Einstein’s Miraculous Argument
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My talk will present an interesting example of the integration of history of science and philosophy of science. It will review how placing what I call Einstein’s 1905 “miraculous argument” for light quanta in its appropriate historical context makes its historical occurrence intelligible as an inspired extension of an evidential strategy Einstein had been using in his work in statistical physics. The philosophical content is the delineation of that evidential strategy, which, I urge, is of great importance in science but has been underappreciated by popular approaches to inductive inference.

Of all the achievements of Einstein’s year of miracles, 1905, one stands out. It is Einstein’s proposal in his light quantum paper that light sometimes behaves as if it consists of independent, spatially localized quanta of energy. The proposal is remarkable in that it stands in direct contradiction with that most perfect product of nineteenth century science, classical electrodynamics. The other achievements of 1905 are better characterized as a completion of works developing in the course of the 19th century. Einstein’s papers on the statistical physics of atoms and molecules vindicate Maxwell’s and Boltzmann’s ambitions. The special theory of relativity brings a better interpretation to the mathematical structure of the Lorentz group that both Lorentz and Poincaré found in classical electrodynamics.

The master stroke of the light quantum paper comes in its sixth section. Einstein takes what looks like a dreary fragment of the thermodynamics of heat radiation, an empirically based expression for the entropy of a volume of high frequency heat radiation. In a few deft inferences he converts this expression into a simple, probabilistic formula whose unavoidable interpretation is that the energy of radiation is spatially localized in finitely many, independent points. We are startled, wondering what happened to the waves of light of the nineteenth century theory and marveling at how Einstein could see the signature of atomic discreteness in the bland formulae of thermodynamics. This inference is what I shall here call “Einstein’s miraculous argument.”

My goal is to place this argument in its historical context. I will suggest that it is entirely unexpected if one locates it against the background of simple wave theories of light. That changes when we locate it in Einstein’s continuing research program in statistical physics. Stretching back to his earliest publication of 1901 and 1902, Einstein had been asking repeatedly how we might infer to the microscopic structure of matter from its macroscopic appearances. In his work of 1901 and 1902, he had sought empirical evidence for a particular law for intermolecular forces in the phenomena of capillarity and electrolysis. The idea that such inferences are possible continued through its later work of 1905 in statistical physics.

The most familiar example was Einstein’s treatment of the ideal gas law. One does not need the complete model of an ideal gas as point-like molecules moving freely in empty space to arrive at it. Very much less is needed. Essentially all one needs is that, microscopically, one has a thermal system of many components that are spatially localized and do not interact with each other, but may certainly interact with components of other systems. Whenever that is the case, the system will exhibit a macroscopic pressure conforming to the ideal gas law.
In his Brownian motion paper of 1905, Einstein gave a quite general derivation of this result. It was used in both his doctoral dissertation of 1905 and the Brownian motion paper. Sugar molecules in dilute solution conform to the conditions; so they exert an osmotic pressure that conforms to the ideal gas law. Indeed a system of small particles suspended in water and visible under a microscope also conforms to the conditions. So they too exert a pressure that conforms to the ideal gas law.

In short, the ideal gas law is a very convenient and powerful macroscopic signature for a particular microscopic structure and one that Einstein used frequently and comfortably.

What Einstein recognized in 1905 was that high frequency radiation carries a macroscopic signature of the microstructure common to ideal gases and dilute solutions: many, independent, spatially localized components. However Einstein did not report the signature of the ideal gas law. (It was there, but much harder to see, since, unlike gases or dilute solutions, the number of components in heat radiation—the quanta—vary in most processes.) What Einstein revealed was a new signature of this microstructure, common to heat radiation, ideal gases and dilute solutions: the logarithmic dependence of entropy on volume.

This use of distinctive signatures is one example of a powerful use of evidence. As my term "signature" suggests, that evidence, once recognized, directly encodes and thus reveals the more theoretical structure. Other examples include Bohr’s decoding from emission spectra of the discrete energies levels of bound electrons; Ehrenfest’s and Poincaré’s decoding from the black body spectrum of discontinuous energies; and Newton’s decoding from orbital motions of the inverse square law of attraction. These examples indicate that, in the right circumstances, the reach of evidence can be great and its power to determine particular results quite strong, in contradiction with pessimistic views of the power of evidence such as are associated with the underdetermination thesis.