share on cash goods, and the richest 10% is in the order of 176%. That is, a poor person is on average willing to forgive 176% more of their total consumption in order to have inflation reduced from 10% to 0. In absolute terms this represents a cost of 2.687% of consumption for the poorest and 0.974% for the richest. I accomplish this by introducing preference heterogeneity in a monetary search model first developed by Lagos and Wright (2005), and calibrate the model to match the expenditure share on cash goods and total expenditures for each individual type using data from the Consumer Expenditure Survey (CEX) for the second quarter of 1996. I also show that this welfare difference increases to 210% (10.522% for the poorest 10% and 3.401% for the richest 10%) whenever frictions in the use of money are imposed (holdup problem). The ability to explicitly model these frictions is the advantage of using this model. Hence, inflation in this framework, as other studies have shown, acts as a regressive consumption tax; and this regressiveness is augmented with the holdup problem.

Key Words: Inflation, welfare, search, and holdup.
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1 Introduction

The topic of welfare cost of inflation has been extensively studied in economics. The idea that holding money has an opportunity cost given by some interest forgone was first introduced by Bailey (1956). Bailey gave the metric for which one could derive how much inflation reduces welfare by calculating the area under an inverse money demand curve between some positive interest rate and zero. This area, known as Bailey's triangle, measures the inefficiency of carrying money whenever nominal interest rates are positive.

Using Bailey's metric, Fischer (1981), Lucas (1981), and Lucas (2000) calculated very small welfare costs of inflation for the U.S. ranging from 0.3% to 1% of income. In these models, a maintained assumption is that markets are competitive. This is also true in Cooley and Hansen (1989) and (1991), where, in both cases, the welfare costs of inflation are less than 1%. In general, under competitive markets, most studies have concluded that moderate inflation is somewhat costless. Wu and Zhang (2000) deviated from competitive markets and have derived much larger welfare costs; in their paper markets are monopolistically competitive.

Lagos and Wright (2005) also deviate from competitive markets by calculating the welfare cost of inflation using a simple model of deep money with bilateral trading and transactional demand for cash, where bargaining frictions are explicitly imposed. In their paper, whenever bargaining power is shifted towards the seller (holdup problem), moderate inflation creates a much larger cost. Hence, deviations from standard real business cycle models can generate higher welfare costs of inflation. The importance of their work is that welfare costs of inflation can be calculated in a model with micro-foundations and no ex-ante assumptions about the use of cash; which differs from cash in advance or money in the utility function models (reduced form models).

Most of the literature, however, have ignored the possible distributional effects of inflation. Easterly and Fischer (2001), using pooled household data on 38 countries have shown that inflation can hurt the poor more than the rich. This result suggests that inflation can act as a regressive tax. Erosa and Ventura (2002), using U.S. household data, calibrate a monetary growth model and show that inflation acts as a regressive consumption tax.

The current paper extends the work by Lagos and Wright[1] (2005) by introducing preference heterogeneity similarly to Bhattacharya, Haslag, and Martin (2005) and calculates the welfare cost of a 10% inflation for each individual type. I calibrate the model to U.S. household cross-sectional data, using the Consumer Expenditure Survey (CEX) for the second quarter of 1996. As with many models with heterogeneous agents tractability becomes a problem, hence, in order to solve the model explicitly and obtain results, I have made some simplifying assumptions about the agent's type distribution.

There are only 10 types of agents grouped by expenditure shares on cash goods. Each group has the same number of agents and represents the 10 different deciles in the expenditure share distribution. Therefore, as we move from agent type 1 to type 10, the average expenditure share on cash goods increases; and since expenditure share on cash goods is negatively correlated with income, I will assume that as agent type increases, the poorer is the individual.

The main findings for the baseline model, where no holdup ($\theta = 1$) and sure probability of a single coincidence meeting ($\sigma = 0.5$) are assumed, are a welfare cost to range between 0.974%, for agent type 1, and 2.687% for agent type 10. This result shows, in line with other studies, that inflation acts as a regressive tax on consumption. When I include the holdup problem ($\theta = 0.5$), the welfare cost increases, as in LW(2005), to range between 3.401% and 10.522% for those same individuals mentioned above. In relative terms this represents a welfare cost of inflation 176% higher for the poorest individuals using the baseline model, and 210% whenever the holdup is imposed.

The facts that the holdup effect increases welfare cost and that inflation acts as a regressive consumption tax are not new in the literature. The contribution of this paper is that it combines these two results in a deep model of money. Also, it shows that the regressiveness of the inflation tax is augmented in the presence of the holdup problem.

In order to obtain my results, I had to use the data and assume which goods were most likely purchased with cash; hence the basket of cash goods used in this paper was: food consumed at and away from home, alcoholic beverages, apparel, gas and motor oil, prescription drugs, fees and admissions to entertainment events, tobacco and smoking supplies, personal care items, and cash contributions. This assumption makes the aggregate expenditure share on cash goods equal to 0.32, which is larger than the

[1] The remaining of this paper will refer to Lagos and Wright (2005) as LW(2005).

expenditure share found in LW(2005) of about 0.10. This might seem inconsistent; however, LW(2005) never use disaggregated data to match expenditure share on cash goods; the value of 0.10 is obtained after the model has been solved using U.S. aggregate data. To make my model comparable to LW(2005) at the aggregate level, I have calculated the welfare cost of inflation for an atomistic agent matching the same expenditure share as in LW(2005). My results are very similar to theirs.

Also, I recalculated the welfare costs using a smaller goods basket (food consumption only), which changes the quantitative results of the model since aggregate expenditure share on cash goods decreases, and found all results to match qualitatively. It is good to notice that by only using food consumption as cash goods, I am underestimating the fraction of consumption purchased with cash (in the data, aggregate expenditure share on food is 0.19) making these results very conservative. I believe that my original choice of basket is closer, if not still smaller, to the actual expenditure share on cash goods by the average American.

Support to the assumption of cash goods consumption can be found on Avery et al. (1987), Kennickell et al. (1997), and Mulligan and Sala-i-Martin (2000), where they conclude that a somewhat large fraction of households do not own a checking account or do not have credit cards. According to Evans and Schmalensee (2005), less than 25% of all consumer expenditures were purchased with credit; this number decreases to less than 15% for the data period studied in this paper. Hence, having individuals interact in a market where only cash is accepted does not constitute a bad assumption here.

The remainder of the paper will be organized in the following matter: first, I will present the model with its set up, interior equilibrium, and solution; second, the model will discuss the data used for calibration, which is the third part; fourth, results will be presented; and the last part will bring the concluding remarks.

2 The Model

This model is a variation of the model in LW(2005) extended by Bhattacharya, Haslag and Martin (2005). Here, agents are allowed to differ with respect to the degree to which they prefer one good over another. As in LW(2005) there will be two types of goods; the day good or cash good and the night good or general good.

2.1 Set Up

Time is discrete with each period divided into 2 subperiods. As a convention these subperiods will be called day and night. There are I types of infinitely lived agents with each type i having mass χ_i, where $\sum_i^I \chi_i = 1$, and discount factor $\beta \in (0,1)$. Each agent type consumes and supplies labor in both subperiods. Let x_i and h_i be the consumption and labor in the day, and X_i and H_i be the consumption and labor in the night for type i agent. Let γ_i be the preference heterogeneity parameter for type i agent[2]. Preferences will be given by:

$$\mathcal{U}_i(x,h,X,H) = \gamma_i u(x_i) - c(h_i) + (1-\gamma_i)\kappa U(X_i) - H_i \quad (1)$$

Where $\gamma_i \in (0,1)$, $\kappa > 0$ and u, c, U are twice continuously differentiable with $u' > 0, c' > 0, U' > 0, u'' < 0, c'' \geq 0, U'' \leq 0$ and $u(0) = c(0) = 0$. Also we assume that $\exists q_i^* \in (0,\infty)$ such that $\gamma_i u'(q_i^*) = c'(q_i^*)$ and $\exists X_i^* \in (0,\infty)$ such that $U'(X_i^*) = \frac{1}{\kappa(1-\gamma_i)}$ with $U(X_i^*) > X_i^*$.

The day market is decentralized with anonymous bilateral matching which gives no scope for IOUs, making money, therefore, essential. Let α be the probability of a meeting. The good x comes in many varieties with each agent only consuming a subset of it. Let's assume that every agent can transform h into x one to one and that every agent does not consume what produces.

For two agents i and j meeting in the day market, there are only four possible outcomes that can be divided into three types of meetings with three different probabilities. First, they are both carrying what each other wants with probability δ, this is called a double coincidence meeting and δ is the double coincidence probability. Second, agent i is carrying what agent j wants but not vice versa; this is called a single coincidence meeting and let σ be the probability for this type of meeting. Symmetrically, σ will also be the probability for the single coincidence case in which j is carrying what i wants but not vice versa. Third, it is possible that both agents meet but neither wants what each other has. This is the case of a no coincidence meeting and the probability of this event happening is $(1-2\sigma-\delta)$.

[2]Note that γ_i enters the utility function in both day and night markets with opposing effects. This is done as to enable the model to match total expenditure data.

The night market is a centralized Walrasian market where agents transform H into X one to one and X varies across individual types. In both night and day markets, goods are perfectly divisible and nonstorable. The only feasible trades in this economy are barter and the exchange of x for money (m) during the day, and the exchange of X for money (m) at night, however, this will only happen if an agent wants to consume more than what it has produced.

One of the important features of the night market is that it resets money holdings for every agent type. Hence, each agent i enters the day market holding m_i, where m_i is perfectly divisible and storable in any quantity $m_i \geq 0$. However, given the type of meeting occurred in the day market, agents of the same type may enter the night market holding different amounts of money. The supply of money M evolves according to $M_t = (1+\rho)M_{t-1}$, where the government changes the money supply with lump-sum transfers or taxes τ on individuals at the beginning of the night market. Therefore, at time t, $\tau = \rho M_{t-1}$.

2.2 Equilibrium

Before formally defining the equilibrium in this model, one needs to characterize both the day and night markets as well as the terms of trade for single and double coincidence meetings in the day market. Let $F_t(\tilde{m})$ and $G_t(\tilde{m})$ be the unconditional measures of money holdings m at time t in the day and night markets respectively. And let $F_{it}(\tilde{m}_i)$ and $G_{it}(\tilde{m}_i)$ be the conditional measures of money holdings m_i of agent type i at time t in the day and night markets respectively. That is, these distributions measures the agent's starting money holdings at the beginning of period t (day or night) where $m \leq \tilde{m}$ for the unconditional distributions and $m_i \leq \tilde{m}_i$ for the conditional distributions. Also, let F_{i0} and G_{i0} be given exogenously for all i.

