CPT and Lorentz Tests with Muons

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Precision experiments with muons are sensitive to Planck-scale CPT and Lorentz violation that is undetectable in other tests. Existing data on the muonium ground-state hyperfine structure and on the muon anomalous magnetic moment could be analyzed to provide dimensionless figures of merit for CPT and Lorentz violation at the levels of $4 \times 10^{-21}$ and $10^{-23}$.

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The minimal standard model of particle physics is CPT and Lorentz invariant. However, spontaneous breaking of these symmetries may occur in a more fundamental theory incorporating gravity [1,2]. Minuscule low-energy signals of CPT and Lorentz breaking could then emerge in experiments sensitive to effects suppressed by the ratio of a low-energy scale to the Planck scale. At presently attainable energies, the resulting effects would be described by a general standard-model extension [3] that allows for CPT and Lorentz violation but otherwise maintains conventional properties of quantum field theory, including gauge invariance, renormalizability, and energy conservation.

In the present work, we study the sensitivity of different muon experiments to CPT and Lorentz violation. Planck-scale sensitivity to possible effects is known to be attainable in certain experiments without muons. These include, for example, tests with neutral-meson oscillations [4,5], searches for cosmic birefringence [3,6,7], clock-comparison experiments [8,9], comparisons of particles and antiparticles in Penning traps [10,11], spectroscopic comparisons of hydrogen and antihydrogen [12], measurements of the baryon asymmetry [13], and observations of high-energy cosmic rays [14]. However, in the context of the standard-model extension, dominant effects in the muon sector would be disjoint from those in any of the above experiments because the latter involve only photons, hadrons, and electrons. Moreover, if the size of CPT and Lorentz violation scales with mass, high-precision experiments with muons would represent a particularly promising approach to detecting lepton-sector effects from the Planck scale.

The standard CPT test involving muons compares the $g$ factors for $\mu^-$ and $\mu^+$, with a bound [15,16] given by the figure of merit

$$r^\mu_g = |g^\mu_- - g^\mu_+|/g_{av} \leq 10^{-8}.$$  

(1)

We show here that data from experiments normally not associated with CPT or Lorentz tests, including muonium microwave spectroscopy [17] and $g - 2$ experiments on $\mu^+$ alone [18], can indeed provide Planck-scale sensitivity to CPT and Lorentz violation.

For the experiments considered here, it suffices to consider a quantum-electrodynamics limit of the standard-model extension incorporating only muons, electrons, and photons. Other terms in the full standard-model extension would be irrelevant or lead only to subdominant effects.

In natural units with $\hbar = c = 1$, the Lorentz-violating Lagrangian terms of interest are

$$\mathcal{L} = -a_{kAB} \bar{l}_A \gamma^k l_B - b_{kAB} \bar{l}_A \gamma^k \gamma^j l_B$$

$$- \frac{1}{2} H_{kAAB} \bar{l}_A \sigma^{kl} l_B + \frac{1}{2} i d_{kAAB} \bar{l}_A \gamma^k l_B$$

$$+ \frac{1}{2} i d_{kAAB} \bar{l}_A \gamma^k \gamma^j l_B.$$  

(2)

Here, the lepton fields are denoted by $l_A$ with $A = 1, 2$ corresponding to $e^-$, $\mu^-$, respectively, and $iD_A = i\partial_A - qA_A$ with charge $q = -|e|$. To avoid confusion with four-vector indices, the symbol $\mu$ is reserved in this Letter solely as a label for the muon.

