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Simulating Sequential Search Models with Genetic Algorithms: Analysis of Price Ceilings, Taxes, Advertising and Welfare

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Abstract

This paper studies advertising, price ceilings and taxes in a sequential search model with bilateral heterogeneities in production and search costs. We estimate equilibria using a genetic algorithm (GA) applied to over 100 market scenarios, each differing based on the number of firms, number of consumers, existence of price ceilings or taxes, costs of production, costs of advertising, consumers' susceptibility to advertising and consumers' search costs. We compare our equilibrium results to those of the standard theoretical consumer search literature and analyze the welfare effects of advertising, price ceilings and sales taxes. We find that price ceilings and uninformative advertising can improve welfare, especially if search costs are sufficiently high.

JEL Classification: C63; D21; D43; D73; D83; M37

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1 Introduction

Taxes, price ceilings and advertising have all been addressed in the consumer search literature in different models with different fundamental assumptions. Advertising is particularly interesting since, despite its natural role in search, the two have not been studied together extensively under general market conditions with heterogeneous agents.¹

¹Robert and Stahl (1993) study advertising in a sequential search model but assume identical search costs and costs of production, the focus being to find price dispersed equilibria in a homogeneous setting. McCarthy (2007) considers bilateral heterogeneities but only with a duopoly.

Through computational intelligence (CI) techniques, this paper addresses these issues. First, we present a unified treatment of taxes, price ceilings and advertising with a common set of underlying assumptions. Second, we study uninformative advertising without directly influencing buyers' utilities or preferences. Advertising is uninformative in the sense that advertisements do not send specific product information to buyers, but the role of advertising remains clear—consumers do not visit firms in any predetermined order, and advertisements facilitate this ordering. Our final contribution addresses two issues in the theoretical search literature. One, we allow for a finite number of firms *and* consumers. Despite Carlson and McAfee's (1983) finding that a finite number of firms can change both the general intuition and some specific comparative statics, many theoretical search models still assume a continuum of consumers. Two, we significantly lessen the computational requirements of firms in that they do not know each others' costs of production or the distribution of consumer reservation prices.

Our underlying model is similar to Carlson and McAfee (1983) and Bénabou (1993) in that we consider a set of firms with heterogeneous marginal costs selling an identical product to consumers with heterogeneous search costs. Buyers know the prices in the market, but not which firm is offering which price, and inelastically demand one unit up to some reservation price. Contrary to the standard setup, however, firms only observe their payoff to different pricing decisions and do not know the other prices or costs in the market, the reservation prices of consumers, or even how many other firms exist. Given this basic setup, the inclusion of sales taxes and price ceilings is relatively straightforward. Taxes directly decrease the margin earned on each unit sold, while price ceilings explicitly alter the population from which firms select their prices. In regard to advertising, we follow a setup similar to Hertzendorf (1993) and treat each ad as a commercial. Given buyers' behaviors (e.g., how many commercials they watch), consumers rank the number of advertisements viewed from each firm and sample firms in the order determined by this ranking. If there are 3 firms and buyer i views 10 ads from firm 1, 8 ads from firm 2, and 12 ads from firm 3, then they visit firm 3 first, firm 1 second, and finally firm 2. The number of commercials seen by consumer i for any given firm is determined by a draw from the hypergeometric distribution.

We search for equilibria of these models using a genetic algorithm (GA), just one technique under the CI umbrella. Adopting concepts from Darwin's theory of evolution, this algorithm evaluates a number of strategies where the best strategies reproduce and

create a new population of strategies. Although these tools remain relatively unadopted in the literature, a growing number of authors have used GAs to develop intuition for models where analytical results have proven untractable or simply to represent their usefulness through equivalence to analytical results. Price (1997), for instance, shows that GA equilibria are essentially identical to analytical solutions in a standard Bertrand or Cournot game. Similar equivalence holds for a model of a vertical chain of monopolies, a pool model and others. Andreoni and Miller (1990) also apply a GA to their study of bidding strategies in auctions (first price, second price, private value and common value). They find that the GA not only converges near the Nash equilibrium but also that the bidding patterns of the computer agents closely relate to observed human bidding strategies, especially with regard to the overbidding often observed in real auctions. Other applications include Albin and Foley (1992) and Keenan and O'Brien (1993), both of which study spatial competition, and Arifovic (1994), which studies firm behavior in a cobweb model. Arifovic shows that the GA can learn equilibrium strategies even when some analytical approaches cannot and that the predicted behavior under the GA differs less from experimental data than many other learning rules.²

Note that our goal is not to strictly analyze the mathematical properties of GAs.³ We also do not analyze the equivalence of our algorithm to those used in standard evolutionary game theory; however, as Reichmann (2001a) shows, the canonical GA is structurally equivalent to the commonly used replicator dynamic. It is also recognized (see Reichmann (2001b)) that replicator dynamics do not cover the whole GA learning process in that they cannot develop completely new strategies. We instead emphasize the application of these tools to consumer search models and, in particular, advertising, taxes and price ceilings. Although we briefly discuss convergence issues in Section 3, a thorough explanation of GAs and their properties and interpretations can be found in Dawid (1999).

Particularly with advertising, our GA approach allows us to study markets in which analytical results have proven difficult. We consider 120 different market combinations, each differing based on the number of firms, number of consumers, costs of production, sales taxes, price ceilings, costs of advertising, consumers' susceptibility to advertising

²For a more thorough description of how GAs have been implemented and many more examples, see Holland and Miller (1991).

³See Goldberg (1989), Davis and Principe (1993) and Mitchell (1996) for a clear explanation of GAs and their mathematical properties.

and consumers' search costs. In total, this provides 2400 simulated equilibrium pricing strategies, 960 simulated equilibrium advertising strategies and 120 total welfare values based on these equilibrium strategies. Using these datasets, we provide three general findings.

One, firms' prices are determined in large part by the number of buyers and firms, the level of search costs and the existence of price ceilings or sales taxes. Specifically, we expect lower prices if the market is subject to a price ceiling or has a high number of consumers or firms, and we expect higher prices if search costs are high or the market is subject to a sales tax. More interestingly, in response to a proportional sales tax, firms increase price by a smaller proportion than the actual tax, as in Carlson and McAfee (1983) and similarly with an increase in marginal cost. Two, in regard to advertising, firms advertise less if charging a higher price or competing with more firms, and firms advertise more intensely in response to higher search costs or more buyers in the market. Note that the amount of advertisements buyers see has no significant effect on the number of ads sent. Finally, with regard to welfare, we find that advertising has a large positive effect on welfare if search costs are sufficiently high. This extends the results of McCarthy (2007) from a duopoly to several firms. A similar welfare result holds for price ceilings. This also supports results from a duopoly search model with price ceilings from Rauh (2004). We want to emphasize, however, that our simulations cannot determine necessity, only sufficiency.⁴ Elegant proofs are therefore unattainable in this case. Nonetheless, our results strongly suggest that price ceilings and uninformative advertising can improve welfare when buyers must search to gain information.

