Signals for CPT and Lorentz violation in neutral-meson oscillations

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Experimental signals for indirect CPT violation in the neutral-meson systems are studied in the context of a general CPT- and Lorentz-violating standard-model extension. In this explicit theory, some CPT observables depend on the meson momentum and exhibit diurnal variations. The consequences for CPT tests vary significantly with the specific experimental scenario. The wide range of possible effects is illustrated for two types of CPT experiments presently underway, one involving boosted uncorrelated kaons and the other involving unboosted correlated kaon pairs.

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I. INTRODUCTION

The notion of studying CPT symmetry to high precision by using the interferometric sensitivity of neutral-meson oscillations to compare properties of mesons and their antiparticles dates from several decades ago [1]. In recent years, experiments with neutral kaons have constrained the CPT figure of merit $r_{K} = |m_{K} - m_{\bar{K}}|/m_{K}$ to about one part in $10^{18}$. The most recent published result is $r_{K} < 1.3 \times 10^{-18}$ at the 90% confidence level from the experiment E773 at Fermilab [2]. A preliminary result corresponding to a value of $r_{K}$ below one part in $10^{18}$ has been announced by the CPLEAR Collaboration at CERN [3]. Experiments now underway such as KTeV [4] at Fermilab or KLOE [5] at Frascati, as well as other future possibilities [6], are likely to improve these bounds. High-precision CPT tests have also been performed with the neutral-B system by the OPAL [7] and DELPHI [8] Collaborations at CERN, and other CPT measurements in the D, B_d, and B_s systems are likely to be feasible in experiments at the various charm and B factories.

On the theoretical front, a purely phenomenological treatment of CPT violation in the neutral-kaon system has also existed for decades [1]. It allows for indirect CPT violation by adding a complex parameter in the standard expressions relating the physical meson states to the strong-interaction eigenstates.

Although a purely phenomenological treatment of this type is necessary in the absence of a convincing framework for CPT violation, it is unsatisfactory from the theoretical perspective. Ideally, CPT violation should be studied within a plausible theoretical framework allowing its existence [9]. The purely phenomenological treatment of CPT violation can be contrasted with the situation for conventional CP violation, where a nonzero value of the phenomenological parameter $\epsilon_{P}$ for $T$ violation can be understood in the context of the usual standard model of particle physics [10–12].

Over the past decade, a promising theoretical possibility for CPT violation has been developed. It is based on spontaneous breaking of CPT and Lorentz symmetry in an underlying theory [13], perhaps at the Planck scale where one might plausibly expect modifications to the conventional theoretical framework arising from string theory or some other quantum theory of gravity. It appears to be compatible both with experimental constraints and with established quantum field theory, and it leads to a general standard-model extension preserving gauge invariance and renormalizability that can be used as a basis for the phenomenology of CPT and Lorentz violation [14,15]. The possibility therefore exists of using CPT tests as a quantitative probe of nature at the Planck scale.

An important feature of the standard-model extension is its applicability to a broad class of experiments. It provides a quantitative microscopic framework for CPT and Lorentz violation at the level of the standard model, allowing the prediction of signals and the comparison and evaluation of experimental bounds. Investigations related to this theory have to date been conducted for neutral-meson systems [7,8,14,16,17], tests of QED in Penning traps [18–22], photon birefringence and radiative QED effects [15,23–25], hydrogen and antihydrogen spectroscopy [26,27], clock-comparison experiments [28,29], muon properties [30], cosmic-ray and neutrino tests [31], and baryogenesis [32].

The strength of the CPT theorem [10] makes it difficult to create a theoretical framework for CPT violation that is plausible and avoids radical revisions of established quantum field theory [9,33]. It is therefore unsurprising that in the context of the standard-model extension the phenomenological parameters for CPT violation turn out to have properties beyond those normally assumed. Indeed, it has been shown that previously unexpected effects appear, including momentum dependence (in both magnitude and orientation) of the experimental observables for CPT violation. The consequences for experimental signals can be substantial and include, for example, the possibility that diurnal variations exist in observable quantities [17].

The present work investigates the theoretical underpinnings of experimental signals for CPT violation in the context of the general standard-model extension. One goal is to illustrate how disparate the implications of the momentum and time dependence can be for different experimental scenarios and thereby to encourage the detailed realistic simulations and data analyses required for a satisfactory extraction of CPT bounds from any specific experiment.

Some theoretical considerations applicable to all relevant neutral-meson systems are presented in Sec. II, along with some results specific to the kaon system. Experiments on CP violation using neutral mesons can be classified according to whether the mesons involved are heavily boosted or not and...
whether they appear as uncorrelated events or as correlated pairs. The possibility of \( CPT \)-violating effects dependent on the momentum magnitude or orientation implies substantially different sensitivities to \( CPT \) violation for these various classes of experiment. In Sec. III, some consequences are developed for two illustrative cases. One is exemplified by the KTeV experiment [4] at Fermilab, in which a collimated beam of highly boosted uncorrelated mesons is studied. The other is exemplified by the KLOE experiment [5] at DAPHNE in Frascati, which involves correlated meson pairs with a wide angular distribution at relatively low boost. The results obtained provide intuition about effects to be expected in various types of experiment, including ones involving other neutral-meson systems.

II. THEORY

This section begins with some theoretical considerations about \( CPT \) violation and its description in the context of the general standard-model extension, applicable to any of the four relevant types of neutral meson. In what follows, the strong-interaction eigenstates are denoted generically by \( P^0 \), where \( P^0 \) is one of \( K^0 \), \( D^0 \), \( B^0 \), \( B^0 \). The corresponding opposite-flavor antiparticle is denoted \( \bar{P}^0 \).

