

LUTE TUNING AND TEMPERAMENT IN THE SIXTEENTH AND SEVENTEENTH
CENTURIES

BY

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Introduction

In 1594, the composer Hercole Bottrigari published *Il Desiderio, overo de' concerti di varii strumenti musicali*, which translated means, “concerning the playing together of various musical instruments.” He wrote it as a dialogue between two fictional characters: Alemanno Benelli and Gratoso Desiderio. Benelli is speaking as the author, and as the story unfolds, Desiderio has sought out Benelli for answers about music and the performance of different instruments together in an ensemble.

In the course of their conversation, Benelli asks Desiderio to tune a lute to the pitches of a harpsichord. He begins by instructing him to tune his open strings, and then asks him to try the other pitches:¹

- Benelli: Now I play F fa ut on the [harpsichord]. Now touch your F fa ut on the first fret of the [E string] of your Lute. Are you tuned in unison with me?
- Desiderio: We are not.
- ...
- Benelli: Now I touch the raised G sol re ut on the [harpsichord]. Touch the fourth fret . . . Are you in unison and tuned with the [harpsichord]?
- Desiderio: No, we are not. And what causes that, M. Alemanno?

Bottrigari is illustrating a common problem at this time: fretted instruments were tuned according to a different standard than keyboards. He eventually goes on to explain to

1. Hercole Bottrigari, *Il Desiderio*, Musicological Studies and Documents, Volume 9, Translated by Carol MacClintock (American Institute of Musicology, 1962), 18.

Desiderio how lutes can play with harpsichords, but the solution to the problem is only one of many that were in use at the time.

Today, Desiderio would not have this kind of problem because all instruments are tuned according to the same standard of equal temperament; however, when performing music from Bottrigari's time, the equal temperament standard does not apply. Although it was known to musicians at the time, equal temperament was not favored during the sixteenth and early seventeenth centuries. Instead, meantone temperament was the preferred choice for instruments, but it was not standardized in the way that equal temperament is today. Many kinds of meantone temperaments existed during this period, and variants depended on the date, geographical location, and even the instrument. These continued into the seventeenth and eighteenth centuries, while newer systems known as well-temperaments were invented. The majority of these applied to keyboard instruments because of the fixed nature of their pitches, but the same aesthetic preference for unequal temperaments was applied to all instruments.

The problem Desiderio faced was that lutes could have their frets set for a variety of different temperaments. In ensembles, they generally had to follow the meantone standard, but its execution differed from other instruments, hence the disagreement between his lute and Benelli's harpsichord. Keyboard instruments, for example, had their own procedures for producing different temperaments, and lutes and similar fretted instruments could produce these as well, but the process and the limitations a particular temperament imposed could differ. The implications at this time were that the lute player would need to alter his or her technique, and in some cases, the instrument itself, in order to accommodate a particular temperament. This raises the question of how performers

dealt with the issue. More importantly, it should give today's performers pause when determining what temperament to use. Just as we cannot apply a universal modern temperament to older music, neither can we apply a historically-informed one either. The principal difference between today's equal temperament and the majority of those found at the turn of the seventeenth century is that the latter had semitones of non-uniform or varied size. This could either be regular, where the distance between the semitones within an octave followed a predictable pattern, or it could be irregular with the distance varying greatly from one to the next. This presents no problem when tuning a keyboard instrument where each string or pipe can be pitched independently of the others. Players of bowed instruments without frets may alter their pitches with the placement of their left-hand fingers, and wind players have fingerings and embouchure adjustments to match a pitch in any temperament.

Fretted instruments place their frets so that each intersects all of the strings on the instrument. The problem with attempting to use semitones of varying size is that when you set the size of a semitone for one string, you have set the same sized semitone for all the other notes on the adjacent strings. This creates unavoidable problems if the temperament you are trying to emulate requires a small semitone on one string but a larger one on the string next to it. The result is that as you progress down the fingerboard and set the size of each of your semitones, you must choose what sized semitone benefits the majority of the pitches on that fret. If the majority of the pitches on a fret are best suited to a small semitone, then it is the same small semitone for all the pitches.

Alternatively, if the majority of the pitches require a large semitone, then they are the same, larger semitone. The inherent problem is that there is usually one pitch on the fret

that requires a semitone of a different size than all the others.

Lute players have dealt with these problems of varied semitone size for centuries, and while there is no perfect solution, there are many combinations of possible solutions that produce good results. What follows is a detailed examination of these solutions. Before we get to these, however, we need a a brief explanation of the history of tuning and temperament. Chapter one provides both context and a working technical knowledge of temperament. The second chapter examines historical fretting systems published during the sixteenth and early seventeenth centuries. Here we will see how lute players at the time dealt with temperament and the shortcomings of their solutions. Chapter three moves beyond historical fretting systems and attempts to provide a unified fretting system for use in performance by combining historical techniques with modern practices. We will see how historical fretting systems must be modified in order to be successful and what other techniques may be used to achieve a workable system. The last chapter provides a conclusion to our predicament and summarizes the many different ways lute players may navigate temperaments in modern performance.

Chapter 1

Tuning and Temperament

The subject of tuning and temperament is one of the most documented and discussed issues in music. Although I cannot present any new information on this subject, it is essential that we have an understanding of it as it applies in the context of this paper.

What follows is a short summary of the history of western tuning methods and systems of temperament. Because there are so many different kinds of temperaments, I will only concentrate on the types that will be used for subsequent discussion and comparison.

These main types are: 1) modern equal temperament and other temperaments with equal semitones; 2) temperaments that utilize unequal semitones such as the regular meantone temperaments called quarter-comma, fifth-comma, and sixth-comma; 3) irregular temperaments referred to as “well temperaments,” which are also comprised of unequal semitones; and, 4) the tuning systems attributed to Pythagoras and Ptolemy.

The majority of my information comes from Murray Barbour’s 1951 book *Tuning and Temperament: A historical survey*, as well as more recent books such as Ross Duffin’s *How Equal Temperament Ruined Harmony (and why you should care)*. As the latter title

suggests, discussions of temperament usually revolve around the concept of equal temperament and whether or not its purpose is justified during certain periods of music history. Barbour's book, while considered one of the most thorough compendiums of information, generally portrays temperaments other than equal temperament as inferior. Authors such as Duffin and others in the historical performance field, feel that equal temperament has degraded the effect of music for which it was not originally intended. Another excellent resource on historical temperaments is Owen Jorgensen's *Tuning*, published in 1991. He discusses virtually every temperament developed for a keyboard instrument during the course of Western music history. Although it is mainly intended for tuners and keyboard players who wish to tune their own instruments, it is nonetheless an invaluable resource to any musician wishing to understand how temperaments work. It is not my purpose to extol the virtues of one temperament over another. The matter is quite subjective, a fact supported by the vast number of temperaments available. While some of these temperaments conformed to the contemporary norms of meantone, others resembled present-day equal temperament. Many historical fretting sources match it quite closely using systems of equal division and other complex methods of dividing a string. While its use on other instruments was discouraged because of the consequences to harmony, it was often the preferred temperament for fretted instruments. Before we address the reasons why, we must first address the history of tuning and temperament, and the main problem it poses to us.

1.1 The Greeks' Debate

The history of tuning begins with the ancient Greek philosophers who proposed the first solutions to tuning notes within the span of an octave. Three of the most influential figures in this area were Pythagoras, Aristoxenus and Claudius Ptolemy. Pythagoras is generally credited with discovering the concept of tuning, and although none of his original writings survive, he established the first mathematical principles that apply to tuning. Later, it was Aristoxenus and Ptolemy who began the debate about tuning systems which continues today. One of the fundamental teachings of the Pythagoreans was that the universe could be explained according to simple numbers and ratios. An example of this numerical simplicity is found in the tetractys, a common symbol associated with the Pythagoreans because it represented the basic sequence of numbers: 1, 2, 3, and 4. Pythagoreans believed that everything in the world could be reduced to these simple numbers. The number four, for example, could be used to explain the four seasons of the

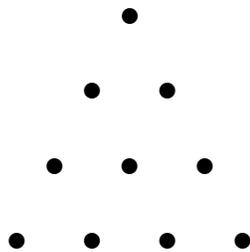


Figure 1.1: The Pythagorean tetractys

year or the four elements of earth, air, fire and water, while the sum of the numbers (1 + 2 + 3 + 4) gave you 10, which was the basis for all arithmetic.¹ For the Pythagoreans, numbers related to everything, and music was no exception.

1. Catherine Nolan, "Music theory and mathematics," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 273.

For the Pythagoreans, the most important intervals were the unison, octave, fifth, and fourth. They are the first pitches in the harmonic series, but the Pythagoreans found they could express them using numeric ratios in the tetractys: 1:1 for the unison, 2:1 for the octave, 3:2 for the fifth, and 4:3 for the fourth. They demonstrated these ratios using the monochord, which was a single string divided into different parts. In order to produce the intervals in a Pythagorean system, the monochord was divided into a certain number of parts and then stopped with either a finger or small bridge. For example, a string divided into two parts and stopped at the first, yielded a distance with the ratio of 2:1. This distance also produced the musical interval of an octave. For the perfect fifth, it was divided into three parts and stopped at the second, creating the ratio 3:2. While other intervals could be produced, they ceased measuring with the perfect fourth, with its ratio of 4:3, because creating other intervals used numbers greater than four. Not only did these numbers fit perfectly into the Pythagorean notion of the tetractys, they also described how music could reflect the physical world. Ratios of pitch translated into ratios of weight and distance. Because their world was so dependent on these numbers, any other intervals in a scale ultimately had to be derived from these original four, regardless of what the actual pitches were.²

In Pythagorean tuning, other intervals were calculated by subtracting or adding the original four intervals in different combinations. In terms of arithmetic, the sum of two ratios meant a product of the two, while subtraction of two ratios meant using division. So

2. Nolan, "Music theory and mathematics," 274.

a Pythagorean would calculate a wholetone by subtracting the fourth from the fifth.

$$3 : 2 \div 4 : 3 = 9 : 8 \quad (1.1)$$

This produced a wholetone with a ratio of 9:8. A semitone was then calculated by subtracting two of these wholetones from the original fourth, resulting in a ratio of 256:243.

$$(4 : 3 \div 9 : 8) \div 9 : 8 = 256 : 243 \quad (1.2)$$

These ratios used numbers that were not part of the tetractys, but that did not matter because they were each created from intervals that were: the fourth, which was a member of the original four intervals; and the wholetone, created from the fourth and fifth.

While the numerical simplicity of using only unisons, octaves, fifths, and fourths fit perfectly with the Pythagorean ideal, using them as the basis for a tuning system produced unacceptable results when applied in a musical context. While the Pythagoreans had found a way to tune their scale, they also uncovered the central problem that has plagued us ever since. Using their system or any other temperament created since then, it is not possible to create a twelve-tone scale that is internally consistent. We can tune a chromatic scale using only Pythagorean fifths by starting with the interval C to G and continuing through the circle of fifths for all twelve pitches, returning to the original note C. After proceeding through twelve fifths from our starting C, the final C is seven octaves above it. If Pythagorean tuning was internally consistent, the C seven octaves above would be the exact same pitch as the C resulting from twelve fifths. We can test this



Example 1.1: The circle of twelve fifths



Example 1.2: The note C spanning seven octaves

mathematically, by adding together twelve ratios of 3:2 and comparing them with the sum of seven octaves using the ratio 2:1.³

$$\frac{3}{2} \times \frac{3}{2} = \frac{531441}{4096} = 129.7463 \quad (1.3)$$

$$\frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} = \frac{128}{1} = 128 \quad (1.4)$$

The mathematical result is that the sum of the twelve fifths is greater than the sum of the seven octaves. If we were to listen to these two pitches, they would not match. The pitch resulting from twelve fifths above our starting C is noticeably sharper than the pitch resulting from seven octaves above the same starting note.

The additional problem with Pythagorean tuning was that it did not accurately reflect the pitches in the harmonic series. The first three pitches in the harmonic series matched Pythagorean ratios perfectly. The pure intervals of the octave, fifth, and fourth in the harmonic series were exactly in tune when compared to the ratios of their Pythagorean

3. Ross W. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)* (New York, NY: W.W. Norton & Company, 2007), 25.

counterparts. However, the fourth pitch in the harmonic series, the major third, when tuned acoustically pure as it naturally occurred in the series, actually vibrated at a ratio of 5:4 and not 81:64, as the Pythagoreans would have calculated by adding together two of their 9:8 wholetones: $9 : 8 \times 9 : 8 = 81 : 64$.

These two tuning discrepancies that result from using only Pythagorean intervals to build a scale of notes are called *commas*. The *ditonic comma* results when building a chromatic scale upon successive fifths, such as the difference between twelve fifths and seven octaves, and the *syntonic comma* results from the difference between the pure harmonic 5:4 major third and the Pythagorean 81:64 major third. The Pythagoreans themselves were well aware of these problems and tried to overcome them, but it was only Aristoxenus and Ptolemy who could offer a solution. Aristoxenus proposal was to ignore the Pythagorean ideals of mathematical simplicity and use tuning systems that divided the string according to parts and not ratios. He described tuning by using equal parts of the string and was the first to develop the concept of tuning using equal semitones which would later be the foundation of equal temperament. Ptolemy's proposal was to modify the Pythagorean notion of ratio by using only ratios that were superparticular in nature, meaning that the first number of the ratio should always be one unit great than the other. So Pythagorean ratios like 2:1, 3:2, 3:4 and even 9:8 were acceptable, but so too were other ratios such as 5:4, the pure major third, and 6:5, the pure minor third. Ptolemy's solution rectified a lot of the Pythagoreans' mathematical ratios with nature's own internal tuning system but it still had problems when it came to musical execution. In a twelve-tone scale, there are three major thirds. Starting on the note C, we can fit three of them within an octave: C to E, E to G \sharp , and A \flat to C. However, adding these intervals



Example 1.3: Three major thirds within an octave

together using Ptolmey's ratios does not get us to a complete octave either. The sum of three thirds should sound the same as an octave with the ratio 2:1, but the mathematical result is slightly less than 2, which sounds flat.

$$\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times = \frac{125}{64} = 1.953125 \quad (1.5)$$

While the note E could remain fixed in relationship to the major third starting from C and the major third ending on G \sharp , the fact that G \sharp and A \flat are used for the other two major thirds implies that they are different notes. Most modern musicians regard the enharmonic respelling of a note as a kind of music homonym: an alternative word to express the same thing. While the different names of G \sharp and A \flat can indicate different functions, such as the third degree of the E major scale or the first degree of A \flat major, they are regarded today as notes having the same vibrating pitch. However, as Ptolemy's tuning problem shows us, the two notes are not only functionally different, they are musically different, and therefore tuned differently.

Ptolemy's solution of tuning with only pure intervals is known as just intonation. While it succeeded in correcting the problems associated with Pythagorean tuning, it made it impossible to tune certain notes in the chromatic scale because they required a different tuning according to the musical context in which they occurred. Instruments of fixed

pitches, such as any keyboard instrument or fretted instrument, whose semitones were fixed and immobile, could not accommodate a tuning of only pure intervals, or at least, not in a scale of twelve tones. Instruments of variable pitch, such as fretless bowed instruments, wind instruments, or the human voice were exempt from this problem because they were able to adjust any note appropriately. Violinists, for example can place their fingers at any position along the fingerboard, and wind players may vary the pitch of their notes using their embouchure.

What about Aristoxenus? While he did invent an alternative solution of equal semitones, he rejected both the notions of Pythagorean numeric purity and the Ptolemaic notion of harmonic purity. His method of tuning divided an octave into equal parts, creating intervals as groups of these individual parts instead of using ratios. His idea of equal parts formed the basis for equal temperament. While this allowed for a tuning of fixed pitches in a chromatic scale, none of these intervals was tuned in a way that precisely matched the tuning in the harmonic series. This was an unacceptable solution for most theorists and musicians during the evolution of western classical music, and it took many years for equal temperament to gain acceptance.

Because the notion of equal semitones did not sit well with early western music theorists and tuning harmonically pure intervals was impossible in a fixed pitch system, they gravitated towards the Pythagorean system of tuning. For one reason, it was simple, using a small number of simple ratios, and for another, it matched the musical tastes of the medieval period such as monophonic chants and polyphonic forms based on fifths, unisons, and octaves. Pythagorean ratios persist even to this very day, but as music changed in the early Renaissance period, composers sought to incorporate more

consonant thirds into their music, which the strict Pythagorean system of ratios did not permit. Thus, the old Greek debate between Pythagoras, Ptolemy, and Aristoxenus resurfaced.

1.2 Temperament

A temperament is a method of tuning a scale that alters an existing system, usually Pythagorean tuning. For most of the Middle Ages, tuning was described according to the ratios that Pythagoras had determined hundreds of years earlier. However, as musical tastes changed during the Renaissance, there was an increasing preference for the sounds of thirds instead of unisons and fifths. This represented the problem of rectifying thirds within the Pythagorean tuning scheme that had plagued the Greeks. The first attempts at a solution appeared at the end of the fifteenth century, and each one used the same approach by changing the size of the fifths within the existing Pythagorean system.

One of the first writers to publish a system that broke with the Pythagorean tradition of tuning was Bartolomeus Ramis de Pareja. In his *Musica Practica* of 1482, he created a chromatic scale using two different groups of fifths. Each group was tuned in pure fifths, just as in Pythagorean tuning, except that one group was slightly sharper than the other.⁴

Although he did not call it a temperament, it technically was not Pythagorean tuning either. Barbour credits Franchinus Gafurius's *Practica musica* of 1496 with the first mention of the idea of temperament. In it, Gafurius says that organists tune their fifths

4. J. Murray Barbour, *Tuning and Temperament: A Historical Survey* (Mineola, New York: Dover Publications, Inc., 2004), 88.

slightly flat, but does not go into any specific details.⁵ Similarly, Arnolt Schlick's 1511 treatise *Spiegel der Orgelmacher und Organisten* only gives us a general idea when he refers to tuning the instrument's fifths flat "as much as the ear will permit."⁶ So while the idea was present as early as 1482, it took several years for it to develop into a system. Tempering is a process of compromise whereby the acoustical purity of one interval is changed slightly in order to accommodate the acoustical impurity of another interval. Specifically, it is the process of changing the size of the fifth in order to accommodate the major third which is not pure in standard Pythagorean tuning. This usually means shrinking the size of the fifth slightly so that it is narrower, or flatter than the pure ratio of 3:2; although, in some cases a fifth may be tuned wider, or sharper than pure. The problem with Pythagorean thirds was that they were much sharper than pure, differing by the syntonic comma, which was the difference between the Pythagorean third at 81:64 and the pure harmonic third at 5:4. Shrinking the size of the fifth resulted in thirds that were flatter than the Pythagorean ratio of 81:54, closer to the harmonically pure ratio of 5:4, and thus more appropriate sounding for music that favored a greater number of thirds. Depending on how the fifth was flattened, you could come very close to a pure 5:4 third, and the listener would not notice the difference. This was at the expense of the fifth, which was now flatter than pure, but it was an acceptable compromise because a flat fifth was less disconcerting than the sharp third of Pythagorean tuning. The strategy was to shrink several fifths by the same or differing amounts. The result was that instead of one set of intervals being completely pure and another unusable, all of the fifths were slightly

5. Barbour, *Tuning and Temperament: A Historical Survey*, 25.

6. Rudolf Rasch, "Tuning and temperament," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 202.

out-of-tune to differing degrees, while some of the thirds came very close to pure. Which thirds approached pure and which fifths were made flat all depended on the particular temperament, and there were many!

Murray Barbour classifies temperaments into four basic types: regular meantone, irregular temperaments, equal division, and equal temperament. Whereas a tuning can be defined as a method of obtaining intervals according to Pythagoras—as in Pythagorean tuning—or according to Ptolemy—as in just intonation—temperaments alter the ratios of the intervals so that they often lie somewhere between the two systems. In regular meantone temperaments, most or all of the fifths in the Pythagorean tuning system are tuned flat by the same amount. Depending on which fifths are flattened, this can result in thirds that are very close to pure, with fifths and fourths that are not. The amount that each fifth was changed from its pure ratio often depended on the musical context. A meantone temperament with very flat fifths resulted in a temperament with thirds quite close to pure, but because of the mutated nature of the fifths, it was usable only in a limited number of keys. Conversely, a meantone temperament that tuned its fifths less flat made more keys playable, but resulted in sharper, less pure thirds.

1.2.1 Regular Meantone and Irregular Temperaments

The first attempts at a systematic application of tempering fifths resulted in the class of temperaments known as “meantone.” No one is entirely sure who invented the first complete meantone temperament, but the general consensus is that it is a shared prize between Pietro Aron and Gioseffo Zarlino. Zarlino is credited with the invention of an

exact system that narrows each fifth by $\frac{2}{7}$ of a syntonic comma. His method, however, was far from practical. Although he provided a monochord diagram, his process of tempering the fifths by that much actually created thirds that were smaller than pure, or too flat. Pietro Aron is credited with the first practical meantone temperament. While not as theoretically exact as Zarlino's, Aron's instructions can be used to create a workable meantone temperament that narrows fifths enough to create pure thirds, while leaving the remainder of the intervals usable. Unlike Zarlino, his instructions are not mathematical in nature, requiring the person to tune by ear and not rely on measurements. His method was to start with pure thirds and build tempered fifths around them. He begins with a pure third between C and E and then proceeds to tune four fifths that are all equally flat with additional pure thirds between A and C \sharp , and D and F \sharp . While Aron says nothing about the division of the comma, Barbour takes these instructions and mathematically proves that if the tuner is able to create Aron's pure thirds by ear and match the size of the other fifths so that they are all tempered by the same amount, each of those fifths will be flatter than pure by $\frac{1}{4}$ of a syntonic comma.⁷ Because this is the amount that each fifth is reduced in size, Aron's temperament is referred to as "quarter-comma meantone temperament." While Aron never referred to it this way, nor did he use mathematics to calculate lengths of a string, later theorists and musicians adopted his ideas and made exact calculations that reproduced Aron's temperament.

Regular meantone temperaments contained eleven fifths that were all narrowed by the same amount, measured as a fraction of the syntonic comma. Narrowing four consecutive fifths in the circle of fifths by $\frac{1}{4}$ of a syntonic comma was enough to make eight thirds in a

7. Barbour, *Tuning and Temperament: A Historical Survey*, 27.

12-tone scale pure while rendering the remaining four unusable.⁸ This was an effective solution for sixteenth-century music because it resulted in more serviceable keys than Pythagorean tuning, and composers simply avoided the keys containing the unusable thirds. This meant that meantone did not solve all of the problems associated with Pythagorean tuning. Just as Pythagorean tuning has a fifth that was too wide to be used, meantone temperaments also had an unusable fifth. The so-called “wolf” fifth existed in every temperament, but its size and location depending on the kind of temperament. As the sixteenth century continued, musicians attempted to correct the problems with quarter-comma meantone, such as its wolf fifth and unusable thirds. These tuning systems narrowed the fifth in smaller amounts, such as $\frac{1}{5}$ or $\frac{1}{6}$ of a syntonic comma. The result was that the thirds were now sharper than pure but much less so than in Pythagorean tuning. While this created more usable thirds in the scale, the wolf fifth remained. Sixth-comma meantone was popular with fretted instruments, as we will see in a later chapter, but even sixth-comma still contained the wolf fifth and unequal thirds.

The next major change in temperaments occurred in the seventeenth century when theorists began using systems that narrowed fifths by varying amounts instead of equal amounts, as was the case with regular meantone temperaments. So called irregular temperaments, or well temperaments, became very popular and widespread in the seventeenth and eighteenth centuries. The advantage to using well-temperaments was that musicians had finer control over which intervals were problematic. For example, in quarter-comma meantone, the fifth between $E\flat$ and $A\flat$ is always too wide to be used.

8. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)*, 33.

Technically, this is because the A^b is really a G^\sharp .⁹ Similarly, the thirds B to D^\sharp , D^b to F and F^\sharp to A^\sharp would also be unacceptable. With irregular temperaments, fifths were narrowed by differing amounts so that the wolf fifth was eliminated, and normally problematic thirds in standard quarter-comma meantone could be made more tolerable by adjusting the size of other intervals instead. These irregular temperaments were similar to regular meantone temperament, except that one fifth might be narrowed one-quarter of a comma, while another might be narrowed only by a sixth or less. As a result, the composer had more control over which keys had better-sounding thirds and which did not.