In regard to sales taxes, price ceilings and advertising, the search literature is relatively thin. Carlson and McAfee (1983) show that proportional sales taxes increase prices by a lesser degree than a per unit tax, which is completely passed on to buyers, but they do not address welfare. Rauh (2004) finds that price ceilings might increase welfare under certain conditions—dependent in part on the reduction in total search cost expenditures and the transfer of production to higher cost firms. For informative advertising, Robert and Stahl (1993) show that equilibrium price dispersion can still arise even without *ex ante* heterogeneities; however, they do not discuss welfare. Janssen and Non (2006) develop a similar model for a duopoly with a small percentage of completely informed consumers, i.e., shoppers. They show that the inclusion of informed consumers

⁴See Marks (2007) for a discussion on simulations, sufficiency and necessity.

has important implications for comparative statics—especially the limiting cases of zero search or advertising costs. McCarthy (2007) also studies informative advertising in a duopoly, allowing for bilateral heterogeneities, and finds that the welfare effects of advertising vary depending on search costs and costs of production and advertising. In all cases, advertising is assumed to be perfectly informative (i.e., each ad reveals perfectly the price charged by a given firm).

The remainder of the paper is organized as follows. In Section 2, we present a brief theoretical background for consumer search models. Section 3 explains the algorithm and presents summary results for different market scenarios. Section 4 discusses comparative statics and welfare effects, Section 5 provides a discussion of GAs and their interpretations and Section 6 concludes. All tables and figures are deferred to the appendix.

2 Basic Model

2.1 Consumers

As is standard in the sequential search literature, consider a buyer who is searching for the best price for a particular good which we assume is homogeneous across all firms. The consumer must also pay some search cost s for every price quote obtained. Denote the lowest current price observed by p^* , in which case the expected gain from an additional search is

$$b = \int_{p_{min}}^{p^*} [V(p) - V(p^*)] dF(p) - s,$$

where the distribution of prices $F(p)$ is common knowledge to all buyers, but individual firm prices are unknown prior to search. If each consumer inelastically demands one unit, then an optimal stopping rule exists so that each consumer is myopic and has a unique reservation price $r(s)$ that solves

$$\int_{p_{min}}^r (r - p) dF(p) = \int_{p_{min}}^r F(p) dp = s.^5$$

Following Carlson and McAfee (1983), an analogous discrete result is to first rank

⁵This extends naturally to more general, downward-sloping demand.

prices in ascending order so that $p_1 \leq p_2 \leq \dots \leq p_N$, in which case a given consumer with search cost s again adopts an optimal stopping rule based on the reservation price p_r that solves

$$\sum_{j=1}^{r-1} (p_r - p_j) f(p_j) = s, \quad (2.1)$$

where $f(p_j)$ is the probability of finding price p_j . Upon seeing any price $p \leq p_r$, the consumer stops searching and purchases the product. If we let $f(p) = \frac{1}{N}$ (i.e., each firm is equally likely to be sampled), then p_r simply equates the difference between itself and the average of the lower prices times the probability of finding a lower price to the cost of search. As is standard in the literature, we assume that buyers enter the market with a free initial search but must pay their search cost to visit additional firms. Note that, due to the structure of equation (2.1), it is impossible for any buyer to search exhaustively and still not find a price below their reservation price. As such, all buyers eventually make a purchase.

2.2 Firms

To ensure that a pure-strategy equilibrium exists, we assume that firms have heterogeneous, constant marginal costs of production.⁶ Since buyers inelastically demand one unit up to their reservation price, each firm's demand is exactly equal to the number of consumers that purchase from their store. Denote by q_n the quantity sold by firm n , in which case each firm's profit is given by

$$\pi_n = (p_n - c_n) q_n.$$

The standard theoretical model assumes a continuum of consumers such that, given the prices in the market and the search cost distribution, a distribution of reservation prices emerges. This distribution provides the expected demand for each firm based on the probability of consumers finding a particular firm—given that firm's pricing decision.⁷ In this model, however, there are a finite number of consumers as well. Each firm's

⁶Heterogeneous marginal costs of production are not necessary for the existence of pure-strategy equilibria. See Rob (1985) for sufficient conditions for pure-strategy equilibria even with identical costs.

⁷In most cases, a continuum of firms is also assumed, in which case we look at a price distribution, $F(p)$ and determine the distribution of reservation prices in terms of this price distribution, the distribution of marginal costs and the search cost distribution. See Bénabou (1993) for more detail. Carlson and McAfee (1983) derive an analogous representation for a finite number of firms.

profit function is therefore highly discontinuous as small price changes may yield large jumps in profits.

Our algorithm derives directly from this consumer and firm behavior. Consumers form their reservation values based on the prices in the market which then determines their stopping rule. If there is no advertising, buyers sequentially visit firms, where each new visit is randomly selected and where each firm has the same probability of being selected. With advertising, buyers search in the order determined by the ranking of the number of advertisements seen from each firm. To discover firms' pricing strategies, we adopt a GA similar to that used in Price (1997).

3 Genetic Algorithm

The standard GA developed as a search algorithm founded strongly on evolutionary concepts.⁸ Agents consider a population of genetic (binary) strings, each of the same exogenously specified length, representing all possible strategies. Based on the relative fitness of the strings, the better-performing strategies are randomly matched together to form sets of parents, which then produce offspring strategies consisting of information from both genetic strings of the parents. These new strategies help form new populations. This requires an exogenously specified fitness function to determine relative fitness, which is just a payoff function mapping strategies to some measure of performance (e.g., profit, utility, etc.). In cases where the payoff depends on other agents, as in our model, the fitness function is said to be state dependent. The process of developing these new populations continues for some specified number of iterations.

Within the general algorithm, the process of updating populations depends on genetic operators—mainly the process of selection, crossover and mutation. Selection determines how strategies are chosen for each new population. Although several types of selection processes exist, we adopt a tournament selection where the best strategies in a given set of strategies replace the worst strategies to form the new population. The tournaments themselves consist of a subset of prices randomly drawn from the population that compete against one another for the highest payoff. This is also a type of elitist selection process, but in our case, the worst strategies from the *population* are not replaced—only the worst strategies selected in the given *tournament*. Crossover

⁸See Arthur (2006) for a review of the first agent-based simulation models used in economics.

determines how new strategies are formed. Again, many crossover designs have been considered in the literature, but in this paper, we adopt the standard one-point crossover. This involves assigning some number (less than the length of the string) where the units of the string beyond that number switch across the parents. For example, if we have a string of length 7 and choose a crossover position of 4, then the 5th, 6th and 7th bits are swapped across parents. This is intended to join the genetic information of both strategies to produce possibly better strategies. Finally, another important aspect of genetic operators is mutation—the process by which certain bits in each gene switch with some probability. Although mutation has only a secondary influence on equilibrium selection, the standard mutation involves randomly selecting a gene to switch from 0 to 1 or vice versa. We adopt this mutation strategy in our algorithm.