The terms associated with the parameters $a_{kAB}$, $b_{kAB}$ are CPT odd, while the others are CPT even. All the parameters in Eq. (2) are assumed small, and they all are Hermitian $2 \times 2$ matrices in flavor space. For example,

$$b_\kappa = \left( \begin{array}{cc} b_\kappa^e & b_\kappa^{e\mu} \\ b_\kappa^{e\mu} & b_\kappa^\mu \end{array} \right),$$  

(3)

where $b_\kappa^e$, $b_\kappa^{e\mu}$ are associated with terms preserving lepton number while the others are associated with terms violating it. Since the usual standard model conserves lepton number, leading-order rates for processes that violate lepton number in the standard-model extension must be quadratic in the flavor-nondiagonal parameters $b_\kappa^{e\mu}$, etc. In contrast, processes violating Lorentz symmetry but preserving lepton number can depend linearly on flavor-diagonal parameters $b_\kappa^e$, $b_\kappa^\mu$, etc. This means that experimental bounds from processes preserving lepton number are typically many orders of magnitude sharper than bounds involving lepton-number violation.

Consider first spectroscopic studies of muonium $M$, which is a $\mu^+ - e^-$ bound state. In experiments at RAL and LANL, precisions of about 20 ppb have been attained both for the $1S$-$2S$ transition [19] and for the ground-state Zeeman hyperfine transitions [17]. However, we restrict attention here to the latter because the hyperfine transition frequencies are much smaller than the $1S$-$2S$ ones, which implies better absolute energy resolution and corresponding sensitivity to CPT and Lorentz violation [20].
The four hyperfine ground states of $M$ can be labeled 1, 2, 3, 4 in order of decreasing energy. The Zeeman hyperfine transitions $v_{12}$, $v_{34}$ have been measured in a 1.7 T magnetic field [21] with a precision of about 40 Hz ($\sim 20$ ppb), and the hyperfine interval has been extracted.

Since electromagnetic transitions in $M$ conserve lepton number, dominant effects in the standard-model extension arise from flavor-diagonal terms in Eq. (2). For the case of an antimuon $\mu^-$, the modified Dirac equation is

$$i\gamma^\mu D^\lambda - m_{\mu^\prime} + \epsilon_{A\lambda} \gamma^\mu - b^\nu_{\lambda\nu} \gamma^\nu\gamma^\lambda + \frac{1}{2} H_{\lambda\nu} \sigma^{\lambda\nu} + 3c_{\kappa\lambda} \gamma^\nu D^\lambda + i d_{\kappa\lambda} \gamma^\nu \gamma^\lambda \psi = 0, \quad (4)$$

where $\psi$ is a four-component $\mu^+$ field of mass $m_{\mu^+}$. A similar equation exists for the $e^+$, containing parameters $a^\nu_{\lambda\nu}, H_{\nu\lambda}^\nu, c_{\mu\lambda}^\nu, d_{\mu\lambda}^\nu$. The associated Hamiltonians are found using established procedures [11]. The Coulomb potential in $M$ is $A^\lambda = (|e|/4\pi r\), 0).

The leading-order Lorentz-violating energy shifts in $M$ can be obtained from these Hamiltonians using perturbation theory and relativistic two-fermion techniques [22]. For the four Zeeman hyperfine levels in a 1.7 T magnetic field, we thereby can determine the corresponding shifts $\delta v_{12}$, $\delta v_{34}$ in the frequencies $v_{12}$, $v_{34}$. We find

$$\delta v_{12} = -\delta v_{34} = -\tilde{b}^\nu_3 / \pi, \quad (5)$$

where $\tilde{b}_3^\nu = b_3^\nu + d_{30} \mu_0 + H_{12}^\nu$. Although in a weak or zero field [23] the results would depend on a combination of both muon and electron parameters for Lorentz violation, only the muon parameters appear in Eq. (5) because in a 1.7 T field the relevant transitions essentially involve pure muon-spin flips. Note that subleading-order Lorentz-violating effects are further suppressed by powers of $\alpha$ or $\mu_0 B / m_\mu = 5 \times 10^{-15}$ and can therefore be neglected.

Since the laboratory frame rotates with the Earth, and since the frequency shifts (5) depend on spatial components of the parameters for $\text{CPT}$ and Lorentz violation, the frequencies $v_{12}, v_{34}$ oscillate about a mean value with frequency equal to the Earth’s sidereal frequency $\Omega = 2\pi/(23 \text{ h 56 m})$. Note that no signal of this type emerges at any perturbative order in the standard model without Lorentz violation. Also, the anticorrelation of the variations of $\delta v_{12}$ and $\delta v_{34}$ could help exclude environmental systematic effects in analyzing real data.