The introduction of irregular temperaments caused an explosion in the number and variety of temperaments available to musicians in the seventeenth and eighteenth centuries. Theorists such as Valloti, Young, Werckmeister, and others all developed their own systems of temperament favoring different intervals in ways that regular meantone temperaments could not. Composers also developed their own temperaments as well, such as J. S. Bach who used his own temperaments in the performance of his keyboard works. His compositions in *The Well-Tempered Clavier* were so named because of the temperament he used for them that allowed use of all keys on the instrument.

1.2.2 Equal Division

All of the methods described thus far are practical ways to create temperaments. For example, Aron's method uses the ear alone and produces a good quarter-comma meantone temperament without the need for complex mathematical calculations. Later

9. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)*, 35.

methods of tuning other meantone temperaments and well-temperaments used the same techniques of narrowing or widening intervals by certain amounts, either by ear or by some process of calculation.

The problem with some of these methods is that they are imprecise. The fifth is narrowed by an indeterminate amount to match a pure third, or intervals are determined by counting the number of beats that occur as pitches clash against one another. Most importantly, the resulting temperaments contain semitones that vary in size according to the placement of the tempered intervals. If we are to compare different temperaments and their specific pitches, we need a more exact way of calculating a given pitch in a given temperament. Recalling Aristoxenus in our survey of Greek tuning methods, his approach consisted of dividing a string or octave into equal parts and creating intervals by grouping tones into sets of these equal parts. Using this same method, we can create several different regular meantone temperaments by dividing our octave into multiple parts and combining wholetones and semitones into a groups. We can then accurately compare and quantify the size of each semitone across different kinds of temperaments. Barbour refers to this process of calculating temperaments as equal division. Equal division was used in the sixteenth century to calculate string lengths for both quarter and sixth-comma meantone, and its principles remained in use into the seventeenth and eighteenth centuries.

The first theorist to use equal division was Vicentino in 1555, who divided his octave into 31 parts using a harpsichord that contained six ranks of keys. He was attempting to create a temperament like quarter-comma meantone, but instead used the principles of equal division instead of Aron's tempered fifths. In order to create an octave of 12 tones out of a total of 31 parts, he combined five parts in each wholetone and made semitones of two

different sizes, a smaller semitone with two parts and a larger semitone with three. The size of the semitone depended on where it occurred within the scale. Musicians during this time were familiar with these differences in semitone size and referred to the larger semitone as the *major semitone* and the smaller semitone as the *minor semitone*. The difference in size between the two semitones was referred to as a comma. We should not confuse this with the diatonic or syntonic comma, which are differences of pitch as well, but have no correlation in systems of equal division.

Vicentino's 31-part system solved the same kinds of problems that meantone temperament did and created pure thirds at the expense of pure fifths. However, it had the added advantage of clearly indicating that semitones differed in size by one comma.

Other approaches to meantone temperament were less specific and would tune certain semitones to a midway point between the wholetone it was dividing. This was usually reserved for the wolf fifth or the chromatic notes which were less likely to be used. As musicians developed systems of equal division, they utilized keyboards that could realize the distinction between semitones more accurately and used split keys that enabled one to play either diatonic or chromatic semitones. The use of enharmonic instruments that had more than twelve notes per octave dates back to the late fifteenth century, and Patrizio Barbieri's recent book documents them extensively.¹⁰ Vicentino's multi-manual keyboard was one example of such an instrument, but was too impractical to use. More practical examples of keyboards with split keys were found later in the seventeenth century, especially in Italy. These instruments had different keys for some or all of the accidentals;

10. Patrizio Barbieri, *Enharmonic: instruments and music 1470-1900*, Tastata. Studi e documenti, vol. 2, Revised and translated studies (Latina: American Institute of Musicology, 2008).

composers and keyboardists such as Handel and Werckmeister continued to use them into the eighteenth century.¹¹

Examining the 31-part system in more detail, let us take the C major scale as an example containing five wholetones: C, D, F, G, A; and two semitones: E and B. In our system, we reach 31 parts by assigning 5 parts to each wholetone and 3 parts to each semitone in the scale. Figure 1.2 shows the arrangement of each pitch in the scale according to its number of parts. Five wholetones at 5 parts apiece makes 25 parts, and adding the remaining

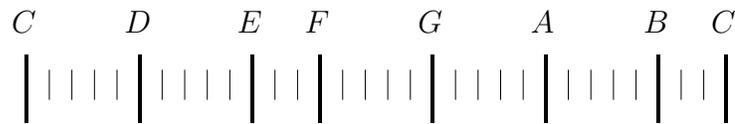


Figure 1.2: The C major scale in a 31-part system

semitones at 3 parts each gives us a total of 31: $25 + 3 + 3 = 31$. Note that semitones E to F and B to C have 3 parts as opposed to 2. As we divide our scale further into chromatic pitches, each wholetone is divided into 2 and 3 parts. Semitones that are diatonic within a scale, such as E and B in a C major scale, always contain 3 parts as opposed to 2. The smaller semitone is reserved for chromatic notes that lie outside the scale. For this reason, the larger semitone is sometimes referred to as the *diatonic semitone* while the smaller of the two is called the *chromatic semitone*. Figure 1.3 shows the rest of our C major scale divided into its chromatic and diatonic semitones. Chromatic semitones represent the shortest distance between two pitches, such as the distance from C to C \sharp , but this is also the same distance as G descending to G \flat . Likewise, the distance ascending from C to D \flat

11. Barbour, *Tuning and Temperament: A Historical Survey*, 108.

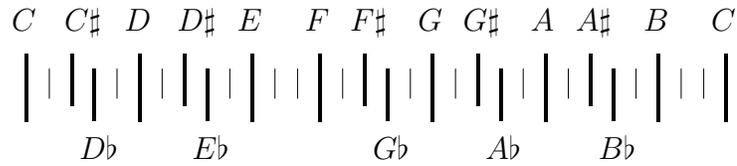


Figure 1.3: The chromatic scale in a 31-part system

is the larger diatonic semitone, as well as the descending distance from G to F \sharp . The term diatonic semitone might be misleading since it implies that D \flat or F \sharp are diatonic to a C major scale. For this reason, the minor and major designations are preferable.

Systems of equal division were not limited to just 31 parts. Zarlino and Salinas describe a 19-division octave in the sixteenth century, and later systems included octaves divided into 43 and 55 parts. Each of these systems grouped their wholetones into an odd number of parts and therefore had their semitones divided unequally with a comma's difference between them. Praetorius, in his *Syntagma Musicum*, describes a 55-part system with nine parts per wholetone and semitones of four or five parts. He describes this system in reference to the "intermediate" nature of the semitones on fretted instruments:

Thus the semitones cannot be either major nor minor, but are, perforce, "intermediate" if anything. For I reckon that each fret [...] contains four-and-a-half commas, whereas the major semitone contains five and the minor semitone only four. Since the error is only half a comma either way, the ear hardly notices it with these instruments [...] Major and minor semitones are both produced by the same fret, both sound in tune, [...] especially since by particular applications of the finger to the string, over the fret, it is possible to have some control over the pitch of the note produced.¹²

As late as 1723, singer and teacher Pier Francesco Tosi expressed the same idea, though not in reference to fretted instruments, in his treatise *Opinioni de Cantori*:

12. Michael Praetorius, *Syntagma Musicum II: De Organographia Parts I and II*, Reprinted 2005, Early Music Series 7, Translated and edited by David Z. Crookes (Oxford: Clarendon Press, 1986), 68.

A Tone, that gradually passes to another, is divided into nine almost imperceptible Intervals, which are called Commas, five of which constitute the Semitone Major, and four the Minor.¹³

Praetorius and Tosi are both describing a system of equal division. Tosi was writing in the eighteenth century, but equal division as a tuning method had already been in use since the sixteenth century when it was used to mimic the same features of quarter-comma meantone.

The basic features of each of these systems is outlined in table 1.1, where the corresponding meantone temperament is given. Of the four that are listed, quarter-comma and sixth-comma are the most significant because they were the most frequently used of the meantone systems. Quarter-comma was used almost exclusively by keyboard and

Number of Divisions	Wholetone	Major semitone	Minor semitone	Meantone equivalent
19	3	2	1	third-comma meantone
31	5	3	2	quarter-comma meantone
43	7	4	3	fifth-comma meantone
55	9	5	4	sixth-comma meantone
67	11	6	5	seventh-comma meantone
79	13	7	6	eighth-comma meantone

Table 1.1: Comparison of systems of equal division

wind instruments, and was the same kind of temperament that Vicentino was advocating with his system. Sixth-comma was the same temperament that Praetorius, and later Tosi, were describing although neither refer to it by that name. I will return to Praetorius's observation about sixth-comma temperament when we examine other specific temperaments for fretted instruments.

Although they each have different parts, each of the above systems function the same way as Vicentino's original 31-division octave. In a 19-division octave, wholetones consist of

13. Pier. Francesco Tosi, *Observations on the Florid Song*, Translated by Mr. Galliard (London: J. Wilcox, 1743), 20.

three parts, and major and minor semitones are two and one parts respectively. 43-part systems contain wholetones with seven parts each, dividing their semitones between four and three parts, and then a 55-part system has nine parts for its wholetones, divided into five and four. Other systems were possible that created seventh and eighth comma meantone; however, these were not used to any great extent since their tunings began to approximate equal temperament, and from a practical standpoint, did not serve as good a purpose.

1.2.3 Equal Temperament

Technically speaking, equal temperament is a type of division in which the octave is divided into twelve equal parts. In terms of its qualities as a temperament, it compensates for the ditonic comma the same way a meantone temperament does by narrowing the fifth. The difference between equal temperament and meantone temperament is that meantone will narrow its fifths much more than that. In equal temperament, all twelve fifths of the scale are narrowed by a slight but equal amount. The ditonic comma is therefore dispersed throughout the entire scale so that every pitch is usable, but this ability has its price. The temperament creates thirds that are as sharp as they possibly can be without having too strong an effect on the listener. While they are not as sharp as the thirds of Pythagorean tuning, they lack the pure quality of a quarter-comma meantone temperament and are too sharp to approximate it in the way that other meantone and irregular temperaments do. However, the homogeneous nature of every third being sharp by the same amount tends to veil this lack of harmonic purity from our ears, so that it is

not as noticeable.

Even with its overly sharp thirds, theorists in the sixteenth century regarded equal temperament as a technical impossibility. Because everyone based their tunings on the Pythagorean 9:8 wholetone, it was mathematically impossible to divide it into two geometrically equal, whole-number ratios. They could only divide the wholetone unequally into a larger semitone of 17:16 and a smaller one of 18:17.¹⁴ If we recall that adding two geometrical ratios together requires a process of multiplication, when 18:17 and 17:16 are “geometrically” added together, it produces the 9:8 ratio.

$$\frac{18}{17} \times \frac{17}{16} = \frac{306}{272} = \frac{\frac{306}{34}}{\frac{272}{34}} = \frac{9}{8} \quad (1.6)$$

The difference between the two ratios is similar to the same differences in semitone size found in meantone temperaments and systems of equal division. The smaller 18:17 ratio is the minor semitone and the larger 17:16 ratio is major semitone.

Although equal temperament was impossible according to geometry, musicians knew how to approximate it very closely without having to worry about the mathematics. Despite this ability, almost everyone preferred meantone temperaments because of their pure thirds. Even later in the seventeenth and eighteenth centuries, irregular temperaments were still favored over equal temperament because they made more keys feasible and were able to produce some thirds that were much closer to pure than their equally-tempered counterparts.

A tuning method very similar to equal temperament appeared in Giovanni Maria

14. Mark Lindley, *Lutes, viols and temperaments* (Cambridge: Cambridge University Press, 1984), 20.

Lanfranco's *Monocordi & Organi* of 1533. His system does not attempt to divide the octave into twelve parts, but instead mimics the quality of the fifth found in systems that do. He describes a system where the fifths are tuned slightly flat and the thirds are made as sharp as the ear can possibly endure.¹⁵ Other writers after him described the same kind of system, sometimes giving him credit and sometimes not. Although Lanfranco's temperament made it possible to use more pitches of the chromatic scale, just as equal temperament does, it still created overly sharp thirds making it inferior to meantone temperament at the time.

One of the first systems to create equal temperament using measurable means was known as the *18:17 rule*. The theorist and lute player Vincenzo Galilei is credited with developing it for practical use.¹⁶ Although technically not true equal temperament, Galilei created twelve semitones of the same size in an octave using a series of Pythagorean minor semitones, or the smaller ratio of the 9:8 wholetone, the 18:17 semitone. His procedure was relatively straightforward: A string is divided into 18 equal parts, and the first part or $\frac{1}{18}$ of the string, is marked as the first semitone. The remaining length or $\frac{17}{18}$ of the string is then redivided into another 18 different parts. The first part of this division is then marked as the second semitone. The process is then repeated with the remaining string length to mark the third semitone, and so on, each time redividing the remaining amount of string into 18 parts and marking the first part as the next semitone.

Since the Pythagorean minor semitone is slightly smaller than an equally tempered semitone, Galilei's system was not the same as modern equal temperament, but was a way

15. Barbour, *Tuning and Temperament: A Historical Survey*, 45.

16. *Ibid.*, 57.

to divide the octave into twelve working semitones. Since each successive semitone is a bit flat, the fifth is slightly flat as well, very close to an equally-tempered fifth, with the octave at the twelfth fret flat as well. This issue seemed to go unnoticed on fretted instruments because of the way in which pitches tend to be sharper at higher frets. Since Galilei was a lutenist, he was using temperaments that he found the best for his instruments. While this worked well for the lute, it did not apply to non-fretted instruments. The same feature of sharp thirds still remained in Galilei's method just as it did with all other temperaments at this time that approximated equal temperament. Despite the predominance of meantone temperament in the sixteenth century, many shared Galilei's opinions and accepted the fact that lutes and other fretted instruments should be tuned with equal semitones. These included such notable musicians as Vicentino, Zarlino, Salinas, Artusi, Praetorius, and Mersenne.¹⁷ This did not mean that lutes were tuned in the same kind of equal temperament that we find today. As we will see later, there were variations in which lutes and other fretted instruments approximated equal frets as well as making their frets unequal to match common meantone temperaments.

Equal temperament was not fully accepted in musical practice until at least the eighteenth century, and even as late as the twentieth. Ross Duffin provides ample evidence that musicians were tuning their fifths flatter than those in today's equal temperament systems well into the nineteenth century, and that true equal temperament really did not become an indisputable fact until 1917 when it was standardized.¹⁸ After that, the advent of modern

17. Lindley, *Lutes, viols and temperaments*, 19.

18. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)*, 138.

electronic tuning devices that enabled tuners and musicians to calculate a correctly tempered fifth precisely, solidified equal temperament's place in the modern musical world.

1.3 Describing Temperaments

Throughout the course of this study, we will need to refer to the qualities of the different temperaments explained thus far as well as compare them to other lute-specific temperaments that we will encounter in later chapters. Audio examples can illustrate the differences between temperaments, but sometimes they are so slight that even the most trained ear might have difficulty in distinguishing them, or be unable to describe the difference accurately. For example, it might be difficult for any of us to distinguish between a fifth that had been narrowed by one-fifth of a comma and one that was narrowed by a sixth. Furthermore, comparing several different temperaments with audio examples in the context of a written paper can impede the reader, and papers, of course, do not have speakers built into them.

Using numerical means to express temperaments is one of the ways to show the exact difference between intervals in a written environment, and we can do this in several different ways. The first, which I have already used, describes intervals as whole-number ratios, as the Greeks did thousands of years ago. This was also the way that most everyone else did so in the sixteenth and seventeenth centuries. For that reason, I will use these whole-number ratios whenever possible to refer to numerical differences between intervals. However, not all intervals are easily defined using these kinds of ratios.

Meantone temperament uses intervals that can be expressed as whole number ratios, but these are not easily derived. We can calculate the ratios of meantone intervals in two ways. The first involves determining the parts of a comma and subtracting them from the fifth, as Aron originally suggested. The only difference is that Aron was using his ear to achieve this, but here we will propose a second, more mathematic method to achieve the same result. Furthermore, we shall also need to precisely articulate the differences of semitone and interval size in a variety of meantone temperaments.

Because each system of meantone temperament has a matching system of equal division, we can express its intervals mathematically with the same system of parts or commas that were used historically. For example, in Vicentino's 31-part octave, the fifth would comprise three wholetones and a major semitone. Referring to figure 1.2, that makes a total of 18 parts out of the total 31. We can calculate this as: $2^{\frac{18}{31}}$. Applying the same ratios of parts as exponential fractions, we can calculate any interval in any meantone system, such as the quarter-comma major semitone: $2^{\frac{3}{31}}$; or the sixth-comma major semitone: $2^{\frac{5}{55}}$. Incidentally, we can also express intervals in equal temperament the same way. The semitone in equal temperament is: $2^{\frac{1}{12}}$; the wholetone $2^{\frac{2}{12}}$; the fifth: $2^{\frac{5}{12}}$ and so on. The obvious problem with these numeric representations is that are more difficult to comprehend than a simple number, and are not readily comparable to standard Pythagorean ratios.

In order to compare all these different kinds of semitones effectively, we need to express either their ratio or their logarithmic formula as a simple decimal number. In this case, the number could be applied to a string length to determine what length of string would produce a sixth-comma meantone major semitone, a quarter-comma meantone fifth, or a

pure third, and it can also determine the vibrating frequency of those particular pitches as well. Referring to table 1.2, if we compare the Pythagorean ratio 3:2 with its quarter-comma meantone equivalent, expressed using logarithmic functions, we can immediately see that the Pythagorean fifth is slightly larger. We also see that the quarter-comma third matches the ratio of the pure third more closely, as opposed to the fifth or sixth-comma third. We know from Aron’s description of tuning meantone

Pythagorean fifth:	$3 : 2 = 1.500$
Quarter-comma meantone fifth:	$2^{\frac{18}{31}} = 1.4955$
<hr style="width: 50%; margin: 0 auto;"/>	
Pure third:	$5 : 4 = 1.2500$
Third-comma meantone third:	$2^{\frac{6}{19}} = 1.2447$
Quarter-comma meantone third:	$2^{\frac{10}{31}} = 1.2506$
Fifth-comma meantone third:	$2^{\frac{14}{43}} = 1.2532$
Sixth-comma meantone third:	$2^{\frac{18}{55}} = 1.2546$

Table 1.2: Comparison of pure and meantone fifths and thirds using decimal equivalents

temperament that the fifth was tuned slightly flatter than pure, and the chart also bears this out, with the quarter-comma fifth smaller than the pure fifth.

The decimal equivalents I am choosing to express these differences do not have any musical significance. However, the useful feature of these decimal numbers is that one can easily rank the different temperaments from smaller to larger, or flatter to sharper. Larger numbers indicate a larger or wider interval that is sharp. Conversely, smaller numbers indicate smaller or narrower intervals that are flat. With the ability to calculate any kind of semitone, whether chromatic or diatonic, in any meantone temperament quarter-comma or otherwise, we can now correctly determine what kind of temperament and semitones lute fretting systems used. That is the matter to which we now turn directly.

Chapter 2

Lute Fretting Systems

In the previous chapter, I set forth several different methods in which we can measure ratios of intervals to determine the quality of their temperament. In this chapter, I will apply these methods to different fretting schemes and determine, from a mathematical standpoint, what kind of tempered interval each fret represents. The overall picture for lute temperaments from the sixteenth through early seventeenth centuries can be outlined as follows: Pythagorean tunings were used exclusively at the beginning of the period, and were retained for certain intervals later in the period. By the mid-sixteenth century, lute players began to add different types of tempered intervals into the existing Pythagorean scheme. Some of these intervals were meantone, some approximating equal temperament, and others were original in their quality. From the end of century onward, fretting systems remained mixed, or else used methods of dividing the octave into twelve equal semitones; however, their semitones did not always match those of modern equal temperament. The important distinction between these systems and keyboard temperaments at the time was that lute temperaments did not fall completely into one kind of tuning or temperament and

were specific to the instrument.

The majority of instructions published during this period provided the player with a practical means of setting frets and did not dwell on theory. Almost all of the sources have very precise rules for determining fret placement that use measuring tools such as a straightedge and compass. These were commonly used for making geometrical divisions, and when used to calculate fret placement could divide a line into any number of equal parts. The remainder of the sources are less precise and instruct the player to place a fret somewhere near another until it sounds agreeable, or to move a fret slightly in one direction or the other, without providing exact measurements.

What follows is a comparison of several fretting systems that appeared in lute and vihuela treatises. For those sources that have instructions on placing frets using a compass, I have determined the exact ratio of the interval using the arithmetic outlined in the previous chapter. The details of the calculations can be found in the appendix.

2.1 Pythagorean Tunings for Lute

Starting in the 9th century, most fretted instruments were tuned according to Pythagorean ratios. The placement of frets on these instruments yielded pure fifths, fourths, and octaves, as well as wholetones with a ratio of 9:8.¹ The result of this, as we saw in the previous chapter, was that major thirds were much wider than pure. What we know of the lute prior to 1500 suggests that it has a four or five strings or courses tuned in fourths.

Such a tuning would accommodate Pythagorean tuning quite well, but after 1500, the

1. Lindley, *Lutes, viols and temperaments*, 9.

lute's repertoire changed and a sixth string was added. The new tuning of the 6-course lute used a major third between the instrument's third and fourth courses which exposes the major limitation of a Pythagorean system of tuning.

Because theory treatises in the early sixteenth century still advocated Pythagorean systems of tuning due to its mathematical principles, lute players were left to forge their own solutions to the tuning dilemma. One of the mainstays of lute repertoire throughout the sixteenth century was intabulated polyphonic music. The first books of music printed by Petrucci in the early 1500s contain pieces of this kind, and composers of lute music included many of them in their publications. Because this music had many harmonies that relied on thirds, a Pythagorean system of tuning would have presented some obvious shortcomings. It is at this point that we see lute players begin a gradual shift away from Pythagorean tunings and towards meantone temperaments.

During the first half of the sixteenth century, at least three different tuning methods for lute were published. Two of these retained the existing Pythagorean tradition of the day, while the third departed from it and employed a kind of meantone tuning similar to that which Pietro Aron and Vicentino were developing during this period. *Epithoma musice instrumentalis*, published in 1530 by Oronce Finé, was one of the sources that retained Pythagorean principles. Finé was a professor of mathematics at the University of Paris who published mostly theoretical works, but it was his personal interest in music that compelled him to write *Epithoma*. Written in Latin, the book includes instructions for tuning a lute, reading tablature, and setting frets. Pierre Attaignant was a friend of Finé's and it was probably this friendship that resulted in the treatise's publication. An additional set of lute instructions appear in Attaignant's publication *Tres breve et familiere*

*introduction pour entendre et apprendre [...] in 1529, this time written in French. Some stipulate that these were instructions from Finé and just translated, but this has never been fully proven.*²

Another example of Pythagorean tuning appeared in a book published about the middle of sixteenth-century. The book was not about music but instead contained various topics concerning France. Appearing in 1556 and carrying the long title of *Discours non plus melancoliques que diverses, de choses mesmement, qui appartiennent a notre FRANCE: & a la fin La maniere de bien & iustement entoucher les Lucs & Guiternes*, it has been attributed to Bonaventure des Periers, but some sources list it as an anonymous author. The subject of tuning lutes and guitars appears in the last chapter and has nothing to do with any of the other chapters in the book. The final chapter of *Discours [...]* contains instructions for fretting and tuning these instruments as well as a diagram of a mesolab, an ancient geometric tool that was first used to solve the problem of dividing a string into equal semitones.

In his assessment of their fretting instructions, Mark Lindley has concluded that both Finé and the anonymous *Discours [...]* preserved the same characteristics of other Pythagorean tuning methods. These included earlier theorists from the fifteenth century such as Henri Arnaut, Johannes Legreze, Nicola Burzio, and Franchino Gafurio, as well as more contemporary theorists like Heinrich Schreiber of Erfurt, and Pietro Aron.³ The features of these fretting schemes resulted in fourths and fifths at ratios of 4:3 and 3:2 respectively, and wholetones with a ratio of 9:8. Semitones depended on which fret the measurements

2. Philippe Vendrix, "Finé, Oronce," *Oxford Music Online*:1, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/09666>.