Total money holdings in this economy is just the sum of all agents money holdings and since each period is divided into two, the total amount of money carried from one subperiod to the other has to be the same. However, the amount of money held by each type of agent can change from one subperiod to the other, that is, in this model money can be transferred from one type of agent to another in the same period.

One important characteristic of this model is that one can show, under certain conditions, that all the conditional distributions of money holdings must be degenerate in equilibrium. This feature is what guarantees the model's tractability. Also, at the beginning of each subperiod, F_{it} and G_{it} are assumed to be given by the individual type i. This implies that individual decisions only depend on money holdings m_i.

Let $V_{it}(m_i)$ and $W_{it}(m_i)$ be the individuals value functions holding m_i in both day and night markets respectively. Let ϕ_t be the price of money in the night market, that is, $1/\phi_t$ is the nominal price of the night good.

Since we have bilateral trading, $x_i = h_i = q_{it}(m_j, \tilde{m})$ where q_{it} is the common value for a trade with $d_{jt}(m_j, \tilde{m})$ being the amount of dollars the buyer pays to obtain q_{it}. In the expression above, m_j is total money holdings of the buyer, while $\tilde{m}$ is total money holdings of the seller. In double coincidence meetings, let $B_{it}(m_i, \tilde{m})$ be the payoff for an agent holding m_i who meets someone with $\tilde{m}$, that is, $B_{it}(m_i, \tilde{m}) = u(q_i) - c(q_i) + W_{it}(m_i)$.

The value function of agent type i beginning the day market at t is given by:

$$\begin{aligned} V_{it}(m_i) = \alpha\sigma \int & \gamma_i u[q_{it}(m_i, \tilde{m})] + W_{it}[m_i - d_t(m_i), \tilde{m}] dF_{it}(\tilde{m}) + \\ \alpha\sigma \int & -c[q_{it}(\tilde{m}, m_i)] + W_{it}[m_i + d_t(\tilde{m}, m_i)] dF_{it}(\tilde{m}) + \\ \alpha\delta \int & B_{it}(m_i, \tilde{m}) dF_{it}(\tilde{m}) + (1 - 2\alpha\sigma - \alpha\delta) W_{it}(m_i), \end{aligned} \tag{2}$$

where the first term represents a single coincidence meeting with the agent being a buyer, that is, it receives utility $\gamma_i u(q_i)$ but only takes $m_i - d_i$ to the night market. The second term is also a single coincidence meeting with the agent being a seller. There, it costs $-c(q_i)$ to produce the good, but the agent is richer at night with $m_i + d_j$. The last two terms are the expected payoffs from bartering and not trading.

In the night market, the value function of agent type i at time t is:

$$W_{it}(m_i) = \max_{X_i, H_i, m_i'} (1 - \gamma_i)\kappa U(X_i) - H_i + \beta V_{it+1}(m_i') \tag{3}$$

subject to

$$X_i = H_i + \phi_t(m_i + \tau) - \phi_t m_i' \tag{4}$$

and $X_i \geq 0$, $0 \leq H_i \leq \bar{H}$, $m_i' \geq 0$. Here $\bar{H}$ represents an upper bound on possible hours worked. Even though the corner solution is possible here, I will follow Lagos and Wright's (2005) approach and assume that X^* and H^* are interior. Lagos and Wright have shown that $0 < H^* < \bar{H}$ will be satisfied in these models under standard conditions.

The terms of trade in this model can be characterized in the following way. In the day market, agent i will choose the optimal d_i and q_i to solve the generalized Nash bargaining problem given by:

$$[\gamma_i u(q_i) + W_{it}(m_i - d_i) - W_{it}(m_i)]^{\theta}[-c(q_i) + W_{it}(\tilde{m} + d_i) - W_{it}(\tilde{m})]^{1-\theta} \tag{5}$$

with $d_i \leq m_i$ and $q_i \geq 0$, $\theta > 0$ being the buyer's bargaining power parameter, and threat points $W_{it}(m_i)$.

For double coincidence meetings, agents will solve a simple symmetric Nash bargaining problem with threat points given by the continuation values $W_{it}(m_i)$. That is, the agents will need to solve:

$$\max_{q_1,q_2,\Upsilon} [\gamma_1 u(q_1) - c(q_2) - \phi\Upsilon][\gamma_2 u(q_2) - c(q_1) - \phi\Upsilon] \tag{6}$$

subject to $-m_2 \leq \Upsilon \leq m_1$, where Υ is the amount of money agent 1 pays agent 2, and q_i is the quantity consumed by agent i.

Hence, an equilibrium in this model can be the defined as follows:

Definition An equilibrium in this model is a list $\{V_{it}, W_{it}, X_{it}, H_{it}, m'_{it}, q_{it}, d_{it}, \phi_t, F_{it}, G_{it}\}$, where for all i and t, $V_{it}(m_i)$ and $W_{it}(m_i)$ are the value functions; $X_{it}(m_i)$, $H_{it}(m_i)$, and $m'_{it}(m_i)$ are decision rules in the night market; $q_{it}(m_i, \tilde{m})$ and $d_{it}(m_i, \tilde{m})$ are the terms of trade in the day market; ϕ_t is the price of money in the night market; and F_{it} and G_{it} are the conditional distributions of money holdings in the beginning of each subperiod. This leads to the following equilibrium conditions for all t:

1. Given ϕ_t, and the conditional distributions F and G; $V_{it}(m_i)$ and $W_{it}(m_i)$ satisfy (2) and (3).

2. $q_{it}(m_i, \tilde{m})$ and $d_{it}(m_i, \tilde{m})$ maximize (5) in the day market.

3. $\phi_t > 0$, which guarantees a monetary equilibria.

4. Night money market clears, that is, $\sum_{i}^{I} m_i G_{it}(m_i) = M$. Walra's law guarantees that the goods market also clears.

5. $\{F_{it}, G_{it}\}$ are consistent with the initial conditions and evolution of money holdings from both day and night markets.

2.3 Solving the Model

In this part, I will present the general solution to the model without assuming most parameter values or functional forms. In order to simplify the solution to this model without loosing its' generality, let's assume that there are no double coincidence meetings, that is, $\delta = 0$. The approach to solve the model will follow the same steps as in Bhattacharya, Haslag and Martin (2005).

Starting with the centralized market, for each agent i, we can solve equation 4 for H_i and substitute it into equation 3 to obtain,

$$W_{it}(m_i) = \max_{X_i, m_i'} (1 - \gamma_i)\kappa U(X_i) - X_i + \phi_t(m_i + \tau) - \phi_t m_i' + \beta V_{it+1}(m_i'). \tag{7}$$

The first order conditions with respect to X_i and m_i' are given by,

$$U'(X_i) = \frac{1}{\kappa(1 - \gamma_i)} \tag{8}$$

and

$$\beta V_{it+1}'(m_i') = \phi_t. \tag{9}$$

The solution to these two equations plus the budget constraint completely solves the consumer problem in the centralized market. It is important to notice here that the solution to H_i requires more work than in LW(2005). In their model $X^* = H^*$, since average money holdings equals M. In this model the average money holdings an agent i exiting the decentralized market carries is $\bar{m}_i = (1 - \sigma)m_i + \sigma M$. Hence it's steady state budget constraint in equilibrium becomes $H_i^* = X^* - \phi(\sigma + \rho)(M - m_i)$, where I have used the fact that $m_i' = (1 + \rho)m_i$ and $\tau = \rho M$. Also, since $M = \sum_i^I m_i G_{it}(m_i)$, in order to completely solve the model one needs to assume some distribution of money holdings across agents.

The price of money ϕ_t can be obtained from the money market clearing condition. Also notice that the choice of m_i' is independent of m_i, which suggests that the distribution of money holdings conditional on i type is degenerate. That is, every agent with the same type enters the decentralized market holding exactly the same amount of money m_i'.

Starting in the decentralized market, first we need to solve the terms of trade problem given by equation 5. From what we know about W_i, it is easy to check that $W_{it}(m_i + d_i) - W_{it}(m_i) = \phi_t d_i$, which reduces 5 to:

$$\max_{d_i, q_i} [\gamma_i u(q_i) - \phi_t d_i]^\theta [-c(q_i) + \phi_t d_i]^{1-\theta} \tag{10}$$

subject to $d_i \leq m_i$.

Lagos and Wright (2005) have shown that the only feasible equilibrium is when $d_i = m_i$ and the solution for q_i can be obtained from the following first order condition:

$$\phi_t m_i = \frac{\gamma_i[(1-\theta)u(q_i)c'(q_i) + \theta c(q_i)u'(q_i)]}{\gamma_i \theta u'(q_i) + (1-\theta)c'(q_i)}. \tag{11}$$

Let $z(q_i) \equiv \frac{\gamma_i[(1-\theta)u(q_i)c'(q_i)+\theta c(q_i)u'(q_i)]}{\gamma_i \theta u'(q_i)+(1-\theta)c'(q_i)}$, we have therefore that $q_i = q_i(m_i)$ is the solution to

$$m_i \phi_t = z(q_i). \tag{12}$$

Note that the solution to the terms of trade problem only depends on the buyer's money holdings and not the seller's.