A general neutral-meson state is a linear combination of the Schrödinger wave function for \( P^0 \) and \( \bar{P}^0 \), which can be represented by a two-component object \( \Psi \). The time evolution of the state is determined by an equation of the Schrödinger form [1]:

\[
i\partial_t \Psi = \Lambda \Psi,
\]

where \( \Lambda \) is a \( 2 \times 2 \) effective Hamiltonian. The eigenstates of this Hamiltonian represent the physical propagating states and are generically denoted \( P_S \) and \( P_L \). The corresponding eigenvalues are

\[
\lambda_S = m_S - \frac{1}{2} i \gamma_S, \quad \lambda_L = m_L - \frac{1}{2} i \gamma_L,
\]

where \( m_S, m_L \) are the propagating masses and \( \gamma_S, \gamma_L \) are the associated decay rates. For simplicity, here and in what follows the subscripts \( P \) are suppressed on all these quantities and on the components of the effective Hamiltonian \( \Lambda \).

Flavor oscillations between \( P^0 \) and \( \bar{P}^0 \) are governed by the off-diagonal components of \( \Lambda \). It can be shown [1] that this system exhibits indirect \( CPT \) violation [34] if and only if the difference of diagonal elements of \( \Lambda \) is nonzero, \( \Lambda_{11} - \Lambda_{22} \neq 0 \). The effective Hamiltonian \( \Lambda \) can be written as \( \Lambda = M - i \Gamma \), where \( M \) and \( \Gamma \) are Hermitian \( 2 \times 2 \) matrices called the mass and decay matrices, respectively. The condition for \( CPT \) violation therefore can in general be written

\[
\Delta M - \frac{1}{2} i \Delta \Gamma \neq 0,
\]

where \( \Delta M = M_{11} - M_{22} \) and \( \Delta \Gamma = \Gamma_{11} - \Gamma_{22} \). Note that the elements \( M_{11}, M_{22}, \Gamma_{11}, \Gamma_{22} \) are real by definition.

In the context of the general standard-model extension, the dominant \( CPT \)-violating contributions to \( \Lambda \) can be obtained in perturbation theory, arising as expectation values of interaction terms in the standard-model Hamiltonian [13]. The appropriate states for the expectation values are the wave functions for the \( P^0 \) and \( \bar{P}^0 \) mesons in the absence of \( CPT \) violation. Since the perturbing Hamiltonian is Hermitian, the leading-order contributions to the diagonal terms of \( \Lambda \) are necessarily real, which means \( \Delta \Gamma = 0 \). The dominant \( CPT \) signal therefore necessarily resides only in the difference of diagonal elements of the mass matrix \( M \). For the general standard-model extension, it follows that the figure of merit

\[
r_p = \frac{|m_p - m_{\bar{p}}|}{m_p} = \frac{\Delta M}{m_p}
\]

provides a complete description of the magnitude of the dominant \( CPT \)-violating effects. This can be contrasted with the usual phenomenological description, for which the effects of \( \Delta \Gamma \neq 0 \) should also be considered. For example, contributions involving the diagonal elements of \( \Gamma \) could in principle play an important role.

To make further progress, it is useful to have an explicit expression for \( \Delta M \) in terms of quantities appearing in the standard-model extension. This has been obtained in Refs. [14,17]. Several factors combine to yield a surprisingly simple expression for \( \Delta M \). Since the eigenstates of both the strong interactions and the effective Hamiltonian \( \Lambda \) are eigenstates of the parity operator, and since charge conjugation is violated by the flavor mixing, any \( CPT \)-violating effects must arise from terms in the standard-model extension that violate \( CPT \) while preserving \( P \). Also, only contributions linear in the parameters for \( CPT \) violation are of interest because all such parameters are expected to be minuscule. Moreover, flavor-nondiagonal \( CPT \)-violating effects can be neglected since they are suppressed relative to flavor-diagonal ones.

The result of the derivation is

\[
\Delta M \approx \beta^\mu \Delta a^\mu,
\]

In this expression, \( \beta^\mu \) is the four-velocity of the meson state in the observer frame: \( \beta^\mu = \gamma(1, \beta) \). Also, \( \Delta a^\mu = r_{q_1} a_{q_1}^{q_1} - r_{q_2}^{q_1} a_{q_2}^{q_1} \), where \( a_{q_1}^{q_1} \), \( a_{q_2}^{q_1} \) are \( CPT \)- and Lorentz-violating coupling constants for the two valence quarks in the \( P^0 \) meson, and where the factors \( r_{q_1} \) and \( r_{q_2} \) allow for quark-binding or other normalization effects [14]. The coupling constants \( a_1^{q_1} \), \( a_2^{q_1} \) have dimensions of mass and are associated with terms in the standard-model extension of the form \( -a_1^{q_1} \gamma^\mu q \), where \( q \) is a quark field of a specific flavor [35]. It is perhaps worth noting that the flavor-changing experiments on neutral mesons discussed here are the only tests identified to date that are sensitive to the parameters \( a_{q_1}^{q_1} \), so bounds from these experiments are of interest independently of any other tests of \( CPT \) and Lorentz symmetry. Note also that the dependence of \( \Delta M \) on the meson four-velocity and hence on the meson four-momentum is difficult to anticipate.
in the context of the usual purely phenomenological description of CPT violation, where it seems reasonable a priori to take $\Delta M$ as independent of momentum. However, the momentum dependence is compatible with the significant changes expected in the conventional picture if the CPT theorem is to be violated.

