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Multifield Reheating after Modular $j$-Inflation

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Abstract.

In the inflationary framework of cosmology the initial phase of rapid expansion has to be followed by a reheating stage, which is envisioned to end in a radiation dominated big bang. Key parameters that characterize this big bang state are the temperature at the end of the reheating stage and the baryon asymmetry. For general interacting theories these parameters are difficult to obtain analytically because of the involved structure of the potential. In this paper multifield reheating is considered for interacting theories in which the inflaton trajectory is weakly curved. This scenario is realized in the model of $j$-inflation, a particular example of modular inflation, allowing an estimate of the reheat temperature.
1 Introduction

In order to understand the physical significance of the CMB observations made over the past two decades in the context of an inflationary picture, it is important to think in terms of a multifield theoretic framework (a recent review can be found in [1]). While few-parameter fits in the context of singlefield inflation are compatible with the data, they provide an incomplete view of the impact of the experimental results on the restrictions of the theory space. An example of such a structured space is given by the framework of automorphic inflation, an approach to the early universe based on symmetry principles, motivated in part by the idea to constrain the potentials with an arbitrary number of fields by retaining a discrete symmetry of the theory as the result of a weakly broken continuous symmetry [2, 3, 4]. This provides a natural embedding of the shift symmetry into a canonical group theoretic framework.

The resulting discrete group $\Gamma$ and the potential $V_{\Gamma}$ associated to $\Gamma$ are specified by numerical characteristics that allow the identification of distinct regions in the curved field theory space of the inflationary models, described by the field space metric $G_{IJ}$ and $V_{\Gamma}$. The restriction of automorphic field theory to two-field theories leads to the class of modular inflation models with a nontrivial geometry given by the hyperbolic Poincaré metric. Subsequent related work on modular inflation was done in [5], while work with...
a focus on the hyperbolic target space of modular inflation includes [6, 7].

After the end of inflation the reheating stage is envisioned to provide the evolutionary link between the inflationary phase and the start of the radiation dominated big bang. Reheating has been discussed mostly in singlefield inflation, following the first papers [8, 9, 10], with a later focus on the early stages of the reheating process, explored first in the references [11, 12, 13, 14, 15]. In the context of multifield theories the analysis becomes complicated and most work has been restricted to separable potentials [16, 17, 18, 19, 20, 21, 22], but interacting fields have been considered for flat target spaces in refs. [23, 24, 25], and a discussion admitting curved inflaton targets can be found in ref. [26, 27].

The most important parameter that results from the reheating period is the reheat temperature $T_{rh}$. To follow the temperature analytically through the whole period of reheating, including preheating, has turned out to be intricate, but the complications of preheating have been recognized to be less essential if the interest is in the temperature at the end of reheating. The observation of the present paper is that in interacting theories with intricate potentials it can happen that the inflationary trajectories are weakly curved, resulting in potentials associated to the inflationary orbit that can be approximated by a simpler singlefield potential. This extends the range of potentials to which the canonical methods can be applied to analyze the reheating stage. The phenomenon of weakly curved trajectories is realized in the model of $j$-inflation, which is defined in terms of a nontrivially interacting inflaton doublet. The simplification obtained in this case allows to compute the reheat temperature and hence the associated baryon asymmetry.

In outline, after describing the general set-up of multifield reheating in Section 2, the paper introduces the general picture of weakly curved trajectories in Section 3. The application to $j$-inflation is given in Section 4 and the paper concludes in Section 5.
2 Multifield reheating set-up

In the framework of reheating after multifield inflation it is standard to consider decays of various types, such as \( \phi^I \to \chi^A \chi^A \) or \( \phi^I \phi^I \to \chi^A \chi^A \), parametrized by couplings of triscalar and bi-quadratic type. The Lagrangian decomposes as

\[
\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}},
\]

where the kinetic energy is determined by, in general nontrivial, metrics \( G_{IJ} \) and \( G_{AB} \)

\[
\mathcal{L}_{\text{kin}} = -\frac{1}{2} G_{IJ} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - \frac{1}{2} G_{AB} g^{\mu\nu} \partial_\mu \chi^A \partial_\nu \chi^B
\]

and the interaction includes self-interactions as

\[
\mathcal{L}_{\text{int}} = -V(\phi^I) - U(\chi^A) - W(\phi^I, \chi^A).
\]