Now we can combine what we know about the terms of trade solution and $W_i(m_i)$ and rewrite the value function in the decentralized market as:

$$V_{it}(m_i) = \alpha\sigma[\gamma_i u[q_{it}(m_i)] + W_{it}[m_i - d_{it}(m_i)]] + \\ \alpha\sigma \int -c[q_{it}(\tilde{m})] + W_{it}[m_i + d_{it}(\tilde{m})] dF_{it}(\tilde{m}) + (1 - 2\alpha\sigma)W_{it}(m_i), \tag{13}$$

assuming no double coincidence meetings. Differentiating the above expression with respect to m_i noting that $W'_{it}(m_i) = \phi_t$ for all m_i, we obtain

$$V'_{it}(m_i) = \alpha\sigma\gamma_i u'[q_{it}(m_i)]q'_{it}(m_i) + [1 - \alpha\sigma d'_{it}(m_i)]\phi_t. \tag{14}$$

The above expression only depends on the buyer's money holdings and type. From the terms of trade problem we have that $d'_{it}(m_i) = 1$ and

$$q'_i(m_i) = \frac{\phi_t}{z'_i(q_i)}, \tag{15}$$

by implicit differentiation. Substituting these expressions into equation 14 and using equation 12 we obtain the following expression:

$$\phi_t = \beta\{\alpha\sigma\gamma_i u'(q_{it+1})\frac{\phi_{t+1}}{z'_{it+1}(q_{it+1})} + (1 - \alpha\sigma)\phi_{t+1}\}. \tag{16}$$

In steady state this expression reduces to:

$$1 + \rho = \beta\{\alpha\sigma\frac{\gamma_i u'(q_i)}{z'(q_i)} + (1 - \alpha\sigma)\}, \tag{17}$$

where q_i is constant and $\phi_t = (1 + \rho)\phi_{t+1}$. We can further simplify equation 17 by defining $\beta \equiv \frac{1}{1+r}$ and defining a nominal interest rate $(1+R)$ as $(1+R) = (1+r)(1+\rho)$, hence equation 17 becomes,

$$1 + \frac{R}{\alpha\sigma} = \frac{\gamma_i u'(q_i)}{z'(q_i)}. \tag{18}$$

In order to explicitly solve this model, I will assume, following LW(2005), that $u(q) = \frac{(q+b)^{(1-\eta)}}{1-\eta} - \frac{b^{(1-\eta)}}{1-\eta}$ for any agent type. This utility form guarantees that $u(0) = 0$; where b was chosen to be a very small number making no change in the model's solution. I will also assume that $c(q) = q$ and $U(X) = lnX$ for every type.

3 Data

The data used for calibration purposes in this paper was taken from the Consumer Expenditure Survey (CEX) for the second quarter of 1996; the data set consists of 3447 observations. Since this data set brings individual expenditure and income information, I will use some of it's features to calibrate γ_i and κ for different agent types. For the purposes of tractability, I will assume that there are 10 types of agents in this model. They will differ with respect to their expenditure shares on cash goods, that is, each agent type corresponds to a different cohort in the expenditure share distribution. In order to define which agent belongs to which cohort, I separated each cohort by expenditure shares on cash goods deciles. This assumption forces each cohort to have the same number of individuals. That is, the first cohort will constitute of all agents in the bottom 10% of the expenditure share distribution; cohort 2 will constitute of agents between the 10th and 20th percentile in the expenditure share distribution, all subsequent cohorts will follow the same pattern making the last cohort of all agents between the 90th and 100th percentiles. My model will be calibrated to match the median expenditure share on cash goods of each cohort.

In order to achieve this, I need to assume the type of goods that can be considered cash goods. The list of chosen goods was given by: food consumed at and away from home, alcoholic beverages, apparel, gas and motor oil, prescription drugs, fees and admissions to entertainment events, tobacco and smoking supplies, personal care items, and cash contributions. These are the goods that agents consume in the day market. Let ES_i^D be the median expenditure share on cash goods across agent type i found in the data, that is, ES_i^D is the agent located at the 50th percentile expenditure share within each cohort. The use of the median as my measure of center is to guarantee that the measure of the expenditure share on cash goods by each cohort will not be sensitive to any particular observation. However, the use of the average expenditure share would not change the results for this particular data set. Table 1 has the average and median expenditure shares by cohort; one can verify that both measures are very similar across every cohort.

Table 1 also has information on the second data feature my model will attempt to match, that is, average total expenditure per type (TE_i^D). The last column of table 1 has the average total expenditure per type relative to type 1 agent (TER_i^D). It is quite obvious that there is a negative correlation between expenditure share on cash goods and total expenditure, in fact this correlation is equal to -0.85. Figure 1 shows this relationship.

The reason why this model will be calibrated to also match total expenditure is because the data suggests that total expenditure and income are correlated. Figure

2 plots total expenditure and after tax income for the average individual type. The correlation coefficient was calculated to be 0.79. By matching the model's solution to total expenditure, I can make some claims about how inflation affects different income individuals. This will allow me to compare my model with other studies, such as Erosa and Ventura (2002), that have investigated this relationship. These correlations suggests that poorer individuals consume cash goods in a larger fraction, in fact the correlation coefficient between expenditure shares and after tax income was calculated to be -0.98. This result is in line with Avery et al. (1987), where they found that high income individuals use a smaller fraction of their transactions on cash goods when compared to lower income individuals.

Also, for comparison purposes, I have calculated the average expenditure share for the entire sample. This aggregated value equals 0.32. Since my model will match each agent's type expenditure share, it will also match the aggregated value. I will use this feature to compare the results of my model to the results in LW(2005), where the expenditure share on cash goods is much lower ranging from about 0.06 to 0.15. The following section will explain the methodology I used to calibrate the model's parameters in detail.

4 Calibration

The list of calibrated parameters is given by: ρ, σ, δ, α, θ, β, η, κ and γ_i. Table 2 has the calibrated values. The values of α and δ were chosen as to simplify the model. The parameter α equaling 1 implies that every individual i will have a meeting in the day market, that is, the probability of having such meeting is a sure event. The parameter δ was set to 0 to eliminate double coincidence meetings. The value of $\beta = 0.99$ was chosen to match most studies that have calibrated this parameter for quarterly data.

The inflation rate ρ was calibrated to 0.01, which matches 1996 second quarter inflation measured by the percentage change in the Consumer Price Index (CPI) for all urban consumers on all items. The bargaining parameter θ was set to 1 as to eliminate the holdup problem where each buyer makes a take it or leave it offer in the day market, which maximizes the buyer's bargaining power; and 0.5 to capture the holdup problem.

Two values were chosen when calibrating σ: 0.3 and 0.5. Since σ is the probability of a single coincidence meeting, by setting $\sigma = 0.5$, we are maximizing the number of successful meetings in the day market. This corresponds to having always an agreement between parties. However, LW(2005) estimated σ using U.S. aggregate data to be 0.3, therefore, for the purposes of checking the sensibility of the results with respect to this parameter, the value of 0.3 was also used. It is important to notice that a σ value of less than 0.5 indicates more friction in the day market, since individuals might not encounter a successful trade. This could potentially increase the welfare costs of inflation. I will use the values of $\theta = 1$ and $\sigma = 0.5$ as my baseline model; all other 3 possible combinations of θ and σ will constitute the different specifications also used.

Finally, η was calibrated to 0.25. I chose this value as an approximation to the range of values LW(2005) used for this parameter. Since in their paper η varies between 0.15 to 0.5, I chose 0.25 and performed sensitivity analysis.

4.1 Calibrating γ_i and κ

Let's define ES_i^M as agent's i expenditure share on the day market good obtained from the model, that is, the percentage of $q_i(\gamma_i)$ consumption relative to $q_i(\gamma_i)+X_i^*(\gamma_i)$. The expression for ES_i^M is therefore given by:

$$ES_i^M = \frac{\sigma q_i^*(\gamma_i)}{\sigma q_i^*(\gamma_i) + X_i^*(\gamma_i)} \tag{19}$$

for $i = 1, 2, ..., 10$; where $X_i^* = \kappa(1 - \gamma_i)$. Note that in the equation above $q_i^*(\gamma_i)$ is being multiplied by σ. The reason for this is that since σ represents the probability of a successful single coincidence meeting, $(1 - \sigma)$ is the probability of an unsuccessful meeting, and whenever this happens $q_i^*(\gamma_i) = 0$. Hence the numerator in the ES_i^M equation should only have the term $\sigma q_i^*(\gamma_i)$. The same reasoning explains the expression in the denominator. Also define TE_i^M as agent's i total expenditure obtained from the model, and TER_i^M as the ratio of TE_i^M relative to agent's 1 total expenditure, that is,

$$TER_i^M = \frac{TE_i^M}{TE_1^M}. \tag{20}$$

Note that $TE_i^M = \sigma q_i^*(\gamma_i) + \kappa(1 - \gamma_i)$, which is the denominator in the ES_i^M equation. The parameters γ_i were chosen as to make $ES_i^M = ES_i^D$ for every i. The scale parameter κ was calibrated as to minimize the sum of $(TER_i^M - TER_i^D)^2$. The solution to these parameters constitutes a system of 11 equations for eleven unknowns. In order to solve this system, one needs to find the solution for q_i^* as a function of γ_i. Whenever $\theta = 1$, $q_i^*(\gamma_i)$ has a closed form solution, however, for $\theta < 1$, $q_i^*(\gamma_i)$ needs to be solved numerically. I have also used numerical methods to solve this system, as none of the equations have closed form solutions[3].

Table 3 has the calibrated values of γ_i and κ. Column 1 has the calibrated values for the baseline model, whereas columns 2 to 4 presents the values of all other specifications of the model. Note that κ varies as we change the set of calibrated parameters in the model, this is because as θ and σ changes, the solution to q_i^* changes and therefore TER_i^M changes for every γ_i. The parameter κ compensates for these effects; γ_i stays very stable for different combinations of σ and θ, however small differences in γ_i generate quite large differences in $1-\gamma_i$. Both of these effects affect the solution to X_i^*. Also, γ_i increases with agent type which implies that agents who spend a higher percentage of their expenditures on cash goods prefer these goods more relative to the general good.

Figures 3 and 4 plot expenditure shares and total expenditure ratios from the data against the expenditure shares and total expenditure ratios found in the model for all different combinations of θ and σ. Both figures show that, for all possible combinations of θ and σ, the model generates expenditure shares and total expenditures very similar to what is found in the data. In fact, aside from rounding errors due to the numerical procedures, this model matches expenditure shares exactly since γ_i was obtained from setting $ES_i^M - ES_i^D = 0$, given κ, for all i. Even though, total expenditure ratios from the model does not match exactly the total expenditure ratio found in the data, figure 5 shows that, for all calibrated values of κ corresponding to different combinations of θ and σ, the model performs quite well. The reason why total expenditure ratios did not match the data exactly is that in the minimization procedure, κ did not produce the sum of squared deviations to be exactly zero.