3. Lindley, *Lutes, viols and temperaments*, 11.

began. In *Discours [...]*, frets 2, 4, and 6 were a series of 9:8 ratios beginning with the nut, while frets 1, 3, 5, and 7 were series of 9:8 ratios beginning with the seventh fret and moving backwards. Finé's method was slightly different, placing frets 6, 8, and 10 at 9:8 ratios starting from the twelfth fret.

The inherent problem with these systems is that the fret placement is based purely on mathematics and maintaining Pythagorean ratios without any deference to practical musicianship. Finé was not a musician, and Lindley suggests that the author of *Discours [...]* was not either.⁴ Furthermore, after attempting to play various pieces from the period, he concludes that "it seems doubtful to me that sensitive players would really have left the pythagorean scheme unaltered."⁵ This becomes apparent in later sources from Juan Bermudo and Sylvestro Ganassi who each began with a Pythagorean tuning scheme and then adjusted it to make it more palatable.

Successive instructions on fretting from the mid-sixteenth century onward de-emphasized Pythagorean systems in favor of tempered intervals and methods of equal fretting. While they certainly acknowledged Pythagorean intervals, and writers such as Bermudo and Ganassi offered complete fretting systems using Pythagorean ratios, they were never intended to be the final solution. Most musicians recognized the importance of Pythagorean intervals for the fourth, fifth, octave, and wholetone; however, they also used non-Pythagorean intervals to create a better solution. The primary distinction between sources that advocated Pythagorean tuning over a tempered kind was that Pythagorean sources tended to focus more on mathematical issues than musical ones.

4. Lindley, *Lutes, viols and temperaments*, 11.

5. *Ibid.*, 13.

2.2 Gerle's Fretting Instructions

The first major break with Pythagorean systems of tuning for lute came from Hans Gerle, a lutenist and composer who published his treatise in 1533. In addition to its specific explanation of fret placement, it is also one of the best sources for lute instruction and practice in the early sixteenth century. It contains detailed explanations of tuning, right-hand and left-hand technique, and tablature. The most important feature of Gerle's treatment of lute fretting, and the others that I will be analyzing in this chapter, is that Gerle was a lute player and not a theorist. From this viewpoint, matters of theory are discarded in favor of practicality. Gerle probably knew very little of the implications of breaking with the tradition of Pythagorean tuning. His emphasis was on finding a practical method of fretting the lute so that it would sound agreeable. The importance of his instructions become more apparent when we see other lutenists such as John Dowland borrowing many of Gerle's methods over 75 years later.

Gerle's instructions are directed towards a player who may or may not have theoretical knowledge of music, but would have had some rudimentary knowledge of arithmetic and geometry. His process involved using a compass and a straightedge, such as a piece of wood or other material that the player cuts to be the same length as the vibrating string length of the instrument. Marks were made on the straightedge, using the compass to divide the distances between each mark into different parts. Once all the appropriate marks were made, they could be transferred to the fretboard and the frets placed accordingly.

He uses a very straight-forward, step-by-step approach for setting each fret, beginning

with the twelfth.

Take a straight-edge that is thin or else a flat piece of wood like a ruler, and make it of such a length that at the top it touches the piece of wood that the strings lie on and also touches the bridge that the strings lie on, and when you have made the ruler so that it touches at both ends (don't make it too short; it must touch as I have said), mark the bottom part at the bridge with an *a*, and the top part with a *b*, so that you will know which end belongs to the bridge. Then lay the ruler on a table, and take a compass and find the middle of the ruler. Mark it with a point or little dot and put an *m* there.

The letter *a* marks the bridge and the letter *b* marks the nut. By placing the twelfth fret in the middle of the string, he divides the string in half yielding $\frac{1}{2}$, or in terms of a ratio, this would be an octave of 2:1. Compasses were widely used at this time to perform all kinds of geometrical divisions and we can assume most learned individuals at this time would know how to use one in order to execute Gerle's instructions. Euclid, the Greek mathematician, described procedures for dividing lines into equal parts in his famous work *Elements*. This book was first translated into Latin in 1482 and appeared in English translation by 1570. Although Gerle was German, it is reasonable to assume that he could have had access to either a copy in Latin or one in German. As far as we know, Gerle was not educated at a university and probably had little or no knowledge of Latin; however, it is still reasonable to assume that with Euclid's work in circulation, he could have come by the knowledge needed to perform the required geometrical calculations.

Gerle continues using pure intervals for the fifth, or seventh fret:

Then divide from the *m* to the *b* [in] three parts; and the first part from the *m* gives you the seventh and lowest fret. Mark it with a dot and put the number 7 there.

The letter *b*, marked in the previous step, denotes the nut at one end of the ruler. Gerle now switches to numbers for frets and indicates the seventh fret with the number 7. The

fret is marked at the first of the three parts starting from the twelfth fret or middle of the string. The resulting fret placement is one-third the length of the string, making the vibrating length of the remaining string two-thirds. We can express this semantically as a string divided into three parts with a vibrating length of two parts, or 3:2, which corresponds to the pure fifth. Gerle could have divided the entire string into three parts and placed the seventh fret at the first part from the nut, but it seemed more important to base successive frets on the location of existing ones, in this case, the twelfth.

Gerle's instructions continue with the first fret, and similar to his instructions for the seventh, he builds on the calculations of the previous fret to find its location.

Then divide elevenfold from the number to the b, and two of the same parts down from the b give you the first fret. Mark this also with a dot and put the number 1 there.

The "number" he is referring to is seven, or the seventh fret that we just marked in the previous step. Here he has us divide the distance from the nut to this fret in eleven parts and mark the second of these parts starting from the nut. This creates a ratio of 33:31, which is not Pythagorean but instead somewhere between the diatonic semitone 16:15 and the ratio 17:16. Mark Lindley has determined that this distance is actually a sixth-comma semitone, but what kind? In chapter two, we examined systems of equal division that divided the octave into multiple parts greater than twelve. Sixth-comma meantone corresponded to an octave divided into 55 parts, and the first fret on the lute would be the first semitone in a 55-division system. If the semitone is chromatic, it has four parts, and if it is diatonic, it has five.

To determine which semitone Gerle was using we can compare our calculations of his fret distance with the known values of different semitones. If we express Gerle's 33:31 ratio

as a decimal by dividing 33 by 31, we get $\frac{33}{31} = 1.0645$ rounded off to the nearest fourth decimal place. Comparing that value to the values of other semitones in table 2.1 we can

Equal temperament semitone:	$2^{\frac{1}{12}} = 1.0600$
Sixth-comma chromatic semitone:	$2^{\frac{4}{55}} = 1.0517$
Sixth-comma diatonic semitone:	$2^{\frac{5}{55}} = 1.0650$

Table 2.1: Equal and sixth-comma semitones

see that Gerle's first fret matches the diatonic semitone in sixth-comma meantone temperament.

Gerle has departed from Pythagorean tuning and opted for a tempered semitone at the first fret. This is not to say that he is advocating a sixth-comma meantone tuning overall.

Gerle still uses a pure fifth at the seventh fret, which is not the same as a fifth tuned in sixth-comma: The fifth is comprised of three wholetones and one diatonic semitone. In a

Pure fifth:	$3:2 = 1.5000$
Sixth-comma fifth:	$2^{\frac{32}{55}} = 1.4967$

Table 2.2: Comparison of pure and sixth-comma fifths

sixth-comma system this makes up 32 parts or *dieses*, and the resulting interval is slightly smaller than the pure Pythagorean fifth, which is what we should expect in any regular meantone temperament (see table 2.2). This will have implications when we examine the internal tuning issues of lutes later.

After having placed the first fret, Gerle continues with the placement of the second, or the wholetone. Here, he returns to the standard Pythagorean 9:8 ratio:

Then divide again from the number 7 to the b threefold, and the first part down from the b gives you the second fret. Mark it also with a dot and put the number 2 there.

Here we have a three part division from the seventh fret to the letter *b*, which was marked in the first step at the nut. By returning to our earlier calculation of the seventh fret, we can formulate the second fret distance and vibrating length accordingly which results in the Pythagorean wholetone.

So far, Gerle has mixed Pythagorean pure intervals with tempered ones, such as the fifth, the major second, and the tempered minor second. As he moves to the perfect fourth at the fifth fret, he writes:

Then divide from the *m* to the *b* in two parts, and the one part gives you the fifth fret. Mark it with a dot and put the number 5 there.

Starting from the twelfth fret, Gerle divides the distance into two parts and takes the first part from the nut as the mark for the fret. The letter *m* is our octave fret, the exact midpoint of the string. Since he is now dividing that again in half, this produces a Pythagorean interval of the fourth with a ratio of 4:3.

Now Gerle turns his attention to the tritone, or sixth fret:

Then put the sixth fret in the middle of the fifth and seventh frets. Make it with a dot and put the number 6 there.

If we assume that Gerle is using “middle” to denote the arithmetic mean, as opposed to the geometric mean, we can calculate the middle distance between the fifth and seventh frets and get a vibrating ratio of 24:17. However, an alternative explanation is that Gerle is being intentionally vague here as if to tell the player to put the fret somewhere between the two frets, and not necessarily exactly in the middle. If we assume this, then it seems reasonable to infer that the player could also alter the placement until it produced a good result.

Regardless of our interpretation of Gerle's instruction, the tritone is neither Pythagorean nor sixth-comma meantone, but it may be close enough to be considered equally tempered. In a sixth-comma meantone with a 55-division octave, the tritone contains either 27 or 28 dieses depending on whether the semitone above the fourth is chromatic or diatonic. For a lute tuned in G, this would be the difference between a C \sharp and a D \flat on the instrument's first course. Calculating the decimal equivalents for these two semitones shows us that Gerle's tritone falls somewhere between the chromatic and diatonic semitones. Although this fret is not identical numerically to the equally-tempered tritone, if we look at a scaled drawing depicting the location of the fret in relation to the other intervals, Gerle's fret and the location of the equally-tempered fret look nearly identical. We must note that Gerle places the fret in the middle of the other two frets without the use of a compass. Furthermore, while Gerle's sixth fret may come close enough to equal temperament that we could call it an equally tempered interval, he is not using equal temperament in the modern sense. In fact, Gerle was using meantone in its original sense, the way in which Aron would have tuned.

Meantone temperaments that did not use equal division had to approximate some of their semitones by dividing the whole tone differently. For example, Aron's original instructions did not consider every semitone in the scale, it only covered a set of thirds that were tuned pure leaving some of the other pitches indeterminate. Semitones that were not included in the scheme were divided, splitting a whole tone into two different parts. Since this was probably done by ear and without the aid of exact measurements, the calculation of this fret is an approximation and we should discount the use of arithmetic to determine the quality of the semitone between the fifth and seventh frets.

In order to better visualize frets that are approximations instead of exact calculations, we can look at a scaled drawing comparing the placement of the different semitones. I will be using many such scaled drawings and each will use the same sample mensur length of 70 centimeters. The purpose of the drawing is not to show the exact placement of frets that would normally be approximate, but to demonstrate how one type of inexact fret may be similar to another type of fret that is exact in its placement by showing how close they are to another. The choice of mensur length is based on what best appears on the page. While 70 centimeters is unusual for a lute, a more common mensur length of 60 centimeters is smaller on the page and does not exhibit the differences of semitone size as clearly.

In turning to Gerle's sixth fret, we can look at one of these scaled drawings comparing his fret with the known placement of the chromatic sixth-comma semitone, the diatonic sixth-comma semitone, and the equally tempered semitone. These frets are shown in red while Gerle's frets are in black. Gerle's fret favors the chromatic side, although given its

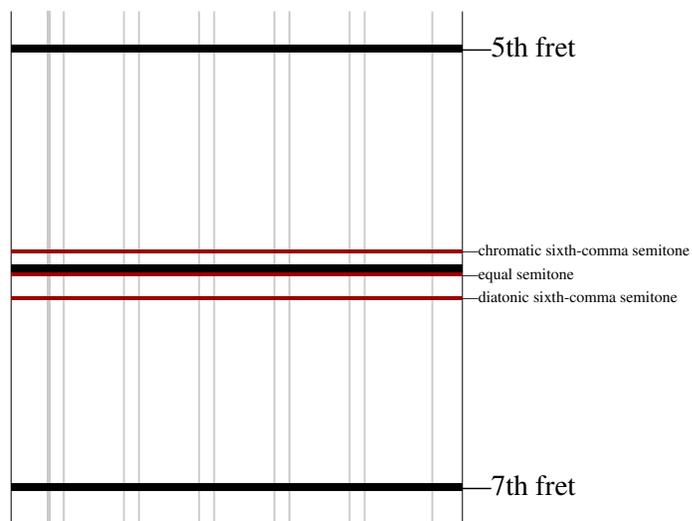


Figure 2.1: Scale drawing of Gerle's sixth fret

proximity, his fret is most similar to an equally tempered semitone. Another point to bear in mind is that as the mensur length decreases, so will the distances between these frets. Conversely, the distances will increase as the mensur increases. This will have more bearing on the theorbo, which can have a mensur length of 80 centimeters or more, and is discussed in later chapters where I focus on executing different types of fret placements. Gerle next instructs us where to place the third fret by taking a portion of the distance from the nut to the first fret.

Then divide from the number 1 to the b [in] three parts, and when you have the three parts then go with the compass unaltered down from the number 1 again five spans; that gives you the third fret. Mark it with a dot and put the number 3 there.

Gerle's uses the term "span" to denote one of the three parts in the distance from the first fret to the nut. Therefore, the total spans from the nut (b) to the third fret is eight: the five spans from the first fret to the third, plus the original three spans from the nut to the first fret. The resulting 99:83 ratio is closest to a diatonic sixth-comma meantone minor third. In looking at the scale drawing, the black line representing the fret completely occludes

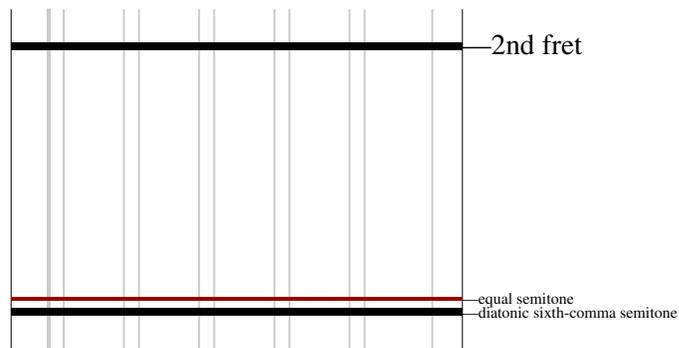


Figure 2.2: Scale drawing of Gerle's third fret

the red line of the diatonic sixth-comma semitone, indicating that for all practical purposes the two semitones are the same. Longer mensur lengths might exhibit a slight

difference in distance but it would be minimal.

The last fret is the fourth which makes the major third. Similar to the sixth fret, he averages the distance between the third and fifth.

Then put the fourth fret between the third and the fifth frets. Mark it with a dot and put the number 4 there.

Averaging the distance between the two frets results in the ratio 792:629, which comes closest to the equal tempered major third. Referring to a drawing of the fourth fret, Gerle's major third is flatter than a true equally tempered third, but sharper than a major third in a sixth-comma temperament. With our sample mensur length of 70 centimeters,

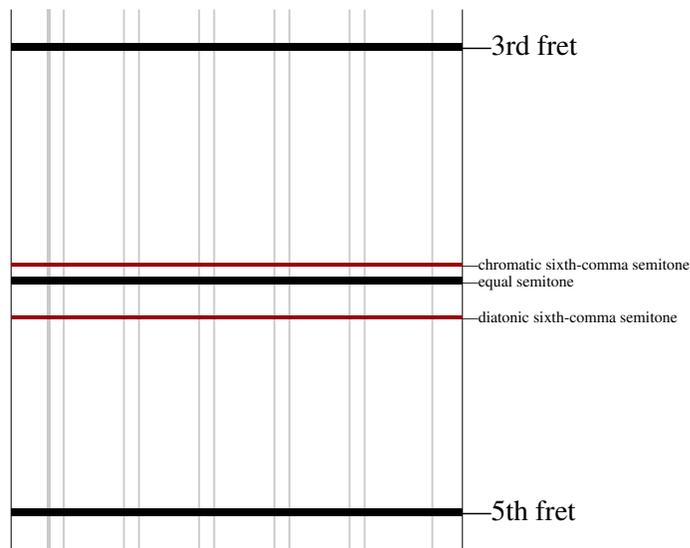


Figure 2.3: Scale drawing of Gerle's fourth fret

the actual difference between Gerle's major third and a sixth-comma meantone major third is about 2 millimeters. The difference between Gerle's third and a quarter comma meantone third is 4 millimeters. So when speaking of comparisons that account for a few thousandths of a decimal, the numbers themselves are small, but when translated into a real instrument, the differences become large.

Gerle limits his fret placement calculations to the seventh fret only; however, he does provide instructions for placing an eighth, but does not leave us any geometrical calculations to do so.

But if on the lute one wants eight frets, then let him make the eighth fret a little closer to the seventh fret than the sixth is.

Taken at face value, we cannot really determine what Gerle exactly means. The distance between frets would naturally decrease as they ascended the fretboard. At this point, we are to assume that the player may use his or her ear to adjust the fret accordingly. This same rule would hold true for the rest of the frets as well.

Taken as a whole, Gerle's fretting instructions are a mixed assortment of different intervals. He uses Pythagorean ratios for the basic intervals, but relies on tempered ones for the others. These are either sixth-comma meantone in nature, or completely unique. His approach seems to have taken hold with other players because we see the same kind of heterogeneous techniques used for many years afterwards.

2.3 John Dowland's Fretting Instructions

Gerle's fretting instructions seemed to have survived for quite some time after his death because they appear in an almost identical form in an instruction method by the English lutenist John Dowland. One of the best known lutenists of his day, Dowland was an important composer of songs and solo music for the lute and remains so to this day. During his lifetime, he published five books for lute and voice, as well as music for viol consort with lute accompaniment. His solo works survive primarily in manuscript, with the notable exception of the *Varietie of Lute-Lessons* which was published by his son,

Robert Dowland, in 1610 and contained music by John Dowland and some of his contemporaries.

As the title suggests, the *Varietie* was intended as an instruction book, however the music in it demands a high level of ability. The book begins with two prefaces, one written by John Baptiste Besard and a second by the elder Dowland. Besard's preface, "Necessarie Observations Belonging to the Lute and Lute-playing," was published three times elsewhere: First, in Latin, in Besard's own *Thesaurus Harmonicus*, a compendium of lute music published in 1603, where it was titled "De modo in testudine studendi libellus." The second time was in 1617, where it was revised slightly and published in his *Novus Partus* with the title "Ad artem Testudinis brevi." Lastly, it was published again that same year, in German, in a pamphlet entitled *Isagoge in artem institutionem*. Each version contained minor differences, but focused on technique rather than tuning or temperament. Dowland's own instructions, entitled "Other Necessary Observations belonging to the Lute" offered additional advice on choosing lute strings and a method for setting the frets on the instrument that followed Gerle's very closely. While his instructions postdate Gerle by more than 70 years, it is obvious that Dowland must have known of Gerle's book because of the similarity of their instructions. It is also possible that they consulted a similar source on geometry and measurement and reached the same conclusions, but it is more likely that Dowland simply knew about Gerle's instructions either through his book directly or from common practice and was repeating them in his own writing.

After giving the usual historical account of Pythagoras discovering harmony by listening to the hammers of blacksmiths, Dowland instructs us to procure a piece of "whitish" wood that is just as long as the distance from the inward side of the nut to the inward side

of the bridge on the lute. From there, his fretting instructions are exactly the same as Gerle's except that Dowland specifies letters instead of numbers since he was using the French style tablature system.

If we account for several printing errors which appear in Dowland's instructions, they are very straightforward.

Wherefore take a thinne flat ruler of whitish woode, and make it just as long and straight as from the inward side of the Nut to the inward side of the Bridge, then note that wnd [*sic*] which you meane to the Bridge with some small marke, and the other end with the letter A, because you may know which belongeth to the one and to the other. then lay the ruler upon a Table, and take a payre of compasses and seeke out the just middle of the Ruler: that note with a pricke, and set the letter N. upon it, which is a Diapason from the A. as appeareth by the striking of the string open.

After describing the same manner of using a ruler in place of the mensur length of the string, Dowland places the twelfth fret, fret N, at the midpoint of the string creating a pure 2:1 octave.

Secondly, part the distances from N. to D. in three parts, then the first part gives you the seaventh fret from the Nut, making a Diapente: in that place also set a pricke, and upon it the letter H.

Dowland makes the first of many typographical errors and uses the letter D instead of A. If we allow for that mistake and read the passage "N. to A." and not "N. to D.", it is obvious that Dowland is marking a perfect 3:2 fifth at the letter H. When he sets the first and second frets, he repeats Gerle's measurements almost verbatim, yielding a 33:31 diatonic sixth-comma semitone and a 9:8 Pythagorean wholetone.

Thirdly, devide [*sic*] the distance from the letter H. to the letter A. in eleaven parts: two of which parts from A. gives the first fret, note that with a pricke, and set the letter B. thereon, which maketh a Semitone. Fourthly, divide the distance from H. to the letter A. in three parts, one of which parts from A. upward sheweth the second fret, note that with a pricke, and set the letter D. upon it, which maketh a wholetone from A.

When he gets to the fifth fret there is another error, mistakenly indicating the first fret instead of the fifth, but otherwise the same 4:3 pure fourth that Gerle uses.

Fifthly, divide the distance from N. to A. into two parts, there the first part sheweth you the first [ie. fifth and not first] fret, sounding a Diastessaron: in that place also set a pricke, and upon it the letter F.

The sixth fret is placed the same as Gerle's, as the mean distance between the fifth and seventh, giving us the same ratio of 24:17.

The sixth fret with is a G. must be placed just in the middle betwixt F. and H. which maketh a Semidiapente.

When Dowland reaches the third fret, he departs from Gerle's geometrical instructions:

Seventhly, divide the distance from the letter B. to A. in three parts, which being done, measure from the B. upwards foure times and a halfe, and that wil give you the third fret, sounding a Semiditone: mark that also with a prick, & set thereon the letter D.

Whereas Gerle would measure five of his spans from the first fret, Dowland only uses four and a half. This results in a ratio of 198:168 which in our sample diagram appears the same as the sixth-comma chromatic semitone. Dowland is intentionally vague here.

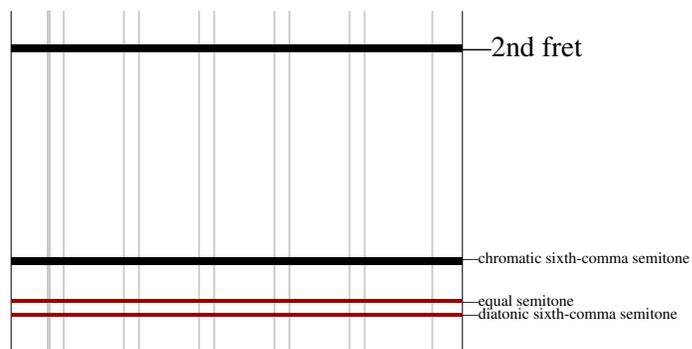


Figure 2.4: Scale drawing of Dowland's third fret

Instead of using whole numbers as everyone else does when making geometrical divisions with a compass, he uses fractions. If we wanted to be exact, we could find the mean of the

original span that is used to find the distance to the third fret and have a true “four and a half spans,” but Dowland does not instruct us to do that.

There are several possible explanations for Dowland’s odd calculation: First, Dowland could have intended to leave the method of calculation to the player. Someone able to execute Dowland’s geometrical calculations could have found the exact number of spans necessary, if he or she wished. Perhaps Dowland simply did not want give the extra detail. A second explanation is that Dowland knew he preferred a chromatic semitone and used the specification of 4.5 spans as an approximation. The compelling case for this explanation is that the 4.5 span measurement is so close to a chromatic semitone in sixth-comma meantone, the same kind of regular meantone temperament that Gerle was using. A third explanation is that Dowland did not know he was expressing the difference between the chromatic and diatonic semitone and arrived at the interval “by ear.” Either by experimentation or estimation, he found a way to represent the fret placement mathematically with an approximate measure of 4.5 spans.