The triscalar and bi-quadratic potentials describing the interactions between the inflaton and the field \( \chi \) providing the decay channels can be generalized to the multifield case as

\[
W(\phi^I, \chi^A) = \sum_{I,A} \left( \frac{1}{2} g_{IA} \phi^I (\chi^A)^2 + \frac{1}{2} \sum h_{IA} (\phi^I)^2 (\chi^A)^2 \right),
\]

where the triscalar coupling parameters \( g_{IA} \) have dimensions of mass and the bi-quadratic coupling parameters \( h_{IA} \) are dimensionless. The oscillatory inflaton is obtained by assuming that the decay field is small at the beginning of the reheating period. The dynamical equations of the inflaton are given by

\[
D_t \dot{\phi}^I + 3H \dot{\phi}^I + G^{IJ}(V + W)_{,J} = 0,
\]

where \( D_t \) is the covariant derivative and the \( \chi^A \) dependence in the interaction \( W \) is via its correlators. The decay products satisfy the equation

\[
\ddot{\chi}^A - \frac{1}{a^2} \Delta \chi^A + 3H \dot{\chi}^A + \delta^{AB}(U + W)_{,B} = 0,
\]

where the inflaton dependence of the interaction potential is as a function of the coherently oscillating background inflaton (here the target space of the decay channel fields.
has been chosen to be flat). The specifics of the oscillations of the components \( \phi^I \) depend on the details of the potential and have been discussed in the literature only in the context of separable potentials. This dependence is nontrivial even in general single field monomial inflation \([28]\), which explains the focus on separable potentials in the literature.

In the perturbative theory of reheating \([8, 9, 10]\) the effect of particle production is encoded in the Klein-Gordon equation by the introduction of a decay term \( \Gamma_{\phi^I} \dot{\phi}^I \) of the inflaton \( \phi^I \) that arises from couplings of the inflaton to other fields that provide the decay channels \( \Gamma^I = n^I \sigma^I v^I \) of the inflaton components \( \phi^I \), leading to the effective dynamics \([29, 30]\)

\[
D_t \dot{\phi}^I + (3H + \Gamma^I) \dot{\phi}^I + G^{IJ} V_{,J} = 0. \tag{7}
\]

The spacetime expansion is determined by the first Friedmann-Lemaitre equation as

\[
H^2 = \frac{1}{3M_{\text{Pl}}^2} \left( \frac{1}{2} G_{IJ} \dot{\phi}^I \dot{\phi}^J + V(\phi^I) + \rho_r \right), \tag{8}
\]

where \( M_{\text{Pl}} \) is the reduced Planck mass, and the potential dominates the kinetic term for large field values. The density of the relativistic decay products, denoted by \( \rho_r \), is determined by \([29]\)

\[
\dot{\rho}_r + 4H \rho_r = \sum_I \Gamma^I (\dot{\phi}^I)^2. \tag{9}
\]

3 Weakly curved trajectories in inflaton target spaces

The strategy adopted below for analyzing reheating after \( j \)-inflation is formulated in the present section in a general conceptual way. The observation is that for some multifield potentials

\[
V(\phi^I) = \Lambda^4 F(\phi^I / \mu) \tag{10}
\]

described by a dimensionless function \( F(\phi^I / \mu) \) that can encode complicated interactions between the multiplet components \( \phi^I \), it can still happen that the inflationary trajectories are only weakly curved and hence traverse a cross section of the potential that is
approximately one-dimensional. Without loss of generality one can characterize such a situation in the multifield topography of the potential \( V(\phi^I), I = 1, ..., N \) as a path for which

\[
\Delta \phi^J \ll \Delta \phi^I, \quad \forall J \neq I. \tag{11}
\]

The resulting cross section of the potential at

\[
V_b(\phi^I) = V_{\phi^I \equiv \text{const}, \forall J \neq I} \tag{12}
\]

leads to a potential that approximates the exact potential in this situation.