The importance of having the parameter κ in the model is that it guarantees that poorer agent's measured by their total expenditure are the ones with higher expenditure shares on cash goods. Figure 5 plots total expenditure ratios obtained from data, the baseline model for $\kappa = 1$, as well as $\kappa = 1$ and $\theta = 0.5$. Compared to the data, it

[3]The methods used to calibrate γ_i and κ were the bisection method and simulated annealing respectively.

obviously performs quite poorly on the quantitative level.

5 Results

In this section, I will present the solutions and welfare costs of inflation for the baseline model and compare it to all other specifications. Using the calibrated parameters mentioned above, table 4 has the solution to q_i^*, X_i^*, and H_i^* for all i and different combinations of θ and σ. Columns 1, 2, and 3 have the solutions for the baseline model. Note that as expenditure shares on cash goods increase (moving from agent 1 to agent 10), willingness to work decreases in the centralized market.

The solutions to q_i^*, regardless of θ and σ, show a positive correlation with expenditure share. That is, as expenditure share on cash goods increase, so does the consumption of that good. This feature is found to be true in the data in general. The opposite is true with respect to the solution to X_i^* as mentioned above. There is a negative correlation between the consumption of the general good and expenditure share on cash goods. Data suggests that this should be the case. Figure 6 plots total expenditure on cash goods, and total expenditures on the general good by agent type for the data. Total expenditure on the general good is definitely decreasing, while total expenditure on cash goods is increasing.

Figures 7 and 8 compare the solutions to q_i^* and X_i^*, for different combinations of θ and σ, with data. Note that in those figures, I am plotting q_i^* and X_i^* relative to q_1^* and X_1^* respectively. I have also performed the same calculations for the data values. The model captures the overall trend in total expenditures on cash goods, regardless of the specification, even though, it does not match the data exactly. However, the model matches the data with respect to the consumption of the general good for all specifications. Overall, the model performs well when compared to the data.

Comparing the results from table 4, q_i^* increases for larger values of σ and θ for all i; this feature is consistent with Lagos and Wright (2004), where they have shown that $\frac{\partial q}{\partial \theta} > 0$ and $\frac{\partial q}{\partial \sigma} > 0$. It is quite trivial to check that these effects are also true in my model. In the case of the bargaining parameter θ, the smaller it becomes the smaller is the buyer's bargaining power, which results in a smaller surplus, decreasing the buyer's incentive to purchase that good. The parameter σ is the probability of a single coincidence meeting, the smaller it becomes the harder it is to accomplish a successful match. Hence, lower σ values should result in lower consumption of cash goods. Since both of these parameters impose more frictions in the market for cash goods and since their effect on q_i^* is qualitatively the same, the consumption of cash good will be the lowest whenever more frictions are imposed at the same time, that is, $\theta = 0.5$ and $\sigma = 0.3$. In order to capture which friction affects q^* the most, we can compare the solutions to q_i^* for two different combinations of the parameters θ and σ: $\theta = 1$, $\sigma = 0.3$, and $\theta = 0.5$, $\sigma = 0.5$. The first combination imposes a larger friction on finding a bargaining partner and no friction in the terms of trade; once a partner has been found, the buyer has all the bargaining power and makes a take-it or leave-it offer to the seller. The second combination imposes the minimum friction on finding a partner, but creates a friction on the bargaining for the buyer. With $\theta = 0.5$, the seller is able to bargain some of the surplus way from the buyer, reducing the buyer's incentive to bring cash to the market. The solution to q_i^* is smaller whenever the bargaining friction is imposed for all i compared to the solution where the meeting friction is imposed. In fact, q_i^* is in general smaller for smaller values of θ regardless of the value assigned to σ. This result suggests that the holdup effect is stronger relative to the single coincidence meeting probability.

Another interesting feature of the solution is the relationship between X_i^*, H_i^*, and money holdings across agents. For any agent i, whenever $X_i^* > H_i^*$ implies that actual money holdings are smaller than average money holdings for the average type i agent, for given ϕ, σ, and ρ. This result can be derived from the agent's budget constraint in the night market. Also, since the agent is consuming more than it produces, it has to use cash to purchase what is left, hence it uses some of the government transfer to accomplish that. This implies that these agents take less than $m+\tau$ to the decentralized market. The opposite is true for the case where $X_i^* < H_i^*$. Hence q_i^* should increase as H_i^* becomes larger than X_i^*. Table 4 shows that agent types 1, 2, 3, and 4 work less than the amount they consume in the centralized market; this situation is reversed for agents type 5 through 10. Also, the difference between H_i^* and X_i^* increases as we move from agent 1 to agent 10, that is, $H_{10}^* - X_{10}^* > H_9^* - X_9^* > ... > H_1^* - X_1^*$. This explains why q_i^* increases with agent type.

5.1 Welfare Cost of Inflation

In this section, the solution to the model will be analyzed with respect to the agent's welfare as we change the inflation rate. Since agent's i utility form is quasi-linear, the welfare cost of inflation calculation becomes quite simple, since consumer surplus can just be measured as the difference in welfare functions when reducing inflation from $x\%$ to zero. That is, let Y_i^* be the steady state welfare function of agent i. Then Y_i^* can be written as,

$$(1-\beta)Y_i^*(\rho) = \alpha\sigma[\gamma_i u(q_i^*(\rho)) - c(q_i^*(\rho))] + \kappa(1-\gamma_i)U(X_i^*) - H_i^*(\rho) \qquad (21)$$

for any inflation rate ρ.

Letting Δ be the fraction of consumption that agent i is willing to forgive in order to maintain the same level of welfare and reduce inflation from some positive value to zero, we have:

$$(1-\beta)Y_i^*(0) = \alpha\sigma[\gamma_i u(q_i^*(0)\Delta) - c(q_i^*(0))] + \kappa(1-\gamma_i)U(X_i^*\Delta) - H_i^*(0) \qquad (22)$$

The welfare cost of inflation will be the value $1-\Delta$ obtained from subtracting equation (22) from (21) and setting the difference equal to zero. Table 5 brings the results in percentage form. Column 1 has the results for the baseline model. Each number on the table corresponds to the percentage amount of consumption forgone that a consumer would allow in order to have inflation reduced from 10% to zero. Figure 9 plots the results from table 5 for every specification as to make the visualization better.

Note that for the baseline model, the welfare cost of inflation ranges between 0.974% and 2.687% and is monotonically increasing. This implies that the poorest 10% can suffer 2.76 times more than the richest 10%. Without a doubt, the regressiveness of the inflation tax is present in this model.

Table 5 also shows that the holdup problem acts as a magnifier with respect to the welfare cost of inflation. As in LW(2005), by introducing the holdup problem, the welfare cost of inflation is larger given any agent type. Since in this paper agents differ with respect to their total expenditures, it is reasonable to ask if the holdup effect also varies with agent type, that is, is the holdup effect different for different individuals? The answer is: most certainly.

Figure 10 plots the difference in welfare cost of inflation, obtained in from table 5, for each agent type between holdup and no holdup. In both cases, regardless of the value of σ, one can detect an increasing relationship. That is, the holdup effect is stronger for individuals who spend a larger percentage of their earnings on cash goods. The model suggests that while the holdup effect is in the order of 3 percentages points for agent type 1, this effect increases to 8 or 11 percentage points, depending on σ, for agent type 10. This result suggests that the holdup effect increases the regressiveness of the inflation tax.

As to make the welfare cost results more tractable, table 6 has the solution to the difference between equations 22 and 21 piece by piece. This table shows that the driving force behind the differences in welfare costs is the difference between the consumption of cash goods and therefore the difference in utility between them. As the difference in utilities increase, more needs to be taken away from consumption whenever inflation is zero in order to have the equation hold; that is, as the difference in utility increases Δ decreases. Note that the largest difference in utilities can be found for the case where $\theta = 0.5$ and $\sigma = 0.3$. This has to be true since the utility function is concave, and optimal q decreases with lower values of θ and σ. Hence, the curvature of the utility function should matter for the welfare calculations. Figures 11 and 12 has welfare cost results for different η values; recall that η measures the degree of risk averseness. In all cases the results remain the same with small changes in welfare costs for given agent types. It appears, therefore, that the results are not sensible to η values in the neighborhood of 0.25.

One last remark on table 6 is that the differences in hours worked at night are similar across agents, however, it is negative for agents 1 to 4, and positive otherwise. Overall, this difference is increasing on agent type. That is, inflation creates an disincentive to work on poorer individuals.

Since this model uses household data to establish a distribution of agent types and calculates welfare cost of inflation for each type, comparing these results to other studies is quite difficult. However, it is possible to aggregate my results given the assumed distribution of agents and calculate the welfare cost of inflation for this average individual. Table 7 has these results for the baseline model and one other specification with $\theta = 0.5$ and $\sigma = 0.5$. I have also reproduced LW(2005) results for comparison purposes.

The last two columns of table 7 brings the expenditure share and welfare cost of inflation for the average individual in the model. With no holdup, the average individual will forgo 1.8% of consumption; this result is somewhat in line with other studies. However, when the holdup effect is introduced, the cost increases to 6.8% of consumption, a much larger value even compared to LW(2005). One possible explanation for this discrepancy is that the average agent in this model consumes a much larger share of the cash good compared to LW(2005). Attempting to resolve this problem, I calculated the welfare cost for an atomistic agent in my model matching it's expenditure share to that of LW(2005). Since LW(2005) use two different time frames in their calculations, their expenditure shares change by a small amount. Hence, for comparison purposes I picked the average of those two numbers when calculating the welfare cost for the atomistic agent. It is quite clear that, the welfare results from this new calculation are very similar to that of LW(2005). This suggests that this model is in line with most studies at the aggregate level, making the results at the disagregated level more robust.

In order to check if the regressiveness of the inflation tax in this model is solely a function of my choice of cash goods basket, I performed welfare cost calculations for a different basket of cash goods. As a conservative measurement, I only used food consumption as cash goods. Table 8 and figure 14 presents these results. It is quite noticeable that even though the results differ in absolute terms, the relativity of them was preserved. This is very important, since no one knows exactly which goods are purchased with cash,and therefore any welfare cost calculation will be influenced by this assumption, however, this paper shows that regardless of your choice, it is possible to assess the cost in relative terms.