As Dowland moves on to the fourth fret, he places it using the same arithmetic mean that Gerle used in his instructions.

then set the fourth fret just in the middle, the which wil[l] be a perfect *ditone*:

Since Dowland’s third fret was slightly smaller than Gerle’s, this will make the placement of his fourth fret different from Gerle’s as well, coming to a ratio of 1584:1266, which is very close to a quarter-comma chromatic semitone. Although this could be an argument in support of quarter-comma meantone on fretted instruments, it is the only interval in this scale that appears to be tempered this way. The other intervals are still a mix of Pythagorean and sixth-comma meantone intervals.

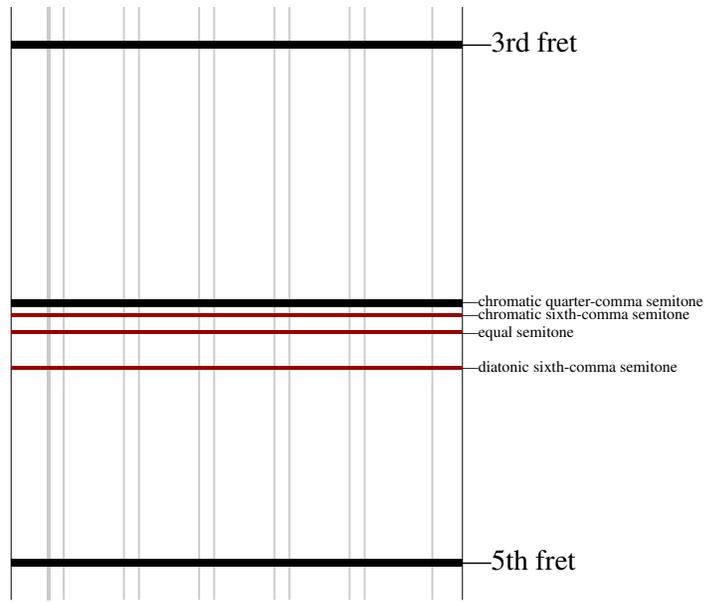


Figure 2.5: Scale drawing of Dowland's fourth fret

Dowland goes on to give us instructions for placing the eighth, ninth and tenth frets instead of stopping at the seventh as Gerle does; however, it is at this point that his calculations go awry. He places the remaining three frets using the same process:

then take one third part from B. to the Bridge, and that third part from B. maketh I. which soundeth *Semitonium cum Diapente*, then take a third part from the Bridge to C. [N.B. He means C. to the Bridge and not the other way around] and that third part maketh E. [N.B. Another misprint here, he means the ninth fret, K.] which soundeth *Tonus cum diapente*, or an *Hexachordio maior*. Then take one third part from D. to the Bridge, and that third part from D. maketh L. which soundeth *Ditonus cum Diapente*.

Apart from the errors with the ninth fret, Dowland is repeating the same pattern for each remaining fret. He seems to be calculating their placement by dividing the total distance between one fret and the bridge into three parts and then setting the fret at one-third of the distance. In looking at the mathematical result of this process, we find the ratios of the higher frets are actually decreasing when they should be increasing. Either we are misinterpreting Dowland's instructions because of further errors in the printed source, or

Dowland has simply got his calculations wrong.

$$\begin{array}{rcl} 99:68 \text{ ratio} & \frac{99}{68} & = 1.4559 \\ 27:19 \text{ ratio} & \frac{27}{19} & = 1.4210 \\ 594:426 \text{ ratio} & \frac{594}{426} & = 1.3944 \end{array}$$

Table 2.3: Decimal ratios of Dowland's eighth, ninth and tenth frets

If we return to the calculations that we obtained following Gerle's scheme, the only differences between those and Dowland's are the third and fourth fret. However, in looking at the semantic instructions themselves the real difference lies with the third fret. Gerle required 5 spans from the first to the third fret while Dowland required slightly less, 4.5. The semantic instructions for the fourth fret were the same in both sources: place the fourth fret in the middle, between the other two. The ratio of the fourth fret was only different because the distance to the third fret was shorter. If we then entertain the possibility that Dowland was essentially using Gerle's original instructions directly, but with only a slight modification to the third fret, perhaps due to personal preference, we could then see that the eighth, ninth and tenth frets were of Dowland's own doing. The last three frets of Dowland's scheme had no precedent, if we assume that Dowland was using Gerle as his primary source. Dowland was unique in his procedure for obtaining the right proportional distance for those last frets. Given the calculations above, we cannot arrive at a usable placement for the last three frets of his scheme, nor can we use Dowland's own fretting instructions as complete source because there is no mention of the eleventh and twelfth frets which figure prominently in his music.

2.4 Ganassi's *Regola Rubertina*

Because Dowland and Gerle's fretting instructions are similar, we can make a strong case for using them as a single unit for comparison, since all but one of their frets align very closely. An important source that would contrast with their methods is Silvestro Ganassi's instruction method for viola da gamba entitled, *Regola Rubertina*. Although he was a gamba player and his method is intended for that instrument and not lute, it does contain fretting instructions that are valuable when applied to the lute.

Ganassi approaches his subject slightly differently than Gerle and Dowland. In the fourth chapter of his treatise, he specifies the locations of each fret using a compass and string division, just as Gerle and Dowland do; however, in the fifth chapter, he adjusts some of these frets to different positions by comparing unisons from one string on the instrument to another. The first, second, third, sixth, and eighth frets are adjusted flatter than their initial placements, while the fourth fret is adjusted sharper. The adjustment of the fourth fret is markedly different from other treatises of the time; although, the frets that make the wholetone, fourth, and fifth, are all kept to their Pythagorean ratios.

For example, Ganassi opens his fourth chapter with the second fret, and uses the same Pythagorean wholetone ratio that everyone else has used at this time:

Please note that the proportion *sesquioctava* produces pitches expressed by these two number, 9:8. This proportion determines the location of the second fret. If you divide the string, beginning from the nut on the fingerboard and ending at the bridge, where the bow is drawn, into nine parts, the first of the nine parts sets the boundary of the second fret.

He also uses the pure 4:3 ratio of the perfect fourth because he says the open strings that are played against it must be in unison.

Then, you divide the string into four parts; the first of these four part will set the location of the fifth fret, which produces the consonance of a fourth, which is created by the proportion of sesquitertia indicated by a ratio of 4:3 [. . .] This produces the consonance of the perfect fourth, because if one then plays the open string, which is at the end of the fourth part of the string length, one achieves the opposite of the 4:3 ratio

Most players would have tuned their open strings using the fifth fret of the adjacent string.

In this case, Ganassi is merely underscoring the fact that the fifth fret should be at the ratio of a pure fourth because the interval of a fourth between the adjacent strings of the viol should be pure also. The seventh fret is kept to a pure ratio as well:

Then you divide the string length into three parts. The first of the three parts will be the end of the seventh fret, thereby producing the consonance of the perfect fifth, of *diapente*, which is formed with the proportion of *sesquialtera* indicated by the ratio of 3:2.

So for these frets, Ganassi is reiterating what most other theorists believed about tuning the whole tone, fourth, and fifth by maintaining their Pythagorean ratios. In chapter five, “Method of Adjusting the Frets”, Ganassi verifies the positions of these three frets by checking the unisons against the various open strings of the instrument. This ensures that he wants all the intervals between open strings to be pure Pythagorean ratios. It is important to note here that this precludes the use of any meantone temperament since the fifths would need to be tempered flatter than pure. We may also say the same thing about Dowland and Gerle’s methods since their fourths and fifths are pure as well.

Once Ganassi has completed setting those frets, he does not revisit them. However, this is not the case for the other frets of the instrument. It is the other intervals that distinguish Ganassi’s ideas about fret placement from some his contemporaries. He departs from certain meantone and Pythagorean ratios in favor of his own that are produced by comparing tones from one string and fret to the tones of a different string or fret.

2.4.1 Ganassi's Non-Pythagorean Frets

Ganassi's first fret differs in size from both Gerle's and Dowland's. He begins by having us place it in a Pythagorean ratio, but eventually moves it to a different kind of ratio. In chapter four, he writes:

After you have positioned the second fret in the manner described above, the first fret should be set halfway between the major and minor semi-tones by their respective proportions. In order not to go into this at length, however, I believe that I have chosen a similar method for finding the first fret which produces a minor semitone, quite easily.⁶

Another translation of the same passage from Mark Lindley's books is slightly different:

Dapoi che hauerai trouato & terminato il ditto secōdo tasto cō il modo ditto di sopra il tasto primo sera terminato al meza tra il scagneleto del manico al secondo tasto ma de piu zoe battēdo di fora la mita della grosseza del tasto . . . & in questo ti haueria possuto resonar il partimento del semiton maior al minor . . . il primo tasto elqual fa leffeto del semiton minor . . .	Now when you have found and located the second fret by the method given above, the first fret is located halfway between the nut and the second fret, but more, i.e. down the neck by half the fret's width. In this regard I could have calculated for you the division of the major and minor semitones. The first fret gives the effect of the minor semitone.
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Taking both translations into account, the placement of the first fret appears to be halfway between the *scagneleto del manico* or nut and the second fret. When the Pythagorean whole-tone (9:8) is divided in half, the resulting ratio is 18:17. This is very close to an equal semitone. It is not exactly the same as a true equally tempered semitone, but is close enough that many theorists and players advocated using a system of semitones based solely on a series of 18:17 divisions.

6. Silvestro Ganassi, "Ganassi's Regola Rubertina (Conclusion)," Translated by Richard D. Bodig, *Journal of the Viola Da Gamba Society of America* 19 (December 1982): 106.

$$\begin{array}{l}
 18:17 \text{ semitone: } \frac{18}{17} = 1.0588 \\
 \text{Equal temperament semitone: } 2^{\frac{1}{12}} = 1.0595
 \end{array}$$

Table 2.4: Comparison of the 18:17 and equally tempered semitones

In the sixth chapter, Ganassi changes the position of the first fret yet again, using the fifth fret of the fourth course as the reference:

Having tuned the open fourth string and having adjusted the fourth fret, you should then check the fifth fret of the third string. Its pitch will provide the means of regulating the first fret of the fourth string.⁷

Ganassi numbers the courses of his instrument in the reverse order that we do today: his “first string” is our sixth course, and his “sixth string” is our first course. On a lute in Renaissance G tuning, the above description would correspond to the unison B \flat found between the fifth fret of the fourth course and the first fret of the third course. Because this unison will not be in tune if the first fret is a 18:17 ratio, the fret must be moved towards the nut, rendering it slightly flat. We can determine the decimal equivalent of this new adjusted first fret by referring to a chart of frets and their placements from Richard Bodig’s article on Ganassi’s *Regola*.⁸ He calculates new values for each fret using Ganassi’s instructions and determining where the frets are moved after they are initially placed. The resulting decimal values occur once the player moves the frets to where they are exactly in unison with one another according to Ganassi’s instructions. Using Bodig’s calculations of Ganassi’s fret adjustments, and comparing them to the fret placements we have seen thus far with Gerle and Dowland, Ganassi’s first fret is much flatter than either Dowland or Gerle. Curiously, Ganassi’s adjusted first fret is almost

7. Ganassi, “Ganassi’s Regola Rubertina (Conclusion),” 114.

8. Richard D. Bodig, “Silvestro Ganassi’s *Regola Rubertina*: Revelations and Questions,” *Journal of the Viola Da Gamba Society of America* 14 (December 1977): 67.

exactly the same as the chromatic sixth-comma semitone. This differs from Dowland and

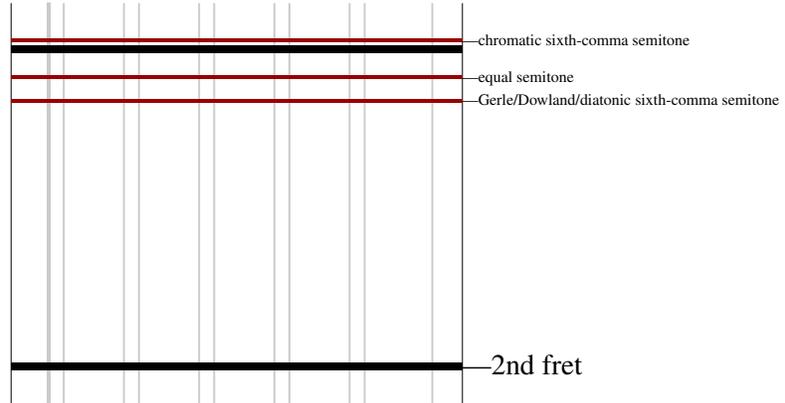


Figure 2.6: Comparison of Ganassi's first fret

Gerle whose first frets come to match the diatonic sixth-comma semitone. For a lute tuned in G, this would be the difference between a $G\sharp$ and an $A\flat$ on the first string. While Ganassi's adjustment renders the first fret in unison with the fourth, it is not a practical solution for a lute because the pitches at the first fret are now $G\sharp$ for the first course, as well as $A\sharp$ for the third.

Ganassi's third fret undergoes a similar transformation. In the fourth chapter, he sets the fret at a pure minor third. Later in the sixth chapter, he adjusts this and tunes it to the octave formed between the first fret of the third course and the third fret of the sixth course. Returning to Bodig's calculations, this results in lowering Ganassi's third fret from its initial pure minor third to something that is flatter than an equally tempered minor third. The drawing for this fret shows that it is approaching the chromatic sixth-comma semitone of Dowland, but not close enough to be considered equal to it, nor to any other kind of semitone that we have seen thus far.

Ganassi's initial setting of the fourth fret is halfway between the third and fifth frets, before the third is adjusted. This gives us a very strange ratio of 48:38. Later, he adjusts

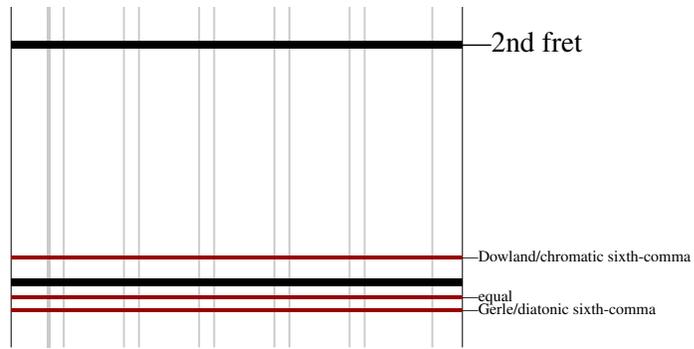


Figure 2.7: Comparison of Ganassi's third fret

the position of the fret so the octave between the second fret of the sixth string and the fourth fret of the fourth string are in tune. As the drawing indicates, his fourth fret is

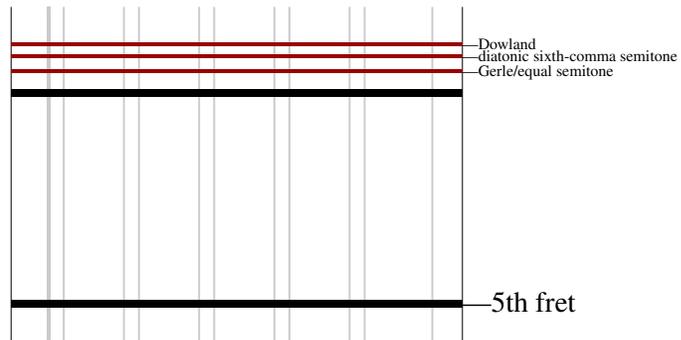


Figure 2.8: Scale drawing of Ganassi's fourth fret

almost unusable since it is yielding a major third that is even sharper than equal temperament. Ganassi provides no further explanation for this, but we must assume that most players would change this to something more suitable and dismiss his instructions for this particular fret.

The initial placement for the remaining sixth and eighth frets is similar to the fourth, and like his contemporaries, Ganassi does not provide any information about frets beyond that. His sixth is the same as Gerle's, the arithmetical mean of the distance between the fifth and seventh, or a ratio of 24:17. His eighth is the most specific we have seen in any source thus far, a 8:5 ratio.

Then the sixth fret is set at the midpoint of the space between the fifth and seventh frets but somewhat less, that is so that the thickness of the fret is within the compass of the distance; that will set its position. The eighth fret will be located so as to have the same spacing as from the fifth to the sixth frets.

Ganassi as well as other sources at this time such as Dowland, mention the thickness of the gut string used as the fret and allude to the fact that this would have some impact on its placement. For example, a particularly thick fret might offset calculations by as much as a millimeter. For modern lute players, this would be an issue for the lower frets, where gut strings of a millimeter or more are often used, but the thickness decreases to less than a millimeter for higher frets. No information exists about the specific thicknesses of the gut strings used for frets on instruments during this time. Ganassi's mention of it here at least indicates that it may have been an issue. His instruction is simply to ensure that no matter how thick the fret is, its entire thickness is *within* the span of the compass.

The final adjustments to Ganassi's fretting scheme include the sixth and eighth frets, which are moved slightly flatter than where they were originally placed during chapter four. The unison between the sixth fret of the third string and the first fret of the second is used to adjust the sixth fret as needed in order to make that sound in tune. The eighth fret is then adjusted so that the octave formed between the open fourth string and the eighth fret of the third string is in tune. Using Bodig's measurements, we can see that the final placement of the sixth fret moves it just slightly past the mark for the chromatic sixth-comma semitone, or C \sharp on an instrument tuned to a relative pitch of G. It is hard to say if this was intentional or simply a coincidence of Ganassi's adjustment procedures. This is perhaps more evident in the placement of the eighth fret, which is moved into a

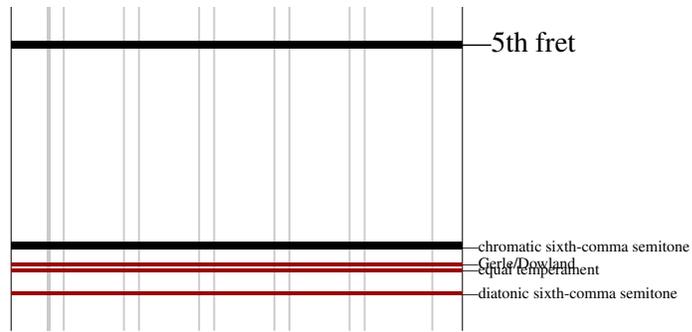


Figure 2.9: Scale drawing of Ganassi's sixth fret

position that is not in any one particular temperament, making it unique.

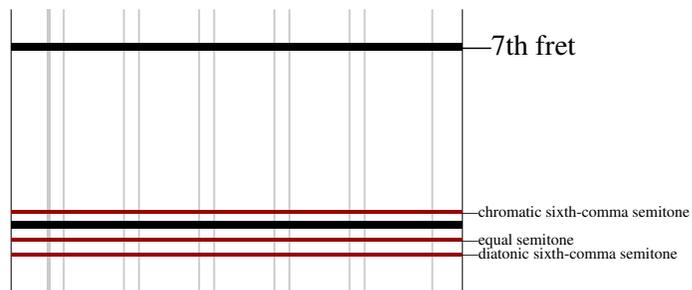


Figure 2.10: Scale drawing of Ganassi's eighth fret

Even though Ganassi spends an entire chapter of this treatise on placing the frets according to geometrical calculations, it is not until the sixth chapter that we see the complete picture of his scheme. Dividing the process as he does seems to acknowledge the dichotomy between a theoretical system of string division and a practical one that works in the realm of performance. Beyond this Platonic differentiation in tuning, we really cannot know what his motivations are.

Ganassi holds many similarities with other sources. The second, fifth, and seventh frets are the same as Gerle and Dowland. His first fret, however, is noticeably flatter than the Gerle/Dowland model, making it closer to a sixth-comma meantone chromatic semitone instead of a diatonic one which Gerle and Dowland favor. Perhaps the most important feature that Ganassi's fretting scheme shares with others from this period is that it

combines different types of semitones from different temperaments. This further indicates that lute temperaments were treated differently than temperaments on other instruments.

2.5 Spanish Vihuela Sources

We can examine sixteenth-century sources on the vihuela for additional information about fretting. The vihuela shared many similar characteristics with the lute and it was evident that players of the vihuela grappled with the same fretting issues that lutenists did.

Despite the two instruments' differences, Spanish vihuelists knew of the lute's repertory and technique. Miguel Fuenllana discusses the right-handed "foreign style" of playing in his *Orphenica lyra* of 1554. This refers to the thumb and index alternation of playing practiced by lutenists at the time.

Information about setting frets on the vihuela come from two sources. The first, and earliest of the two is Luis Milan's *El Maestro*. Milan was a well-known vihuela composer and poet who seems to have spent most of his active life around Valencia. Although his birth and death dates are not known, it is believed he died sometime after 1561, due to a reference in one of the composer's books of poetry, *El Cortesano*.⁹ Milan is best described as both a poet and bard who would sing or recite his poetry to his own music. He was active mostly at the court of Valencia where he made his living working for the dukes of Calabria. *El Maestro* is his only musical publication but remains one of the most important vihuela publications we have, not only for its musical content but for its information about performance practice such as tempo and interpretive elements that are

9. Luis Gásson, *Luis Milán on Sixteenth-Century Performance Practice*, ed. Thomas Binkley, Publications of the Early Music Institute (Bloomington, Indiana: Indiana University Press, 1996), 6.

absent from other contemporary treatises.

While *El Maestro* contains only a few references to fret placement, they are very significant. Milan's own remarks were not as exact as the others we have seen. He gave no measurements with which to mark frets on a straightedge, but he did give some clues as to which frets he wanted adjusted to achieve a different quality for certain notes. In Gasser's monograph on performance practices in Milan's music, he translates the following section from *El Maestro* where Milan discusses changing the position of the fourth fret when performing Fantasia 14.

Whenever you play the fourth and third tones in those places through which the fantasia moves, raise the fourth fret of the vihuela a little, so that the note of the fret becomes strong and not weak. [Siempre que tañeráis el cuatro y tercero tono por estos términos que est fantasia anda, alzaréis un poco el cuarto traste de la vihuela para que el punto del dicho traste sea fuerte y no flaco]¹⁰

Additionally, there is another piece of information concerning the same fret before the beginning of the music for *Con pavor recordó el moro*.

Playing in these pieces on the vihuela, you have to raise the fourth fret a bit toward the pegs. [Tañdo por estas partes en la vihuela habéis de alzar un poco el cuarto traste hacia las clavijas de la vihuela]¹¹

Both references concern the fourth fret, and both offer an adjustment that raises the fret closer to the pegs, making the interval smaller. This would equate to a narrower major third; however, it also assumes that the fret has been placed in some manner prior to being moved. It is not possible to know by what means Milan would have first placed his fourth fret, but it seems that for certain pieces, he preferred it to be slightly flatter than its initial placement. This could corroborate Dowland's use of a quarter-comma chromatic

10. Gásson, *Luis Milán on Sixteenth-Century Performance Practice*, 156.

11. *Ibid.*

semitone for his fourth fret, which is substantially flatter than either an equally-tempered or Pythagorean semitone. However, the problem here is that Dowland achieves a flatter fourth fret by making his third fret flat as well. Milan mentions no other fret to adjust, so we might assume that all his frets were placed according to standard Pythagorean proportions. Still, both Dowland and Milan do show a preference for flatter fourth frets. Perhaps the most complete discussion on vihuela fretting appears in Juan Bermudo's *De tañer vihuela*, published in 1555. Bermudo's publication is one of the most extensive on vihuela performance practice, including several chapters on intabulation for the vihuela, and fret placement. Bermudo provides a total of three different fretting schemes. The first of these, which appears in chapter 77, is Pythagorean. The second scheme attempts to correct the semitones in the first scheme by adjusting the first, sixth, and eighth frets while leaving the others in their original Pythagorean form. The third and last of Bermudo's schemes is unrelated to either of the previous two and is the closest to today's equal temperament.