In the next section, a few experimental consequences of the dependence on the meson four-momentum magnitude and orientation are considered, and illustrations of these consequences for specific experiments are given. These examples involve kaons, for which a widely used variable for $\delta_K$ [1]. It can be introduced through the exact relation between the eigenstates of the strong interaction and those of the effective Hamiltonian:

$$ |K_S\rangle = \frac{(1+\epsilon_K + \delta_K)|K^0\rangle + (1-\epsilon_K - \delta_K)|K^0\rangle}{\sqrt{2(1+|\epsilon_K + \delta_K|^2)}}. $$

$$ |K_L\rangle = \frac{(1+\epsilon_K - \delta_K)|K^0\rangle - (1-\epsilon_K + \delta_K)|K^0\rangle}{\sqrt{2(1+|\epsilon_K - \delta_K|^2)}}. \tag{6} $$

In the kaon system, indirect $T$ violation is small and any CPT violation must also be small. This ensures that $\delta_K$ is effectively a phase-independent quantity. However, $\epsilon_K$ does vary with the choice of phase convention.

Under the assumption that CPT and $T$ violation are both small, $\delta_K$ can in general be expressed as

$$ \delta_K = \Delta \lambda / 2 \Delta \lambda, \tag{7} $$

where $\Delta \lambda$ is the difference of the eigenvalues (2) of $\Lambda$. In terms of the mass and decay-rate differences $\Delta m = m_K - m_S$ and $\Delta \gamma = \gamma_S - \gamma_L$, one has

$$ \Delta \lambda = \lambda_S - \lambda_L = - \Delta m - \frac{1}{2} i \Delta \gamma = - i \frac{\Delta m}{\sin \phi} e^{-i\phi}. \tag{8} $$

In this expression, $\phi = \tan^{-1}(2 \Delta m / \Delta \gamma)$ is sometimes called the superweak angle. Note that a subscript $K$ is understood on all the above quantities.

In the context of the standard-model extension, $\Delta \lambda \equiv \Delta M$ is given by Eq. (5). For a meson with velocity $\beta$ and corresponding boost factor $\gamma$, Eqs. (5), (7), and (8) imply

$$ \delta_K = i \sin \phi e^{i \phi} \gamma (\Delta a_0 - \beta \cdot \Delta a) / \Delta m. \tag{9} $$

The figure of merit $r_K$ in Eq. (4) becomes

$$ r_K = \frac{|m_K - m_{\bar{K}}|}{m_K} \approx \frac{2 \Delta m}{m_K \sin \phi} |\delta_K| \approx \frac{\beta \mu \Delta a_\mu}{1 \text{ GeV}}. \tag{10} $$

Using the known experimental values [36] for $\Delta m, m_K$, and $\sin \phi$ gives

$$ r_K \approx 2 \times 10^{-14} |\delta_K| \approx \frac{\beta \mu \Delta a_\mu}{1 \text{ GeV}}. \tag{11} $$

A bound on $|\delta_K|$ of about $10^{-4}$ therefore corresponds to a constraint on $|\beta \mu \Delta a_\mu|$ of about $10^{-18}$ GeV.

In the above expressions, the explicit momentum dependence arises from the dependence of $\Delta M$ on $\beta$. Since the eigenfunctions and eigenvalues of $\Lambda$ depend on $M_{11}$ and $M_{22}$, the possibility exists that there is also hidden momentum dependence in the parameter $\epsilon_K$, in the masses and decay rates $m_S, m_L, \gamma_S, \gamma_L$, and in the associated quantities $\Delta m, \Delta \gamma, \phi$. However, all such dependence is suppressed relative to that explicitly displayed above because the CPT-violating contribution to $M_{22}$ is the negative of the contribution to $M_{11}$. The sole linearly independent source of variation with momentum is therefore the difference $\Delta M$, and only the parameter for CPT violation $\delta_K$ is sensitive to $\Delta M$ at leading order. For example, $\epsilon_K$ depends on $\Delta M$ at most through correction factors involving the square of the ratio $\Delta M/\Delta \lambda$, which is of order $\delta_K^2$, so for all practical purposes conventional indirect CP $(T)$ violation displays no momentum dependence in the present framework [37]. The same is true of the other quantities, basically because a small difference between diagonal elements of a matrix changes its eigenvalues only by amounts proportional to the square of that difference.

### III. Experiment

The implications of the four-velocity and hence momentum dependence in the parameters for CPT violation can be substantial for experiments with $P^0$ mesons. The possible effects include ones arising from the dependence on the magnitude of the momentum and ones arising from the variation with orientation of the meson boost [17]. Consequences of the dependence on the momentum magnitude include the momentum dependence of observables, the possibility of increasing the CPT reach by changing the boost of the mesons studied, and even the possibility of increasing sensitivity by a restriction to a subset of the data limited to a portion of the meson-momentum spectrum. Consequences of the variation with momentum orientation include a dependence on the direction of the beam for collimated mesons, a dependence on the angular distribution for other situations, and diurnal effects arising from the rotation of the Earth relative to the constant vector $\Delta a$.

Since in real experiments the momentum and angular dependence are often used experimentally to establish detector properties and systematics, particular care is required to avoid subtracting or averaging away CPT-violating effects. However, observation of signals with momentum dependence would represent a striking effect that could help establish the existence of CPT violation. It can also suggest new ways of analyzing data to increase the sensitivity of tests. For example, data taken with time stamps can be binned according to sidereal (not solar) time and used to constrain possible time variations of observables as the Earth rotates [17].