6 Conclusion

This paper calibrated a simple monetary search model to U.S. household expenditure data and calculated the welfare cost of 10% inflation for different individual types grouped by expenditure share on cash goods deciles. These individuals varied with respect to the degree they preferred cash goods over the general good. The main findings of this exercise, in line with other studies, were that: first, inflation acts as a regressive consumption tax since individuals with smaller total expenditure and therefore less earnings suffer more with inflation than individuals with higher total expenditure. Second, the introduction of the holdup problem generates higher relative welfare costs, augmenting; therefore, the regressiveness of the inflation tax.

The main contribution of this paper was to combine two known results from the literature - inflation is regressive and holdup increases it's cost - in a deep, but simple, model of money with micro foundations. The simplicity of this model makes it useful for applications to different data sets as well as different countries.

Hence, some interesting extensions to this work would be to apply this model to developing countries who have experienced periods of large inflation. In these countries, poorer individuals use cash for almost every purchase with very little bargaining power; hence inflation could potentially be relatively more harmful to these individuals compared to the wealthier. Also, one could introduce a competing media of exchange in the model in the form of a credit good, where only a fraction of the population have access to it at some cost. This good would protect individuals against inflation, making it's cost smaller relative to the fraction of the population that does not have access to credit. This scenario is probably applicable to most countries in the world, but specially poorer countries with high degrees of inequality.

Lastly, one could introduce heterogeneity in the labor productivity in the centralized market together with preference heterogeneity. This feature could match the results from this paper, however, it would provide us with better insights with respect to labor decisions under inflation. The current paper displays different labor choices as inflation changes, however, workers are paid the same, which might not be the best assumption, since differences in wage rates could potentially provide some explanation, other than preferences, for how much people decide to work.

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A Tables

Table 1: Data Summary by Agent Types

Agent Type	**Rel. Freq**	**Average** ES	**Median** ES^D	**Average** TE^D **($)**	TER^D
1	0.1	0.127	0.136	16125	1
2	0.1	0.20	0.203	9262	0.574
3	0.1	0.241	0.242	8782	0.544
4	0.1	0.274	0.273	7788	0.483
5	0.1	0.303	0.303	7469	0.463
6	0.1	0.332	0.332	7060	0.438
7	0.1	0.364	0.363	6572	0.407
8	0.1	0.402	0.402	6220	0.386
9	0.1	0.452	0.450	5571	0.345
10	0.1	0.563	0.547	4702	0.292

Table 2: Calibrated Values of Model Parameters

Parameter	**Value**
σ	0.3 and 0.5
α	1
δ	0
β	0.99
θ	0.5 and 1
η	0.25
κ	chosen to minimize $\sum_{i=1}^{10}(TER_i^M - TER_i^D)^2$
γ	chosen to match ES_i^D

Table 3: Calibrated Values of γ and κ

	$\beta = 0.99$ $\eta = 0.25$			
	$\sigma = 0.5$		$\sigma = 0.3$	
	$\theta = 1$	$\theta = 0.5$	$\theta = 1$	$\theta = 0.5$
κ	200.37	173.39	88.84	66.02
γ_1	0.98706	0.98975	0.98438	0.98706
γ_2	0.99194	0.99359	0.99023	0.99194
γ_3	0.99353	0.99487	0.99207	0.99353
γ_4	0.99451	0.99561	0.99329	0.99451
γ_5	0.99518	0.99619	0.99414	0.99517
γ_6	0.99579	0.99667	0.99487	0.99579
γ_7	0.99634	0.9971	0.99554	0.99634
γ_8	0.99687	0.99753	0.99619	0.99687
γ_9	0.99742	0.99796	0.99686	0.99742
γ_{10}	0.99825	0.99861	0.99786	0.99824

Table 4: Model's Solution to q_i^*, X_i^*, and H_i^* for Different Parameter Combinations of θ and σ

	$\theta = 1$						$\theta = 0.5$					
	$\sigma = 0.5$			$\sigma = 0.3$			$\sigma = 0.5$			$\sigma = 0.3$		
Agent Type	q^*	X^*	H^*	q^*	X^*	H^*	q^*	X^*	H^*	q^*	X^*	H^*
1	0.810	2.592	2.579	0.723	1.388	1.379	0.559	1.778	1.769	0.445	0.854	0.849
2	0.826	1.614	1.609	0.740	0.867	0.864	0.567	1.111	1.108	0.454	0.532	0.529
3	0.831	1.296	1.294	0.746	0.705	0.703	0.570	0.889	0.887	0.457	0.427	0.426
4	0.834	1.101	1.100	0.750	0.5965	0.5962	0.572	0.762	0.761	0.458	0.3626	0.3624
5	0.836	0.966	0.967	0.752	0.520	0.521	0.574	0.661	0.662	0.459	0.3183	0.3187
6	0.839	0.843	0.845	0.755	0.455	0.456	0.575	0.577	0.578	0.461	0.278	0.279
7	0.841	0.733	0.736	0.757	0.396	0.397	0.576	0.502	0.504	0.462	0.241	0.242
8	0.843	0.626	0.630	0.759	0.339	0.341	0.577	0.428	0.431	0.463	0.206	0.208
9	0.845	0.516	0.521	0.761	0.279	0.282	0.578	0.354	0.357	0.464	0.170	0.172
10	0.848	0.351	0.358	0.764	0.189	0.194	0.579	0.241	0.245	0.465	0.116	0.118

Table 5: Welfare Costs of 10% Inflation, by Agent Type, Measured as the Percentage of Total Consumption Forgone

	$\beta = 0.99$ $\eta = 0.25$			
	$\sigma = 0.5$		$\sigma = 0.3$	
Agent Type	$\theta = 1$	$\theta = 0.5$	$\theta = 1$	$\theta = 0.5$
1st	0.974	3.401	1.967	5.699
2nd	1.276	4.711	2.616	7.807
3rd	1.438	5.421	2.954	8.942
4th	1.572	5.965	3.262	9.863
5th	1.698	6.520	3.521	10.592
6th	1.812	7.011	3.772	11.361
7th	1.957	7.560	4.070	12.221
8th	2.115	8.208	4.388	13.194
9th	2.317	8.987	4.793	14.316
10th	2.687	10.522	5.641	16.561

Table 6: Welfare Cost of Inflation Piece by Piece

	$\theta = 1$													
	$\sigma = 0.5$							$\sigma = 0.3$						
Type	Δ	$q(0) - q(\rho)$	$H(0) - H(\rho)$	$u(q(0))$	$u(q(\rho))$	$u(q(\rho)) - u(q(0))$	$\kappa(1-\gamma_i)$	Δ	$q(0) - q(\rho)$	$H(0) - H(\rho)$	$u(q(0))$	$u(q(\rho))$	$u(q(\rho)) - u(q(0))$	$\kappa(1-\gamma_i)$
1	0.990	0.450	-0.006	1.207	0.702	-0.505	2.592	0.980	0.556	-0.005	1.151	0.494	-0.657	1.387
2	0.987	0.459	-0.002	1.225	0.713	-0.513	1.615	0.974	0.569	-0.002	1.172	0.503	-0.669	0.868
3	0.985	0.463	-0.001	1.231	0.716	-0.515	1.296	0.970	0.574	-0.0008	1.179	0.505	-0.673	0.704
4	0.984	0.465	-0.0001	1.235	0.718	-0.517	1.100	0.967	0.576	-0.0001	1.183	0.507	-0.675	0.596
5	0.983	0.466	0.0003	1.237	0.719	-0.518	0.966	0.964	0.578	0.0003	1.186	0.509	-0.677	0.520
6	0.982	0.467	0.0008	1.240	0.721	-0.519	0.844	0.962	0.580	0.0008	1.189	0.510	-0.679	0.455
7	0.980	0.468	0.001	1.242	0.722	-0.520	0.733	0.959	0.582	0.0011	1.191	0.511	-0.680	0.396
8	0.978	0.469	0.0016	1.244	0.723	-0.5204	0.627	0.956	0.583	0.0015	1.193	0.512	-0.681	0.338
9	0.977	0.470	0.002	1.246	0.725	-0.521	0.517	0.952	0.585	0.002	1.196	0.513	-0.683	0.279
10	0.973	0.472	0.003	1.249	0.726	-0.523	0.351	0.943	0.587	0.0024	1.199	0.514	-0.684	0.190
	$\theta = 0.5$													
	$\sigma = 0.5$							$\sigma = 0.3$						
Type	Δ	$q(0) - q(\rho)$	$H(0) - H(\rho)$	$u(q(0))$	$u(q(\rho))$	$u(q(\rho)) - u(q(0))$	$\kappa(1-\gamma_i)$	Δ	$q(0) - q(\rho)$	$H(0) - H(\rho)$	$u(q(0))$	$u(q(\rho))$	$u(q(\rho)) - u(q(0))$	$\kappa(1-\gamma_i)$
1	0.966	0.508	-0.006	0.981	0.331	-0.649	1.778	0.943	0.515	-0.005	0.892	0.180	-0.711	0.854
2	0.953	0.515	-0.002	0.992	0.335	-0.657	1.111	0.922	0.526	-0.002	0.905	0.182	-0.722	0.532
3	0.946	0.519	-0.0008	0.996	0.336	-0.660	0.889	0.910	0.529	-0.0008	0.909	0.183	-0.726	0.427
4	0.940	0.520	-0.0001	0.998	0.337	-0.661	0.761	0.901	0.531	-0.0002	0.912	0.184	-0.728	0.362
5	0.935	0.521	0.0001	0.999	0.3375	-0.662	0.661	0.894	0.533	0.0003	0.914	0.1844	-0.729	0.318
6	0.929	0.522	0.001	1.001	0.338	-0.663	0.577	0.886	0.535	0.0009	0.916	0.1847	-0.731	0.278
7	0.924	0.523	0.0014	1.002	0.3383	-0.664	0.503	0.877	0.536	0.0011	0.917	0.1851	-0.732	0.242
8	0.918	0.524	0.0018	1.003	0.3387	-0.665	0.428	0.868	0.5363	0.0014	0.918	0.1853	-0.733	0.206
9	0.910	0.5244	0.002	1.005	0.339	-0.6654	0.354	0.856	0.538	0.002	0.920	0.1857	-0.735	0.170
10	0.895	0.526	0.003	1.007	0.340	-0.667	0.241	0.834	0.540	0.0026	0.922	0.1861	-0.737	0.116

Table 7: Expenditure Shares and the Welfare Costs of 10% Inflation Comparisons with $\sigma = 0.5$