3.1 Base Model, Sales Taxes and Price Ceilings

With genetic strings of length 7, our selection, crossover and mutation operations are as follows. From the full population of 300 prices, we select 250 prices for each firm to enter the tournament stage. The best 40 strategies for each firm then combine to form new pairs from which offspring prices arise. These prices replace the worst prices of the 250 in the given tournament. The children are determined by the crossover position, which we set to 4, and mutation follows the standard setup where we randomly select a gene at each construction of a new population and switch this from 0 to 1 or vice versa. Our specific algorithm consists of the following steps:

1. Set an initial population of prices of size P for each of N firms (identical for all firms) and assign each firm a marginal cost of production c . Prices are coded as binary numbers so as to represent a genetic string.
2. Assign each of K consumers a search cost drawn from an exogenously specified distribution.
3. Each firm draws T prices from their population.
4. Compute the payoffs for each of the T prices for all firms. This is the tournament stage of the algorithm where, for a given firm, each of the T prices compete against one another to progress to the next stage's population. To compute payoffs, we do the following:

- (a) Convert prices from binary code to integers. Given prices and search costs, each consumer forms T reservation prices—one for each tournament—following equation (2.1).
 - (b) Given their reservation values, each consumer randomly selects a firm. If that firm’s price is below their reservation price, the buyer stops searching and buys the product. Otherwise, they randomly select another firm. This process continues until all consumers find a firm from which to buy.
 - (c) This forms the quantity sold for each firm for each of the T tournaments, q_{nt} for $t = 1, \dots, T$. The profit for each t^{th} tournament is therefore $(p_{nt} - c_n) q_{nt}$. Note that each firm’s marginal cost, c_n , remains unchanged throughout the algorithm. This is also true for each consumer’s search cost.
5. Convert prices back to binary code, where the best-performing prices randomly match with one another to spawn new prices. These new prices are formed by the crossover and mutation operations discussed previously.
 6. The new prices replace the worst-performing prices and form a new population.
 7. Repeat until each firm’s population consists of the same prices (prices differ across firms but not within a population for a given firm).

The above setup is our base model, and we consider 24 variants of this. These variants consist of three different numbers of consumers ($K = 30$, $K = 100$, and $K = 500$), two different numbers of firms ($N = 10$ and $N = 30$), two levels of search costs and two levels of production costs. We study each variant with a population of size 300 and allow firms to update prices for 250 generations. In all cases, search costs follow a log-normal distribution with standard deviation of $\log(3)$. The two different levels of search costs are defined by two different means ($\log(5)$ and $\log(25)$). Marginal costs of production follow a uniform distribution on $[0, 20]$ and $[0, 60]$ for low and high marginal cost markets, respectively.

Sales taxes and price ceilings enter naturally into our algorithm. We impose a \$60 price ceiling by adjusting each firm’s price population prior to each tournament. If any firm draws a price greater than \$60 in a given tournament, this price is set at \$60. Any price below \$60 is left alone. We also introduce an 8% proportional sales tax (separately from the price ceiling) by adjusting each firm’s profit function to $(p_{nt}(0.92) - c_n) q_{nt}$.

For a general sense of how our algorithm works, the draws and the path of prices for base markets (without taxes, price ceilings or advertising) are provided in Figure 1. In the sub-figures, the two columns are based on low and high search costs respectively, while the three rows represent 10, 100 and 500 consumers. For clarity, these graphs only contain draws for the lowest and highest cost firms (the red and blue lines respectively) as well as the average price across all firms (the black line). Note also that, at each generation, there is not just one price for a given firm—there is a population of prices. To condense the population to one price, we approximate the price distribution for each firm and take the price associated with the most mass in this distribution. We estimate each distribution using kernel density estimation with bandwidths computed through Maximum-Likelihood Cross-Validation. Each price plotted in the figures is the price associated with the highest mass from this distribution.⁹

Recall that convergence is achieved when all prices in each firm’s population are identical. In an evolutionary sense, this means that one genetic string has overpowered all others.¹⁰ From Figure 1, we see that all prices converge relatively quickly, which is indicative of most of our models; however, it is difficult to notice any obvious trends simply by these graphs. As such, we combine all of our simulated data and formally study the effect of firms, buyers, search costs, etc. in Section 4 using regression analysis.

3.2 Advertising

Advertising enters our algorithm primarily as a mechanism to direct consumer search. It does not directly affect firm profit functions except through the cost of advertisements but can attract consumers depending on how much the firm advertises relative to its rivals. We have in mind television commercials that offer little or no vital information about the product. By advertising more intensely, however, consumers are in some sense persuaded to visit that firm earlier in the search process. Specifically, we extend the algorithm above in three important ways. One, each consumer draws a number of commercials to watch. This comes from an exogenously specified distribution, and we adjust the mean of this distribution to consider changes in tv-watching behavior. Two,

⁹We also include plots of price variances and welfare for each generation in Figure 2. For these figures, the black line (left axis) represents welfare values, and the red line (right axis) represents price variances.

¹⁰As noted by Dawid (1999), convergence to a uniform state does not necessarily mean we have found an equilibrium and could be a byproduct of the coding used. However, we have run this model several times with different coding mechanisms and find that convergence states are consistent across all models.

we add a second strategic variable that firms must choose. This enters the algorithm the same as price in that we consider a population of genetic strings representing different numbers of commercials. In addition to price, each firm now draws several numbers of commercials to compete against one another in a tournament. Note that payoffs in each tournament depend on the (price, commercial) pair drawn by the firm. Three, given the number of commercials each buyer watches and the number of commercials each firm purchases, each consumer watches some number of commercials from each firm as determined by a random draw.¹¹

We can think of advertising as balls in an urn. If there are 100 total advertisements and firm A buys 20, then they have purchased 20 of the 100 balls. Firms can all purchase advertisements at the same cost per ad, and we consider two different marginal costs (\$3 per ad and \$10 per ad). Each buyer then chooses some number of balls from the urn, which represents their television watching intensity. Given this number, determining the number of commercials consumers see from each firm is just a draw from the hypergeometric distribution.

Buyers then rank the number of commercials viewed in descending order and search firms in this order, so if consumer i sees the most commercials from firm B and the second most commercials from firm A, they visit firm B first and firm A second. Note that buyers still stop searching whenever they find a price below their reservation price, so this consumer would only go to firm A if firm B's price exceeded their reservation price.

3.3 Summary Statistics

We present basic summary statistics in Tables 1 through 5. Table 1 shows the mean, min, max and standard deviation for the 60 simulated markets with low production and advertising costs, while Table 2 shows similar statistics for high production and advertising costs. Some basic trends from these tables are: (1) prices generally increase with higher search costs, (2) if firms can advertise, they tend to price less but the range of prices tends to increase, and (3) with high search costs, the price ceiling is binding for most all firms. (3) is particularly interesting as, despite sometimes large

¹¹The structure of equation (2.1) remains unchanged. Since advertising does not affect buyers' anticipation of lower prices, it also has no effect on reservation prices (i.e., each firm remains equally likely to price below a given consumer's reservation price).

cost advantages, firms do not undercut their rivals. Buyers therefore do not search as much in equilibrium and are more likely to purchase from whichever firm they randomly select; however, this is not necessarily a welfare improvement since reservation prices also decrease, and buyers receive less consumer surplus for each purchase.

From Table 3, we see that reservation prices behave somewhat as we would expect. High search costs and high costs of production (yielding higher prices) each increase mean reservation values. Finally, notice from Table 5 that in some cases, all firms advertise with nearly identical intensities. This represents somewhat of an arms race and is clearly a welfare loss as demand is dictated entirely by random search but firms still spend money on advertising. In these cases, outright banning of advertising may improve overall welfare.