With the notation \( \phi = \phi^I \) the potential \( V(\phi) \) along the inflaton trajectory can be approximated by truncating the Taylor expansion around its minimum \( \phi_0 \) at some order. For small \( (\phi - \phi_0) \) the lowest order is most relevant for the oscillatory behavior of the inflaton and higher orders are suppressed. The potential thus will be of monomial form

\[
V_b(\phi) = \Lambda_b^4 \left( \frac{\phi - \phi_0}{\mu} \right)^n \tag{13}
\]

for some energy scales \( \Lambda_b \) and \( \mu \). This allows to apply results from the analysis of monomial inflation based reheating discussions. For quadratic and quartic inflation the oscillating inflaton \( \phi_{\text{osc}} \) can, for example, be described explicitly in terms of trigonometric and Jacobi elliptic functions, respectively [28, 31]. The standard couplings considered are the triscalar, bi-quadratic, and Yukawa couplings

\[
\mathcal{L}_{\text{int}} = -g \phi \chi^2 - \frac{1}{2} h \phi^2 \bar{\chi}^2 - h \phi \bar{\nu} \psi, \tag{14}
\]

with the associated perturbative decay rates \( \Gamma_{\text{tri}}, \Gamma_{\text{bi}, q} \) and \( \Gamma_Y \). These determine the decrease of the amplitude \( \mathcal{A}(t) \) of \( \phi_{\text{osc}} \) in addition to the damping that results from the Hubble-Slipher expansion. The decay into radiation \( \rho_r \) can then be parametrized as

\[
\dot{\rho}_r + 4H \rho_r = \Gamma \dot{\phi}^2. \tag{15}
\]

While the bi-quadratic potential has been the focus of much of the preheating literature, it cannot complete reheating, and the triscalar coupling plays an important role toward
the end of the process. The fermionic decay channel will be considered further below. In quadratic inflation with mass $m_{\phi}$ one finds for the triscalar coupling the decay constant $\Gamma_{\text{tri}} = g^2/8\pi m_{\phi}$ and the amplitude of the oscillating inflaton decreases as

$$A(t) \equiv \frac{M_{\text{Pl}}e^{-\Gamma_{\text{tri}}t}}{m_{\phi}t}.$$  \hspace{1cm} (16)

The result for $\Gamma_{\text{tri}}$ assumes that the computation can be done with perturbative quantum field theory, neglecting potential Bose enhancements in the presence of other particles \cite{11, 12, 13, 14, 15}. Since the focus in the following is on the reheat temperature, such preheating effects will be neglected.

Given the decay rate $\Gamma$ of the inflaton the reheat temperature $T_{\text{rh}}$ is usually computed under the assumption of thermalization at the time $t_\Gamma \approx \Gamma^{-1} \approx (3H)^{-1}$. With the density $\rho_r$ of a distribution of relativistic particles in thermal equilibrium

$$\rho_r = \frac{\pi^2}{30} g_* T^4,$$  \hspace{1cm} (17)

where $g_* = n_b + (7/8)n_f$ is a count of the number of relativistic degrees of freedom, and evaluating the Friedman-Lemaitre equation at $t_\Gamma$

$$\rho_r = 3M_{\text{Pl}}^2 H^2 \approx \frac{1}{3} \Gamma^2 M_{\text{Pl}}^2,$$  \hspace{1cm} (18)

leads to the reheat temperature \cite{28}

$$T_{\text{rh}} = \left( \frac{10}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma M_{\text{Pl}}}. \hspace{1cm} (19)$$

The reheat temperature is weakly constrained by nucleosynthesis, which leads to a lower limit of a few MeV. If supersymmetric models are considered, there is an upper bound that arises from the gravitino abundance \cite{32, 33, 34}. The set-up just described of an effective reduction of a multifield inflationary dynamics to a one-dimensional reheating epoch is realized in the model of $j$-inflation, to which this is applied below.