Dawn Espinosa translated Bermudo's work, with the original Spanish printed alongside the English translation, and prefaced her translation with an informative discussion. In it, she provides a table of Bermudo's fretting schemes and the ratios she has calculated for each fret. For my analysis, I have used these to compare each of Bermudo's fretting schemes with the others that we have seen so far.

Similar to Ganassi, Bermudo gives us a simple Pythagorean-based fretting system but then offers two alternative systems as well. This attests to the changes in tuning that were occurring during the time. Prior to the sixteenth century, Pythagorean tuning had been the dominant system, but it had its inadequacies. By starting with a Pythagorean system and

then providing alternatives, writers could now acknowledge its importance and at the same time move past its insufficiencies. For example, Bermudo's second fretting system presents a solution that corrects some of the problems with a Pythagorean system and arrives at the exact same placement for the third fret as Ganassi does. Also similar to

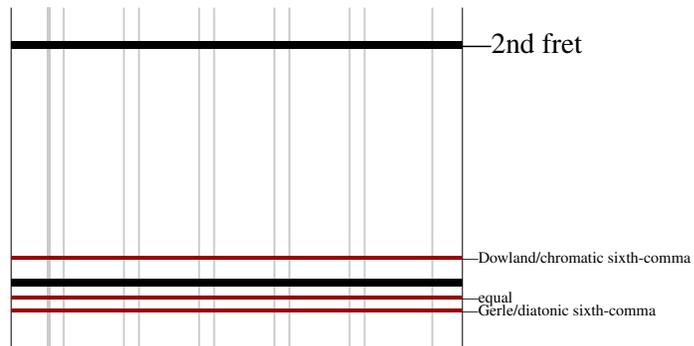


Figure 2.11: Bermudo's third fret

many of his contemporaries, his solution for the first fret clearly advocates the diatonic semitone. However, it is slightly sharper than the sixth-comma diatonic semitone of Gerle

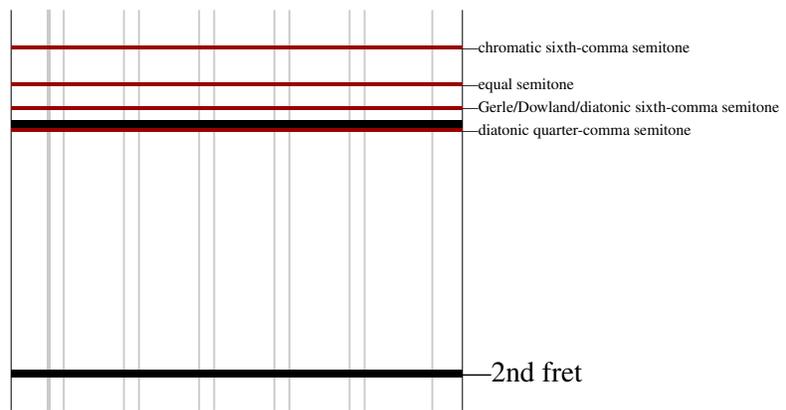


Figure 2.12: Comparison of Bermudo's first fret

and Dowland.

Bermudo refers to the "faults" found in fretting systems between notes that are *mi* or *fa*.

In terms of sixteenth-century theory, this was the modern equivalent of a sharp note versus a flat note. After setting the frets in his first scheme, the Pythagorean one, he

discusses the problems found on the eighth fret:

On the eighth fret there seem to be three faults. This fret should be fa for the seventh, fourth and first strings, but is it [made] mi for all the strings.¹²

Bermudo is highlighting the central problem about how to set the frets: the distance of one semitone on one string is the same for all the other semitones on that string.

Depending on the temperament and semitone, a minor semitone might be required for one string while a major semitone would be needed for another.

One of Bermudo's solutions, which we also see with Milan, is to adjust the frets according to the key of the piece. Milan, for example, would adjust the fourth fret when playing the third and fourth tones, while Bermudo describes players who move their frets according to their ears:

We have seen players who, with their frets set for the sixth, want to play the fourth mode, but are unable to do it without moving the frets as their good ear tells them. What I intend to do here is to give the measurements by which to place the frets, so that those who are not [such good] musicians will be able to place them with ease and exactitude, and thus the vihuela will be more perfect.¹³

Bermudo seems to suggest that good players are capable of moving their frets for different modes, while inexperienced or lesser skilled players should rely on Bermudo's calculations to place the frets for them.

For the latter, Bermudo devised a scheme that approximates modern equal temperament, essentially splitting the whole tone into two equal halves, although the composer himself acknowledges that it is not his aim to do so:

12. Dawn Astrid Espinosa, "Juan Bermudo 'On Playing the Vihuela' ('De tañer vihuela) from *Declaración de instrumentos musicales* (Osuna, 1555)," *Journal of the Lute Society of America* 38-39 (1995-96): 95.

13. *Ibid.*, 78.

All the [theorists] agree that the whole tone cannot be divided into two equal semitones, but that is what we are presuming to do. The above being the most agreed and true thing among theorists, on the vihuela we find the opposite in practice.¹⁴

For Bermudo, the issue was a question of theory versus practice. Espinosa summarizes this issue succinctly: “for him, theory has more authority than practice, but he concedes that practice precedes theory.”¹⁵

In table 2.5, the difference between each of Bermudo’s frets and modern equal temperament is expressed in both cents and in three different mensur lengths: 600mm, 700mm, and 800mm. In this instance, we use millimeters for the mensur length because they can express the differences with more accuracy. 700mm is the same as 70cm, which has been the reference mensur length in our other diagrams. Cents, a logarithmic measurement, are used as another point of comparison. Cents are often used with pitches in equal temperament, where the octave is divided into 1200 cents giving each semitone an equal amount of 100 cents each. Subjectively speaking, the human ear can notice differences of a few cents, but near or less than one cent would be difficult. The negative numbers in the table show a fret that is flatter than equal temperament, while a positive number indicates a fret sharper than equal temperament.

As the table shows, all but two of these frets are within a few millimeters of an equally-tempered semitone, in a variety of different mensur lengths. The only exceptions are at the third and eighth frets which are flatter than equal temperament. When looking at the differences in cents, these frets are quite flat, and most listeners would notice a

14. Espinosa, “Juan Bermudo ‘On Playing the Vihuela’ (‘De tañer vihuela) from Declaración de instrumentos musicales (Osuna, 1555),” xx.

15. Ibid.

<i>Fret</i>	<i>In cents</i>	<i>At different mensur lengths in mm</i>		
		<i>600mm</i>	<i>700mm</i>	<i>800mm</i>
1	0.50	0.16	0.19	0.22
2	0.31	0.09	0.11	0.13
3	-5.87	-1.71	-2.00	-2.28
4	0.62	0.17	0.20	0.23
5	-1.96	-0.51	-0.59	-0.68
6	-1.46	-0.36	-0.42	-0.48
7	-1.65	-0.38	-0.44	-0.51
8	-7.82	-1.72	-2.00	-2.28
9	-1.33	-0.27	-0.32	-0.37
10	-3.91	-.76	-0.89	-1.02

Table 2.5: Differences between Bermudo's third scheme and true equal temperament

difference of 5 to 8 cents; however, the differences of fractions of one cent would be virtually undetectable. For what reason Bermudo chose to make these particular frets flatter than the others, by comparison, we cannot know. However, an interesting aspect to Bermudo's scheme is that, after the fourth fret, each one is gradually flatter. This may be to counteract a problem common to lutes and fretted instruments where pitches at the higher frets tended to sound sharper. The effect was greater at the highest frets, such as the tenth, which is perhaps why this fret is almost four cents flat in Bermudo's scheme.

2.6 Sources of Equal Fretting

The last category of fretting systems that we will examine come from the late sixteenth and early seventeenth centuries and advocated dividing the octave into equal semitones. This is not to say that these systems produced the kind of equal temperament that we have today. Creating a temperament with semitones that are exactly equal to one another requires a special class of mathematical functions called logarithms, which were not used

in tuning until later in the seventeenth century. Despite their ability to divide the octave correctly into equal semitones, equal temperament did not become a true standard until the twentieth century. As Ross Duffin has argued, even during the nineteenth century musicians still tempered their fifths more than those in equal temperament.

Prior to these advances in mathematics, equal semitones were approximated using other methods. Sources during this time describe several different systems of measurement that produced temperaments very close to modern equal temperament, but which were achieved through a means of calculation that did not involve logarithms. The first of these we have already seen with Bermudo's third method of fretting, which uses standard geometrical string divisions common to most lute sources at the time. Additional information on fretting in equal semitones comes from Vincenzo Galilei and Marin Mersenne, as well as numerous other anecdotes collected by Mark Lindley in which contemporary musicians refer to the lute's ability to tune equally.

One of the simplest ways in which a lute player could achieve equal semitones was to divide the octave into a series of Pythagorean minor semitones, at a ratio of 18:17.

Mersenne, Galilei, and other sources refer to it as the *18:17 Rule*, and it could create equal semitones on any instrument, but the technique seems to be solely reserved for fretted instruments.

The idea originates with the problem of dividing the Pythagorean 9:8 whole tone into 2 geometrically equal parts, which I discussed in the previous chapter. A series of minor semitones produced fairly acceptable results, but as the frets progressed further down the fingerboard, the distances became increasingly smaller, making each fret flatter than the next.

Looking at table 2.6, we can see the 18:17 rule compared with equal temperament when executed with a variety of different mensur lengths as well as how it compares in terms of cents. The differences between the two systems for the first five frets are very slight: only

<i>Fret</i>	<i>In cents</i>	<i>At different mensur lengths in mm</i>		
		<i>600mm</i>	<i>700mm</i>	<i>800mm</i>
1	-1.05	-0.34	-0.40	-0.46
2	-2.09	-0.65	-0.75	-0.86
3	-3.14	-0.91	-1.07	-1.22
4	-4.18	-1.15	-1.34	-1.54
5	-5.23	-1.36	-1.59	-1.81
6	-6.27	-1.54	-1.80	-2.05
7	-7.32	-1.70	-1.98	-2.26
8	-8.36	-1.83	-2.14	-2.44
9	-9.41	-1.94	-2.27	-2.59
10	-10.45	-2.04	-2.38	-2.72
11	-11.50	-2.12	-2.47	-2.82
12	-12.54	-2.18	-2.55	-2.91

Table 2.6: Comparison of fret placement using the 18:17 system

less than two millimeters between the measurements of these frets, and at most a five cent difference in pitch. At the higher frets, however, the differences become larger. By the time we reach the octave, the fret is more than twelve cents flatter than the pure octave found in equal temperament. This would be very noticeable to most individuals.

Despite the decrease in distances, using a series of equal minor semitones in the 18:17 rule provided a practical temperament because its problems were mitigated by two factors: First, most musicians at this time were using some form of temperament that flattened fifths, and at the seventh fret where the fifth occurs, the fret is already flattened by a few millimeters. Second, the tension created by stopping a string at a given fret, especially if the fret itself is quite thick, or doubled as is the case with the viola da gamba,

can raise the pitch slightly.¹⁶ This effect is compounded by the fact that changes in finger pressure or position have a more noticeable effect on pitch at high positions. Making the frets gradually flatter as they move to higher positions would help counteract this effect. Another method of determining equal fret positions used a device called a mesolab, which was an instrument of Greek origin that could determine mean proportionals, or the average distance between two lines. In 1558, Zarlino published a fretting scheme for lute that relied on such a device.¹⁷ There is also a picture of one in the *Discours non plus melancoliques que diverses, de choses mesmement, qui appartiennent a notre FRANCE: & a la fin La maniere de bien & iustement entoucher les Lucs & Guiternes*, published in 1556, which was mentioned earlier. However, use of the mesolab did not appear in any practical fretting guides and seems to have been relegated to music theory sources where equal temperament was a kind of puzzle to be solved and not really taken seriously in a musical context.

By the seventeenth century, advances in mathematics employed decimal numbers and logarithms to express musical ratios. Johannes Faulhaber was the first to use logarithms for a fretting scheme in his 1630 publication.¹⁸ In order for it to work, the string had to be divided into 20,000 parts. This idea later found its way into an appendix to a translation of Rene Descartes' *Musicae compendium*. Mark Lindley has concluded that its translator is William Brouncker, who also wrote the appendix, which contains several fretting schemes for lute, including our modern $\sqrt[12]{2}$ method, as well as others of his own devising.

Despite the accuracy of logarithmic calculations in placing frets at equal temperament, the

16. Lindley, *Lutes, viols and temperaments*, 21.

17. *Ibid.*, 26.

18. *Ibid.*, 21.

method was largely ignored except in a few treatises. While many authors spoke of the lute's ability to play with equal semitones, the idea of an "equal temperament" was somewhat less certain. From the available evidence, systems approximating equal temperament and utilizing the idea of equal semitones certainly existed during this period, but they still recognized the difficult issue of how to reconcile unequal semitones found in the temperaments of all the other instruments. The central crux of the lute player's struggle was how to navigate between these two poles.

2.7 Summary

Based on this lengthy description of the available sources on fretting from the sixteenth century and seventeenth centuries, one can see that some of them do agree on the placement of certain frets (see table 2.7). The second, fifth, and seventh are all uniformly Pythagorean in nature, making the fretted intervals of the perfect fourth and fifth pure. When players tuned their open strings to one another by using the fifth fret of the adjacent strings as a reference, this made the interval between the two strings a perfect fourth as well. This is corroborated by every source at this time that contained instructions for tuning the lute's open strings.

The use of Pythagorean intervals puts these fretting systems at odds with a meantone temperament. The second fret seems like a good choice at first glance, since it would be used in most scalar passages as the wholetone above the open strings. However, when used in chords, the second fret often holds the third of the chord. In meantone temperaments, it would need to be lower in pitch or moved closer to the nut in order to

<i>Fret</i>	<i>Dowland</i>	<i>Gerle</i>	<i>Ganassi</i>	<i>Bermudo II</i>
1		diatonic $\frac{1}{6}$	unique/chromatic $\frac{1}{6}$	diatonic $\frac{1}{4}$
2		Pythagorean 9:8 wholetone		
3	chromatic $\frac{1}{6}$	diatonic $\frac{1}{6}$	unique	
4	chromatic $\frac{1}{4}$	unique/equal	unique	equal
5		Pythagorean 3:2 fourth		
6		unique/equal	unique/chromatic $\frac{1}{6}$	unique
7		Pythagorean 2:3 fifth		
8	n/a	n/a	unique	

Table 2.7: Comparison of fretting schemes

accommodate either a sixth or quarter-comma meantone temperament. Furthermore, the presence of a fretted pure fourth would create pure fourths between open strings during the tuning process. This discounts the use of any meantone temperament because of the wide third that would result between the instrument's third and fourth courses.

It is clear that lutenists were experimenting with meantone intervals for some of their frets. We can see some general agreement that the first fret should be diatonic in nature.

This would mean that for instruments tuned in either G or A, their first frets would have to be A \flat or B \flat respectively. However, there is disagreement over the remaining chromatic frets. For example, each source has its own technique for placing the fourth fret, some of which are very close to equal temperament, but each is essentially unique to their system.

Dowland and Gerle seem to favor sixth-comma meantone for their third frets, but disagree over the quality. Ganassi and Bermudo, on the other hand, arrive at the same placement for their third fret, but it does not match any interval in the meantone scheme of temperaments.

Given the issues with the extant fretting resources presented in this chapter, we can conclude that most of them are problematic at best. It is evident that players embraced

a variety of different techniques when addressing the issue of fretting, and we should use that as a guiding principle when creating our own, but that means we must inevitably make our own choices. Lastly, the historical solutions were not uniform and none consisted entirely of Pythagorean intervals or meantone intervals of one type or another. With that in mind, in the next chapter I shall propose more accurate solutions for fretting that retain some of the principles found in these historical resources. I will then examine my solutions in a musical context, using examples from the period.

Chapter 3

Modern Lute Fretting

If we recall the excerpt from Hercole Bottrigari's *Il Desiderio* at the beginning of this paper, he demonstrated that not all the pitches of a lute match those of a harpsichord. This was because the standard tuning in the sixteenth and early seventeenth centuries was meantone temperament, a preferred temperament among instruments due to its harmonious, pure thirds. In the previous chapter, we found that none of the major sources on lute fretting specified a uniformly meantone fretting system, quarter-comma or otherwise. Today, lute players who wish to participate in ensembles that are employing meantone temperaments are going to find themselves in the same situation in which Desiderio found himself, trying to match the pitches on his lute with those of the harpsichord.

It seems obvious that if ensembles were using a meantone temperament, then lutes must have as well, but it is less obvious how they accomplished this. Because of the arrangement of semitones on a lute, meantone temperaments must be executed differently than on other instruments. Bottrigari and others recognized these problems and proposed

practical solutions for dealing with them by ultimately deciding that the lute was an equal semitone instrument. Others observed that it was unique in its temperament and music sounding bad on one instrument seemed to sound better on a lute.¹ However, the fact remains that, in large and small ensembles, lutes were combined with other instruments which very likely used quarter-comma and other varieties of meantone temperament. So it stands to reason that players had found ways to reconcile the problem.

Since different historical schemes set forth in the previous chapter fall short of a comprehensive solution to meantone fretting, we are left to find our own solutions. In this chapter, I will address the issue of meantone temperaments in ensembles, specifically quarter-comma meantone because it is the most common temperament associated with music of this period and is also one of the more problematic temperaments to realize on the lute. I will examine it in three different contexts: 1) a continuo ensemble with theorbo; 2) a continuo ensemble with lute; and 3) ensembles with lute or theorbo in tablature notation.

The solutions proposed here will build on ideas we have taken from historical sources, but will also incorporate alternative ones, some of which are modern and contradict the approach of historical treatises. They include the use of *tastini* or “little frets” made of wood or ivory that only span one string and can be glued to the fret board. These small frets make it possible to have a different kind of semitone on one string, such as a chromatic semitone, while the rest of the strings all have the diatonic semitone. Another alternative involves the use of frets placed at an angle so that they are not parallel to the nut or bridge. This results in a fret that could be a diatonic semitone at one end and a

1. Lindley, *Lutes, viols and temperaments*, 45.

chromatic one at the other. The arguments found in historical sources against such solutions support the notion that the practice was commonly used, even if these sources sought to rally against it.

Any attempt to use meantone temperament on the lute is a kind of compromise. The least drastic solutions include alternative fingerings for certain notes and varying the finger pressure of the left hand to alter the pitch. Adjustments such as these could make pitches more agreeable in a tempered setting and enable the instruments to function in a given meantone context. A lutenist could have also omitted offending notes from a realization, or remained completely tacet if the temperament did not agree with the rest of the ensemble. There are many different combinations of techniques that can enable a lute to play in a quarter-comma meantone temperament, but as we shall see, each combination has a different effect on the instrument's capabilities.

3.1 The Lute in Ensembles

If we overlook the problems with historical fretting instructions, it is possible to realize quarter-comma meantone temperament on a lute or theorbo, but it has some consequences. When a lute is put into a meantone temperament, its compass may be limited and some of its idiomatic qualities may suffer. For example, common left-hand chord shapes that provide the most resonance may not be possible. These issues can be mitigated if the instrument is used for basso continuo, where the choice of chord shape is at the player's discretion. Since the player has more flexibility and control over the number of voices in the chord and its location on the fretboard, it is easier to overcome

some of meantone's limitations in an ensemble setting. In solo situations, where the exact location of each pitch is dictated by the tablature, a meantone temperament could produce undesirable results and necessitate the use of a different temperament. It is important to note that there is no apparent evidence for quarter-comma fretting in any solo literature from the seventeenth century onwards. Therefore, it is possible that for later solo repertoire lutenists employed less problematic meantone temperaments such as sixth-comma, or utilized the quasi-equal and custom temperaments that were featured in the previous chapter. This leaves issues with quarter-comma meantone mostly relegated to ensemble literature throughout the period of this study, and to a lesser extent, solo literature in the sixteenth century.

Meantone fretting on a lute imposes a uniform semitone type for each fret. The main feature of meantone temperaments are thirds that are much closer to pure than Pythagorean tuning or equal temperament. The two consequences of this are narrow fifths, or fifths that are flatter than pure, and unequal semitones, namely the diatonic and chromatic semitone. For the keyboard, this means choosing between an $F\sharp$ and a $G\flat$, or a $D\sharp$ and an $E\flat$. In any meantone temperament, these are two different notes, rather than enharmonic equivalents. When tuning a keyboard, the choice between a chromatic or a diatonic semitone can be independent of any other note on the instrument. It is possible to have an $F\sharp$, $A\flat$ and $C\sharp$ all on the same octave. On the lute, however, this is not possible because fret placement dictates the size of the semitone for all notes along that fret. For example, the choice of a chromatic semitone on one course to yield a $C\sharp$ might force another course to have $G\sharp$ instead of $A\flat$.

In temperaments with unequal semitones, lutenists must choose between a chromatic or a

diatonic semitone when placing frets on their instrument. For example, if we were fretting our instrument in quarter-comma meantone, the first fret would either be a chromatic semitone or a diatonic one. The choice of semitone is up to the player, but the main consideration the player should take into account is whether any pitches on that fret must be chromatic or diatonic. For a lute in standard G tuning, the second course is tuned to D; therefore, the first fret of the D course could either be $E\flat$ or $D\sharp$, the second fret of the course E and the third is F. The distance between E and F is a diatonic semitone, which makes the distance between the second and third fret diatonic. Because of this one requirement in tuning between two notes on a single course, the distance of a diatonic semitone between these two frets will apply to all other courses as well. Given these requirements, Dowland's chromatic sixth-comma fret in table 2.7 seems impossible, because this would create something flatter than F. Gerle's diatonic fret, on the other hand, makes more sense in this context.

When choosing semitone size, players must also consider that some chromatic notes are more common than others. It is more common to find an $F\sharp$ in the seventeenth century than it is a $G\flat$. Returning to our previous example of the D course, the fourth fret makes better sense as a chromatic semitone, giving us an $F\sharp$, instead of a $G\flat$. If we use this same logic and move back to the first fret of that course, we could choose an $E\flat$ over a $D\sharp$ by reasoning that we are probably more likely to encounter an $E\flat$ than a $D\sharp$, although this is not always the case.

For the sake of argument, let us say that we have settled on the semitone choices we have discussed above: diatonic first and third frets, followed by a chromatic fourth fret. The result of these semitone sizes are summarized in figure 3.1, and as we can see, this has an

interesting impact on the pitches on the other courses of the same frets. Since we have

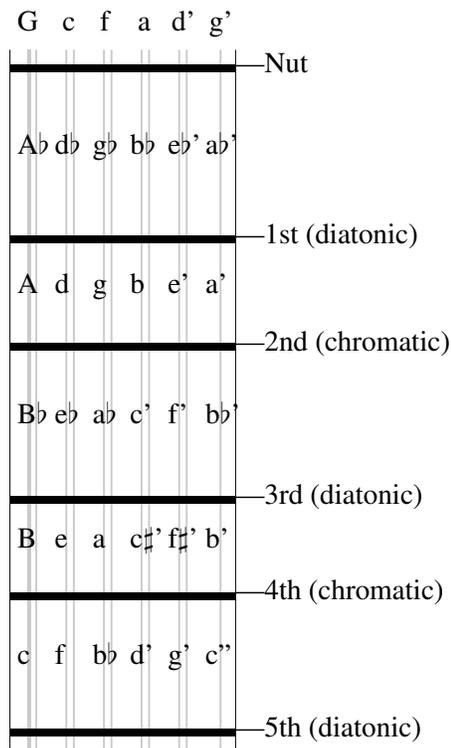


Figure 3.1: Standard quarter-comma fretting

chosen $E\flat$ over $D\sharp$ for the first fret, this results in a $B\flat$ for the third course, which is a good choice; however, the fourth and fifth courses are $G\flat$ and $D\flat$, instead of the more likely $F\sharp$ and $C\sharp$. This also creates a problem because the $G\flat$ found on the first fret of the fourth course will not match the $F\sharp$ on the fourth fret of the second course. Similarly, the $D\flat$ on the fifth course will not match the $C\sharp$ on the third.

The alternative solution is to have a chromatic semitone for the first fret instead of a diatonic one, as seen in figure 3.2, but this only makes things worse. It allows for an $F\sharp$, $C\sharp$ and $G\sharp$ on the lower courses, matching the $C\sharp$ and $F\sharp$ of the fourth fret, but the $D\sharp$ and $A\sharp$ on the upper courses prove to be a problem. $A\sharp$ is an especially unlikely chromatic note and does not match the $B\flat$ found on the third fret of the sixth course.