4 Results

Although we notice some trends from the summary statistics, we need to be a little more thorough to garner any definitive results. Especially with regard to advertising and market size (e.g., numbers of consumers and numbers of firms), general impacts on price and welfare are not clear. To study these effects in more detail, we combine the 120 market scenarios consisting of 2400 different price and advertising equilibria into one dataset, and we build another dataset consisting of the 120 welfare and price dispersion values from each market setup. We take as our measure of welfare the average sum of consumer and producer surpluses across the final tournament, at which point all price populations have converged to a uniform state. Because populations have converged, welfare values have also stabilized, save for the randomness inherent in the search process. For every product sold (recall there is one product for every consumer), the consumer surplus is the buyer's reservation price minus the price they pay and search costs they accrue, while the producer surplus is the price received minus costs of production and cost of advertising if relevant.

Treating price, advertising intensity and average welfare as our dependent variables, our independent variables are marginal costs of production and several dummy variables indicating the general market setup. Specifically, we have a dummy variable for search costs (0 for low, 1 for high), number of consumers (0 for low, 1 for middle, 2 for high), number of firms (0 for low, 1 for high), consumer television behavior (0 for little time

watching television, 1 for more time), cost of advertising (0 for low, 1 for high), as well as dummy variables for whether sales taxes, price ceilings or advertising are present in the market.

4.1 Comparative Statics

Column 1 of Table 6 presents results for basic OLS regressions of price on number of consumers and firms, level of search costs, marginal cost and whether the market is subject to a price ceiling or tax. We only look at the 1440 cases where advertising is not an option. Results are generally not surprising, but note that the effect of marginal cost on price, although positive, is statistically significantly less than one. This implies that firms respond to a \$1 increase in marginal cost with a less than \$1 increase in price. This differs from Carlson and McAfee (1983), who find that marginal cost increases are completely passed on to buyers; however, in their analysis, firms had convex cost functions while we consider constant marginal costs of production. A similar relationship exists between price and sales taxes. Consistent with Carlson and McAfee, we find that the expected price increases by less than 8% in response to an 8% sales tax.

To study advertising intensity, we restrict our sample only to the 48 markets (960 equilibrium advertising levels) where advertising is permitted and use instrumental variables in a regression of advertising level on price and the full set of dependent variables. Since marginal production costs only affect advertising through price, we use marginal costs as an instrument for price. Results from these regressions are included in column 2 of Table 6. We see that the number of firms has a significant negative effect on the number of advertisements. We also see that firms advertise higher prices with less intensity and advertise with much more intensity if search costs are high.

Note also that reservation prices have a positive effect on the price variance, and through this relationship, it follows that search costs have a positive effect on the variability of prices. This relationship between variance and search costs is also found in the theoretical model developed by Reinganum (1979). Also, consistent with MacMinn (1980) and Carlson and McAfee (1983), we find that an increase in the number of firms leads to an increase in price variability.

4.2 Welfare

We are particularly interested in the welfare effects of sales taxes, price ceilings and advertising. Since these three mechanisms do not exist together in any one market, we cannot study the full set of 120 market scenarios due to collinearity. Therefore, for sales taxes and price ceilings, we restrict our analysis to the 72 markets with either price ceilings, taxes or neither. Results are presented in column 4 of Table 6. We see that the interaction between price ceilings and search cost levels has a significant positive effect on welfare. This supports the theoretical findings in Rauh (2004), as discussed in Section 1. Intuitively, high search costs allow for considerably higher equilibrium prices, in which case a given price ceiling is more likely binding than if search costs were lower. Price dispersion therefore diminishes as many firms (in some cases, all firms) price at or near the ceiling. Search intensity then decreases as most consumers will find a price below their reservation price on their first search.

For advertising, we consider only the 48 markets where advertising is allowed. We also include interaction terms between advertising intensity and search cost levels (low or high). Results are in column 3 of Table 6. The most important result here is that advertising levels by themselves may have a negative effect on welfare, but for high search costs, advertising can create a welfare gain. This supports the theoretical results in McCarthy (2007). The intuition is that advertising can facilitate more efficient search by attracting buyers to firms from which they are more likely to purchase. If search costs are sufficiently high, this avoids significant welfare losses otherwise incurred through costly search.

5 Discussion of Agent-based Simulations

Although simulation-based results are a growing trend in economics, the role of agent-based simulations remains minimal. This has been attributed to two primary reasons: One, it is difficult to interpret the dynamics of the model and to generalize results, and two, the fitness functions and general simulation process have no clear statistical properties.¹² In this sense, support that our results are a valid equilibrium rather than a highly unlikely realization of some random process is difficult to obtain. We address

¹²See Leombruni and Richiardi (2005) and Leombruni et al. (2006) for more detail.

each of these issues in turn. Note, however, that in simpler cases where analytical results are tractable, the standard GA has proven to work very well in estimating the analytical equilibrium.¹³

5.1 Interpretation

In a purely mathematical sense, the GA's equilibrium search method is a strong optimization procedure. As Holland (1992) shows, the GA thoroughly explores both the set of proven successful strategies as well as new strategies. But it remains unclear as to what the resulting equilibrium, as well as the path to the equilibrium, actually represent.

Price (1997) terms the convergent equilibrium a “reflexive equilibrium,” the convergence to which depends strongly on the percentage of the population replaced after every generation.¹⁴ Through a comparison of analytical results, he finds that selecting more than the top 20% for reproduction tends to force the algorithm into non-Nash equilibrium states. This can be interpreted as a firm that acts too whimsically in response to temporarily good strategies, not allowing appropriate time to consider alternatives. Conversely, selecting less than 12% for reproduction can yield cyclical strategies that may not represent any equilibrium at all (e.g., indecisive firms constantly switching between several different strategies).¹⁵

In interpreting the specific components of the GA, Dawid (1999) provides a clear analysis. We can think of the population of strategies (i.e., all the genetic strings) as a set of people all assigned a strategy and all competing against one another. Some of them do poorly and therefore do not survive to the next round, while those that do survive exchange information with the other survivors (e.g., through the crossover operation).¹⁶ Finally, the mutation operator introduces some slight mistakes or attempted innovations by the agents.

In general, the dynamics of the GA represent a type of social learning through interactions with other agents and allows us to study equilibrium behaviors without

¹³See Price (1997) for applications of GAs to Bertrand and Cournot competition and other simpler market settings.

¹⁴Although there are a few forms of convergence, in our case this represents the point at which the population has adapted to a uniform state.

¹⁵See Chattoe (1995) for extensions of the standard GA where this reproduction rate is endogenous.

¹⁶We acknowledge that, through crossover, some amount of information is taken from one strategy and given to another, but as Dawid emphasizes, this does not mean that this information is lost. In his words, “the information about the success of other actions is not preserved by the single individual but by the whole population.”

the strong rational expectations assumption. It also explicitly incorporates evolutionary phenomena such as natural selection, spreading of genes throughout a population and genetic mutations. In our case, the path to equilibrium represents the evolution of strategies for a given firm, where each tournament consists of a series of hypothetical strategies. Eventually, poor strategies are dismissed, the process by which depends on all other strategies, and each firm chooses one strategy in particular.

