I am very honored to be here this evening. In fact I am so honored that I have written a little poem for the occasion. It's not a very good poem, I'm afraid. In fact, it's pretty corny. But it has the compensating virtue of being short. And in any case, it will serve to introduce my topic. Here is it:

I think that I shall never see
A circle perfect as can be.
So tell me please, Professor Paul
(that's me),
How circles can be thought at all!

"A circle perfect as can be." Here is the problem. The concept of a perfect circle is an example of what I want to call an ideal concept.

It is idealized because of course we do not encounter any really perfect circles in our ordinary, day to day trafficking with dinner plates, manhole covers, and other more or less circular objects. A perfect circle is an abstract geometrical figure on a Euclidean plane.

There are lots of other examples of such idealized, "limit"-concepts. Geometry provides a number of examples: the notion of a geometrical point, or of a perfectly straight line that has no thickness at all. But there are other kinds of examples too:

(a) moral concepts - for example, absolute justice;

(b) scientific concepts - an ideal gas, a point mass, a frictionless surface;

(c) and, of course, a conspicuous case - the concept of God.

Now my question this evening is: Where do we get such ideas? I raise this question not because I have
any pet theory of my own about how we get such concepts — I do not — but because I think there is a real problem here, a problem that is too often not asked, or else dismissed as being trivial or easily solved. I do not think it is trivial, or that it can be easily solved.

So my purpose this evening is to focus on a problem, not to offer a solution.

The history of this problem goes way back — at least to Plato.

Socrates, as everyone knows, went around asking people what justice was and what virtue was, and so on.

Plato also wanted to know what these things were, but he wanted to know something else too: Where do we get our concepts of justice and virtue? And note — these are ideal concepts he was worried about.

Plato was struck by a curious fact. When we describe something as just or as virtuous to a greater or lesser extent, we are not just appealing to its own intrinsic characteristics or "properties", as we are for instance when we describe something as "red". We are measuring the thing, with its various characteristics and properties, against some kind of standard or ideal.

Thus, for example, when we describe an actual political state as a more or less "just" state, we are measuring it against some kind of standard of what it is to be a "just" state. Our notion of this ideal need not be articulated very well. It may in fact even turn out to be incoherent and contain incompatible elements, if we ever get around to making it explicit and analyzing it in detail. But no matter. I am not trying to recommend the concept, but only to point out that we have it.

Likewise, when we describe someone as a "good" person, what a "real person" ought to be like, we are judging him or her against a standard of what people ought to be. Here again, our standard may be confused and incoherent; it may be only implicit. But it is there.

Yet again, when we judge a painting or a poem as more or less "beautiful" — for instance, the poem I read you just a moment ago — we are applying some kind of standard of aesthetic beauty.
Now Plato recognized that something funny is going on in such judgments. They require two kinds of information. (They are after all relational judgments, with two poles.)

(a) First, they require information about the object being evaluated. There is no special problem about how we get this information - that is, no special problems over and above the general epistemological questions that one can always ask. We can see the object, or otherwise encounter it in our direct experience. We get our information about its characteristics and properties by observing the object itself.

(b) But second, we also need information about the standards or ideals against which we are measuring the object. We need this information if we are going to say whether and to what extent the object measures up to those standards.

Where do we get this latter kind of information? While the objects we are describing and evaluating are no problem - they are all around us - the standards in terms of which we are describing and evaluating them are not all around us. We are living, I am sorry to say, in a conspicuously imperfect world, one that is far from ideal.

So where do we get those concepts?

It was considerations like these that led Plato to his theory of Forms and to his so called "Theory of Recollection".

Now, as a matter of fact, it seems that we use ideal concepts like this in two main realms:

(a) in mathematics - primarily, it seems, but not exclusively, in geometry - and in physics treated as a "mathematicized" science;

(b) and in matters of values - in ethics, aesthetics, and so on.
No doubt this is why, historically, Platonism (in a suitably broad sense of the term) has always been most at home, most plausible, in the realms of mathematics and values. Mathematicians, for instance, have historically tended to be "Platonist" about their discipline. And conversely, Platonists have historically always had a strong attraction to mathematics -- even when they weren't especially good at it. There is a strong Pythagorean strain in Platonism, as in mathematics.