	LW(2005)				**Atomistic Agent**		**Average Agent**	
	1900-2000		**1959-2000**					
	ES	1-Δ	ES	1-Δ	ES	1-Δ	ES	1-Δ
no holdup ($\theta = 1$)	0.12	0.014	0.10	0.008	0.11	0.009	0.32	0.018
holdup ($\theta = 0.5$)	0.09	0.032	0.06	0.025	0.075	0.022	0.32	0.068

Table 8: Welfare Cost of 10% Inflation using Food as the only Cash Good with $\sigma = 0.5$

	$\theta = 1$, $\kappa = 130$		$\theta = 0.5$, $\kappa = 80$	
Agent Type	γ	(1-Δ)*100	γ	(1-Δ)*100
1	0.9589	0.663	0.9550	2.154
2	0.9731	0.751	0.9702	2.685
3	0.9785	0.815	0.9760	3.014
4	0.9816	0.859	0.9799	3.340
5	0.9843	0.914	0.9826	3.628
6	0.9863	0.958	0.9848	3.909
7	0.9880	1.000	0.9868	4.195
8	0.9898	1.072	0.9888	4.638
9	0.9918	1.155	0.9909	5.156
10	0.9944	1.348	0.9939	6.263

B Figures

Figure 1: Expenditure Share on Cash Goods and Total Expenditure Ratios by Agent Types

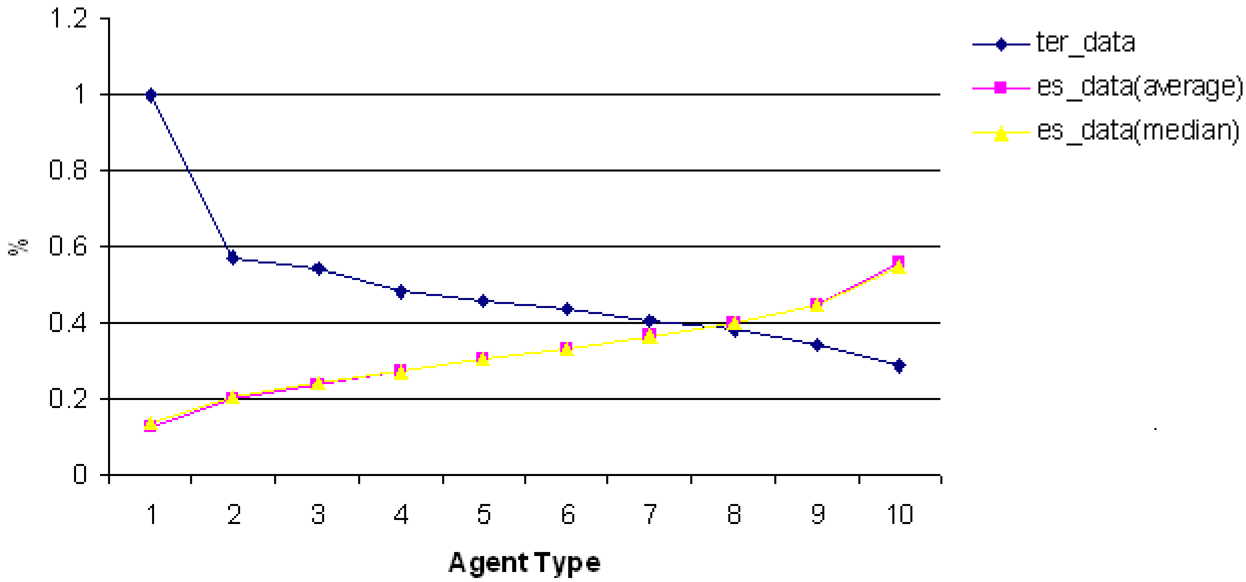

Figure 2: Relationship between Total Expenditure and After Tax Income by Agent Types

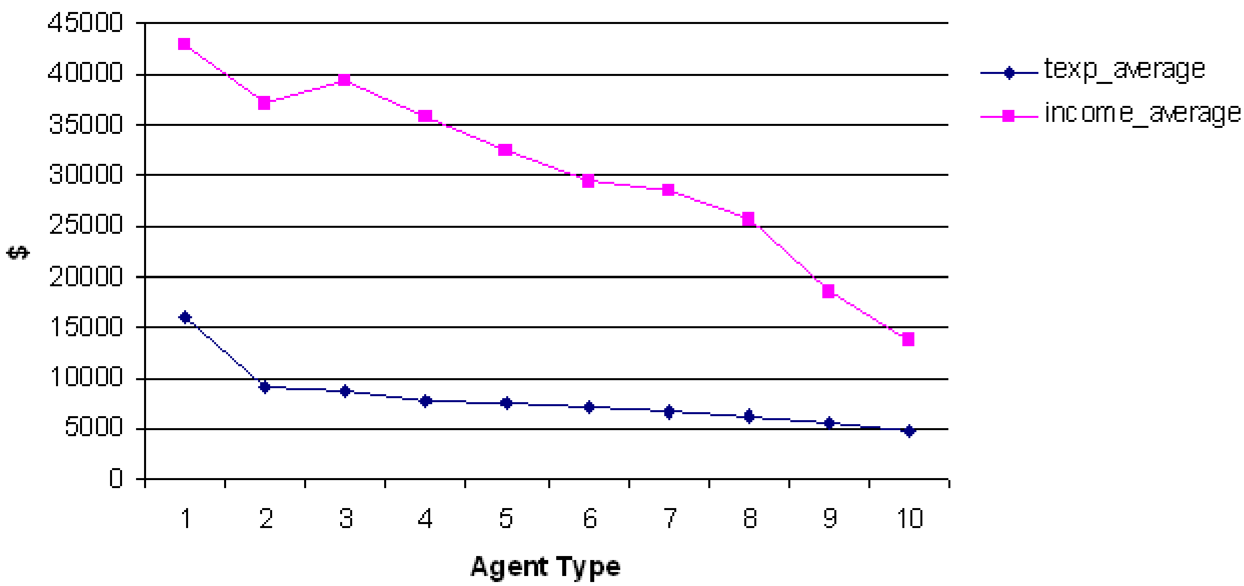

Figure 3: Comparison between Expenditure Shares on cash Goods from the Model with Data