Although this process is intuitively quite different than the common replicator dynamic, there are some important similarities.¹⁷ Both are driven by evolutionary processes where weaker strategies have a lesser chance of survival. In fact, a GA with proportional selection is defined by almost an identical difference equation as that which defines the replicator dynamic. Further, Riechmann (2001a, 2001b) shows that GAs and the replicator dynamics are structurally equivalent and provides a clear analogy of the GA with an evolutionary game. One important difference, however, is the initial population, which is essentially infinite (i.e., a continuum) in replicator dynamics. While in the GA, there is a finite set of strategies considered by each agent, although final equilibrium strategies are not constrained to lie in this set of initial strategies. But as Reichmann shows, agent-based models can provide good approximations for models too complex to be treated analytically. More importantly, the standard replicator dynamic cannot generate strategies outside of those considered in the original population. When considering price ceilings, for example, this would be problematic as the price ceiling would never be binding unless it was exogenously set below the upper limit of the initial price population. This is not a problem in our evolutionary algorithm.

5.2 Validating Results

One important aspect of any simulation is the possibility of replication. Marks (2007) discusses three specific aspects in this regard: “Numerical identity,” “distributional equivalence” and “relational equivalence.” Numerical identity, the strongest measure of replication, requires identical results across different simulations. Distributional equivalence, a slightly weaker measure, exists when results do not differ in any statistically significant way. Finally, relational equivalence, the weakest measure, holds when qualitative results match across simulations. Throughout several different simulations, our

¹⁷See Hofbauer and Sigmund (1988), Cressman (1992), Samuelson (1997) and Fudenberg and Levine (1998) for good expositions of replicator dynamics.

replications consistently satisfy distributional and relational equivalence. Relational equivalence holds even for more drastic changes to our algorithm (e.g., completely dismissing crossover and using only selection and mutation to build new populations).

Unfortunately there are no rigorous tests that ensure validity. Appealing to one measure given by Leombruni et al. (2006), program validity represents the validity of the simulation relative to the model. Our simulation certainly satisfies this criterion. We also stress that our results satisfy in large part perhaps the strongest supporting argument of any simulation—replicability. In general, our simulation adopts consumer behavior and firm profit functions identical to the standard theoretical search model. In the absence of advertising, buyers’ search behaviors are dictated completely by random sampling, and their purchasing rule is identical to that of the theoretical literature. Moreover, our algorithm converges to a uniform state in all 120 settings. As such, the resulting price and advertising strategies are, at the least, one of perhaps many equilibrium results and, as an extension, so are our welfare results.

6 Conclusion

In this paper, we simulate equilibria for sequential consumer search models and show how these equilibria change under different market settings. Although the computational issue itself is of some relevance, the simulations here are particularly useful as they provide a unified framework to study markets where analytical results have proven difficult. More than anything, our simulations serve an exploratory purpose. We study the general behavior of firms in different markets, their responses to external forces such as taxes and price ceilings, their responses to more fundamental changes such as search costs and their responses to more endogenous forces such as the ability to advertise. Although we admit that, by use of simulations, we are excluding any possibility of rigorous mathematical proofs of necessity, we also stress the usefulness of simulations in providing general intuition for strategic behavior.

In particular, we find that advertising and price ceilings can improve welfare, controlling for the market size, search costs, costs of production and costs of advertising. The intuition is that advertising can facilitate more efficient search by attracting buyers to firms from which they are more likely to purchase. If search costs are sufficiently high, this avoids significant welfare losses otherwise incurred through costly search. The

intuition for price ceilings is slightly different. In this case, price dispersion diminishes substantially as many firms (in some cases, all firms) price at or near the ceiling. Search intensity then decreases as most consumers immediately find a price below their reservation price, again avoiding the welfare loss of costly search.