There are two critically important reasons why the alternative solution of a chromatic first

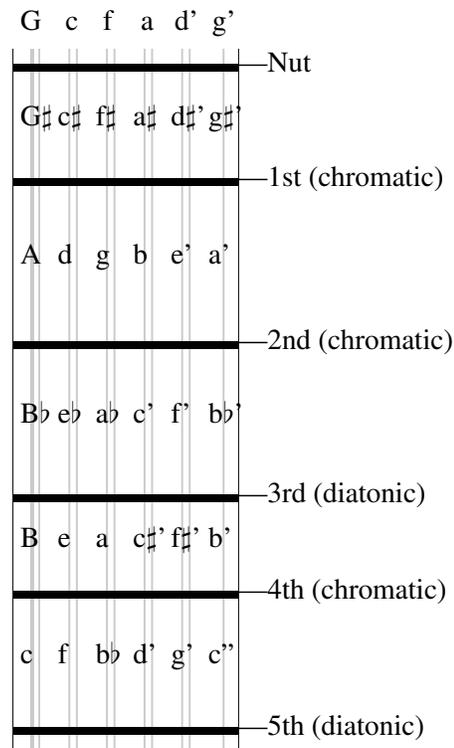
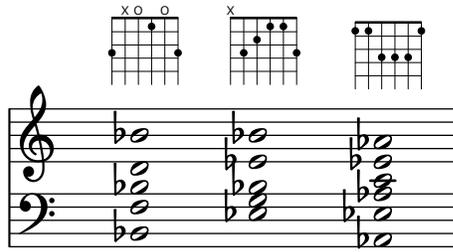


Figure 3.2: Alternate quarter-comma fretting for the lute

fret is undesirable. First, it seems that most of the historical sources agree that the first fret was some kind of diatonic semitone. Referring back to table 2.7, Dowland, Gerle, Ganassi, and Bermudo all agreed that the first fret was diatonic in nature. Although their system was closer to sixth-comma than quarter, it indicates a preference for the diatonic semitone, or notes that have the \flat accidental as opposed to a \sharp .

The second reason is that a chromatic first fret would create false unisons between the $B\flat$ and the $A\sharp$, as well as the $E\flat$ on the fifth and the $D\sharp$ on the second. These unisons are quite common in lute tablature, and comprise some of the most common chord shapes used in continuo playing. (see example 3.1) A chromatic first fret would render these chords unusable without additional adjustments, making it unlikely that a chromatic semitone would be used in either a solo or ensemble context.

While these figures demonstrate that a lute can be placed in a meantone fretting, it does



Example 3.1: Common chords on the lute using the first fret

not imply that it is a requirement. Composers such as Bottrigari seemed to think that the lute was simply an instrument that played in equal semitones, even in ensembles with instruments that played un-equally. In the discussion between his two fictional protagonists in *Il Desiderio*, Benelli tells Desiderio:

Therefore I do not wish either to affirm or deny, or even to dispute, whether or not the semitone said to be minor is minor, or if indeed it has a position between the greatest and the least; it will suffice to demonstrate by its effects – i.e. the Clavicembalo, the Organ, and their like, sound two unequal semitones, one larger than the other. The Lute and the Viols sound two equal semitones, that is, a tone divided into two equal semitones according to the idea of Aristoxenus.²

Bottrigari was comfortable with this arrangement because he classified the instruments into different categories. According to him, instruments fell into three different types: 1) stable; 2) stable but alterable; and, 3) completely alterable. The determining factor for an instrument was its ability to alter pitch. Instruments in the first category were stable because their pitches could not be altered. These included all keyboard instruments. The third category included instruments whose pitches were entirely changeable, such as the trombone and violin. Their pitches existed continuously, either along the slide of the trombone, or at any point on the fingerboard of a violin. In the middle, where instruments

² Bottrigari, *Il Desiderio*, 17.

are “stable but alterable”, is where we find lutes, viols, recorders, and transverse flutes. In Bottrigari’s second category of instruments, pitches existed at fixed points but were variable to certain degree. Flute players could vary pitch by changing the placement of their fingers, or controlling their breath. Lute players, according to Bottrigari, could “touch their frets a little higher or a little lower” in order to vary their pitches.³

The suggestion here is that lutes played in ensembles using a different temperament than the rest of the instruments. Keyboard instruments would have used a meantone, or other kind of unequal temperament, and the lutes would have played with these unequally-tuned instruments using their native quasi-equal temperament. In order for such an ensemble to be successful, Bottrigari suggests that only the instruments of the first two categories, stable and stable but alterable, or those from the first and third, stable and entirely alterable, be used together, and never all three together. The problem with using all three, according to him, is that the middle group will not be able to match pitches with the first and third groups at the same time. To this, he also adds: “I think it best to add that no concert of instruments should ever be given without the addition of a human voice.”⁴

For Bottrigari, the issue was not with temperament, but with arrangement and orchestration. Certain instruments should only play with one kind of instrument and not another. While this provides players some latitude when choosing temperaments, it still begs the question as to how players could have adjusted their instruments to match other instruments tuned in a meantone temperament. For that, we should examine a kind of lute

3. Bottrigari, *Il Desiderio*, 15.

4. *Ibid.*, 23.

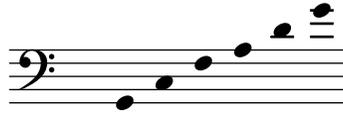
that was specifically designed for ensemble use.

3.2 The Theorbo

As lutes were used more and more in ensembles towards the end of the sixteenth century, a new type of lute was invented specifically intended to be played in ensembles. The theorbo, as it was called, was a much larger instrument and had additional bass strings that extended beyond the length of the neck. Although all kinds of lutes, including the theorbo, were generally fretted in the same way, the tuning of the theorbo presented other alternative fretting solutions that were not available on lutes using standard tuning. The theorbo's nominal pitch was almost always A, instead of the G as with other lutes of the time. Technically, any lute or theorbo can be tuned to any key, and it was not uncommon to find lutes pitched to F, G, A, and D. This applied to lutes in a consort where each instrument existed in a variety of sizes. The pitch could also result from the instrument's mensur length. The top string would be tuned as high as possible without breaking and that point determined the overall pitch of the instrument.

In an ensemble, the pitch of a lute or theorbo had to be standardized so that it could play with other instruments. In English consort music as well as most lute song publications in England, the standard lute pitch was G. In Italy, however, the theorbo was usually pitched to A. Regardless of the pitch of a lute, it was tuned so that the preceding course was lower than the one following it. (see example 3.2) This was not the case for the theorbo, which had first and second courses that were an octave lower. (see example 3.3)

Since theorbos were designed for accompaniment and needed to provide more volume



Example 3.2: Standard lute tuning in G



Example 3.3: Theorbo tuned in A with re-entrant first and second courses

than other lutes of the day, the body size was much larger and the strings longer. Because of the increased mensur length, it was not possible to preserve the low to high arrangement of courses as they were on the lute. Players found that as they tried to tune the upper strings to their normal lute pitches, they would break and it was not possible to fashion a gut string thin enough to hold the pitch at that length. To solve the problem, they tuned the strings to the same pitch but at an octave lower, thus preserving the same intervallic relationships between strings as they were on the lute. This made chord shapes identical between instruments and only altered the voicing of the chords. Since the theorbo was primarily a continuo instrument, the change in voicing did not present a problem; in fact, it became more of an advantage. The re-entrant tuning kept the overall tessitura of the instrument lower and away from that of the accompanied singer or instrumentalist.

All theorbos had eight additional bass strings that descended diatonically in pitch from the A on the sixth course. Therefore, the seventh, eighth, and ninth courses would be G, F, and E, continuing on to an octave G on the fourteenth course. The disposition of these lower courses varied somewhat from instrument to instrument. Praetorius discussed two

kinds of theorbos: the first he called a Roman style theorbo, which had six courses on the fretboard; and a second, which he called the Paduan-style theorbo that had eight courses on the fretboard.⁵ Because of this variation, players today may opt to have their seventh and eighth courses on their fretboards before the additional strings on the extended neck.

See figure 3.3 below. The advantage to having these additional courses on the neck is that

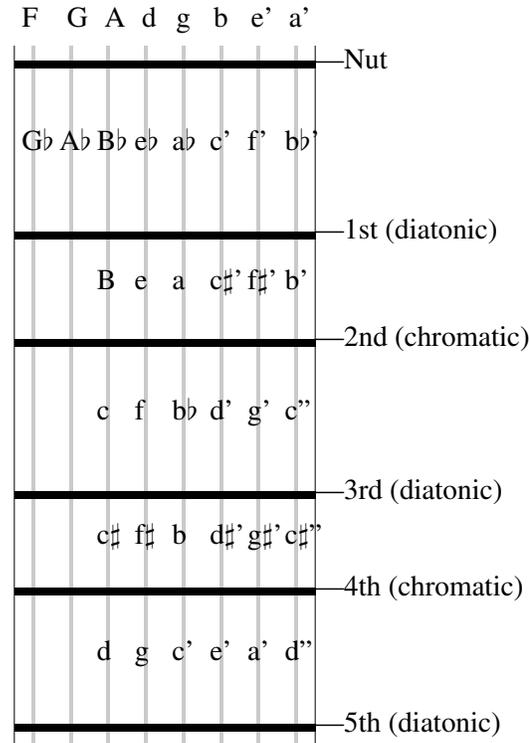


Figure 3.3: Theorbo with extend courses

a player is able to fret additional chromatic notes with the left hand. On the longer strings that are attached to the extension this is not possible and any chromatic changes in the pitches of those strings must be done using the tuning pegs prior to playing.

While it might have been possible to tune a lute's first course to a chromatic semitone, this was impossible on the theorbo. For example, the first fret had to be diatonic because of the open E♯ and B♯ on the second and third courses so that the pitches on those courses

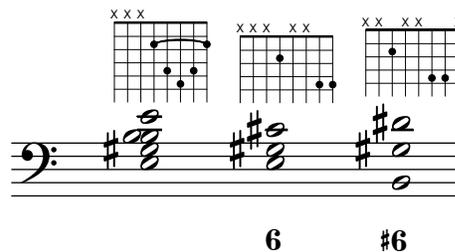
5. Praetorius, *Syntagma Musicum II: De Organographia Parts I and II*, 59.

on the first fret would be F \sharp and C \flat . If a chromatic semitone was used, an E \sharp and B \sharp would result, making this type of semitone unusable. Similarly, the presence of a B \flat at the third fret of the fourth course dictates that the next fret must be chromatic to create a B \sharp at the fourth fret of the same course. Even the sixth fret is determined to be diatonic because of the E \flat to F \flat that occurs on the third course. These restrictions could explain why Praetorius's theorbo had all of its extended courses on the long neck, and off of the fretted short neck. This would enable the player to set a fixed F \sharp and G \sharp on the seventh and eighth courses and leave the pitches on the first fret in their preferred diatonic semitones. Essentially, the frets of a theorbo tuned to meantone temperament were "fixed" in their positions because the location of the diatonic semitones between B and C, and E and F, determined whether the fret was either chromatic or diatonic. If we also take into consideration the similar issue of octaves affecting the position of frets on the lute in G, it is apparent that a theorbo should have its initial five frets arranged the same way as a lute, with a diatonic semitone for the first fret and alternating diatonic and chromatic semitones for successive frets. If we were to blend successfully with quarter-comma meantone, we will need to limit some of the semitones that are available on our instruments and find alternative methods of playing them.

3.2.1 Solutions Utilizing Re-entrant Tuning

The theorbo has the advantage of using both chromatic and diatonic semitones within the same octave. Because of the re-entrant nature of the theorbo's tuning, certain notes that are an octave apart on the lute are unisons on a theorbo. This offers a possible solution to

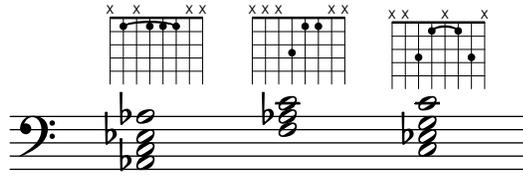
some of the problems of semitone size because a particular pitch could be a diatonic semitone at one fret while being chromatic at a different fret, and still be in the same octave. Referring to figure 3.3, the $A\flat$ on the first fret of the fourth course and the $G\sharp$ found on the fourth fret of the second course are in the same octave, whereas on a lute in standard G tuning they are an octave apart. Other notes are still an octave apart, such as the $E\flat$ on the fifth course of the first fret and the $D\sharp$ on the third course of the fourth fret. However, in continuo playing octave displacement does not matter and players are able to substitute different octaves as needed. Therefore, all the player has to do is choose the appropriate fingering for the left hand to obtain either the chromatic or diatonic semitone. For chords that require chromatic semitones, such as a $G\sharp$ or a $D\sharp$, the player can use pitches found on the fourth fret. Some of the more common left-hand chord patterns that use this fret include the E major triad and chords with a sixth above the bass. For others



Example 3.4: Chords using chromatic semitones on the fourth fret

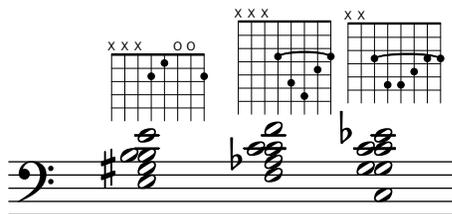
requiring diatonic semitones, such as the $A\flat$ or $E\flat$, the player may use pitches on the first fret. These include $A\flat$ major, F minor, and C minor. Although instances of an $A\flat$ major triad are rare, all the needed pitches are on the first fret, and F minor and C minor triads are both possible using a limited number of voices.

While re-entrant tuning makes it possible to play chords with different kinds of semitones,



Example 3.5: Chords using diatonic semitones on the first fret

sometimes the left-hand chord fingerings that result are not the easiest to execute, nor are they as idiomatic to the instrument as other more commonly used fingerings. More ideal chords for the theorbo are easier to execute, have more potential voices, and favor open strings whenever possible. The fingerings for F minor and C minor listed in example 3.5 are not commonly found in existing theorbo tablatures of the time, nor are they used very often among modern players. The more common fingerings for these chords use the fourth fret. Additionally, the E major chord in example 3.4 uses the chromatic semitone on the fourth fret, but ignores the open E and B on the third and second courses. More common chord fingerings below in example 3.6 indicate that a player would more likely use a fully-voiced F minor or C minor chord with a barre at the third fret than those listed previously. An E major chord that makes use of the open B and E strings sounds much more resonant and is easier for the left hand. The obvious problem with these more



Example 3.6: Common theorbo chord shapes

idiomatic chord shapes is that if they are used on a theorbo in meantone temperament,

their semitones are the opposite of what they should be. The E major chord shown above would have a diatonic A \flat instead of the chromatic G \sharp and the thirds of the F minor and C minor chords would be chromatic in nature instead of diatonic.

The question raised here is: how did players manage in a meantone temperament without the use of their common chord patterns? In a small ensemble, without keyboards or other instruments with meantone requirements, we might have the option of using a non-meantone temperament; however, if this is not desirable, we should look at other available modifications to make meantone more successful on our instruments. Selective left-hand fingerings alone are not enough to overcome the problems imposed by meantone temperaments.

3.2.2 Tastini

The advantages that re-entrant tuning offers can render more idiomatic chord shapes unusable in a meantone temperament. If we are to use these shapes, yet still be able to play in meantone, we need the ability to apply different semitones locally within a fret instead of being forced to have all the pitches at one fret of a certain size. In other words, we need to mix both chromatic and diatonic semitone sizes within the same fret. For example, consider the pitches of the extended bass courses: the first chromatic note on the seventh and eighth courses is determined by the quality of the first fret. Since the first fret on a theorbo is a diatonic semitone, this would make the pitches on this fret for these two lower courses A \flat and G \flat , respectively. It is far more likely that a G \sharp and an F \sharp are needed, but shifting the entire first fret to a chromatic semitone would alter the rest of the notes

and result in a B \sharp on the third course instead of a C.

To correct this problem and apply a chromatic semitone localized only to one or two courses, lute players during this time employed the use of *tastini*. The diminutive form of *tasto*, the Italian word for fret, these “little frets” were small pieces of wood that were glued to the fretboard to create a chromatic semitone on one or two courses while the remainder of the courses on the fret were diatonic. Courses beyond the sixth on a theorbo were used for bass support, and any that were on the fretboard, such as the seventh and eighth course, were only stopped at the first fret. This made the use of *tastini* an ideal choice since it only affected the first fret. Players now had the ability to use an F \sharp and G \sharp while keeping the rest of the pitches on the first fret at their original diatonic position. See figure 3.4.

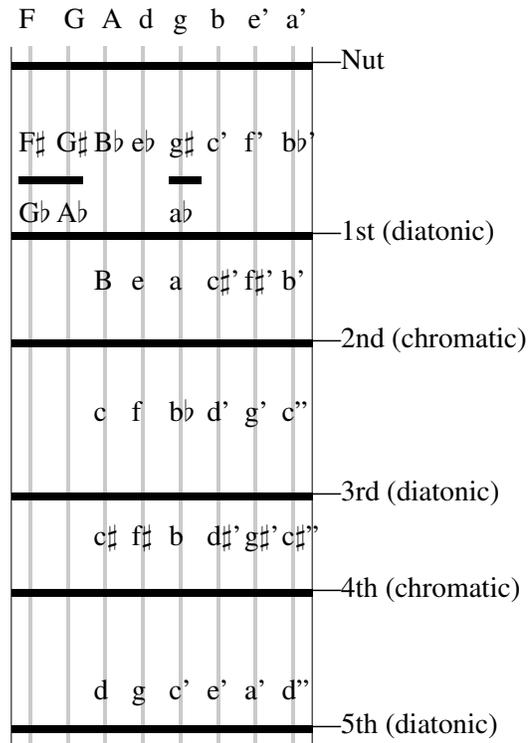


Figure 3.4: Theorbo with added tastini

Players today have employed *tastini* on other frets as well, which can help solve the

previous problems of idiomatic chord shapes in meantone frettings. Referring again to figure 3.4, an additional tastino on the fourth course can provide us with a G \sharp , which enables us to play the more common E major chord shape described in example 3.6 using the open strings on courses two and three. The problem of C minor and F minor chords, however, still remains. We could switch the entire fourth fret to a diatonic semitone, thereby giving us the needed pitches, but we would lose the ability to play some of the sixth chords described in example 3.4. One could argue that additional tastini at the fourth fret could correct this problem, but such a solution might become unwieldy. Also, it means sacrificing several chromatic semitones, in this case C \sharp , F \sharp and G \sharp , for the sake of two diatonic ones: E \flat and A \flat .

While modern players have embraced the use of tastini, there are no surviving instruments with their tastini intact. Yet, it is obvious they were in use because of the different historical accounts that describe them. The earliest of these comes from Vincenzo Galilei's *Fronimo*, where he did not have very good things to say about them. Galilei's description of tastini suggests that players were using them in different places on the instrument and not just the first fret.⁶ Today, tastini are found usually only on theorbos, almost always at the first fret, and rarely elsewhere. There are exceptions to this, and players today are free to put tastini anywhere, but the most common location seems to be at the first fret.

Galilei's main disagreement over the use of tastini was that it made adjustments to one fret only in a certain pitch context, for example when F \sharp is wanted instead of a G \flat , but that one adjustment does not work in other pitch contexts or match the same pitch at a

6. Vincenzo Galilei, *Fronimo*, *Musicological Studies and Documents*, volume 39, Translated and edited by Carol MacClintock (Rome: American Institute of Musicology, 1985), 165.

different location on the fingerboard. He also maintained that the lute was tuned in equal semitones and that a well-placed fretting system was sufficient to play all the pitches necessary. In his mind, tastini ruined that sort of system because if the lute were tuned in equal semitones, there would be no need for a tastino: the fret would function correctly as either a chromatic or diatonic semitone.

Whether or not we follow Galilei's advice, his attitude towards tastini is the most important indicator that they were in use. Some players must have used them at this time, otherwise Galilei would not have mentioned it. However, we must note that the manner in which Galilei describes their usage appears to have no direct application to correcting the problems found in quarter-comma meantone. After all, since Galilei tuned equally, meantone was not an issue for him. All of this points to the fact that if we are to use tastini on our theorbo or lute, it must be in a way Galilei had not envisioned, and so we are left to create our own solutions.

Bermudo has a brief account of tastini in the context of the vihuela. His description of their use fits very closely with our current usage of them:

For faults that may arise, take the advice given before of looking for the notes on other frets, or with the pressure of the finger when stopping the note, of by placing another fret in front of the principle fret, which, when placed for this purpose, should be thicker than the first fret so that it does not rub against the string. This [extra] fret can be placed by dividing the distance from the third fret to the bridge into eight parts, and wherever the compass reaches [downward from the third fret] will be the first fret, which will form *fa*.⁷

Recalling table 2.7, Bermudo's first fret is a diatonic semitone or what he calls *fa*. In order to obtain the chromatic fret, or *mi*, Bermudo proposes an additional fret that is front of the first fret, or placed between the nut and the first fret. He says the fret should be

7. Espinosa, "Juan Bermudo 'On Playing the Vihuela' ('De tañer vihuela) from Declaración de instrumentos musicales (Osuna, 1555)," 115-116.

slightly thicker, which we can infer is so that the *fa* fret does not prevent it from working properly. Although these are not true tastini, insofar as his fret spans the entire

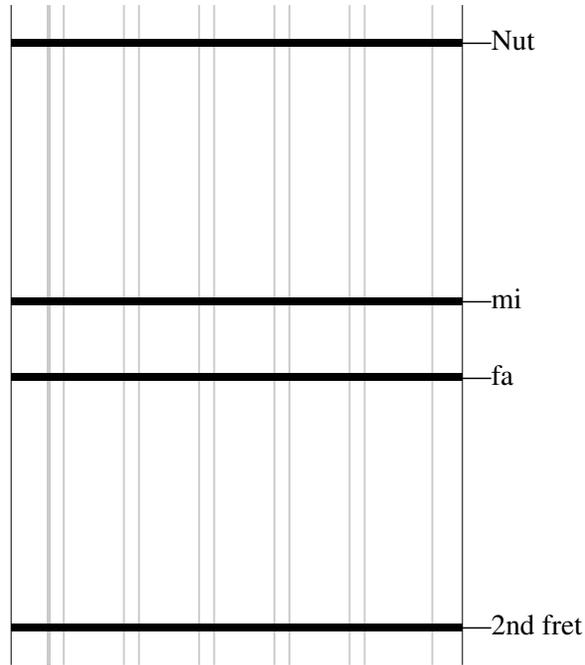


Figure 3.5: Bermudo's *mi* and *fa* first frets

fingerboard instead of just the affected course, it is the strongest evidence we have in support of using any kind of additional fret to create both chromatic and diatonic semitones.

A later reference to tastini from the seventeenth-century comes from Jean Denis who was a harpsichord builder during the first half of the the century. He refers to “staggered” frets on the lute which could be made of ivory. The reference appears in Lindley’s book, and the context in which Denis was discussing tastini was that someone had tuned a harpsichord in equal temperament. Denis criticizes this approach stating that someone should perfect the lute and viola da gamba so that they may accommodate unequal semitones instead of “ruining a good and perfect tuning in order to accommodate

imperfect instruments.”⁸

Another reference comes from Christopher Simpson’s *A Compendium of Practical Music* and appears to describe tastini as they are commonly used today on the theorbo:

I do not deny but that the slitting [*sic*] of the keys in harpsichords and organs, as also the placing of a middle fret near the top or nut of the viol or theorbo where the space is wide, may be useful in some cases for the sweetening of such dissonances as may happen in those places; but I do not conceive that the enharmonic scale is therein concerned, seeing those dissonances are sometimes more, sometimes less, and seldom that any of them do hit precisely the quarter of a note.⁹

The first part of Simpson’s description matches Bermudo’s additional fret exactly, as he describes an additional fret between the first fret and the nut. He does not state whether or not this middle fret spans the entire fretboard. The second part of his statement refers to the difference between the diatonic and chromatic semitone. Simpson calls these “quarter notes” as we today might refer to quarter tones, or half of a semitone. He seems to think that, from a practical standpoint, these semitones are seldom precisely what they are supposed to be, either diatonic or chromatic. Simpson’s opinions aside, he does provide us with evidence that tastini or additional frets were used historically in the same places on the fretboard of a lute as players might use them now. Beyond tastini, there are more possibilities to overcome some of the difficult issues pertaining to quarter-comma meantone temperament and frets.