Now while I do not want to recommend Plato's solution to the question of the origin of our ideal concepts -- the theory of Forms and the theory of Recollection -- I do want to recommend the question itself. It is a good question.

And it is my topic this evening.

Let me at the outset head off two kinds of responses that are frequently made at this stage of the discussion, and that I think completely miss the point. In so doing, I will try to sharpen the issue a bit.

(1) The first response is a "relativist" one. It goes like this: Judgments of values, it says, judgments about goodness, beauty, etc., are entirely subjective. They are culture-dependent, and even within a given culture they vary from person to person in an entirely idiosyncratic way.

Reply: Maybe so. But this is no objection to my question, and it is certainly not an answer to it. Indeed, the observation only aggravates the problem. We now have more ideal concepts to find the origin of.

I want to stress again that I am not at all concerned with justifying any given ideal concepts. I am not concerned with their authority, their reliability, or their "truth". I am interested only in where we get them. Certain answers to that question may commit you to certain views about the authority of these concepts. But that is another matter, and I am not interested in it here. I am asking only about the origin of these concepts, not about their correctness or authority.

Furthermore, this first objection may be plausible in the case of ideal concepts about values. But it is harder to make the objection in the case of ideal concepts about mathematics and mathematicized physics.

(2) The second response is related to the first one. Here it is: We get our ideal concepts, this re-
sponse says, from society, from conditioning; we learn them "on our mother's knee".

Reply: This is no doubt true. Our Western conception of beauty, for instance, is quite different from that of other cultures. For that matter, it is quite different from what it was five hundred years ago. But again, this doesn't answer the question; it only delays an answer. If I got my ideal concepts from my mother, from my teachers, from society at large—where did they get them? We are looking for the ultimate origins of these concepts.

The problem—Plato's problem, in fact—is how could anyone have got an ideal concept like these in a world where there are no ideal things to match those concepts?

My claim this evening then is that there is a special problem about the origins of these ideal concepts. I want to drive that point home, and perhaps convince you of it, by looking at several attempts to deny that there is any special difficulty with these concepts and to explain how we get them in a more or less straightforward way. Those attempts I think fail, and their failure is instructive. For convenience, I will put these attempts in the framework of the "classical modern" dispute between the seventeenth-century Rationalists and the eighteenth-century Empiricists.

Descartes, in Meditation III, divides ideas into three kinds according to their origin. Two of his three divisions correspond roughly to the more modern distinction between knowledge "by acquaintance" and knowledge "by description". They are

(a) adventitious ideas (as he calls them). These are the ideas that we get from the senses by just observing things. They are, roughly, the ideas we get "by acquaintance". We don't have to do anything on our own part to get such ideas; we are passive in the case of these ideas. Of course, we are probably not completely passive in our acquiring of even these ideas. But that does not spoil the division; there is still a contrast between these "adventitious" ideas and:

(b) factitious ideas. These are ideas that we construct out of more basic ideas. For example, I have never been to Cairo. But I have
an idea of the city, an idea gathered from photographs, descriptions, news accounts, and so on. My idea of Cairo is obtained not "by acquaintance" (as if I had actually been there) but "by description".

The kind of case Descartes had in mind under class (b) is the kind of idea we get by combining other ideas, as in my idea of a "golden mountain", made up of the idea of gold and the idea of a mountain, or as in my composite idea of Cairo. The results of this kind of combining activity would be "complex" ideas, molecular ideas (so to speak) made up out of more basic, "atomic" ingredients.

But, while that is what Descartes had in mind, the general notion of a factitious idea does not require this combining activity. The general notion is that a factitious idea is one we produce by doing something to another idea, or perhaps to several other ideas. This "doing something" may involve combining ideas, but it may be something else too. The notion then is a very liberal one.

So far the picture seems fairly straightforward. But Descartes thought there was also a third kind of idea, his notorious innate ideas. He needed a third kind of idea because he thought there were some ideas that did not come directly from the senses, and could not be produced from ideas that did come from the senses.

Not surprisingly, these ideas are for the most part just the ideal concepts that so bothered Plato — and that so bother me. (Descartes also allowed certain other innate ideas that perhaps do not fit this description — for instance, the idea of the ego or self. But these result from peculiarities of his own theory, and do not affect my point here.)