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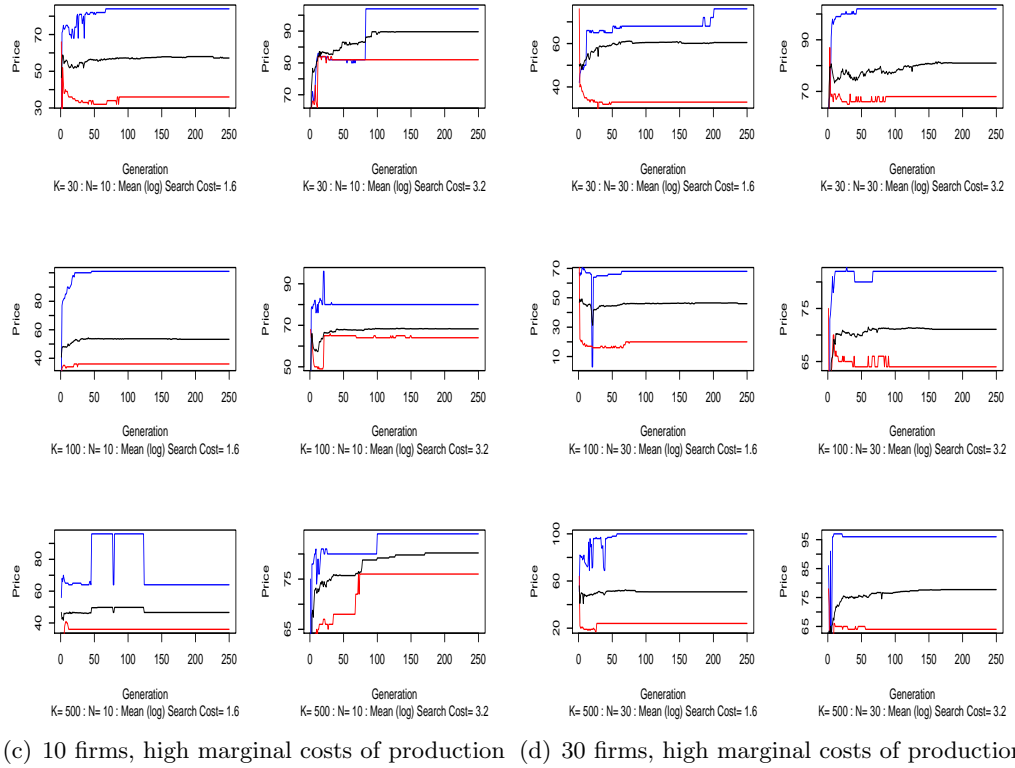
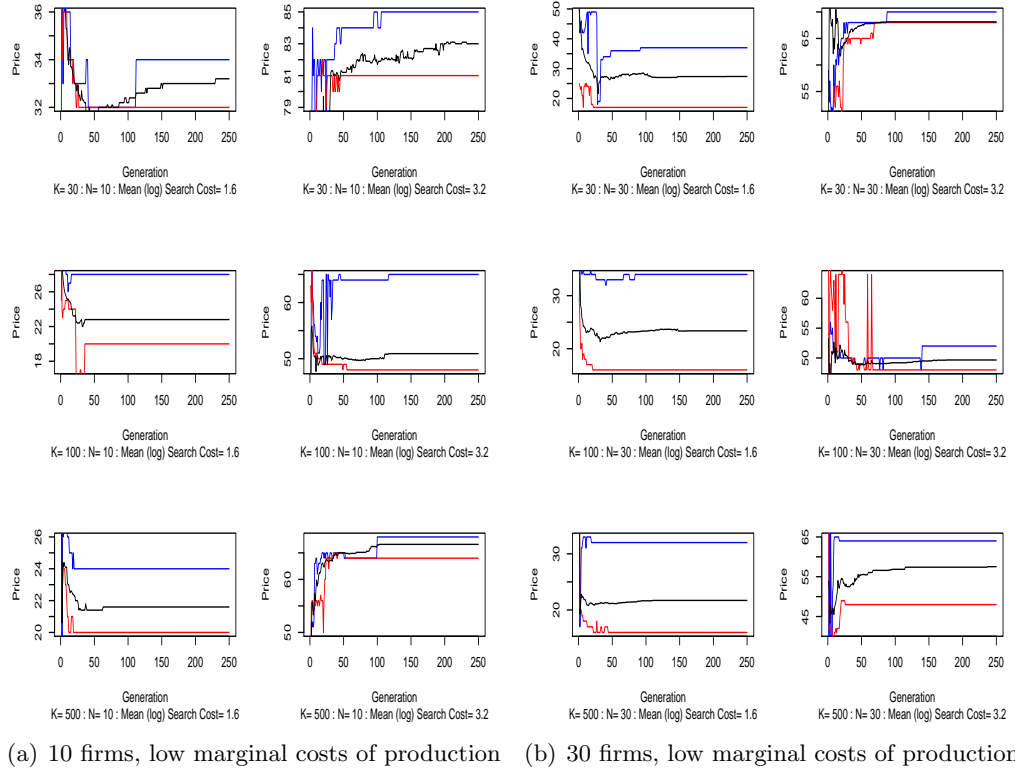
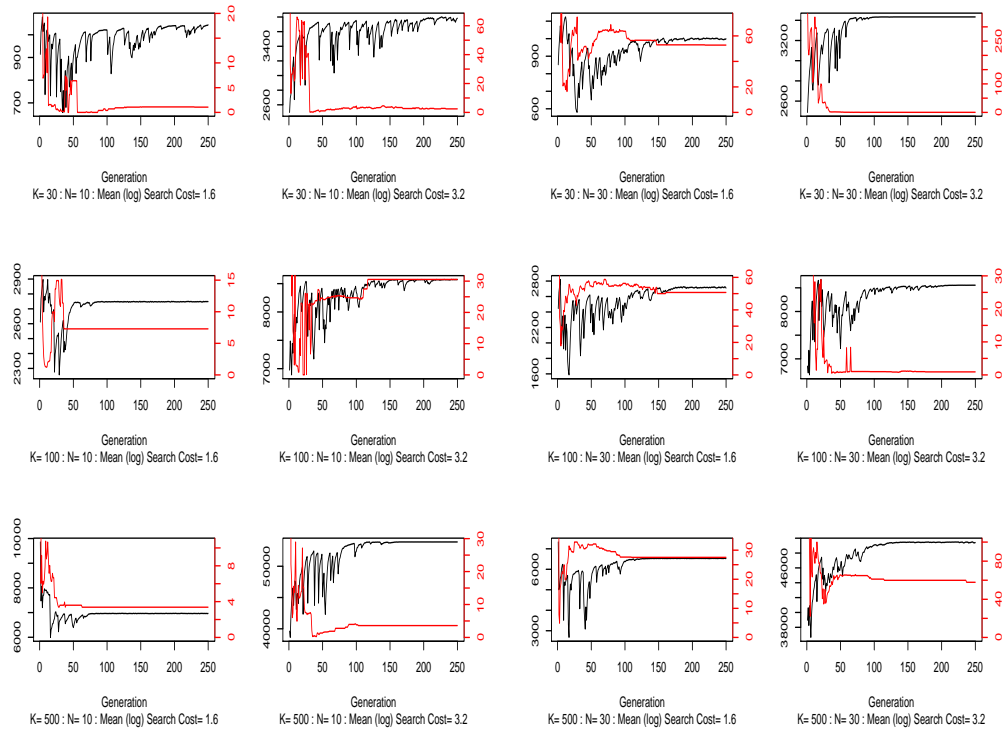
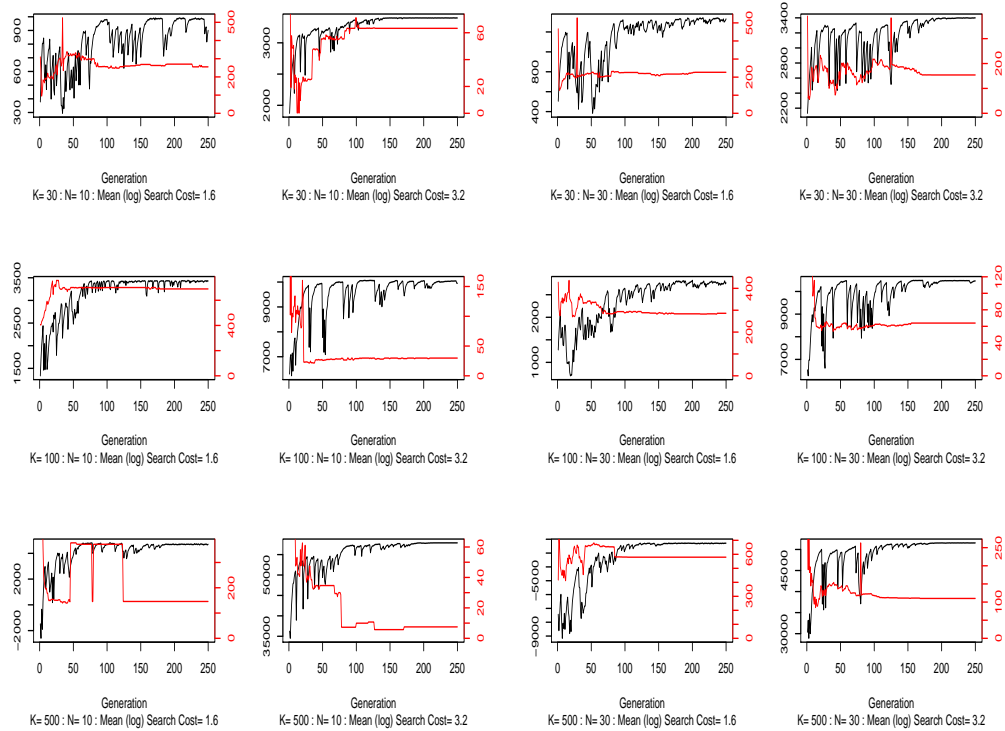


Figure 1: Simulated Prices for Base Models (No Price Ceilings, Taxes or Advertising)



(a) 10 firms, low marginal costs of production (b) 30 firms, low marginal costs of production



(c) 10 firms, high marginal costs of production (d) 30 firms, high marginal costs of production

Figure 2: Welfare and Price Variances for Base Models (No Taxes, Price Ceilings or Advertising)