Given the reheat temperature the baryon asymmetry can be determined in terms of the parameter $\epsilon$ characterizing the strength of the CP violation. This is usually defined in
the context of nucleosynthesis via the photon density \( n_\gamma \) as \( \eta = n_B/n_\gamma \), or in terms of the entropy density

\[
s = \frac{2\pi^2}{45} g_* T^3 = \frac{\pi^4 g_*}{45 \zeta(3)} n_\gamma \tag{20}
\]
as \( n_B/s \), leading to \([9, 29]\)

\[
\frac{n_B}{s} \approx \frac{3}{4} \frac{T_{\text{rh}}}{m_\phi}. \tag{21}
\]

Here \( m_\phi \) is an effective inflaton mass, which might be the Lagrangian mass, or can be given in terms of other fundamental parameters that characterize the inflatonary model.

### 4 Reheating after \( j \)-inflation

Automorphic inflation is a framework that is based on symmetry considerations, motivated in part by the goal to provide a unified conceptual context for the shift symmetry as part of discrete symmetry groups that leaves the field theory invariant \([2, 3]\). The smallest possible groups of interest are congruence subgroups \( \Gamma(N) \) at level \( N \) of the full modular group \( \text{SL}(2, \mathbb{Z}) \), leading to the subclass of modular inflation theories \([2, 4]\).

In modular inflation the potential is determined in terms of a modular function \( F \) that is constructed from modular forms \( \Phi_i \) relative to a congruence subgroup \( \Gamma(N) \). The defining congruence relation for a given integer \( N \) is not unique and different types of groups \( \Gamma(N) \) can be considered. A natural class of potentials is given by

\[
V_d = \Lambda^4 |F(\Phi_i)|^{2d} \tag{22}
\]
for some integer \( d \).

The simplest case is obtained at level \( N = 1 \), in which case the group \( \Gamma(1) \) is the full modular group and the symmetry imposes the strongest possible constraints on the potential in the modular context. For this group the set of all modular forms is generated by only two functions, the Eisenstein series of weight four and six. For general weight \( w \) the Eisenstein series \( E_w \) can be defined in a computationally convenient way in terms
of the Bernoulli numbers $B_w$ and the divisor function $\sigma_w$, given by

$$\sigma_w(n) = \sum_{d|n} d^w,$$

as

$$E_w(\tau) = 1 - \frac{2w}{B_w} \sum_n \sigma_{w-1}(n) q^n,$$

where $q = e^{2\pi i \tau}$. The $E_w$ are modular forms of weight $w$ for $w \geq 4$, but $E_2$ is only a quasimodular form of weight two, characterized by a nonhomogeneous transformation behavior (see below).

While the potentials of the above type relative to the modular group thus can be written in terms of the modular forms $E_4$ and $E_6$, the phenomenological analysis of any modular inflation models also involves the Eisenstein series $E_2$ because it enters in the derivatives of modular forms. The space of $SL(2,\mathbb{Z})$-modular forms and their derivatives is therefore generated by $E_2, E_4$ and $E_6$. CMB observables such as the spectral index involve derivatives of the potential, hence of modular forms, and therefore involve $E_2$. This raises the issue of the precise modular nature of such observables. It was shown in [4] that the nontrivial geometry of the target space provides the necessary structure to turn the observables into functions whose fundamental building blocks include almost holomorphic terms. Physical observables in modular invariant inflation models are therefore in general not modular, but almost holomorphic modular.