The previous section established the momentum depen-
dence of various observables. From the experimental perspective, the expressions obtained can be regarded as defined in the laboratory frame. To display explicitly the time dependence arising from the rotation of the Earth, it is useful to exhibit the relevant expressions in terms of parameters for CPT violation expressed in a nonrotating frame. In what follows, the notation and conventions of Ref. [29] are adopted, with the spatial basis in the nonrotating frame denoted $(\hat{X}, \hat{Y}, \hat{Z})$ and that in the laboratory frame denoted $(\hat{x}, \hat{y}, \hat{z})$.

In this coordinate choice, the basis $(\hat{X}, \hat{Y}, \hat{Z})$ for the nonrotating frame is defined in terms of celestial equatorial coordinates [38]. The rotation axis of the Earth defines the $\hat{Z}$ axis, while $\hat{X}$ has declination and right ascension $0^\circ$ and $\hat{Y}$ has declination $0^\circ$ and right ascension $90^\circ$. This right-handed orthonormal basis is independent of any particular experiment. It can be regarded as constant in time because the Earth’s precession can be neglected on the time scale of most experiments, although care might be required in comparing results between experiments performed at times separated by several years or by decades.

In the laboratory frame, the most convenient choice of the $\hat{z}$ axis depends on the experiment but typically is along the beam direction. For example, if the experiment involves a collimated beam of mesons the $\hat{z}$ direction can be taken as the direction of the beam. If instead the experiment involves detecting mesons produced in a symmetric collider, the $\hat{z}$ direction can be taken along the direction of the colliding beams at the intersection point. Since time-varying signals are absent or reduced if $\hat{z}$ is aligned with $\hat{Z}$, in what follows these two unit vectors are taken to be different. Then, $\hat{z}$ precesses about $\hat{Z}$ with the Earth’s sidereal frequency $\Omega$, and the angle $\chi \in (0, \pi)$ between the two unit vectors given by $\cos \chi = \hat{z} \cdot \hat{Z}$ is nonzero. For definiteness, choose the origin of time $t = 0$ such that $\hat{z}(t = 0)$ lies in the first quadrant of the $\hat{X}-\hat{z}$ plane. Also, require $\hat{x}$ to be perpendicular to $\hat{z}$ and to lie in the $\hat{z}-\hat{Z}$ plane for all $t$: $\hat{x} = \hat{z} \times \hat{Z} \csc \chi$. Completing a right-handed orthonormal basis with $\hat{y} = \hat{z} \times \hat{x}$ means that $\hat{y}$ moves in the plane of the Earth’s equator and so is always perpendicular to $\hat{Z}$.

Figure 1 shows the relation between the two sets of basis vectors. For ease of visualization only, the basis $(\hat{x}, \hat{y}, \hat{z})$ has been translated from the laboratory location to the center of the globe. Note, however, that at the location of the labora-

![FIG. 1. Bases in the laboratory and nonrotating frames.](image)

ory $\hat{z}$ may lie at a non-normal angle to the Earth’s surface. Similarly, the angle $\chi$ is unrelated to the colatitude of the experiment unless the $\hat{z}$ axis happens to be normal to the Earth’s surface in the laboratory.

Conversion between the two bases can be implemented with a nonrelativistic transformation, given by Eq. (16) of Ref. [29]. This assumes relativistic effects due to the rotation of the Earth can be disregarded, an assumption valid to about one part in $10^9$ on the Earth’s equator. The time variation of a parameter $a = (a^x, a^y, a^z)$ for Lorentz violation can then be directly obtained in terms of its nonrotating-frame components $(a_x, a_y, a_z)$:

\[
\begin{align*}
\delta_x(t) &= a_x \cos \chi \cos \Omega t + a_y \cos \chi \sin \Omega t - a_z \sin \chi, \\
\delta_y(t) &= -a_x \sin \Omega t + a_y \cos \Omega t, \\
\delta_z(t) &= a_x \sin \chi \cos \Omega t + a_y \sin \chi \sin \Omega t + a_z \cos \chi.
\end{align*}
\]

The above expressions permit the time variation of the quantity $\beta \cdot \Delta a$ and hence the time variation of various CPT observables to be extracted.

The explicit form of the momentum and time dependence of the parameter $\delta_K$ in the kaon system can be found in the general case of a kaon with three-velocity $\beta = \beta(\sin \theta \cos \phi \sin \theta \sin \phi \cos \theta)$ in the laboratory. Here, $\theta$ and $\phi$ are conventional polar coordinates defined in the laboratory frame about the $\hat{z}$ axis. If the $\hat{z}$ axis is the beam axis, these polar coordinates can be identified with the usual polar coordinates for a detector. In terms of these quantities, the above expressions yield

\[
\delta_K(p, t) = \frac{i \sin \phi e^{i \phi}}{\Delta m} \gamma(p) \left[ \Delta a_0 + \beta(p) \Delta a_2 \left( \cos \theta \cos \chi \sin \theta \cos \phi \sin \chi \right) + \beta(p) \left[ -\Delta a_2 \sin \theta \sin \phi + \Delta a_1 \left( \cos \theta \sin \chi \sin \theta \cos \phi \cos \chi \right) \right] \sin \Omega t + \beta(p) \left[ \Delta a_1 \left( \cos \theta \sin \chi \sin \theta \cos \phi \cos \chi \right) + \Delta a_2 \sin \theta \sin \phi \right] \cos \Omega t \right],
\]

where $\gamma(p) = \sqrt{1 + \frac{p^2}{m_K^2}}$ and $\beta(p) = |p|/m \gamma(p)$, as usual.