Table 1: PRICE SUMMARY STATISTICS FOR LOW PRODUCTION AND ADVERTISING COSTS

	Model Type	Market														
		33.20	22.80	21.60	27.37	23.37	21.70	83.00	50.90	66.60	68.13	49.67	57.53			
Mean	Base Model	33.20	22.80	21.60	27.37	23.37	21.70	83.00	50.90	66.60	68.13	49.67	57.53			
	Sales Tax	32.80	23.20	22.80	29.10	24.80	22.33	81.90	52.60	67.20	68.00	49.83	60.80			
	Price Ceiling	32.80	22.80	21.40	28.13	23.33	21.47	58.00	55.20	60.00	52.00	48.97	54.40			
	Advertising ^a	28.20	25.20	26.40	20.50	21.40	21.93	40.70	64.00	67.40	55.37	49.40	52.87			
Min	Advertising ^b	21.60	21.40	22.40	11.20	16.87	20.03	84.80	64.10	53.20	64.27	65.60	52.53			
	Base Model	32.00	20.00	20.00	17.00	16.00	16.00	81.00	48.00	64.00	68.00	48.00	48.00			
	Sales Tax	32.00	16.00	20.00	17.00	16.00	16.00	80.00	48.00	64.00	64.00	48.00	48.00			
	Price Ceiling	32.00	20.00	20.00	20.00	16.00	16.00	48.00	48.00	60.00	52.00	48.00	48.00			
Max	Advertising ^a	24.00	16.00	24.00	8.00	16.00	16.00	32.00	64.00	66.00	32.00	40.00	52.00			
	Advertising ^b	18.00	16.00	19.00	4.00	8.00	16.00	80.00	64.00	50.00	16.00	64.00	52.00			
	Base Model	34.00	28.00	24.00	37.00	34.00	32.00	85.00	65.00	68.00	70.00	52.00	64.00			
	Sales Tax	34.00	28.00	28.00	40.00	48.00	32.00	85.00	65.00	68.00	70.00	65.00	64.00			
Std	Price Ceiling	34.00	28.00	24.00	36.00	34.00	32.00	60.00	60.00	60.00	52.00	56.00	56.00			
	Advertising ^a	36.00	36.00	32.00	40.00	36.00	37.00	48.00	64.00	68.00	81.00	80.00	64.00			
	Advertising ^b	24.00	24.00	32.00	16.00	32.00	26.00	96.00	65.00	58.00	72.00	80.00	56.00			
	Base Model	1.03	2.70	1.84	7.27	7.12	5.25	1.56	5.53	1.90	0.51	0.96	7.61			
	Sales Tax	1.03	3.16	3.16	7.03	8.07	5.36	1.73	6.69	1.69	0.91	4.10	6.51			
	Price Ceiling	1.03	2.70	1.90	6.22	7.09	4.89	4.32	3.16	0.00	0.00	1.94	3.25			
	Advertising ^a	5.12	7.96	3.86	9.91	4.72	4.95	6.80	0.00	0.97	15.54	9.05	3.05			
	Advertising ^b	2.07	2.99	3.95	4.09	3.39	3.59	7.73	0.32	2.70	9.68	4.88	1.17			
Market Setup	Firms	10	10	10	30	30	30	10	10	10	30	30	30			
	Consumers	30	100	500	30	100	500	30	100	500	30	100	500			
	Search Costs	Low	Low	Low	Low	Low	Low	High	High	High	High	High	High			
	Costs	Low	Low	Low	Low	Low	Low	High	High	High	High	High	High			

^aConsumers watch fewer commercials in this case.

^bConsumers watch more commercials in this case

Table 2: PRICE SUMMARY STATISTICS FOR HIGH PRODUCTION AND ADVERTISING COSTS

	Model Type	Market											
Mean	Base Model	57.20	53.30	46.60	60.47	45.97	50.80	89.80	68.30	80.20	80.97	71.07	77.73
	Sales Tax	60.90	55.60	51.20	65.70	47.87	52.17	87.40	72.90	81.30	83.07	72.07	81.80
	Price Ceiling	51.00	44.10	44.80	51.73	41.73	44.40	60.00	59.60	60.00	60.00	56.53	59.87
	Advertising ^a	30.20	26.80	22.50	15.67	19.57	23.83	54.40	45.20	61.60	35.60	48.67	62.17
	Advertising ^b	21.60	22.70	19.20	18.93	16.97	27.23	21.80	65.20	55.70	24.80	37.33	55.13
Min	Base Model	36.00	36.00	36.00	33.00	20.00	24.00	81.00	64.00	76.00	68.00	64.00	64.00
	Sales Tax	44.00	36.00	36.00	32.00	21.00	24.00	81.00	65.00	72.00	68.00	64.00	64.00
	Price Ceiling	42.00	36.00	36.00	32.00	24.00	24.00	60.00	56.00	60.00	60.00	48.00	56.00
	Advertising ^a	24.00	16.00	16.00	8.00	16.00	16.00	52.00	36.00	52.00	32.00	40.00	56.00
	Advertising ^b	16.00	11.00	16.00	11.00	9.00	12.00	16.00	64.00	52.00	16.00	34.00	48.00
Max	Base Model	84.00	101.00	64.00	76.00	68.00	100.00	97.00	80.00	84.00	102.00	82.00	96.00
	Sales Tax	84.00	101.00	96.00	76.00	68.00	100.00	97.00	97.00	96.00	102.00	82.00	96.00
	Price Ceiling	60.00	60.00	60.00	60.00	60.00	60.00	60.00	60.00	60.00	60.00	60.00	60.00
	Advertising ^a	36.00	44.00	33.00	64.00	24.00	38.00	64.00	52.00	64.00	96.00	52.00	64.00
	Advertising ^b	24.00	32.00	33.00	32.00	32.00	36.00	32.00	68.00	64.00	96.00	48.00	64.00
Std	Base Model	15.98	26.26	12.08	15.06	16.91	24.07	7.96	5.46	2.74	12.41	8.00	10.49
	Sales Tax	15.52	25.66	18.84	16.48	18.27	22.49	7.35	9.61	5.96	11.95	7.83	11.38
	Price Ceiling	7.96	9.34	10.96	11.31	13.52	13.71	0.00	1.26	0.00	0.00	5.01	0.73
	Advertising ^a	3.94	11.66	4.45	9.70	2.30	7.97	5.06	6.55	4.30	13.17	3.94	3.38
	Advertising ^b	3.24	7.24	5.05	6.89	6.07	9.18	4.54	1.93	4.97	16.51	4.75	6.43
Market Setup	Firms	10	10	10	30	30	30	10	10	10	30	30	30
	Consumers	30	100	500	30	100	500	30	100	500	30	100	500
	Search Costs	Low	Low	Low	Low	Low	Low	High	High	High	High	High	High

^aConsumers watch fewer commercials in this case.

^bConsumers watch more commercials in this case.