The theory of innate ideas, then, is Descartes' answer to the same problem that Plato tried to answer with his theory of Recollection.

Now the eighteenth-century Empiricists rejected the Rationalist theory of innate ideas. And for good reason — there are grave difficulties with the theory, as any good history of philosophy will be glad to point out. But in rejecting the theory of innate ideas, the Empiricists had very little of their own to say about the problem that gave rise to that theory in the first
Empiricism, in fact, has remarkably little to say about ideal concepts.

Empiricism tends to think there is no special problem here, and that all our ideas, including our ideal concepts, can be accounted for more or less straightforwardly in terms of what Descartes called adventitious or factitious ideas — ideas that either come directly from the senses or else are the products of doing things to ideas that do come from the senses.

Well, is that right?

Let's give Empiricism a run for its money, and see how it fares. By "Empiricism" now, I no longer mean only the eighteenth-century Empiricists exclusively. I mean anyone in the broad "Empiricist" tradition who apparently thinks there is no special problem about ideal concepts.

If they are right, then our ideal concepts must either come from simply observing the world around us — in the way that we get the concept "dog", let us say — or else they are the results of our doing something to concepts we get in that way.

To begin with, let us focus on the concept of a perfect circle, the concept I celebrated in my little poem at the beginning of my talk.

At first, you perhaps wouldn't think that the "adventitious" alternative is a very attractive one. After all, the fact that we do not observe any perfectly circular things in the world is the fact that got this whole discussion going.

But surprisingly, some "Empiricists" have held that we do get ideal concepts — at least mathematical ones — by simply abstracting them from the senses, just like any other ordinary concept.

For example, Aquinas held this. Here is the background: Aristotle, in Metaphysics VI, had held that there were three main kinds of theoretical sciences: (a) natural sciences, (b) mathematical sciences, and (c) metaphysics or what he called "theology". In the Middle Ages, the division was drawn by the Aristotelians — including Aquinas — as follows:

(a) The "natural" sciences (Aristotelian physics) deal with certain kinds of forms of things, forms that not only exist in matter but are also defined in rela-
tion to matter as well. For instance, the famous example of the "snub". There are no "snub" things that are immaterial; there are only snub noses, and noses are pretty material kinds of things. Moreover, for the Aristotelians, the notion of "the snub" is defined (on those rare occasions when we would ever care to do so) as a kind of curvature of the nose, so that its very definition refers to matter.

(b) On the other hand, metaphysics or theology deals with forms that do not exist in matter, and are not defined in relation to matter either. For example, God and the angels — or, in Aristotle's own theory, the so-called "separated substances".

(c) But between the natural sciences and metaphysics or theology, the Aristotelians located mathematics. Mathematics, according to them, treated of forms that exist in matter, but are not defined in relation to matter. For example, the geometrical notion of a curve, unlike the "physical" notion of "the snub", is not defined in terms of matter — in terms of what it is that is curved; the curve is just the shape itself, quite apart from whatever has that shape. On the other hand, the Aristotelians agreed that, like "the snub", curves do not exist apart from material objects.

Note the claim here. Mathematical concepts are concepts of things that exist in matter. The concepts are abstractions from our experience of material objects in pretty much the same way other concepts are abstracted, according to the Aristotelians. In short, there really are perfect circles in the world, there really are perfectly straight lines with no thickness at all, and so on.

I find this theory astonishing, and altogether unconvincing. To me, it is hard to avoid regarding this theory as an indication that its holders simply hadn't thought about the problem very much.

Note, however, that while Aquinas (along with other Aristotelians) says that mathematical ideal concepts are abstractions from our experience of material objects, he has a different story to tell about values — the other great area where we encounter ideal concepts. In his Summa theologiae, I, 2, 1, obj. 1 and reply, Aquinas is getting ready to present his five famous "proofs" for the existence of God. As a preliminary, he asks whether one needs to prove the existence of God. (After all, if the existence of God is
somehow obvious, then there is hardly any point in going on to give five laborious "proofs" of the fact.)

The first objection Aquinas considers says no, we do not need to prove the existence of God because St. John Damascene (an early Christian author) says, the knowledge of God is naturally implanted in all of us. In other words, it is innate.