The expression (13) is directly relevant for specific experimental and theoretical analyses, including those in the following subsections. One feature of this expression is that the complex phase of $\delta_K$ is determined by $i \exp(i \phi)$, which is independent of momentum and time. The real and imaginary parts of $\delta_K$ therefore exhibit the same momentum and time dependence. For instance, $\Re \delta_K$ and $\Im \delta_K$ scale together if a meson is boosted. Another feature is that the nature of the CPT-violating effects
experienced by a meson varies with its boost. For instance, if \( \Delta a = 0 \) in the laboratory frame then a boosted meson experiences a CPT-violating effect greater by the boost factor \( \gamma \) relative to a meson at rest. In contrast, if \( \Delta a = 0 \) in the laboratory frame then there is no CPT-violating effect for a meson at rest but there can be effects for a boosted meson. The angular dependence in Eq. (13) plays an important role in the latter case. The variation of the size of \( \delta_K \) according to sidereal time \( t \) adds further complications, including the possibility of effects averaging to zero if, as usual, data are taken over extended time periods and no time analysis is performed.

Evidently, the momentum and time dependence displayed in Eq. (13) implies that details of the experimental setup play a crucial role in the analysis of data for CPT-violating effects. Next, some issues relevant to two specific and substantially different experiments are discussed. Section III A examines some aspects of an experiment involving collimated uncorrelated kaons with a nontrivial momentum spectrum and high mean boost. Section III B considers a few issues for an experiment producing correlated kaon pairs from a high mean boost. The examples provided are chosen to improve intuition about the disparate consequences of the momentum and time dependence of the type (13) in CPT observables and to motivate their careful experimental study.

### A. Boosted uncorrelated kaons

In this subsection, a few implications of the momentum and time dependence are considered for a particular experimental scenario involving CPT tests with boosted uncorrelated kaons. Among possible experiments of this type is the KTeV experiment presently underway at Fermilab [4]. The kaon beam in this experiment is highly collimated and has a momentum spectrum with an average boost factor \( \tilde{\gamma} \) of order 100, so \( \tilde{\gamma} \approx 1 \). The geometry is such that \( \tilde{\epsilon} : \tilde{\epsilon} \equiv \cos \chi = 0.6 \).

In experiments of this type, expression (13) for the momentum and time dependence of \( \delta_K \) simplifies because the kaon three-velocity reduces to \( \tilde{\beta} = (0,0,\tilde{\beta}) \) in the laboratory frame. One finds

\[
\delta_K(p,t) = \frac{i \sin \tilde{\phi} e^{i \phi}}{\Delta M} \gamma [\Delta a_0 + \beta \Delta a_2 \cos \chi + \beta \sin \chi (\Delta a_1 \sin \Omega t + \Delta a_3 \cos \Omega t)].
\]

All four terms in this expression depend on momentum through the relativistic factor \( \gamma \). The first two exhibit no time dependence, while the last two oscillate about zero with the Earth’s sidereal frequency \( \Omega \). Note that a conventional analysis seeking to constrain the magnitude \( |\delta_K'| \) while disregarding the momentum and time dependence would typically be sensitive to the average value

\[
|\delta_K'| = \frac{\sin \tilde{\phi}}{\Delta M} |\Delta a_0 + \beta \Delta a_2 \cos \chi|.
\]

where \( \tilde{\beta} \) and \( \tilde{\gamma} \) are the weighted averages of \( \beta \) and \( \gamma \), respectively, taken over the momentum spectrum of the data.

In many experiments, including KTeV, \( \delta_K \) is indirectly reconstructed from other observables. It is therefore of interest to identify the momentum and time dependence of the quantities measured experimentally. These include, for example, the mass difference \( \Delta m \), the \( \delta_K \) lifetime \( \tau_K = 1/\gamma_S \), and the ratios \( \eta_{+-}, \eta_{00} \) of amplitudes for \( 2\pi \) decays, defined as usual by

\[
\eta_{+-} = \frac{A(K_L \rightarrow \pi^+ \pi^-)}{A(K_S \rightarrow \pi^+ \pi^-)} = |\eta_{+-}| e^{i \phi_{+-}} = \epsilon + \epsilon',
\]

\[
\eta_{00} = \frac{A(K_S \rightarrow \pi^0 \pi^0)}{A(K_S \rightarrow \pi^0 \pi^0)} = |\eta_{00}| e^{i \phi_{00}} = \epsilon - 2 \epsilon'.
\]

In the Wu-Yang phase convention [39], it can be shown that \( \epsilon \approx \epsilon_K - \delta_K \) [40,41]. Note that \( |\epsilon| \approx 2 \times 10^{-3} \) [36] and that \( |\epsilon'| \approx 6 \times 10^{-6} \) [42].