For the full modular group there is a distinguished modular invariant function that is fundamental in the sense that every other modular invariant function of this group can be expressed in terms of this function as a polynomial quotient. This function is the absolute invariant $j(\tau)$ with $\tau$ in the upper halfplane, which therefore provides the basic building block of all functions invariant under $SL(2,\mathbb{Z})$. Since $E_4$ and $E_6$ are generators of the modular forms, the $j$-function can be written purely in terms of these functions in a computationally convenient way as

$$j(\phi^I) = 1728 \frac{E_4^3}{E_4^3 - E_6^2},$$
where \( \tau = \tau^1 + i\tau^2 \) is the complex dimensionless inflaton defined as \( \tau = \phi/\mu \), with \( \mu \) an energy scale. Here the Bernoulli numbers that enter \( E_4 \) and \( E_6 \) are given by \( B_4 = -1/30 \) and \( B_6 = 1/42 \). A different view of the \( j \)-function can be obtained in terms of a counting function associated to the harmonic oscillator because the denominator in the above definition is related to the Dedekind eta function \( \eta(\tau) \) as \( (E_4^3 - E_6^2)/1728 = \eta^{24} = \Delta \), where \( \Delta \) is the Ramanujan cusp form of weight twelve.

Given the fundamental nature of the \( j \)-function, it is natural to consider an inflationary model based on \( j \), leading to the potential of \( j \)-inflation as [2, 4]

\[
V(\phi^I) = \Lambda^4 |j(\tau^I)|^2,
\]

(26)

where \( \Lambda \) is a second energy scale.

The definition of modular inflation models is completed by noting that the target space spanned by the inflaton multiplet is the complex upper half-plane \( \mathcal{H} \). This space has a nontrivial metric geometry \( ds^2 = G_{IJ}d\tau^I d\tau^J \), determined by the continuous Möbius group \( \text{SL}(2, \mathbb{R}) \), and given by the Poincaré metric

\[
ds^2 = \frac{(d\tau^1)^2 + (d\tau^2)^2}{(\tau^2)^2}.
\]

(27)

A more detailed discussion of the framework of modular inflation can be found in [2, 4] and a more in-depth discussion of the automorphic generalization to multifield inflation is given in [3].

In \( j \)-inflation the scenario described in the previous section is realized in that one of the components of the inflaton multiplet is distinguished because of its larger variation along the orbit. In the case of \( j \)-inflation it is the first component \( \phi^1 = \mu\tau^1 \) that varies much more than the imaginary component. This suggests an approximation of the potential by a monomial fit. As a first approximation it is useful to consider a simple quadratic fit because this allows to apply results known from the simplest chaotic inflation model to the reheating theory of \( j \)-inflation. By far the most effort in the reheating literature has been focused on this model, with just a few prominent references given by [14, 15].
The approximation of the potential along the trajectory can, with \( \phi = \phi^1 = \mu \tau^1 \), be written as

\[
V_j(\phi) = \frac{\Lambda^4}{\mu^2} \left( \phi - \frac{\mu}{2} \right)^2,
\]

hence degree two chaotic results can be adopted by introducing the effective \( j \)-inflation mass as

\[
m_j^2 = \frac{2\Lambda^4}{\mu^2},
\]

(29)

where \( \Lambda_j = \Lambda \alpha_j \) with \( \alpha_j \approx 59 \). This mass determines the frequency of the oscillating background inflaton that is usually coupled to its decay channel \( \chi \) by either triscalar couplings \( g\phi\chi^2 \) or bi-quadratic couplings \( h\phi^2\chi^2 \). As noted above, the latter coupling is not sufficient to complete reheating. The decay rate \( \Gamma_{\text{tri}} \) in \( j \)-inflation takes the form

\[
\Gamma_{\text{tri}}^j = \frac{g^2 \mu}{8\sqrt{2\pi}\Lambda^2}
\]

(30)

and therefore the reheat temperature is given by

\[
T_{\text{rh}}^j = \frac{1}{2\pi} \left( \frac{5}{4g_*} \right)^{1/4} \left( \frac{g}{\Lambda_j} \right) \sqrt{\mu M_{\text{Pl}}}
\]

(31)

With \( T_{\text{rh}} \) the baryon asymmetry after \( j \)-inflation is given in terms of the fundamental parameters of the model as

\[
\frac{n_B}{s} = \frac{3}{16\pi} \left( \frac{5}{4g_*} \right)^{1/4} \left( \frac{g\mu}{\Lambda_j^2} \right) \sqrt{\mu M_{\text{Pl}}} \epsilon.
\]

(32)