In his reply to this objection, Aquinas admits that in a sense the knowledge of God is innate in us, because God is the "good" of man, and we all naturally desire the good. We cannot desire what we do not have at least some concept of. But of course we don't always know that God is the good of man, so that we still have to prove the existence of God.

No doubt Aquinas' answer suffices to disarm the objection. But note what he has admitted here in passing: we all naturally desire the good, and therefore all naturally have a concept of the good. Aquinas even uses the term 'innate' here. Not even he would go so far as to claim that our concepts of ideal values are "adventitious". I do not think he should have done so for mathematically ideal concepts either. I do not find perfect circles in the world around me, any more than I find perfect justice.

But, if ideal concepts are not "adventitious", then if Empiricism is right, they are the results of our doing something to adventitious concepts. Thus, the challenge for Empiricism is to tell us what we do to adventitious ideas to yield "ideals".

Well, what about it? Let us look at some actual suggestions, and see how they fare.

(1) Here is a theory that is always one of the first to be proposed. Let's call it the "negation" theory. It goes like this: We form the concept of a perfect circle, for instance, by observing the imperfect circles in the world around us, and then mentally removing their imperfections.

In other words, we observe figures on a blackboard, dinner plates, manhole covers, and other such imperfectly circular things, and we notice that they are all a little lopsided or lumpy. They aren't quite right. So we mentally smooth them out in our imagination, firm them up, "perfect" them. The result is the concept of a perfect circle.
But this theory won't work. Watch carefully: We observe figures on a blackboard, dinner plates, manhole covers, and so on, and we notice WHAT? We notice that they are "not quite right". Not quite WHAT? Isn't this just another way of saying that we notice they are not quite perfectly circular? How could we "remove their imperfections" unless we recognized that they had "imperfections" in the first place—in short, how could we do this unless we already had the concept of a perfect circle?

The problem with this theory, then, is that it describes a process that comes too late. It does not explain how we produce a concept of a perfect circle; it presupposes that we already have one.

The point can be generalized. The notion of something bounded, limited, finite in some way, always presupposes that we can already refer to what is on the other side of that boundary or limit. Consider a fence, or a closed door, for example. This doesn't mean that we know what is on the other side of the fence or beyond the door. But we do have the notion "beyond the fence", "through the door". The very notion of a fence or a door as a boundary implies a reference to what is beyond the boundary, what is cut off from us on this side.

In short, the concept of the finite or bounded is constructed by negating the concept of the infinite—not the other way around. And this is what is wrong with the "negation theory" as an account of the origin of our ideal concepts.

David Hume, the arch-Empiricist, held a version of this "negation theory". Listen to what he says in his *Enquiry concerning Human Understanding*, section II about how we get our ideal concept of God:

The idea of God, as meaning an infinitely intelligent, wise, and good Being, arises from reflecting on the operations of our own mind, and augmenting, without limit, those qualities of goodness and wisdom.

Oh, that's how we do it, is it? "Augmenting without limit", that's the process. Descartes had already sufficiently refuted this suggestion in his third Meditation.
And I must not imagine that I do not conceive infinity as a real idea, but only through the negation of what is finite in the manner that I comprehend rest and darkness as the negation of movement and light. On the contrary, I see manifestly that there is more reality in infinite substance than in finite substance, and my notion of the infinite is somehow prior to that of the finite, that is, the notion of God is prior to that of myself. For how would it be possible for me to know that I doubt and that I desire— that is, that I lack something and am not all perfect—if I did not have in myself any idea of a being more perfect than my own, by comparison with which I might recognize the defects of my own nature?

Without wishing to commit myself to the details of Descartes' theory, I should say that he has won this round.

(2) Well, let's try another theory. This one we'll call the "approximation" or "convergence theory". It says that when we observe all those dinner plates and manhole covers and the like, we notice that they are all approximations of a certain shape, which we then call a "perfect circle".

Or put it like this: When we observe all those dinner plates and manhole covers, we can mentally arrange them side by side in our imagination in various ways. If we should happen to arrange them in order from the most "lumpy" and "lopsided" to the least "lumpy" and "lopsided" (even though at the time we wouldn't know enough to recognize them as "lumpy" or "lopsided"—that is what we just learned from considering the "negation" theory), then perhaps we can come to see that the series of shapes tends to converge on a certain limit. In that case, we form the idea of a "perfect circle" by taking the limit of the series. The fact that we might mentally arrange these same shapes in other orders as well is no objection; that just means that we might derive other concepts too from the same "adventitious" data.