Consider first the case where \( |\epsilon_K| > |\delta_K| > |\epsilon'| \). This corresponds to the current experimental situation, since \( |\delta_K| \) is presently constrained to about \( 10^{-4} \). Neglecting \( \epsilon' \) then gives

\[
|\eta_{+-}| e^{i \phi_{+-}} = \eta_{00} e^{i \phi_{00}} = \epsilon \approx \epsilon_K - \delta_K = (|\epsilon_K| + i |\delta_K|) e^{i \phi},
\]

where the last expression follows because the phases of \( \epsilon_K \) and \( \delta_K \) differ by \( 90^\circ \) [43]. Then, it follows that

\[
|\eta_{+-}| = \eta_{00} \approx |\epsilon_K| [1 + O(\langle \delta_K / |\epsilon_K| \rangle^2)], \quad \phi_{+-} = \phi_{00} = \phi + |\delta_K / |\epsilon_K||.
\]

The above expressions and the results in the previous section show that at leading order in CPT-violating parameters the only observable quantities exhibiting leading-order momentum and time dependence are the phases \( \phi_{+-} \) and \( \phi_{00} \). In terms of experimental observables and parameters for CPT violation, one finds

\[
\phi_{+-} = \phi_{00} \quad \sin \phi = \frac{\Delta m}{\gamma_{++}} \sin \chi (\Delta a_0 + \beta \Delta a_2 \cos \chi + \beta \sin \chi (\Delta a_1 \sin \Omega t + \Delta a_3 \cos \Omega t)) \quad \text{(19)}
\]

The other experimental observables \( |\eta_{+-}|, |\eta_{00}|, \epsilon', \Delta m, \phi, \tau_S = 1/\gamma_S \) either have no momentum and time dependence or have it suppressed by the square of the parameters for CPT violation.

The result (19) shows that an experiment using collimated kaons with a momentum spectrum can in principle place independent bounds on each of the components \( \Delta a_0, \Delta a_2, \Delta a_1, \Delta a_3 \). The variation with sidereal time in Eq. (19) is a sum of sine and cosine terms, equivalent to a pure sinusoidal variation of the form \( A_{+-} \sin(\Omega t + \alpha) \) with amplitude \( A_{+-} \) proportional to \( a_\perp = (\Delta a_2)^2 + (\Delta a_3)^2 \) and phase \( \alpha \) determined by the ratio \( \Delta a_2 / \Delta a_3 \).
\[ A_{+} = \beta \gamma \sin \frac{\phi}{3} \sin \frac{\chi}{3} \left[ \Delta m a_{+} \right], \quad \tan \alpha = \Delta a_{X} / \Delta a_{Y}, \] (20)

Binning in time would in principle allow independent constraints on \( \Delta a_{X} \) and \( \Delta a_{Y} \). A time-averaged analysis would provide a bound on the combination \( \Delta a_{0} + \beta \Delta a_{Z} \cos \chi \), from which \( \Delta a_{0} \) and \( \Delta a_{Z} \) could be separated if the momentum spectrum is sufficiently broad to permit useful binning in \( \beta \).

In the specific case of the KTeV experiment, it may be feasible to disentangle \( \Delta a_{X} \) and \( \Delta a_{Y} \), but separation of \( \Delta a_{0} \) and \( \Delta a_{Z} \) may be difficult to achieve.

Although the derivation of Eq. (19) neglects the role of \( \epsilon' \), the small size of the latter ensures that the results remain valid to leading order in CPT-violating quantities provided \( |\delta K| \) is not too small. Since the observable CPT violation is predicted to occur only in the mixing matrix, \( \epsilon' \) itself is not directly affected. This restricts its role to momentum- and time-independent corrections to the above equations or to suppressed contributions controlled by the product \( \epsilon' \delta K \). For example, if \( \epsilon' \) is included one finds Eq. (19) acquires a correction \( -\Delta \phi/3 \):

\[ \phi_{+} = \phi_{00} - \frac{1}{3} \Delta \phi + \frac{\sin \phi}{\eta_{+} - |\Delta m|} \gamma [\Delta a_{0} + \beta \Delta a_{Z} \cos \chi] 
+ \beta \sin \chi (\Delta a_{Y} \sin \Omega t + \Delta a_{X} \cos \Omega t), \] (21)

where \( \Delta \phi = \phi_{00} - \phi_{+} \). However, this difference contains at most higher-order CPT-violating effects [44].

The occurrence of momentum and time dependence in the observables for CPT violation has a variety of consequences for the analysis of data in experiments of the type considered in this subsection [17]. These range from direct implications such as distinctive CPT signals correlated with the momentum spectrum and time stamps to more indirect consequences such as a CPT reach proportional to the boost factor \( \gamma \) under suitable circumstances. There are also some more exotic implications, such as the possibility in principle of improving the CPT reach under certain circumstances by examining only a specific momentum range of a subset of the available data. Evidently, a careful experimental analysis allowing for the effects of possible momentum and time dependence has the potential to yield interesting results.

B. Unboosted correlated kaon pairs

In this subsection, some effects of the momentum and time dependence are considered in an experiment testing CPT using correlated kaon pairs from \( \phi \) decay in a symmetric collider. The KLOE experiment [5] at DAPHNE in Frascati provides an example of this kind. These experimental circumstances differ substantially from those of the example in the previous subsection, and as a result the CPT reach is controlled by different factors. The origin of the kaon pairs in the decay from the \( \phi \) quarkonium state just above threshold implies a line spectrum in the laboratory-frame momentum of about 0.1 GeV for each kaon, so the momentum dependence of the CPT observables is relatively uninterest-

In contrast, significant implications for the experiment arise from the wide angular distribution of the kaon momenta in the laboratory frame.

Consider first the general situation of a quarkonium state with \( \mathcal{J}^{PC} = 1^- \) decaying at time \( t \) in its rest frame into a correlated \( P - \bar{P} \) pair. Since the laboratory frame coincides with the quarkonium rest frame, the time \( t \) can be identified with the sidereal time introduced earlier. For \( P = K \) the relevant quarkonium is the \( \phi \) meson, while for \( P = B_{s} \) it is \( Y(4S) \), for \( P = B_{s} \) it is \( Y(5S) \), and for \( P = D \) it is \( \psi(3770) \).