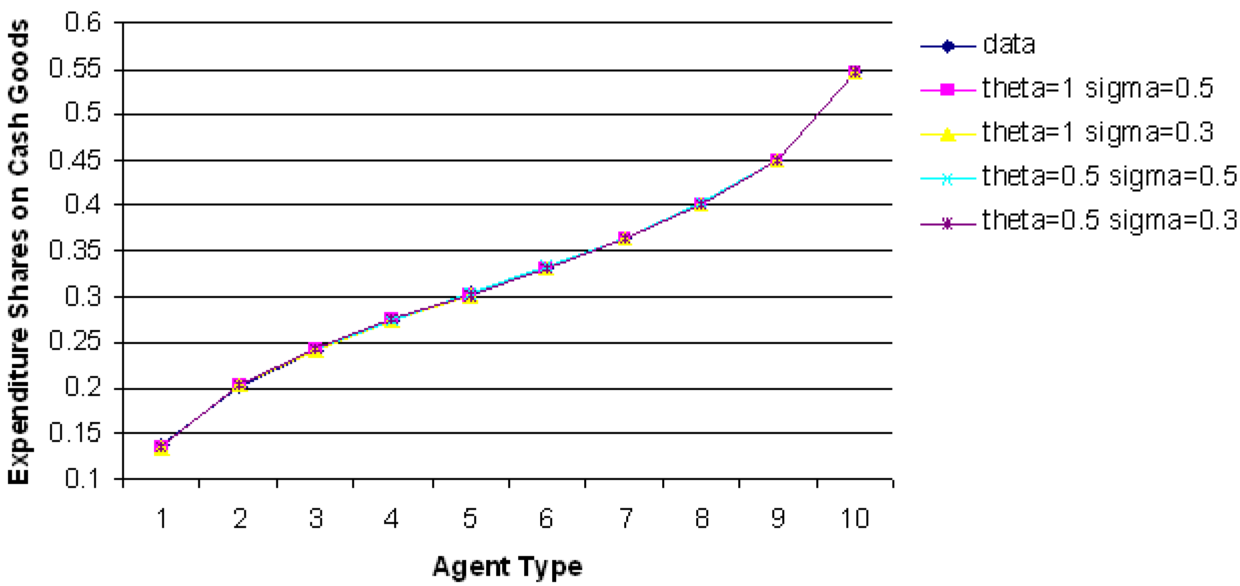

Figure 4: Comparison between Total Expenditure Ratios from the Model with Data

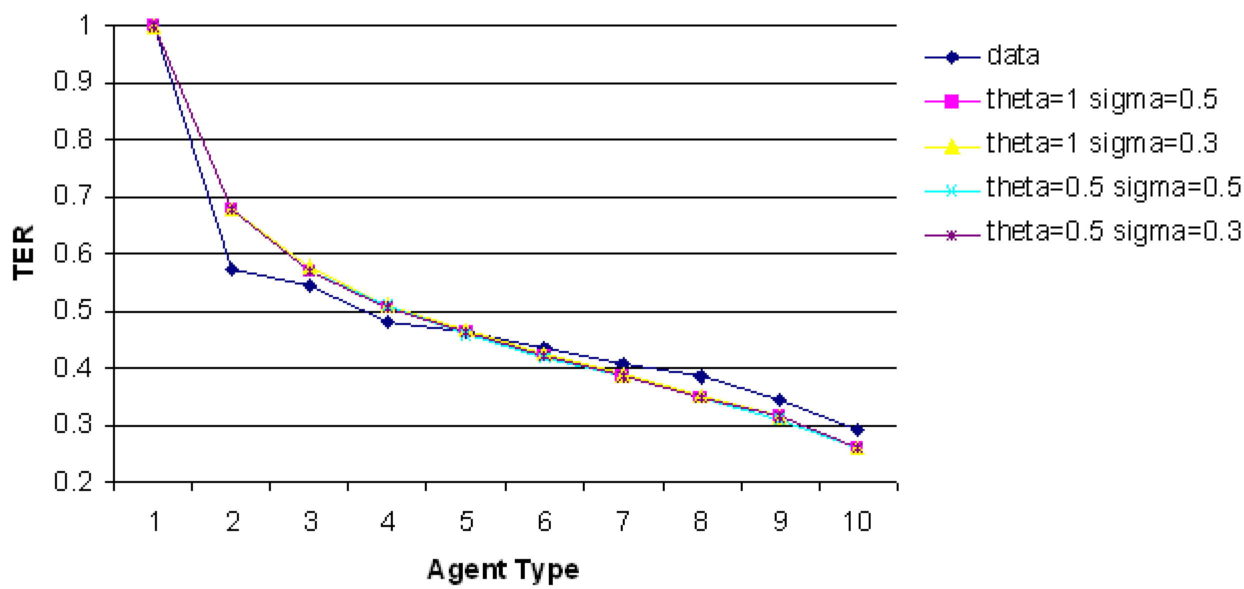

Figure 5: Sensitivity Check of Total Expenditure Ratios with $\kappa = 1$

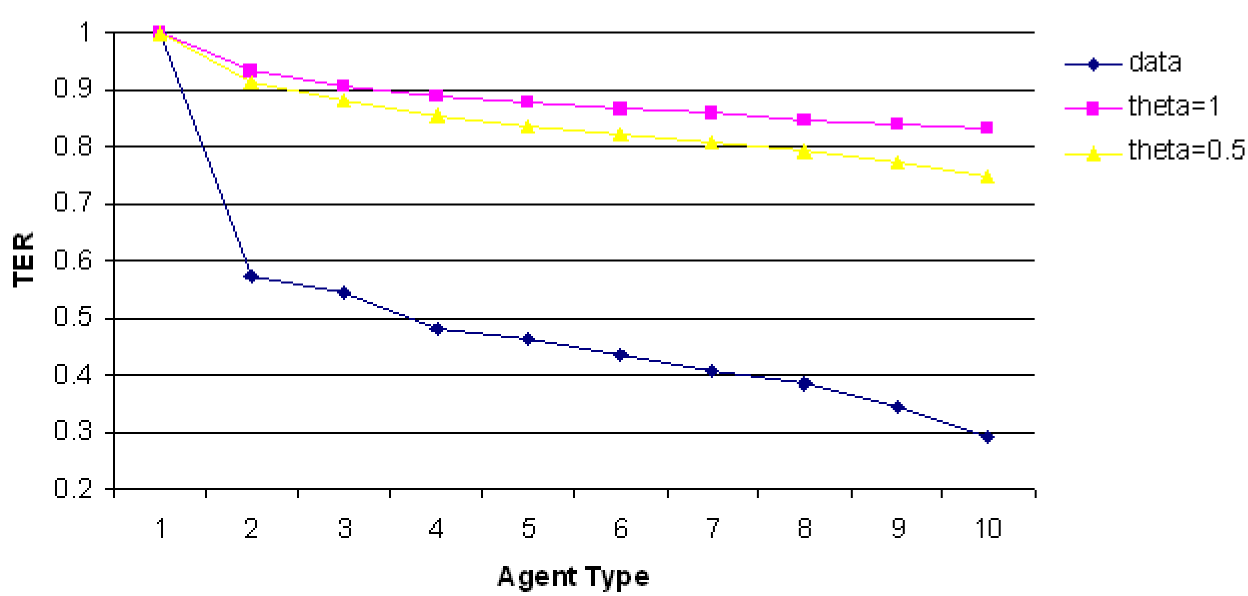

Figure 6: Total Expenditure on Cash and General Goods from Data by Agent Type

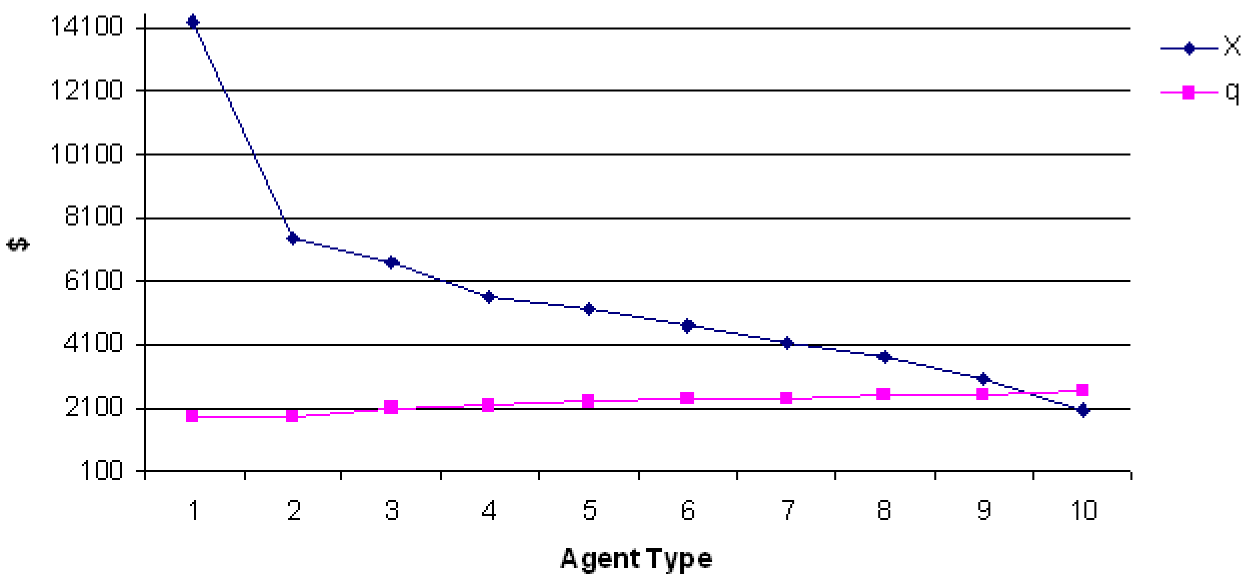

Figure 7: Total Expenditure on Cash and General Goods from Data compared to Model by Agent Type

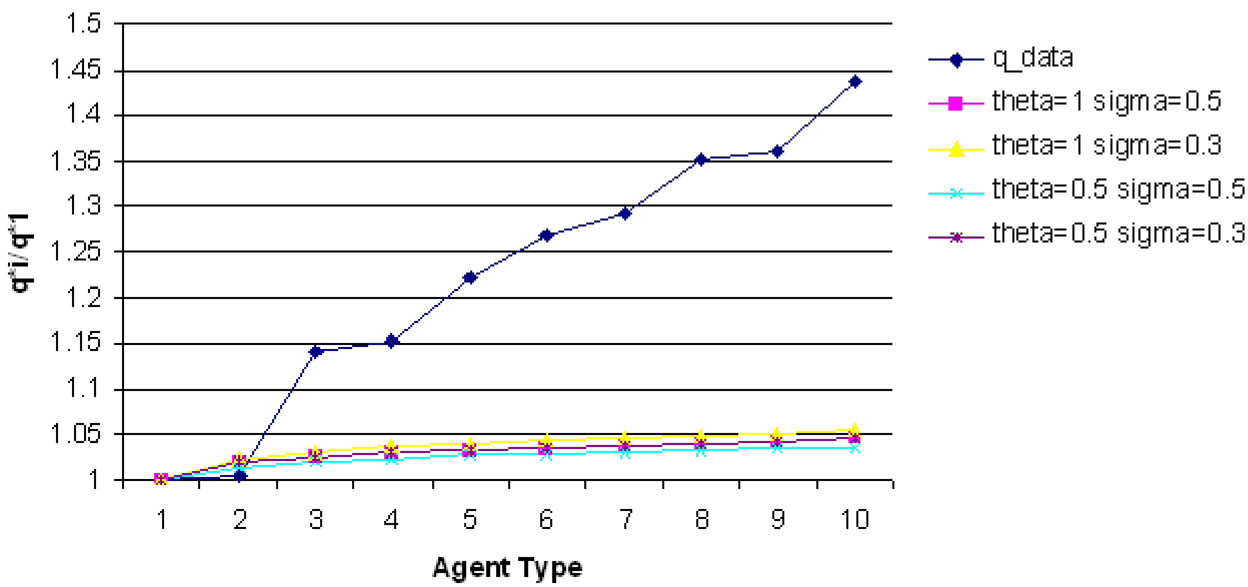

Figure 8: Total Expenditure on Cash and General Goods from Data compared to Model by Agent Type