8. Lindley, *Lutes, viols and temperaments*, 47.

9. Christopher Simpson, *A Compendium of Practical Music in Five Parts*, Reprinted from the Second Edition of 1667, Edited and with an introduction by Phillip J. Lord (Oxford: Basil Blackford, 1970), 51.

3.2.3 Other Solutions

Aside from tastini, there were diverse methods by which lutenists were able to coax meantone temperaments from their fretting. One of these involved placing frets at an angle so that a fret could be diatonic on one side of the fingerboard and chromatic on the other. For the theorbo, this could be used as a substitute for tastini. It is possible to use an angled first fret, for example, to achieve the chromatic semitones necessary on the seventh and eighth courses. Instead of placing a tastino at the left side of the fingerboard, the first fret could be slanted so that it angled towards the chromatic side of the semitone as it moved towards the lower courses.

The problem with this is that pitches in the middle of the fret are somewhere between chromatic and diatonic. Juan Bermudo discusses the practice of angled frets on the vihuela and comes to the same conclusions:

[...] some players hope to fix the abovementioned faults by putting the frets where the said faults occur at an angle, taking them out of line. This is not a solution but a cover-up [...] Take a fret where there is a fault (where it is *mi* for strings but needs to be *fa* for others) and you will find that, by slanting the fret, it does not hit any string in the right place.¹⁰

While we might be able to achieve a chromatic semitone at the eighth course, each successive course would be slightly sharper until reaching the top course. Only the first and last courses would be truly either chromatic or diatonic, the courses in the middle would be something in between and not in a specific temperament.

Despite what Bermudo and other writers of the time have said about angled frets, we can find use for them. Angled frets work best when they are strategically located and used for

10. Espinosa, "Juan Bermudo 'On Playing the Vihuela' ('De tañer vihuela) from Declaración de instrumentos musicales (Osuna, 1555)," 112-113.

frets that might have only one or two useful pitches on them. In a “standard” quarter-comma meantone fretting system, as shown in figure A.2 of the appendix, the sixth fret is diatonic and duplicates the E^b and A^b found at the first fret. A common left-hand fingering for the first inversion triad, or a chord with a 6 above the bass, uses the bass on the fifth and sixth courses. These include the first-inversion D major triad with the F^\sharp as well as the first-inversion A major triad with the C^\sharp , both found on the fourth fret. However, we lack the G^\sharp or D^\sharp for either E major or B major tonalities. If we move our sixth fret, which is commonly diatonic, so that it is placed at an angle, it is possible to get a very close approximation of a chromatic semitone for these pitches (see figure 3.6). The slanted sixth fret allows us to play the needed triads with G^\sharp and D^\sharp in the bass, and disadvantages are minimized because the sixth fret is not commonly used in other left-hand chord shapes. Alternatively, we could employ a tastino underneath these two

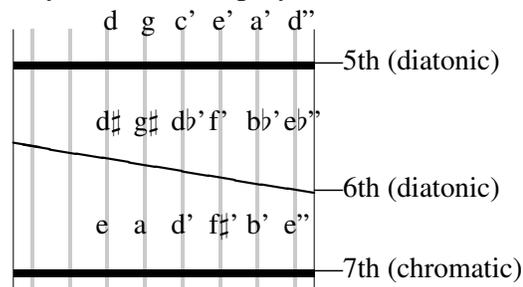


Figure 3.6: Theorbo with angled sixth fret

courses and avoid the slanted fret altogether; however, such a solution would be at the player’s discretion.

Other solutions for achieving a successful quarter-comma meantone temperament do not involve adjusting frets at all, and advocate positioning the left hand so that individual courses are pulled in one direction or another to raise their pitch slightly and compensate for frets that are using an incorrect semitone. Praetorius describes such a method in his

Syntagma Musicum:

Thus the semitones cannot be either major nor minor, but are, perforce, “intermediate” if anything. For I reckon that each fret [...] contains four-and-a-half commas, whereas the major semitone contains five and the minor semitone only four. Since the error is only half a comma either way, the ear hardly notices it with these instruments [...] Major and minor semitones are both produced by the same fret, both sound in tune, [...] especially since by particular applications of the finger to the string, over the fret, it is possible to have some control over the pitch of the note produced.¹¹

It is clear that Praetorius is describing meantone temperament, because he refers to chromatic (minor) and diatonic (major) semitones as having different numbers of commas. However, Praetorius is actually referring to sixth-comma temperament which divides its wholetones into nine commas, split four to five, versus a quarter-comma which contains five commas per wholetone and is divided two to three.

Syntagma was published in 1619, so we might assume that sixth-comma meantone had begun to replace quarter-comma in some musical circles, but it is still a meantone temperament. More importantly, Praetorius describes fretted instruments as having equal semitones, divided exactly in the middle between chromatic and diatonic, but essentially played unequally. According to him, the player has the ability to change the quality of the semitones so that they might be close to chromatic or diatonic as the music requires.

Additional evidence of using finger pressure to correct pitch problems is found in Bermudo’s treatise. He advises players either to locate the note somewhere else on the fingerboard or to use finger pressure to alter it, and refers to this method several times in his treatise, indicating that it might have been preferable to the addition of extra frets.¹²

Bottrigari also describes this same technique, when he says players “touch their frets a

11. Praetorius, *Syntagma Musicum II: De Organographia Parts I and II*, 68.

12. Espinosa, “Juan Bermudo ‘On Playing the Vihuela’ (‘De tañer vihuela) from Declaración de instrumentos musicales (Osuna, 1555),” 106.

little higher or a little lower”, as I mentioned earlier.¹³

Bending pitches using variable pressure in the left-hand, such as Praetorius, Bermudo, and Bottrigari describe, is possible, but not easily done. Another common way for a lutenist to bend pitches is to pull the course with the finger to one side or the other. This technique is very common in twentieth-century classical guitar literature where extreme fluctuations of pitch are exploited for various compositional reasons. Although there is no evidence of its use in historical lute tablatures or other musical sources for lute or theorbo, it is possible to conjecture that players during that time might have found a way to utilize it in one form or another in order to adjust to meantone in ensemble playing.

3.3 Meantone Fretting in Tablature Sources

Thus far, we have discussed the methods by which lute players can adapt to the problems of quarter-comma meantone fretting systems in ensemble situations. For the most part, these methods require the player to place certain notes on certain frets. In ensemble music, where basso continuo is used, the player has the ability to do this because the music is in staff notation and no left-hand pitches are dictated anywhere. In fact, it is customary for players to move bass notes into different octaves when necessary and even use reduction methods that omit repetitive notes. In essence, the player may re-compose sections of his or her part to fit the instrument’s compass and make it sound as idiomatic as possible.

In lute tablature sources, the placement of notes in the left-hand is precisely dictated. The

13. Bottrigari, *Il Desiderio*, 15.

player usually does not move notes to different locations on the fretboard in order to compensate for any potentially incorrect semitones in a meantone temperament. Because of this fact, tablature sources alone indicate that quarter-comma meantone temperament is not a viable temperament for solo lute literature in the late sixteenth and early seventeenth centuries. Additionally, ensemble music for lute and voice, where the lute is an accompanying instrument using tablature rather than basso continuo notation, further indicates that quarter-comma meantone is not usable, and a different kind of temperament would be preferable, such as sixth-comma meantone or another temperament specific to the lute.

The lute song repertory is a unique genre featuring a lute part written in tablature, one or more voices, and sometimes a bowed bass. Just as with solo music, players usually do not take the same liberties with pitch placement as they do when playing basso continuo. In 1597, John Dowland published the first book of music written in this new genre. Entitled *The First Booke of Songs or Ayres*, his accompaniments were very complete, and equal in caliber to his solo works for lute. The songs were composed in keys idiomatic for the lute in Renaissance tuning, such as G minor or major. Near the end of Dowland's first book of Ayres is a well-known song entitled "Come heavy sleep" that opens in G major but has a very striking key change to B major about mid-way through the song at measure 9. Such a change of tonality befits the subject matter of the song, but if we look closely at Dowland's placement of the pitches on the lute, they are at odds with a quarter-comma fretting system. In example 3.7, there are repeated instances of F \sharp and D \sharp in the first measure of our example, which are represented in the tablature part by the character *b*. English lute music was written in the French system of tablature, so the frets are indicated

Example 3.7: Dowland, "Come heavy sleep" from *The First Booke of Songs or Ayres* (1597), m. 14

Example 3.7: Dowland, "Come heavy sleep" from *The First Booke of Songs or Ayres* (1597), m. 14

with letters. The tablature character *a* is the open string and the character *b* is the first fret. If we were trying to follow the meantone fretting system I outlined earlier, these notes would be $G\flat$ and $E\flat$, and would sound quite strident against the B. Recalling Dowland's own choice for the first fret, as shown in table 2.7, the quality of semitone is diatonic, but is in sixth-comma and not quarter-comma. A sixth-comma fret would certainly be more palatable in this case and perhaps explains why Dowland himself was advocating a sixth-comma diatonic semitone instead of a quarter-comma one.

Other examples from Dowland's works highlight the central problem with employing meantone fretting systems on lutes. Because of the way in which fretted instruments are tuned, there are cases when both the diatonic and chromatic semitone are required at the same fret. For example, in his song *Sorrow stay*, there are both G minor chords and D major chords, which require a $B\flat$ and an $F\sharp$. However, looking at an excerpt from the song below, we can see that these notes are both placed on the same fret. The notes and their corresponding tablature characters are highlighted in red. Both notes occur on the

doe not, O doe- not my heart poore heart af-fright

a a e a d d c a e a c c a a a a

b a c e b d a b c d f e d c

c c d a d a c d a c c e a

Example 3.8: Dowland, “Sorrow stay” from *The Second Booke of Songs or Ayres* (1600), mm. 9–10

first fret, where there is the tablature character *b*. If we were to use a meantone fretting system, the $B\flat$ would be true but the $F\sharp$ would be a $G\flat$ instead, unless we were using a tastino to correct the problem. This kind of issue results when frets determine the quality of the semitone regardless of what the particular pitch might be. Keyboard and other instruments are exempt from this kind of issue because their semitones can be tuned independent of one another. A keyboard or organ can very easily have $F\sharp$ and $B\flat$ existing at the same time.

Referring back to Dowland’s own fretting instructions from the previous chapter, he described one kind of fretting system that remained fixed once it was set. While he seems to use a sixth-comma diatonic semitone at the first fret, which would ease the problem of having the $F\sharp$ and $B\flat$ on the same fret, he does not mention the use of tastini or any other corrective frets. Although it would not sound quite as pronounced as in quarter-comma, the difference between $G\flat$ and $F\sharp$ would still remain, even in sixth-comma meantone.

Since other composers and lute players were subject to the same tuning constraints, it seems Dowland was trying to create his own temperament that satisfied both semitone requirements in his works. How he managed to do this is still somewhat of a mystery, since his own fretting instructions are problematic. His contemporaries might have opted for an equal semitone approach by splitting the difference between frets or opting for positions that simply satisfied their ears. Yet, the ensemble dilemma would have remained, and applying an equal semitone solution, a sixth-comma meantone solution, or a completely original system of fretting, would mean that the lute would be attempting to play in tune with an ensemble that was using a different temperament.

The same issues that affect fretting systems for Renaissance lute also affect the theorbo as well. Although they are less common, there are tablature accompaniments for the theorbo, and just as we studied lute tablatures for clues regarding choices of temperament, we can examine theorbo accompaniment tablatures for the same information. An example of an early seventeenth-century accompaniment comes from Girolamo Kapsberger, a theorbo player and composer who was active in Rome. In addition to publishing several books of music for solo theorbo, he published four books of villanelle written for one, two, and three voices, with written accompaniment for guitar and theorbo. Additional instruments could have been used, doubling the vocal parts, but Kaspberger is non-specific as to which kinds. The vocal and bass parts are written in staff notation, while the guitar and theorbo parts are written in their own specialized notation. In the case of the guitar, alphabetic notation is used, where a series of different letters indicate which chord to play. The theorbo part is in Italian tablature, where numbers are used in place of letters to indicate fret placement and the order of strings is actually inverted from French tablature, placing

the top string of the theorbo on the bottom line of the tablature staff. For the sake of consistency, I have transcribed Kapsberger’s tablature part into French tablature so we can compare the examples with lute and theorbo.

The song “All’ ombra”, from his first book of villanelle, contains a cadence in A major towards the end of the piece. The G \sharp and C that are used are highlighted in red. Similar to

The image shows a musical score for the piece "All' ombra" by Kapsberger. It consists of four staves. The top two staves are vocal lines in 3/2 time, with lyrics "fa di per lei sa do- ri" and "fa di per lei sa do- - ri". The third staff is a bass line. The fourth staff is a French-style lute/theorbo tablature with six lines labeled a, c, d, b, c, a from top to bottom. The notes are: a c d c | a c d c | a c d c | a c d c. The second measure of the second system has a red 'b' on the d line and a red 'b' on the c line.

Example 3.9: Kapsberger, “All’ ombra” from *Di Villanelle*, bk. 1 (16??), mm. 21–22

our previous example from Dowland, these two notes are found on the same fret, indicated with the tablature character b highlighted in red. If we were employing quarter-comma meantone on our theorbo, the G \sharp would instead be an A \flat . If this indeed was the case, this difference in semitone quality could simply have been accepted. On the other hand, it seems more likely that an alternative temperament was chosen that would have made the G \sharp more usable.

If Kapsberger’s theorbo was definitely in quarter-comma meantone, a tastino would have

been the only solution to avoid the A^b issue on his first fret. An alternative solution that I proposed earlier would be to change the left-hand fingering of the E major chord so that the G^\sharp on the fourth fret is used resulting in a chromatic semitone and not a diatonic one. However, Kapsberger's tablature clearly indicates that the G^\sharp on the first fret is to be used. While these examples represent only a fraction of the types of problems that lute players faced when tuning to meantone temperaments, the issue of chromatic versus diatonic frets was so pervasive in the literature that it was impossible to ignore. According to historical evidence, some of the techniques lute players employed in their attempts to address the conundrum were controversial. When we are playing in meantone temperaments today, we must be willing to do the same and apply our own solutions, controversial or otherwise. In the concluding chapter of this study, I will summarize all of the findings presented here and how we might apply them in today's performances.

Chapter 4

Summary of Solutions

Lute players today who want to perform music from the sixteenth and seventeenth centuries must reconcile the use of historically appropriate temperaments on their instruments. Because of the nature of fretted instruments, using temperaments with unequal semitones requires a different approach than is used with keyboards or other instruments. In some ways, lutes have greater flexibility with temperaments than keyboards do, but this flexibility results in a multitude of choices and options that players must consider when fretting their instruments. Such choices can also reflect personal opinions rather than historical accuracy.

To realize temperaments with unequal semitones on the lute, there are two basic approaches, both of which are supported by historical evidence. The first is the “fixed semitone” method where each fret is one type of semitone: chromatic, diatonic, equal, or something unique. The second approach treats the lute as an enharmonic instrument, similar to harpsichords with split keys. In this method, two kinds of semitones, such as the diatonic and chromatic, are available at certain locations on the fretboard using split or

slanted frets, or with the aid of tastini. The choice is entirely at the discretion of the player, and either technique can work regardless of the context in which it is used.

In this chapter, I shall elaborate on the flexibility of these approaches and describe scenarios in which either can be employed successfully. I shall also discuss methods for adjusting frets in order to perform with keyboards and other instruments in ensembles.

Temperaments during the Baroque period were not universally applied. While the modern musical world relies on established standards of pitch as well as temperament, such standards did not exist at this time in musical history. Pitch could vary from city to city, and even from ensemble to ensemble. This same variation applied to temperaments as well, not only between different ensembles, but between different instruments in the same ensemble. This is perhaps a reason that the lute held such an important place: it could conform to the different requirements of temperament more easily than keyboards; however, this is not to say that specialized lute temperaments were limited to ensemble music.

4.1 Frets with Fixed Semitones

As lute players, we have the option to “fix it and forget it”, or in other words, set our frets to a certain position and leave them. Players such as Dowland probably used this option, as well as vihuela composers like Bermudo. This requires a customized kind of fretting system in which we accept the limitations of the temperament and either set our frets in a way that makes the entire fretboard available to us, or select which semitones we want to play on which fret. The historical sources presented in chapter two showed a predominant

use of these kinds of customized temperaments consisting of different kinds of semitones used throughout the octave. Neither do they conform to a particular kind of meantone, such as quarter-comma or sixth-comma, nor are they consistently Pythagorean or equal in nature.

Evidence clearly indicates that fretting systems approximating equal temperament had their place and purpose on the lute, while most other instruments preferred to use a non-equal temperament. So why is it that fretted instruments held this exception? The reason seems to be that it was simply easier for a lutenist to divide the octave into twelve approximately equal semitones than it was for a keyboardist. A keyboardist would temper intervals aurally, counting beats between notes, while a lute player could visualize the fret distances and create quasi-equal semitones by making a good visual approximation between two existing frets and placing the fret somewhere in the middle.

The visual method of approximating equal semitones could also take an iterative approach as well, wherein the fret is adjusted several times while playing to find the position at which the semitones sound the best. Whether by visual placement or trial and error, the semitones that resulted from such methods were not today's standard of "Equal"

Temperament, but an "equal-ish" temperament. These kinds of customized temperaments would be irregular in nature, with semitones of varying size throughout the octave.

Depending on the semitone, however, they would be much closer to modern equal temperament than any of the existing meantone temperaments of the time.

The use of temperaments that could approximate equal semitones was a feature of the lute that would have appealed to amateurs. The treatises from which we take our fretting instructions were often written for amateur musicians and intended for their education. A

simple fretting plan that yields a quasi-equal temperament would simplify matters for someone who was still learning how to play the instrument. As players became more experienced, they could start moving frets around to their liking, creating a temperament that could selectively use different sized semitones. Bermudo describes this when he refers to musicians who move their frets according to their ears, but he instead wants to make the vihuela “more perfect” with equal semitones for inexperienced players.¹

Temperaments which are almost equal, as well as other customized schemes, would also be suitable for solo lute music and repertoire for small ensembles such as the lute and voice. For example, the Dowland song *Come heavy sleep* discussed in the previous chapter would benefit from a temperament with fifths that were not too narrow. In such a temperament, the D \sharp and F \sharp would not sound quite as strident as they would if were in quarter-comma meantone. Some of the fretting proposals discussed in the previous chapter could work in this regard, with fifths wider than quarter-comma, although yet another solution is to use a different type of meantone.

Such varieties of meantone were presented in chapter one, and in chapter two we saw that many of the historical fretting sources preferred sixth-comma meantone for certain frets, and that it figured quite prominently for the first, third, and sixth frets (see table 2.7). A similar temperament was also typical for other fretted instruments such as cittern, which was commonly tuned somewhere between quarter-comma meantone and equal temperament.² Sixth-comma meantone is essentially the midway point between quarter-comma meantone and equal temperament, because some of its thirds are tuned

1. Espinosa, “Juan Bermudo ‘On Playing the Vihuela’ (‘De tañer vihuela) from *Declaración de instrumentos musicales* (Osuna, 1555),” 78.

2. Peter Forrester, “Wood and wire: cittern building,” *Lute News* 75 (2005): 12.

slightly wider than pure, but not as wide as in equal temperament. Other meantone temperaments whose thirds were wider than pure work well in early sixteenth-century repertoire such as the music of vihuela composers Alonzo Mudarra and Luis Milan.³ Lastly, we can refer to Praetorius, in his *Syntagma Musicum*, describing frets with $4\frac{1}{2}$ commas.⁴ Although he is describing a semitone divided equally, it is still sixth-comma in nature because the whole tone has 9 parts, indicating that the frets were initially set in that kind of temperament.

Sixth-comma meantone is an excellent choice for fretted instruments because it does not restrict the instrument as much as quarter-comma does, and generally does not require many modifications to make it successful. At the same time, it makes a good compromise between equal semitones and the pure thirds of quarter-comma meantone, allowing the lute to play all the semitones in the octave without much difficulty. I can say both personally and anecdotally that many lutenists prefer to use sixth-comma meantone as their default temperament for most music. It works better on the lute than quarter-comma, and also retains the qualities of affect and expression that are lacking in equal temperament.

Both custom fretting schemes and sixth-comma meantone are ideal for the lute in either solo music or small ensembles. For example, in lute song repertoire and in ensembles with other fretted instruments, violins, flutes, or other instruments without fixed semitones can conform to the temperament of the fretted instruments. Problems can arise when fretted instruments are combined with keyboards, which are not tuned in sixth-comma, or

3. William Bernard Hearn, *Performing the Music of Alonso Mudarra: An Investigation into Performance Practice in the Music of the 'Vihuelistas'* (Doctor of Musical Arts, University of Arizona, 1995), 56.

4. Praetorius, *Syntagma Musicum II: De Organographia Parts I and II*, 68.

if the members of the ensemble prefer quarter-comma meantone.

It is possible to play a lute in quarter-comma meantone using a fixed system with one semitone per fret; however, players must restrict the placement of certain semitones on the fretboard. They should choose the semitones needed for a particular piece, and place them in their designated location on the fretboard. For a theorbo in A, for example, we can set our frets using the standard quarter-comma fretting pattern for lute described in chapter three (see figure A.2 in the appendix for the complete fretting). In this pattern, C \sharp and F \sharp are found on the fourth fret, and if we slant our sixth fret as described in figure 3.6, we would also have the G \sharp as well. Depending on the piece of music, that might suffice for our needs.

If the semitones vary from piece to piece, or between sections of a larger work such as an opera, players can change the position of the frets if they have enough time when not playing. Alternative suggestions would be to slant certain frets, either before or during performance, or to tie a double fret so that it could be split and one side moved higher or lower than the other. Using varied pressure with the left-hand to alter the pitch is another possibility, but this has a limited effect as it will also reduce the resonance of the stopped note, muffling it slightly. Beyond these types of surface fixes, we must use extended methods if we want to have more than one kind of semitone at a fret.

4.2 Enharmonic Fretting

If we truly need two different kinds of semitones available to us, and fixed frets do not work, then we must use either tastini or additional frets tied to the fingerboard. This offers

the best possible solution and treats the lute as an enharmonic instrument, allowing both chromatic and diatonic semitones to be played. The drawback is that it increases the complexity of the instrument substantially, and requires added technical skills on the part of the player. Players who work in ensembles that use temperaments such as quarter-comma will opt for enharmonic fretting if they wish to have a completely workable solution. Players can also choose enharmonic fretting in solo literature. Generally speaking, one tastino will make a chromatic semitone for a single course, while the remaining courses are diatonic. A common example of this are the tastini found between the nut and first fret, as we saw in figure 3.4. Whether stopping a course using a fret or tastino, the best resonance is created on the instrument when the finger of the left-hand stops the course just behind the fret or a tastino, being as close to it as possible without going past it. Usually, this process results in one kind of semitone per course so that, if a tastino were used at one course, that course would contain a chromatic semitone, while the others would be diatonic. However, some players cultivate the skill of playing both the diatonic and chromatic semitones on a course where a tastino is used. In that case, the diatonic semitone is played with the finger placed between the tastino and main fret, and the chromatic with the finger placed just behind the tastino. Skill is required to navigate the space between the fret and tastino, having enough space between the two—and a small enough finger—to stop the course correctly in order to achieve a good sound.

Tastini can be affixed to the fretboard in various ways. Ivory was the material of choice during this time, but since it is illegal today, a toothpick and a piece of double-sided tape will work just as well. The main disadvantage of tastini, once a player is accustomed to

using them, is that they may slip or break. More permanent solutions are possible, such as using glue and a more durable wood, but this forces the instrument into a particular temperament which would needlessly restrict its use when professional needs dictate tuning in a variety of temperaments. Since permanent tastini are therefore not an ideal solution, the “tape and toothpick” approach might be the best option.