Immediately following the strong decay of the quarkonium, the normalized initial state \( |i\rangle \) has the form [45]

\[ |i\rangle = N |P_{S}(+P_{L}(-))-|P_{L}(+)P_{S}(-)||, \] (22)

where \( N \) is a normalization. In this expression, the eigenstates of the effective Hamiltonian are denoted by \( |P_{S}(\pm)| \) and \( |P_{L}(\pm)| \), where \( (+) \) means the particle is moving in a specified direction and \( (-) \) means it is moving in the opposite direction.

Suppose at time \( t + t_{1} \) in the quarkonium rest frame the meson moving in the \( (+) \) direction decays into \( |f_{1}\rangle \), while the other decays into \( |f_{2}\rangle \) at \( t + t_{2} \). Define for each \( \alpha \) the ratio of amplitudes

\[ \eta_{\alpha} = |\eta_{\alpha}| e^{i \phi_{\alpha}} = A(P_{L} \rightarrow f_{\alpha}) / A(P_{S} \rightarrow f_{\alpha}), \] (23)

Note that some of these quantities may depend on the momentum and time through a possible dependence on \( \Delta M \). Then, the net amplitude \( A_{12}(p, t, t_{1}, f_{2}) \) for the correlated double-meson decay into \( f_{1} \) and \( f_{2} \) is

\[ A_{12}(p, t, t_{1}, f_{2}) = \hat{N} \left( \eta_{2} e^{-i (m_{S_{1}} + m_{L_{1}} + \gamma_{S_{1}} + \gamma_{L_{1}}) / 2} - \eta_{1} e^{-i (m_{S_{1}} + m_{L_{1}} + \gamma_{S_{1}} + \gamma_{L_{1}}) / 2} \right), \] (24)

where \( \hat{N} = N A(P_{S} \rightarrow f_{1}) A(P_{S} \rightarrow f_{2}) \). In this expression, the possible dependence on the three-momenta \( p_{1} = -p_{2} = p \) of the two mesons and on the sidereal time \( t \) has been suppressed in the right-hand side of Eq. (24), but if present it would reside in \( \eta_{\alpha} \) and \( \hat{N} \).

Taking the modulus squared of the amplitude (24) gives the double-decay rate. In terms of \( t = t_{1} + t_{2} \) and \( \Delta t = t_{2} - t_{1} \), the double-decay rate \( R_{12}(p, t, \Delta t, \Delta t) \) is

\[ R_{12}(p, t, \Delta t, \Delta t) = |\hat{N}|^{2} e^{-\gamma / 2} \left| \eta_{1} \right|^{2} e^{\Delta \gamma / 2} \left[ \left| \eta_{2} \right|^{2} e^{\Delta \gamma / 2} \right. 
- \left. 2 \left| \eta_{1} \eta_{2} \right| \cos (\Delta m \Delta t + \Delta \phi) \right], \] (25)

where \( \gamma = \gamma_{S} + \gamma_{L} \) and \( \Delta \phi = \phi_{1} - \phi_{2} \).

A detailed study of the CPT signals from symmetric-collider experiments with correlated mesons requires simulation with expressions of the type (25) for various final states \( f_{1}, f_{2} \). Given sufficient experimental resolution, the dependence of certain decays on the two meson momenta \( p_{1}, p_{2} \) and on the time \( t \) could be exhibited experimentally by ap-
propriate data binning and analysis. However, some caution is required because different asymmetries can be sensitive to distinct components of \( \Delta M \).

Consider, for example, the case of double-semileptonic decays of correlated kaon pairs in a symmetric collider. Neglecting violations of the \( \Delta S = \Delta Q \) rule, for the state \( f_+ = l^+ \pi^+ \nu \) one finds \( \eta_+ \approx 1 - 2 \delta_K \), while for \( f_- = l^+ \pi^- \bar{\nu} \) one finds \( \eta_- \approx -1 - 2 \delta_K \). Inspection of Eq. (25) shows that the double-decay rate \( R_{+ \to -} \) can be regarded as proportional to an expression depending on the ratio

\[
\left| \frac{\eta_+}{\eta_-} \right| = 1 - 2 \Re \delta_K(+) - 2 \Re \delta_K(-) = 1 - \frac{4 \Re (i \sin \phi e^{i \phi})}{\Delta m} \gamma(p) \Delta a_0.
\]  

(26)

In the first line of the above expression, the \( CPT \)-violating contributions from each of the two kaons have been kept distinct because they differ in general for mesons traveling in different directions. All the angular and time dependence in Eq. (13) cancels from the second line because in the present case of a symmetric collider \( \beta_1 \cdot \Delta a = -\beta_2 \cdot \Delta a \).

The proportionality factor for the double-decay rate \( R_{+ \to -} \) is \( |\hat{N} \eta_+|^2 \), which depends on the full expression (13) for \( \delta_K \). However, this factor plays the role of a normalization. Unless it is carefully tracked, which could be a potentially challenging experimental task, no angular or time dependence would be manifest in the double-semileptonic decay mode. For instance, the normalization factor would play no role in a conventional analysis to extract the physics using an asymmetry, for which normalizations cancel. Moreover, as mentioned above, the line spectrum in the momentum means that the dependence on \( |p| \) is also unobservable. Indeed, \( \gamma(p) = 1 \). The double-semileptonic decay is therefore well suited to placing a clean constraint on the timelike parameter \( \Delta a_0 \) for \( CPT \) violation, and the experimental data can be collected for analysis without regard to their angular locations in the detector or their sidereal time stamps.