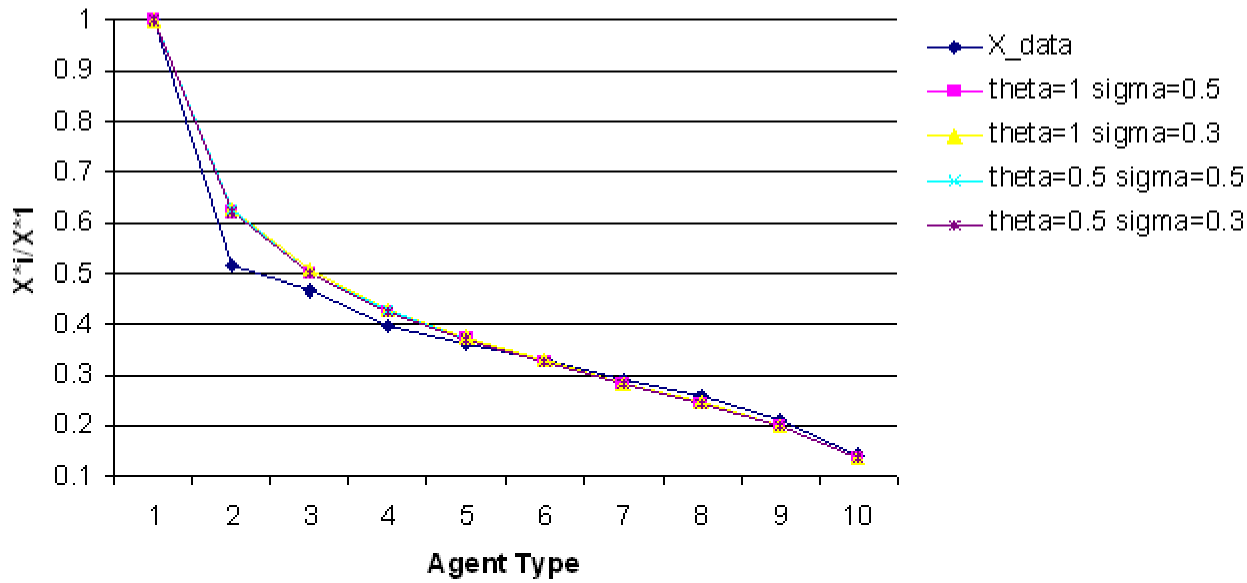

Figure 9: Welfare Costs of 10% Inflation by Agent Type

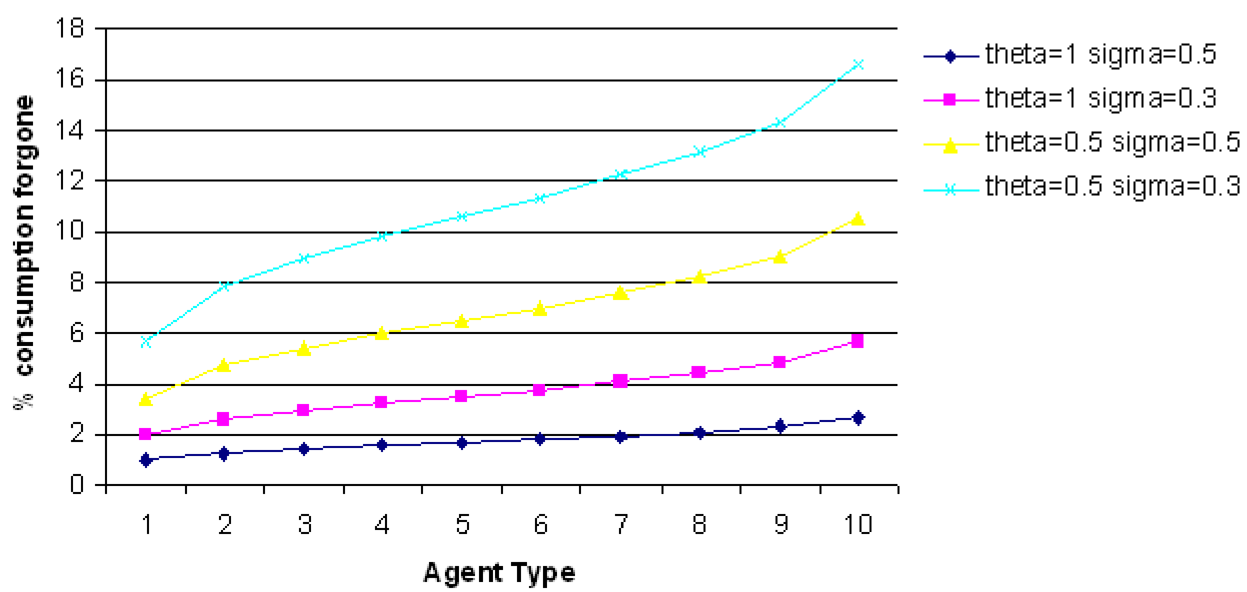

Figure 10: **Holdup Effect Measured as the Difference in Welfare Cost of 10% Inflation**

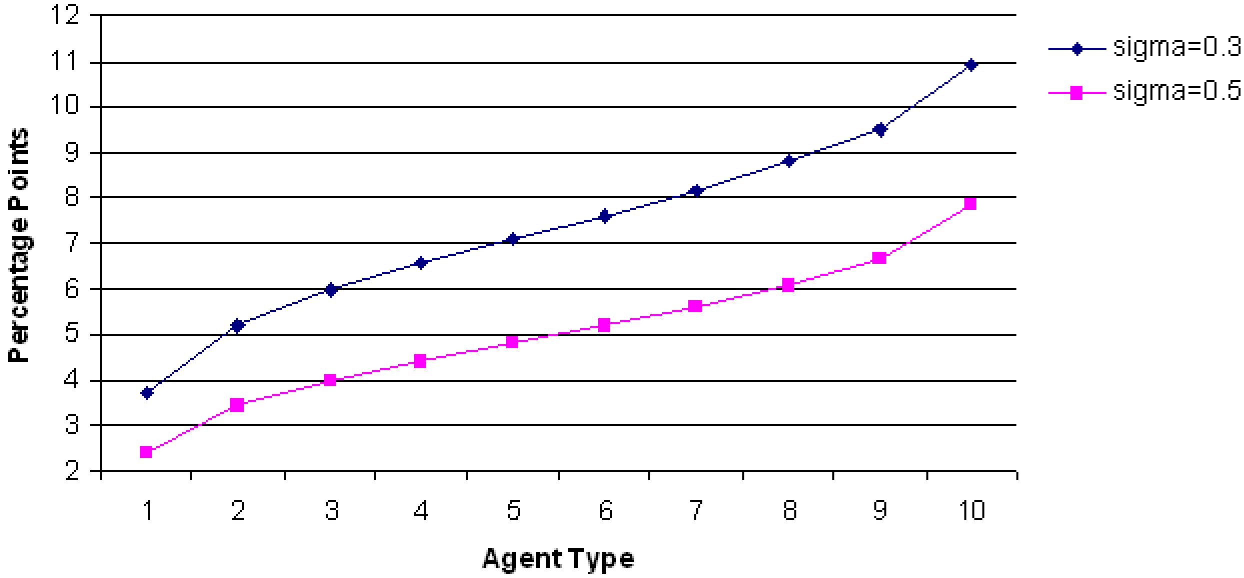

Figure 11: Welfare Costs of 10% Inflation for different η values with $\theta = 1$, $\sigma = 0.5$ by Agent Type

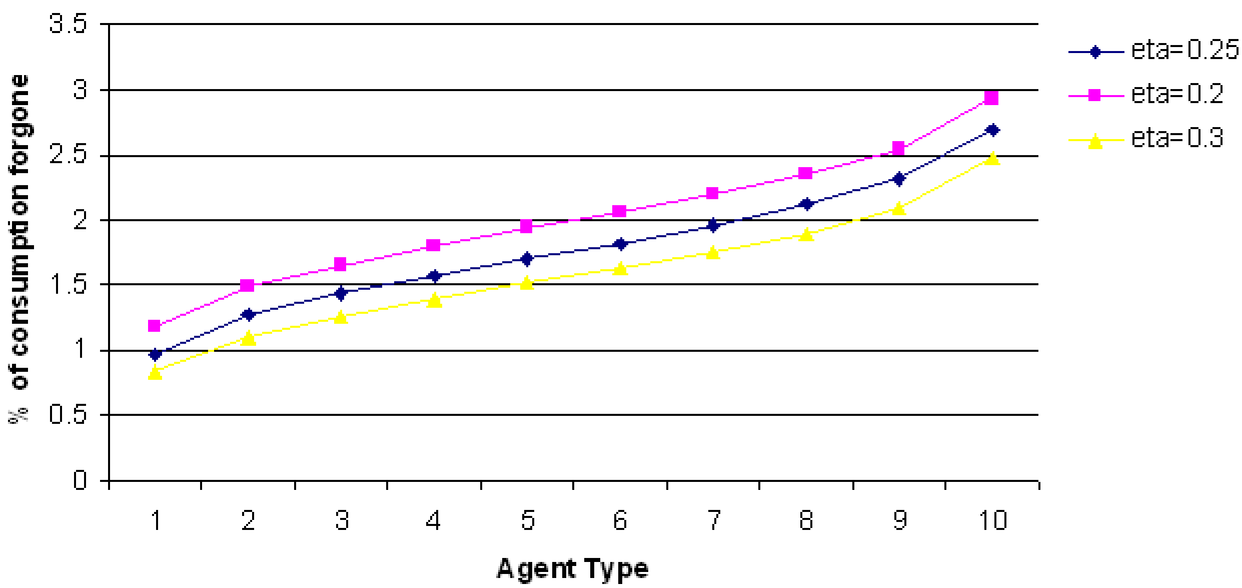

Figure 12: Welfare Costs of 10% Inflation for different η values with $\theta = 0.5$, $\sigma = 0.5$ by Agent Type

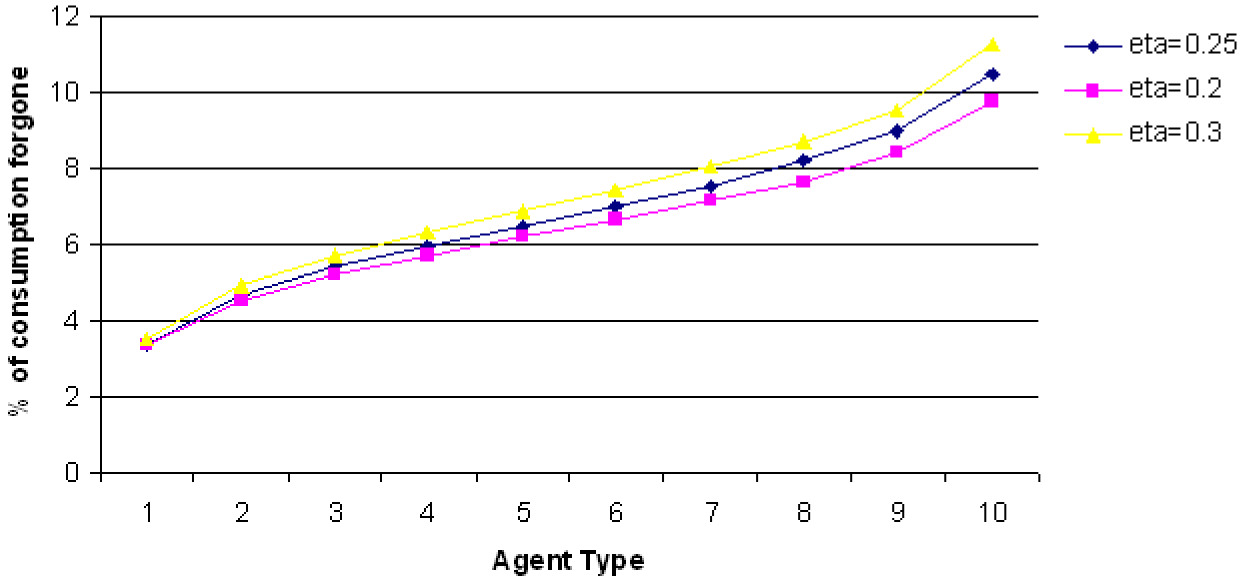

Figure 13: Comparing Welfare Costs of 10% Inflation for different Basket of Cash Goods with $\sigma = 0.5$ by Agent Type