Table 3: MEAN RESERVATION PRICE

		Market											
Low Costs	Base Model	46.50	33.13	30.06	37.61	31.21	30.03	136.45	87.42	110.56	120.27	85.65	106.70
	Sales Tax	46.11	33.55	30.96	39.33	32.34	30.59	135.46	89.03	111.16	120.13	85.78	110.59
	Price Ceiling	46.11	33.13	29.84	38.70	31.20	29.95	111.98	91.94	103.96	104.13	84.94	104.31
	Advertising ^a	40.77	30.25	34.48	29.41	31.34	30.47	69.44	106.18	110.43	93.33	91.96	97.27
	Advertising ^b	34.88	30.09	30.23	18.12	26.61	27.88	132.52	109.78	99.35	130.59	111.78	103.39
High Costs	Base Model	63.58	51.86	50.23	63.82	46.50	47.26	143.62	104.52	124.15	131.74	106.55	126.73
	Sales Tax	68.17	53.78	52.09	67.67	46.96	47.57	141.18	107.95	125.06	133.16	107.39	130.59
	Price Ceiling	62.51	51.56	49.27	58.80	45.41	46.81	113.98	96.35	103.96	112.13	92.46	109.79
	Advertising ^a	43.20	34.11	29.51	22.93	29.77	30.71	99.67	93.23	103.65	119.23	94.95	113.61
	Advertising ^b	34.80	30.11	27.72	28.77	26.90	32.44	52.67	117.36	108.84	78.05	84.25	98.49
	Firms	10	10	10	30	30	30	10	10	10	30	30	30
	Consumers	30	100	500	30	100	500	30	100	500	30	100	500
	Search Costs	Low	Low	Low	Low	Low	Low	High	High	High	High	High	High

^aConsumers watch fewer commercials in this case.^bConsumers watch more commercials in this case.

Table 4: MEAN WELFARE

		Market											
Low Costs	Base Model	1043.58	2748.33	6965.83	998.98	2708.99	6528.69	3782.79	8571.25	53880.29	3433.63	8555.04	49406.23
	Sales Tax	950.84	2567.48	6605.96	984.57	2665.57	5506.42	3564.93	8278.07	51617.62	3266.43	8158.78	49043.66
	Price Ceiling	1032.44	2748.33	6866.41	1048.83	2728.76	6538.77	3059.98	9074.79	50585.87	2949.63	8478.87	48669.24
	Advertising ^a	608.64	1018.41	12716.87	626.66	1431.65	10876.60	1829.72	9404.32	49714.91	2163.45	7232.31	40714.77
	Advertising ^b	440.51	2146.13	9163.86	145.47	1736.96	8783.39	3308.45	10741.30	42220.51	3766.54	9927.43	46696.10
High Costs	Base Model	900.06	3428.53	4701.56	1330.75	2763.41	-2285.78	3397.78	9950.19	57873.20	3399.51	10464.66	51554.74
	Sales Tax	975.75	3224.76	3415.14	1285.93	2412.50	-5159.10	3124.37	9644.79	55358.44	3273.39	10034.80	50122.23
	Price Ceiling	938.41	3458.90	3890.51	1298.20	2973.67	-2175.11	2521.18	9293.41	47795.84	2840.92	9151.43	44506.09
	Advertising ^a	673.08	2604.70	13925.25	419.56	2228.55	10000.81	2690.14	8151.72	47551.11	3101.00	8542.81	49794.57
	Advertising ^b	443.78	2638.57	11979.43	406.60	1824.24	14803.98	1385.06	10108.44	52799.33	1820.15	7220.33	45071.98
	Firms	10	10	10	30	30	30	10	10	10	30	30	30
	Consumers	30	100	500	30	100	500	30	100	500	30	100	500
	Search Costs	Low	Low	Low	Low	Low	Low	High	High	High	High	High	High

^aConsumers watch fewer commercials in this case.^bConsumers watch more commercials in this case.

Table 5: ADVERTISING SUMMARY STATISTICS

		TV Intensity	Market															
Low Costs	Mean	Low	15.40	31.00	52.60	5.00	10.83	39.27	27.20	51.60	54.20	21.33	37.33	53.47				
		High	13.60	26.70	53.00	2.90	13.60	33.30	48.80	52.50	54.10	28.20	49.60	53.97				
	Min	Low	0.00	0.00	46.00	0.00	0.00	0.00	0.00	48.00	54.00	0.00	0.00	52.00				
		High	0.00	0.00	46.00	0.00	0.00	0.00	48.00	48.00	54.00	0.00	44.00	52.00				
	Max	Low	44.00	54.00	54.00	32.00	52.00	55.00	48.00	54.00	55.00	48.00	54.00	55.00				
		High	38.00	51.00	55.00	36.00	54.00	55.00	52.00	55.00	55.00	52.00	52.00	55.00				
	Std	Low	17.69	26.72	2.67	9.62	17.83	22.08	23.69	1.58	0.42	13.83	18.06	1.01				
		High	17.83	23.38	3.46	8.80	21.84	25.83	1.69	1.96	0.32	19.20	2.25	0.67				
High Costs	Mean	Low	5.50	9.60	27.50	0.83	3.70	13.77	13.60	36.60	52.90	4.03	13.40	50.67				
		High	3.80	12.00	40.10	1.33	3.37	16.33	7.20	52.20	54.20	3.53	13.13	53.07				
	Min	Low	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	52.00	0.00	0.00	46.00				
		High	0.00	0.00	0.00	0.00	0.00	0.00	0.00	52.00	53.00	0.00	0.00	48.00				
	Max	Low	20.00	32.00	55.00	6.00	33.00	55.00	36.00	54.00	54.00	32.00	54.00	54.00				
		High	20.00	44.00	55.00	20.00	47.00	53.00	28.00	54.00	55.00	32.00	54.00	54.00				
	Std	Low	7.59	15.46	24.74	1.80	9.51	23.26	17.61	25.32	0.74	8.41	22.66	2.20				
		High	7.57	19.18	21.42	4.57	11.77	23.61	11.22	0.63	0.92	7.82	22.44	1.36				
	Market Setup	Firms	10	10	10	30	30	30	10	10	10	30	30	30				
		Consumers	30	100	500	30	100	500	30	100	500	30	100	500				
		Search Costs	Low	Low	Low	Low	Low	Low	High	High	High	High	High	High				

Table 6: REGRESSION RESULTS

	Dependent Variable			
	Price	Advertising Intensity	Average Welfare	
Constant	30.275*** (0.888)	72.381*** (8.403)	7625.972 (11392.620)	-29236.130*** (6627.455)
Number of buyers	-2.710*** (0.327)	24.181*** (1.746)	12710.300*** (3388.878)	15650.180*** (1708.242)
Number of firms	-3.084*** (0.621)	-24.045*** (2.991)	-1662.124 (3241.278)	2376.385 (2655.880)
Level of search costs	27.121*** (0.518)	93.463*** (10.169)	-1579.179 (5853.812)	-15066.340* (8775.012)
Price ceiling	-7.800*** (0.624)			-2420.271 (4332.191)
Sales tax	1.869*** (0.656)			-1763.784 (3085.378)
TV intensity		-1.309 (2.175)	-1428.421 (2493.791)	
Price		-2.377*** (0.318)		
Marginal cost	0.751*** (0.021)			
Advertising cost		-27.514*** (2.785)	983.658 (3522.704)	
Average marginal cost			-613.628 (901.895)	-556.828** (236.542)
Average advertising level			-253.428 (223.667)	
Average price			-205.639 (191.210)	883.477*** (223.695)
Advertising×search costs			739.683*** (173.522)	
Price ceiling×search costs				10368.560* (6065.408)
Price dispersion			17.061 (26.150)	-40.155*** (13.261)
R^2	0.787		0.808	0.693
N	1440	960	48	72