In contrast, certain mixed double-decay modes are sensitive to \( \delta_K \) only in one of the two decays. This is the case, for instance, for double-decay modes with one semileptonic prong and one double-pion prong. In these situations, the double-decay rate \( R_{12} \) in Eq. (25) can be directly sensitive to the angular and time dependence exhibited in Eq. (13), and in particular it can provide sensitivity to the parameters \( \Delta a \) for \( CPT \) violation. In a conventional analysis, \( CPT \) violation in a given double-decay mode of this type is inextricably linked with other parameters for \( CPT \) violation \[43,46,47\]. However, in the present case the possibility of binning for angular and time dependence means that clean tests of \( CPT \) violation are feasible even for these mixed modes.

Consider, for example, a detector with acceptance independent of the azimuthal angle \( \phi \). The distribution of mesons from the quarkonium decay is symmetric in \( \phi \), so the \( \delta_K \) dependence of a \( \phi \)-averaged data set is governed by the expression

\[
\delta_K(p, \theta, t) = \frac{1}{2 \pi} \int_0^{2\pi} d\phi \, \delta_K(p, t) = \frac{i \sin \phi e^{i \phi}}{\Delta m} \gamma[\Delta a_0 + \beta \Delta a_2 \cos \chi \cos \theta + \beta \Delta a_y \sin \chi \cos \theta \sin \Omega t + \beta \Delta a_\chi \sin \chi \cos \theta \sin \Omega t].
\]  

(27)

Inspection shows that by binning in \( \theta \) and in \( t \) an experiment with asymmetric double-decay modes can in principle extract separate bounds on each of the three components of the parameter \( \Delta a \) for \( CPT \) violation. This result holds independent of other \( CP \) parameters that may appear, since the latter have neither angular nor time dependence. Combining data from asymmetric double-decay modes and from double-semileptonic modes therefore permits in principle the extraction of independent bounds on each of the four components of \( \Delta a_\mu \).

The same reasoning applies to other experimental observables. For example, one can consider the standard rate asymmetry for \( K_L \) semileptonic decays \[36\],

\[
\delta = \frac{\Gamma(K_L \to l^+ \pi^- \nu) - \Gamma(K_L \to l^- \pi^+ \bar{\nu})}{\Gamma(K_L \to l^+ \pi^- \nu) + \Gamma(K_L \to l^- \pi^+ \bar{\nu})} = 2 \Re \epsilon_K - 2 \Re \delta_K(p, t),
\]  

(28)

where the symbol \( \Gamma \) denotes a partial decay rate and \( \Delta S = \Delta Q \) has been assumed. In principle, this asymmetry could also be studied for angular and time variation, leading to constraints on \( \Delta a_\mu \).

IV. SUMMARY

This paper has considered some issues involving momentum- and time-dependent experimental signals for indirect \( CPT \) violation in a neutral-meson system. Effects of this type are predicted in the context of a general standard-model extension allowing for \( CPT \) and Lorentz violation. Their consequences for data analysis vary substantially with the specifics of a given experiment.

Following a theoretical description of the momentum and time dependence applicable to any neutral-meson system, specific theoretical results are obtained for kaons. Some \( CPT \) observables depend explicitly on the magnitude and orientation of the meson momentum, which leads to diurnal variations in observables.

Illustrations of the consequences of these results for experiments are provided using two types of scenario already adopted at Fermilab and Frascati, one with collimated boosted uncorrelated kaons and the other with uncollimated correlated kaon pairs from \( \phi \) decay at rest. The implications described for these scenarios also provide intuition about possible effects in other experiments with kaons and in experiments with \( D \) or \( B \) mesons.

The primary message of this work is that a complete ex-
traction of CPT bounds from any experiment allowing for possible momentum dependence and diurnal variation of the observables is a worthwhile undertaking and one that has the potential to yield further surprises from the enigmatic neutral-meson systems.

Note added. The KTeV Collaboration has reported [48] a preliminary bound $A_{1+} \equiv 0.5^\circ$ on the amplitude (20) of variations of $\phi_{1+}$ with sidereal time, corresponding to a constraint $a_{1} \lesssim 10^{-20}$ GeV.

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[9] A number of recent theoretical efforts along these lines are discussed in CPT and Lorentz Symmetry, edited by V. A. Kostelecky (World Scientific, Singapore, 1999).
[33] It has been suggested that in the kaon system CPT-violating effects might be generated through an unconventional quantum mechanics in which the Schrödinger equation is replaced with a density-matrix formalism. See J. Ellis et al., Phys. Rev. D 53, 3846 (1996), where it is also shown that the resulting effects can be disentangled from the CPT-violating parameter $\delta_K$ of interest here.
[34] Direct CPT violation in the decay amplitudes is neglected here because it is suppressed in the standard-model extension and is expected to be unobservable [14].
[35] A field redefinition can be used to shift equally the values of all the parameters $a_\mu$ without physical consequences [15]. For a particular $P^\mu$ meson, one could therefore replace $\Delta a_\mu$ with a single $a_\mu$ factor. For clarity, this redefinition is avoided in the present work.


[44] A careful analysis involving the use of the magnitude and phase of $\eta_{+-}$ and $\eta_{00}$ can yield a $CPT$ test, but it is necessary to account for contributions to the effective Hamiltonian arising from $3\pi$ decays and from violations of the $\Delta S \neq \Delta Q$ rule in semileptonic decays. See L. Lavoura, Mod. Phys. Lett. A 7, 1367 (1992).