Nice theory. But will it work? I don't think it will. Just how are we supposed to recognize what shape it is that all these other shapes "approximate"? How are we supposed to "take the limit"?

The metaphor of "taking the limit" comes from mathematics. Perhaps it is circular question-begging to
try to explain how we get our ideal concepts, including those in mathematics, by appealing to a mathematical metaphor. But, whatever you think about that, the appeal of the mathematical metaphor comes from the fact that frequently the process of "taking the limit" is a matter of simply applying an algorithm, a clear and strictly defined procedure. The attractiveness of the mathematical metaphor then lies in its appeal to rigor.

But it is after all only a metaphor. What do we get when we try to go behind the metaphor? What is the process by which we come to recognize the shape towards which the series of all these other shapes converges? What is the algorithm here? What is the rigorous process, the clear and strictly defined procedure? The metaphor suggests there is one, but what is it?

And, as far as I can see, that's the end of it. The "approximation" or "convergence theory" promises us that there is a nice, clear procedure here, but it never gets around to telling us exactly what it is. This is not an explanation; it is simply a reaffirmation of one's Empiricist faith.

Here, as in many other treatments of our problem, there is a remarkable tendency to avoid facing the question, to dismiss the problem by coming up with a nice name for whatever is supposed to solve it ("approximation", "convergence", "taking the limit"), but with no real theory behind that name. These might well be called "Black Box" approaches. In one end go the adventitious ideas; out the other end come the ideal concepts. What goes on inside—nobody knows! But if nobody knows, why don't we just say so, and stop pretending we have a theory?

(3) Here's another contender. Actually, this theory is not really an attempt to explain how ideal factitious concepts are produced from adventitious concepts. Instead, it is a defense of the view that our ideal concepts are adventitious ones after all. (Recall the discussion of Aquinas earlier.) But no matter; let's discuss it here anyway.

This theory we shall call the "indistinguishability" theory. It says: Look, it's true that we never observe any perfectly circular objects in the sensible world around us. But we do observe things that are perceptually indistinguishable from perfectly circular objects. We see things that are so circular that their "lumpiness" is simply not observed. The concept of a
perfect circle is then just an "adventitious" idea derived from the experience of such objects.

The objection against the view that ideal concepts are adventitious ones was that there simply are no ideal objects from which those concepts could be abstracted or copied. But, this theory now says, that objection fails. Ideal adventitious concepts would not have to be copied from ideal objects; they can be copied from objects that are perceptually indistinguishable from ideal objects. And while there aren't any of the former, there certainly are plenty of the latter.

Reply: This theory is perhaps plausible for the concept of a perfect circle. But it is not so plausible for other geometrically ideal concepts. What about the notion of a geometrical point, for example? If something is big enough to observe at all, it's already distinguishable from a geometrical point; it's too big. So too, any "line" that I can see may be perceptually indistinguishable from being straight. (That is, perhaps I cannot see any curvature in it.) But, if I can see it at all, it is too thick to be a genuinely straight line, and so is "distinguishable" from one.

And, whatever you may think about those cases, it is quite clear that the theory is totally unacceptable for ideal concepts of values. There may be observable shapes that are perceptually indistinguishable from perfect circles, but there are definitely not any human beings that are indistinguishable from ideal humans, or any political entities that are indistinguishable from the ideally just state.

You may well say that this doesn't matter. All it means is that we don't have a single theory to account for the origin of all ideal concepts. But why should we? Is there any reason in advance to assume that all ideal concepts are derived in the same way? We have a theory now, the "perceptual indistinguishability" theory, that can account for the origin of the concept of a perfect circle. It is no criticism of that theory to say that it will not account for the origin of other concepts.

That is a fair reply, and makes a good point. But it won't save the theory, because I have another objection to the theory as applied to any ideal concept. There may very well be objects that are perceptually indistinguishable from perfect circles. But those same objects are also perceptually indistinguishable from
slightly lumpy shapes - in fact, they just are slightly lumpy shapes. Why then do we get the one concept from them and not the other?