The result (5) could directly be used to place a bound on $\text{CPT}$ and Lorentz violation in the laboratory frame.

$$\hat{H} = \gamma_0 \hat{\gamma} \cdot \hat{\pi} + m_\mu + \frac{1}{2}(g - 2)\mu_\mu \gamma_0 \sum \cdot \hat{B} - a^{00}_\mu - \tilde{c}^{\mu}_0 - \tilde{c}^{\mu}_j + \tilde{d}^{\mu}_j \gamma_j + [b^{\mu}_j + (d_{jk}^{\mu} + d_{jk}^{\mu})] \gamma_j + \frac{1}{2} \epsilon_{jkl} H^{\mu}_{kl} + m d_{jk}^{\mu} \gamma_j \gamma_k$$

where $\Sigma^j = \gamma_j \gamma^0 \gamma^j$, $\mu_\mu$ is the muon magneton, and $\hat{\pi} = \hat{p} - qA$, with $q = +|e|$ for $\mu^+$. This Hamiltonian contains no terms that provide leading-order corrections to the $g$ factors for $\mu^+$ or $\mu^-$. Instead, the dominant sensitivity to $\text{CPT}$ violation results from the sensitivity to small frequency shifts associated with the spin precession. The conventional figure of merit $r_\mu^\mu$ therefore is zero at leading
order despite the presence of explicit CPT violation, which means alternative figures of merit are needed [11].

A Foldy-Wouthuysen transformation [24] can be used to convert the Hamiltonian $\hat{H}$ to another Hamiltonian $\hat{H}'$ in which the $2 \times 2$ off-diagonal blocks contain only first-order terms in the magnetic field $\vec{B}$ [25] and in the parameters for CPT and Lorentz violation. We find $\hat{H}' = \exp(\gamma' \vec{\gamma}' \phi) \exp(-\gamma' \vec{\gamma}' \phi)$, with $\tan 2\phi = (\Sigma \cdot \vec{\pi})/m_\mu$ and $|\Sigma \cdot \vec{\pi}|^2 = \vec{v}^2 - q \Sigma \cdot \vec{B}$. The off-diagonal blocks in $\hat{H}'$ are irrelevant at leading order since here they produce effects that are at least quadratic in small parameters.

The upper-left $2 \times 2$ block of $\hat{H}'$ is the relevant relativistic Hamiltonian for the $\mu^+$ in the laboratory frame [26]. It has the form

$$\hat{H}' = \mathcal{E}_0 + \mathcal{E}_1 + \frac{1}{2} \vec{\sigma} \cdot (\vec{\omega}_{s,0} + f_1 \vec{\beta} + \vec{\gamma}_s),$$

(10)

where $\mathcal{E}_0 = \gamma m$ and $\gamma = (1 - \beta^2)^{-1/2}$ with three-velocity $\vec{\beta}$. The term $\mathcal{E}_1$ contains irrelevant spin-independent corrections. The quantity $\vec{\omega}_{s,0} = (g - 2 + 2/\gamma) \mu_\mu \vec{B}$ is the usual spin-precession frequency. The term $f_1 \vec{\beta}$ is proportional to $\vec{\beta}$, and its contributions average to zero since the detectors in the $(g - 2)$ experiments are spread around the ring and their data are summed. The term $\vec{\gamma}_s$ depends on the parameters for CPT and Lorentz violation and partially on $\vec{\beta}$, but again only the $\vec{\beta}$-independent terms are relevant here.