Only weak constraints are known about the coupling \( g \), but the standard suppression of quantum corrections of the potential via \( g \ll m_\phi \) is conveniently parametrized as \( g = 10^{-\ell} m_j \), where \( \ell \geq 1 \). This leads to estimates of the reheat temperature and the baryon asymmetry in terms of the basic parameters as

\[
T_{\text{rh}}^j = \frac{1}{2\pi} \left( \frac{5}{4g_*} \right)^{1/4} 10^{-\ell} \Lambda_j \sqrt{\frac{M_{\text{Pl}}}{\mu}}
\]

\[
\frac{n_B}{s} = \frac{3}{8\pi} \left( \frac{5}{4g_*} \right)^{1/4} \left( \frac{10^{-\ell}}{\Lambda_j} \right) \epsilon \sqrt{\mu M_{\text{Pl}}}.
\]

(33)
Inflationary trajectories with $\mu \approx 10 M_{\text{Pl}}$ typically lead via the CMB constraints of the scalar amplitude [35, 36] to $\Lambda \approx 10^{-5} M_{\text{Pl}}$. Using these ingredients and $g_* \approx 100$ we obtain for the reheat temperature after $j$-inflation the estimate

$$T_{\text{rh}}^j \approx 10^{13-\ell} \text{GeV} \left( \frac{\Lambda}{10^{13} \text{GeV}} \right) \left( \frac{M_{\text{Pl}}}{\mu} \right)^{1/2},$$

and for the baryon asymmetry

$$\frac{n_B}{s} \approx 10^{2-\ell} \epsilon \left( \frac{10^{13} \text{GeV}}{\Lambda} \right) \left( \frac{\mu}{M_{\text{Pl}}} \right)^{1/2}.$$

By adjusting the parameters it is possible to obtain the observed value $n_B/s \approx 10^{-10}$.

A similar computation can be done for the Yukawa coupling $h_{\psi \phi \overline{\psi} \psi}$ with decay rate for $m_{\psi} \ll m_{\phi}$ as

$$\Gamma_{\phi \psi} = \frac{h_{\psi}^2 m_{\phi}}{8 \pi}.$$

The inclusion of this coupling does not raise the reheat temperature because the fermionic reheat temperatures associated to $\Gamma_{\phi \psi}$ is suppressed by a factor

$$s_p \approx \frac{\Lambda_j}{\sqrt{\mu M_{\text{Pl}}}}.$$

The above results for the reheat temperature and the baryon asymmetry show that $j$-inflation can accommodate a low reheating temperature. This makes it natural to consider nonthermal leptogenesis as a mechanism to generate the baryon asymmetry in this framework, obtained via the inflaton decay into heavy right-handed neutrinos [37]. The decay of these Majorana neutrinos into lepton and Higgs fields creates a lepton asymmetry $n_L/s$, which is partially converted into the baryon asymmetry via sphaleron processes [38]. This mechanism can avoid abundance problems that one encounters in baryogenesis models with high reheat temperature.

5 Conclusion

The general case of multifield inflation models involves interacting fields, hence potentials that are not separable in the inflaton components, making some aspects of their analysis
difficult for an analytical approach. A special class of such models is given by theories which exhibit a simplifying feature in that the topology in field space is such that along the inflaton trajectory the different components of the inflaton multiplet cover different ranges in field space, leading to weakly curved trajectories that admit a quasi one-dimensional treatment. In this case the potential $V_b$ along the trajectory depends essentially on a single component of the inflaton multiplet.

A systematic class of highly interacting inflation models is obtained in the context of twofield inflation by the framework of modular inflation. The model of $j$-inflation provides an example of the above phenomenon, which allows to derive the reheat temperature $T_{rh}$ and its associated baryon asymmetry $n_B/s$ in terms of the model parameters and a parameter that tracks the scaling of the coupling parameter. The latter parameter is constrained in order to suppress quantum corrections to the potential, leading to an upper bound of the reheating temperature after $j$-inflation of the order of $T_{rh}^j \sim 10^{13-\ell}\text{GeV}$.

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