If you say that we can get both concepts from such objects, then what do we do differently in the two cases? What do we do when we derive the concept of a perfect circle that we do not do (or the other way around) when we derive the concept of the "slightly lumpy shape" from the very same observable objects? Plainly there must be something different in our procedure, or else - starting from the same starting point, and applying exactly the same procedure - we would end up with exactly the same concept in each case.

Once again, the explanation evaporates once one looks at it closely. It begins with soothing words and reassurances, but never really gets around to answering the question.

(4) Let's try one more theory. This one we shall call the "definition" theory. It says: We get the concept of a perfect circle by definition. A perfect circle is defined as a "locus of points on a Euclidean plane equidistant from a single point called the center", or something like that. That's after all the way the concept was taught to us in geometry classes.

Reply: Very good, but let's look more closely. A circle is defined as a locus of WHAT on a WHAT equidistant from a single WHAT called the center? Look at all the ideal concepts in that definition.

The theory makes the valid observation that some ideal concepts can be defined in terms of others. But the problem is: How do we get started? Where do we get the original concept of a "point", of a "Euclidean plane", and so on?

Well, there we are. We have looked at four "Empiricist" attempts to explain the origin of ideal concepts, and we have found all of them wanting. Are there any other theories? No doubt there are, but I don't know of any that fare any better than these.

The moral of the story then is not that Empiricism is wrong here, and that there is no way to account for ideal concepts in a theory that allows only what Descartes called "adventitious" and "factitious" ideas. The moral of the story is rather that there is a problem here, a problem to which we don't yet have the
answer. The problem cannot be simply waved away or "solved" with a mere reaffirmation of one's Empiricist faith. It has to be faced squarely.

Historically, there have been three main theories that I know of that did recognize the special difficulties of ideal concepts and tried to solve them. They were: (a) the Platonic theory of Forms, together with the theory of Recollection, (b) Descartes' theory of innate ideas, and (c) the Augustinian theory of illumination. This is not the place to rehearse those theories in detail. Each of them has grave difficulties, of course, difficulties that Empiricists are only too happy to point out. But each of them does have the virtue of honestly recognizing a problem that is often just ignored.

Finally, let me look briefly at one other theory that is frequently heard in this connection. This theory we shall call the "psychologico-evolutionary" theory. It goes roughly like this: The nature of our minds (or "brains", if you must) is such that it spontaneously tends to view the world in terms of these ideals. We just automatically tend to see dinner plates and such, not as perfect circles perhaps, but at least in terms of perfect circles. This natural tendency is part of the built in "wiring" of the mind. We are pre-programmed to simplify our experience by organizing it around these ideal reference points. Perhaps there is some evolutionary advantage to doing so, so that this tendency is the result of long evolutionary development. But whatever the details, the fact is that these ideal concepts are the results of purely natural factors. There is no need to appeal to spooky entities like Platonic Forms, no need to appeal to innate ideas (with all their problems), no need to resort to God, as the Augustinian theory of illumination did, to explain how we get our ideal concepts.

I suspect some approach like this is probably correct. But notice what the theory really says. It is not the hard-headed Empiricism it perhaps likes to pretend to be. It pooh-pooh's Platonism and the other traditional attempts to take the problem of ideal concepts seriously. But in fact the theory abandons Empiricism altogether, and embraces about as strong a version of the Platonic-Cartesian-Augustinian point of view as I can imagine.

This theory takes the epistemological problem of the origin of ideal concepts, and answers it by making some very strong metaphysical claims. Nature itself has
put these tendencies into our minds; there is some "evolutionary advantage" to doing so. How could nature have done that? How could it have built in a tendency to see the world in terms of ideals? How could there be any evolutionary advantage at all to seeing the world in terms of things that don't exist?

Yet it is clear that something like this is true. There are no ideal gases, no frictionless surfaces, no perfectly elastic bodies, no point masses. And yet it is a strange fact about reality that the best way to deal with it is in terms of such ideal things. It is as if nature itself contained an implicit reference to an ideal version of itself. Those ideals, whether they exist or not, have left their traces all around us.

I said that I think something like this approach is correct, and I do. But it is only the beginning of a theory. And, far from being a rejection of Platonism, of innatism, of illumination, this approach may be regarded as merely a latter-day version of them.