The spin-precession frequency $\omega_s$ is calculated as $\omega_s = i[\hat{H}', \vec{\sigma}] = \vec{\omega}_s \times \vec{\sigma}$. Since the detectors are in the $\hat{x}\hat{y}$ plane in the laboratory frame, only the vertical component $\omega_z$ is measured. Substituting for $\hat{H}'$ and keeping only the velocity-independent terms along the $\hat{z}$ direction gives for $\mu^+$ the result $\omega_z = \omega_{s,0} + 2 \tilde{b}_z$, where $\tilde{b}_z = b_z^\mu / \gamma + m_\mu d_{30} + H_2^\mu$. Note that $\tilde{b}_z$ reduces to $b_z$ in the nonrelativistic limit [27].

The cyclotron frequency $\omega_c$ is obtained from $[\hat{H}', \vec{r}] = \vec{\pi} / \mathcal{E}_0$, which contains a term $\vec{\omega}_c \times \vec{r}$. However, no leading-order corrections to the usual cyclotron frequency appear: $\omega_c = \omega_{c,0} = 2 \mu_\mu \vec{B} / \gamma$. Subleading-order terms do in fact contribute but are of lower order than those in $\omega_z$ and therefore can be ignored.

Combining the above results and converting to the nonrotating frame as in Eq. (6), we find the correction to the $\mu^+$ anomaly frequency $\omega_a = \omega_z - \omega_c$ due to CPT and Lorentz violation is

$$\Delta \omega_\mu^+ = 2 \tilde{b}_z^\mu \cos \chi + (2 \tilde{b}_x^\mu \cos \Omega t + \tilde{b}_y^\mu \sin \Omega t) \sin \chi,$$

(11)

where $\chi$ is now the colatitude of the experiment. The corresponding expression $\Delta \omega_\mu^-$ for $\mu^-$ is obtained by the substitution $b_j^\mu \to -b_j^\mu$ in the expressions for $\tilde{b}_X, \tilde{b}_Y, \tilde{b}_Z$.

These results suggest two interesting types of experimental signal. The first involves the difference $\Delta \omega^\mu = \delta \omega_\mu^+ - \delta \omega_\mu^-$, which is $\Delta \omega^\mu = 4 \tilde{b}_z^\mu / \gamma$ in the laboratory frame [28]. It is impractical to measure $g - 2$ for both $\mu^+$ and $\mu^-$ simultaneously, so instead one can directly consider the time-averaged difference $\Delta \omega^\mu$. In the nonrotating frame,

$$\Delta \omega^\mu = \frac{4}{\gamma} b_Z^\mu \cos \chi.$$

(12)

An appropriate figure of merit $r_{\Delta \omega^\mu}$ here is the relative energy difference between $\mu^+$ and $\mu^-$ caused by their different spin precessions:

$$r_{\Delta \omega^\mu} = \frac{\Delta \omega^\mu / m_\mu}{b_Z^\mu}.$$

(13)

The CERN $g - 2$ experiments compared average $\mu^+$ and $\mu^-$ anomaly frequencies, finding [15] $\Delta \omega^\mu / 2 \pi = 5 \pm 3$ Hz. This gives a value of $r_{\Delta \omega^\mu}$ on the order of $2 \times 10^{-22}$, corresponding to $b_Z^\mu = (2 \pm 1) \times 10^{-22}$ GeV. A subsequent measurement at BNL [18] provides a $\mu^+$ result within 1 standard deviation of the CERN $\mu^-$ result. If the BNL experiment eventually limits the frequency difference to 1 ppm, it would provide a sensitivity at the level of $r_{\Delta \omega^\mu} \approx 10^{-23}$, corresponding to $b_Z^\mu \lesssim 10^{-23}$ GeV.

The second interesting type of experimental signal involves sidereal variations in the anomaly frequency. It can be studied using $\mu^+$ alone, in which case time stamps on frequency measurements would permit a bound on sidereal variations of $\omega_\mu$. An appropriate figure of merit $(r_{\omega_\mu})_{\text{sidereal}}$ is the relative size of the amplitude of energy variations compared to the total energy. Assuming a precision of 1 ppm, we estimate an attainable bound of

$$(r_{\omega_\mu})_{\text{sidereal}} = |\Delta \omega_\mu^+| / m_\mu \lesssim 10^{-23}.$$

(14)

The associated bound on parameters in the nonrotating frame is

$$|\sin \chi| \sqrt{(b_X^\mu)^2 + (b_Y^\mu)^2} \lesssim 5 \times 10^{-25} \text{ GeV},$$

(15)

which again represents sensitivity to the Planck scale. Note that this test involves different sensitivity to CPT violation than the previous one: the two figures of merit $r_{\Delta \omega^\mu}$, $(r_{\omega_\mu})_{\text{sidereal}}$ depend on independent components of parameters for CPT and Lorentz violation.

In addition to effects in flavor-diagonal processes, off-diagonal terms of the type in Eq. (3) arising in the standard-model extension allow Lorentz-violating contributions to flavor-changing processes. For example, precision searches have been performed for the radiative muon decay $\mu \to e \gamma$, which has a branching ratio below $5 \times 10^{-11}$ [29]. This decay has previously been analyzed using a CPT- and rotation-invariant model with Lorentz and lepton-number violation that involves terms equivalent (up to field renormalizations) to those of the form $c_{00}^e \mu$ and $d_{00}^e \mu$ in Eq. (2) [14]. The results of this analysis indicate that combinations of the dimensionless parameters $c_{00}^e$ and $d_{00}^e$ are bounded at the level of about $10^{-12}$ by rest-frame muon decays or by muon lifetime measurements in the CERN $g - 2$ experiments, and at the level of about $10^{-19}$ by constraints from horizontal.
air showers of cosmic-ray muons. As expected from the discussion following Eq. (3), these bounds are several orders of magnitude weaker than those from lepton-number preserving processes. An extension of this analysis to include all types of term in Eq. (2) would provide the best existing bounds on the flavor-nondiagonal parameters in the electron-muon sector of the standard-model extension. Useful constraints on these parameters could also be extracted from other future experiments. These include the proposed tests for muon-electron conversion [30], which have an estimated sensitivity to the process $\mu^- + N \to e^- + N$ of $2 \times 10^{-17}$, and the various precision tests that might be envisaged at a future muon collider.

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[2] For recent discussions of experimental and theoretical approaches to testing CPT and Lorentz symmetry, see, for example, CPT and Lorentz Symmetry, edited by V. A. Kostelecký (World Scientific, Singapore, 1999).


[20] In hydrogen and antihydrogen, a 1S-2S frequency resolution of about 1 mHz might be attainable in principle. However, leading-order sensitivity to CPT and Lorentz violation requires magnetic mixing of the 1S and 2S spin states, which introduces experimental difficulties associated with Zeeman field broadening [12]. Similar issues would be relevant for experiments measuring the 1S-2S transition in muonium.

[21] In these and $g - 2$ experiments, the magnetic field is determined in terms of the proton NMR frequency. In principle, this could introduce additional Lorentz- and CPT-violating effects involving the proton. However, clock-comparison experiments [8,9] suggest any such effects are too small to affect the muon tests considered here.

[22] See, for example, G. Breit, Phys. Rev. 34, 553 (1929).

[23] Even in a zero field, the hyperfine interval is split.


[26] An expression for the full Hamiltonian $\hat{H}$ in the nonrelativistic limit is presented in Ref. [9].

[27] Intuition about this result can be obtained by considering an instantaneous observer Lorentz boost from the muon rest frame to the laboratory. The frequency $\omega^\mu$ scales as $\gamma$, $b^\mu_3$ is unaffected since it is perpendicular to the boost, and $d^\mu_0$ and $H^\mu_2$ each scale as $\gamma$. The net effect produces the stated $\gamma$ dependence in the definition of $b^\mu_3$.

[28] Note that $\Delta \omega^\mu$ involves only the CPT-violating parameter $b^\mu_3$ instead of the combination $b^\mu_1$. The CPT-preserving parameters $d^\mu_0$ and $H^\mu_2$ cancel in particle-antiparticle comparisons.