For players who wish to extend a lute’s capabilities even further, and avoid some of the problems associated with tastini, we can make available chromatic and diatonic semitones on all courses. Recalling Bermudo’s example of an additional *mi* fret in figure 3.5, if an extra fret spans the entire width of the fretboard, we have the option of playing either semitone on every course. Bermudo seems to suggest that players at the time were comfortable doing this with their left hand. The scale of the instrument would have a direct effect on the difficulty of such a technique. For example, on smaller lutes with a mensur of 60 centimeters or less, the distance between a diatonic and chromatic semitone in quarter-comma meantone at the first fret is just over a centimeter. Depending on the size of one’s fingers, it might be challenging to finger the diatonic semitone in this space. On instruments with a longer mensur length, such as the theorbo where distances of 85 centimeters or larger are not uncommon, this distance increases to almost two centimeters.

Instruments with longer mensur lengths have a greater distance between their semitones and therefore could hold a small advantage in situations where different semitones are required. The larger distance would make it easier for an experienced player to choose the diatonic semitone over the chromatic one, and vice-versa. The thickness of the fret would also be a factor because the chromatic fret would need to be slightly larger in diameter

than the diatonic one. If this were not the case, and the chromatic fret were smaller in diameter, then the thicker diatonic fret would vibrate against the course if it were stopped using the chromatic fret. While a larger diameter chromatic fret would prevent this, it would also decrease the available fretboard space between pitches, making larger instruments with a longer mensur an ideal choice for enharmonic fretting solutions.

Ensemble situations are often the main cause of problems reconciling temperaments with fretted instruments. Theorbos and archlutes, which also have longer mensur lengths, can enjoy preferred status not only because of their increased volume due to their body size and string length, but also because they may navigate issues of semitone size better than some of their smaller counterparts.

4.3 Playing with Ensembles

When performing with large ensembles, or groups with keyboards and other fretted instruments, players must take special care to ensure that each musician's tuning and temperament matches everyone else's. Typically, this means that the ensemble must agree about the temperament, and tuning and fret placement will be done according to that choice. For example, let us suppose that the temperament of choice for an ensemble is quarter-comma meantone. Prior to rehearsal, we would choose either fixed frets or an enharmonic strategy, and use measurements or an electronic tuner to place our frets and tastini in the correct positions. At rehearsal time, we would then tune our instrument to the appropriate pitch level, and hope for the best! However, in an actual rehearsal situation, things usually do not always work this way.

In ensembles with keyboard instruments, you are at the mercy of the keyboard tuner. I have been fortunate to have worked with many excellent tuners that can tune an instrument in any temperament. However, keyboards sometimes are not tuned regularly, and heat or humidity changes in the room in which the instrument is kept can have drastic effects on the pitch and stability of the instrument. No matter if a keyboard was tuned correctly to quarter-comma meantone, a day or two later it will have changed slightly with any change in the weather or climate conditions, and we as lute players must adapt to those changes.

The tenuous nature of keyboard tuning, especially harpsichords, means that lutenists must be able to tune their instruments to the keyboard, and more importantly, adapt to a temperament that may not necessarily be found in a chart of fret measurements or on the dial of an electronic tuner. If we arrive at a rehearsal with our instrument correctly tuned to quarter-comma meantone, but the keyboard is not, then we and the rest of the ensemble must adjust our frets to match its tuning, whatever that is. In order to do this, it is important to remember the general concepts of temperaments on a lute, rather than focus on technical specifics, such as determining the exact temperament of the keyboard.

Knowing the name of the keyboard's temperament is not as important as being able to play in tune with it.

When faced with a keyboard in an unknown or unspecified temperament, the first thing to do is ascertain the overall pitch level of the instrument. Is its A tuned to 415 Hz, to 440 Hz, or somewhere in the middle? You will need to match the pitch level of your instrument to the keyboard. Next, examine the more common semitones such as F \sharp , C \sharp , and B \flat , then compare them to those found on your instrument or to an electronic tuner.

Do they match your pitches, or the quarter-comma semitones as indicated on the electronic tuner? If they do not, then you will need to adjust your frets accordingly. You will also need to determine the qualities of the remaining semitones. Does the keyboard have an E \flat instead of a D \sharp , or a G \sharp instead of an A \flat , and where is its wolf fifth? Finally, and most importantly, are the semitones consistent across all the octaves of the instrument? This can help you and the other members of the continuo section determine who is best equipped to play specific accidentals in the music.

If the keyboard's tuning no longer matches yours, then you must discard all of the fret placements you calculated earlier; however, this is not as drastic as it may seem. Even if the keyboard has morphed into some unknown temperament, whether by climate change or a tuning error, adjusting a few frets on your instrument will probably be enough to correct the situation. Assuming you are using a theorbo, tune your open A courses to the keyboard. Since the first fret is always a diatonic semitone, adjust the first fret of the theorbo to match the B \flat of the keyboard. Because this will affect the tuning of the seventh and eighth courses, you will need to make additional corrections. If your eighth course is on the long neck of the instrument, you can simply tune the course to either F or F \sharp to match the keyboard. If your seventh and eighth courses are on the fingerboard, then you can either slant the fret at an angle or adjust your tastini to produce a G \sharp and F \sharp that match the keyboard. If you are short on time, or your tastini do not cooperate, you can elect to use an A \flat for the seventh course and adjust your sixth fret so that you have the G \sharp available on the fifth course (see figure 3.6). You can make similar adjustments to the fourth fret to produce an F \sharp or you may tune your eighth course to F \sharp , even if it is on the fretboard. You will have to remember, however, that the F \natural is no longer available. This is

perhaps why some seventeenth-century theorbists used a F \sharp for their lowest course, instead of the low octave G, so that they could have both the F \natural and the F \sharp available as open courses.

After setting your first fret and determining how you will fret chromatic semitones of the lower courses on the fingerboard, only a few other frets remain to check and adjust, if necessary. The second fret will be chromatic because it contains the F \sharp and C \sharp ; you will need to ensure that these match the keyboard as well. Finally, check all the open strings of your instrument and tune them to the keyboard. With your frets properly adjusted, if the keyboard is in a consistent temperament, then your instrument should match the keyboard.

In an ideal world, the keyboard is kept in the proper conditions and tuned regularly so that its temperament will be stable and you will not have to make any drastic adjustments to your frets other than tuning your instrument's overall pitch level to that of the keyboard. Even if this is the case, there will still be certain pitches, chords, and voicings that simply sound better on one instrument and not the other. In this scenario, one solution is for the continuo section to divide the responsibilities of each player so that problematic figures go to the instrument with the best tuning for that particular figure. A lutenist can also change the voicing of a chord so that offending notes are not placed in the highest tessitura of the chord, where they are easily heard, but instead, placed in the lower registers where they are obscured from the listener.

In cases where your instrument's chromatic notes or accidentals do not agree with another instrument's, despite your corrective steps, you can always omit them. Andreas Werckmeister, whose irregular temperaments are an excellent choice in

seventeenth-century repertoire, published a treatise on accompaniment which is partially translated by F. T. Arnold:

It is also not advisable that one should always just blindly play, together with the vocalists and instrumentalists, the dissonances which are indicated in in the Thorough-Bass, and double them⁵

Often, these dissonances are semitones that may be either diatonic or chromatic in nature; therefore, if there is any disagreement between you and the other instruments, it is best solved by omitting the offending note. This technique can be applied to other notes which may not match among different instruments, such as the thirds of chords. In that case, the lute player or keyboardist can elect to play an open fifth, and allow another instrument or the soloist to complete the harmony with the third. In this way, disagreements of temperament are solved by avoiding the confrontation altogether.

Above all else, the most important aspect of utilizing temperaments when performing with ensembles is that your instrument should sound its best. For example, even with all the instruments of an ensemble tuned precisely to quarter-comma meantone, there still may be disagreements in pitch simply due to the proclivities of each instrument. This is why all our practical examples and the historical evidence support a great amount of flexibility in applying tuning and accompaniment techniques. There is no doubt that musicians at this time employed unequal temperaments, but they were also very aware of the difficulties that these temperaments could create. We must, then, acknowledge that they too grappled with them and were still able to create exquisite and beautiful music.

5. F. T. Arnold, *The Art of Accompaniment from a Thorough-Bass*, vol. 1, Introduction by Denis Stevens (Mineola, New York: Dover Publications, Int., 1965), 210.

4.4 Conclusion

Regardless of the instrument's size, tuning, or repertoire, lute players have many different options when it comes to choosing a temperament. One can review all the historical evidence, and propose as many different fretting solutions as one is able, but the principle will always remain the same: the choice of fretting rests with the player. The sources are quite clear that there were a multitude of different choices at one's disposal, some of which aroused heated arguments between musicians. The same is true today and one can find very spirited discussions of tuning and fretting in various online forums dedicated to topics on tuning and lute playing. Similar arguments can be found in print too, and that will never change.⁶

What is certain is that temperaments with unequal semitones were an aesthetic choice for musicians during this time. Theoretical arguments against the use of equal temperament existed, such as the Pythagorean wholetone which could not be divided equally into two geometric halves. The appearance of logarithmic mathematical functions in the seventeenth century allowed theorists to solve this particular problem, and divide a wholetone into two identically equal parts. Still, musicians favored their unequal semitones because they found the resulting temperaments more aesthetically pleasing, and congruent with the musical ideals of affect and dissonance. Methods for approximating equal temperament were widely known as early as the fifteenth century, yet the resistance to adopting them was a clear indication that they were considered inferior to other temperaments that created purer thirds, and more pungent dissonances, at the expense of

6. David Dolata, "Tunings and Temperaments: Theory in the Service of Practice," *LSA Quarterly* (February 1995): 26–29.

limiting key choices.

Another fundamental aspect of unequal temperaments is the quality they give to different pitches and keys. In meantone temperaments, the difference between F \sharp and B \flat can be distressing, an effect that composers utilized. They chose certain keys based on the intervals and dissonances within them, and used them to excite the senses and provoke the passions. Unequal temperaments gave composers the expressive power to do this. Today's modern ideals do not cultivate these effects and the differences between semitones are homogenized, making all keys and accidentals equal. However, in the sixteenth and seventeenth century, differences between semitones created differences between keys and spawned perceptions of the "characters of the keys" or the idea that C major has a quality that is fundamentally different than A major, beyond the simple issue that one is higher or lower in pitch than the other.

Temperament was a concept that pervaded all music during this time, and we who have chosen to specialize in the field must recognize its importance in imbuing music with special expressive qualities. This requires careful consideration and a deep understanding of the issues at work. As lute players, we have additional obligations because we have the option of curtailing some of these aesthetic goals for practical solutions. It seems that lutes and other fretted instruments were granted a kind of immunity and were allowed to tune equally. Bottrigari seemed to think so, and he was probably not alone in his beliefs.⁷ However, this does not absolve us from attempting to meet these aesthetic goals in the first place.

Modern conveniences, such as electronic tuners, can make it seem easy to choose one

7. Bottrigari, *Il Desiderio*, 19.

temperament over another, and then move between them; however, our colleagues in the past had no such tools and relied on their ears. This required a flexible approach from lute players who needed to match with different kinds of ensembles. The sources examined here bear this out, with multiple methods for setting frets, and iterative techniques that instruct the player to try different positions and decide on placements that sound the best. Even these treatises disprove the notion of an exact placement scheme. For example, every source instructs players to place the fifth fret at a pure ratio, making it an un-tempered perfect fourth. Because players use the fifth fret to tune the open strings, such an interval would create havoc when tuning all the courses. The major third between the fourth and third courses would be intolerably wide! Tempered keyboard instruments could have offending intervals, such as wolf fifths or other unusable thirds, hidden away in less obvious places. Lutes did not always have this ability.

Setting temperaments on the lute requires diligence and a great amount of experimentation to yield effective results. While we are allowed to use a kind of equal temperament, it is never the preferred choice. As lute playing evolves through the 21st century, we will continue to re-examine these arguments, and hopefully reach the same conclusions. The lute can and did play in a variety of temperaments, both in solo and in ensemble contexts, but the effectiveness of its performance shall always rest with the skill of the player. No one would ever question the choice of temperament in performance unless it distracted the listener from the quality of the music because it was badly tuned or was the wrong choice for the repertoire. Therefore, the choice of temperament should be based upon what would best serve the music, by enhancing its expression and giving it the warmth and brilliance it justly deserves.

Appendix A

Complete Fretting Diagrams

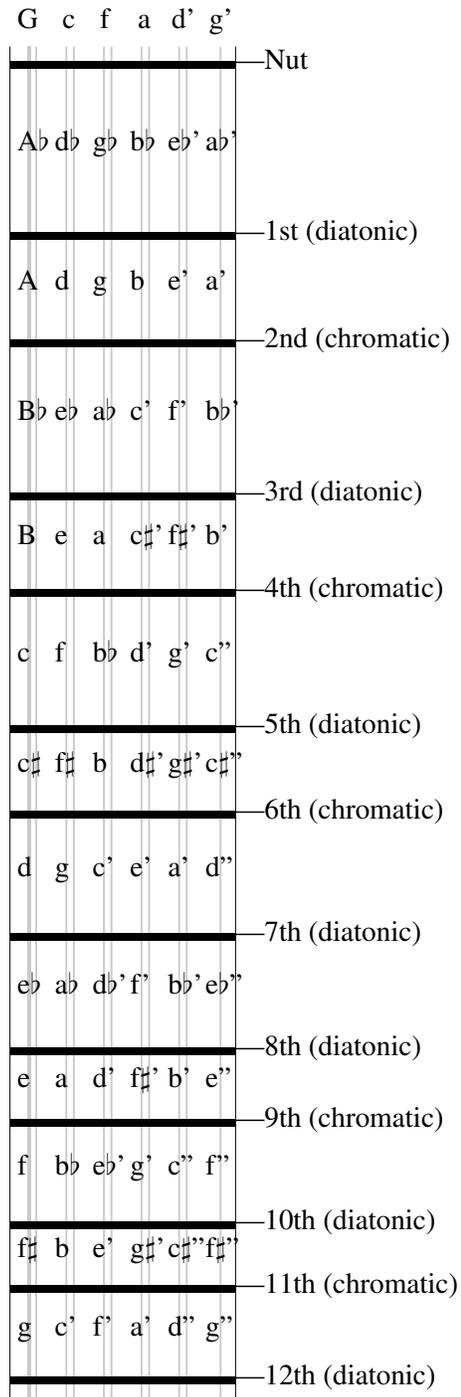


Figure A.1: Complete quarter-comma fretting for lute in G

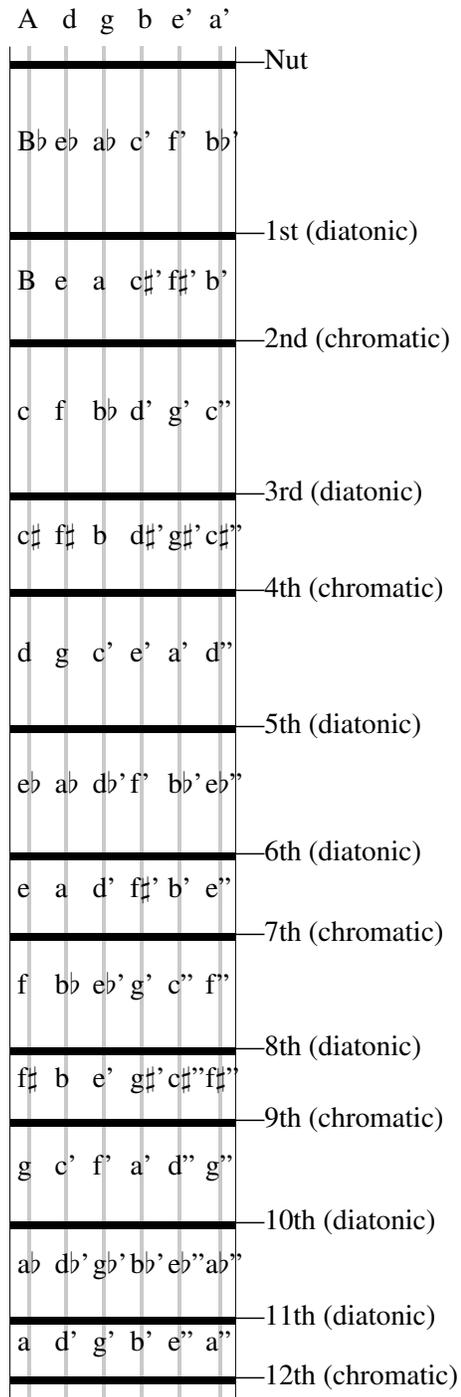


Figure A.2: Complete quarter-comma fretting for theorbo (extended courses not shown)

Appendix B

Fret Placement Guide

Below is a set of tables for determining the placement of a fret or pitch for any of the temperaments discussed in this paper. Table B.1 covers fretting systems with 12 individual frets. Table B.2 covers enharmonic fretting systems where a total of 19 frets are possible, and assumes a standard Renaissance lute pitch of G. Because of this assumption, frets for B \sharp and E \sharp are provided so we can also place frets for lutes tuned in F, A, or other nominal pitches.

To determine the placement of a given fret, find the coefficient for the pitch you wish to use and multiply it by your mensur length. Use the resulting value, and measure from the nut to find the exact position of the fret. For example, if a lute has a mensur of 65 centimeters, and we want to use Dowland's first fret, multiply 65 by 0.0606 for a value of 3.939. That is the distance, in centimeters, from the nut to the first fret using Dowland's own placement.

Temperament / Fret	1	2	3	4	5	6	7	8	9	10	11	12
Pietro Aron	0.0429	0.1056	0.1641	0.2000	0.2523	0.2844	0.3313	0.3600	0.4018	0.4409	0.4661	0.5000
Hans Gerle	0.0606	0.1111	0.1616	0.2058	0.2500	0.2917	0.3333	n/a	n/a	n/a	n/a	0.5000
John Dowland	0.0606	0.1111	0.1515	0.2008	0.2500	0.2917	0.3333	n/a	n/a	n/a	n/a	0.5000
Mersenne 1	0.0625	0.1111	0.1667	0.2000	0.2500	0.2889	0.3333	0.3750	0.4000	0.4371	0.4667	0.5000
Mersenne 2	0.0556	0.1070	0.1576	0.2044	0.2476	0.2930	0.3298	0.3670	0.4022	0.4354	0.4667	0.5036
Ganassi (initial)	0.0555	0.1111	0.1667	0.2084	0.2500	0.2917	0.3333	0.3750	n/a	n/a	n/a	n/a
Ganassi (final)	0.0508	0.1111	0.1563	0.2099	0.2500	0.2881	0.3333	0.3672	n/a	n/a	n/a	n/a
Bermudo I	0.0636	0.1111	0.1563	0.2099	0.2500	0.2977	0.3333	0.3757	0.4074	0.4375	n/a	n/a
Bermudo II	0.0617	0.1111	0.1563	0.2099	0.2500	0.2963	0.3333	0.3745	0.4074	0.4375	n/a	n/a
Bermudo III	0.0564	0.1093	0.1563	0.2066	0.2500	0.2923	0.3319	0.3672	0.4049	0.4375	n/a	n/a
17:18 Rule	0.0556	0.1080	0.1576	0.2044	0.2486	0.2903	0.3298	0.3670	0.4022	0.4354	0.4667	0.4964
Equal Temperament	0.0561	0.1091	0.1591	0.2063	0.2508	0.2929	0.3326	0.3700	0.4054	0.4388	0.4703	0.5000

Table B.1: Table of coefficients for 12-fret systems

Temperament	G \sharp	A \flat	A	A \sharp	B \flat	B	B \sharp	C	C \sharp	D \flat	D	D \sharp	E \flat	E	E \sharp	F	F \sharp	G \flat
Third Comma	0.0358	0.0704	0.1037	0.1358	0.1667	0.1966	0.2254	0.2531	0.2799	0.3057	0.3305	0.3545	0.3777	0.3999	0.4214	0.4422	0.4622	0.4814
Quarter Comma	0.0437	0.0649	0.1058	0.1449	0.1638	0.2004	0.2353	0.2522	0.2849	0.3008	0.3313	0.3606	0.3747	0.4021	0.4282	0.4409	0.4653	0.4771
Fifth Comma	0.0472	0.0624	0.1067	0.1489	0.1625	0.2020	0.2397	0.2519	0.2872	0.2986	0.3317	0.3632	0.3734	0.4030	0.4312	0.4403	0.4667	0.4752
Sixth Comma	0.0492	0.0611	0.1072	0.1511	0.1617	0.2030	0.2421	0.2516	0.2884	0.2973	0.3319	0.3647	0.3727	0.4035	0.4328	0.4399	0.4675	0.4741
Seventh Comma	0.0504	0.0602	0.1076	0.1526	0.1613	0.2036	0.2437	0.2515	0.2892	0.2965	0.3320	0.3657	0.3722	0.4039	0.4339	0.4397	0.4680	0.4735
Eighth Comma	0.0513	0.0596	0.1078	0.1536	0.1609	0.2040	0.2448	0.2514	0.2898	0.2960	0.3321	0.3663	0.3719	0.4041	0.4347	0.4396	0.4683	0.4730

Table B.2: Table of coefficients for 19-fret systems

Appendix C

Calculations

Here are the mathematical details used to calculate whole-number ratios for each fret. Formulas are grouped according to source and the order of frets according to how they appear in each source. Each instruction method focuses on where to place the fret on the neck of the instrument, but in order to determine the ratio we must find the vibrating length. Therefore, each passage will have two calculations associated with it. One denoted F which represents the location of the fret and another denoted V which represents the vibrating length. In all cases, the mensur length of the string will be represented by the constant m .

The calculation of fret placement (F) is determined according to the given passage from the instructions, while the vibrating length (V) is calculated as the difference between the fret distance and mensur length.

$$V_x = m - F_x$$

As calculations proceed through a given source, the source often refers back to frets that have already been placed. For this, each fret calculation is referred to by name, such as F_3 which would refer to the fret placement calculation for the third fret.

C.1 Hans Gerle

Fret 12

$$F_{12} = \frac{m}{2}$$

$$V_{12} = m - F_{12} = m - \frac{m}{2} = \frac{2m}{2} - \frac{1m}{2} = \frac{1m}{2} \rightarrow 2 : 1$$

Fret 7

$$F_7 = 2 * \left(\frac{F_{12}}{3}\right) = 2 * \left(\frac{\frac{m}{2}}{3}\right) = \frac{\frac{2m}{2}}{3} = \frac{m}{3}$$

$$V_7 = m - \frac{m}{3} = \frac{2m}{3} \rightarrow 3 : 2$$

Fret 1

$$F_1 = 2 * \left(\frac{F_7}{11}\right) = 2 * \left(\frac{\frac{m}{3}}{11}\right) = \frac{\frac{2m}{3}}{11} = \frac{2m}{33}$$

$$V_1 = m - \frac{2m}{33} = \frac{31m}{33} \rightarrow 33 : 31$$

Fret 2

$$F_2 = \frac{F_7}{3} = \frac{\frac{m}{3}}{3} = \frac{m}{9}$$

$$V_2 = m - F_2 = m - \frac{m}{9} = \frac{8m}{9} \rightarrow 9 : 8$$

Fret 5

$$F_5 = \frac{F_{12}}{2} = \frac{\frac{m}{2}}{2} = \frac{m}{4}$$

$$V_5 = m - F_5 = m - \frac{m}{4} = \frac{3m}{4} \rightarrow 4 : 3$$

Fret 6

$$F_6 = \frac{F_5 + F_7}{2} = \frac{\frac{m}{4} + \frac{m}{3}}{2} = \frac{\frac{7m}{12}}{2} = \frac{7m}{24}$$

$$V_6 = m - F_6 = m - \frac{7m}{24} = \frac{17m}{24} \rightarrow 24 : 17$$

Fret 3

$$F_3 = (3 + 5) * \left(\frac{F_1}{3}\right) = 8 * \left(\frac{\frac{2m}{33}}{3}\right) = 8 * \frac{2m}{99} = \frac{16m}{99}$$

$$V_3 = m - F_3 = m - \frac{16m}{99} = \frac{83m}{99} \rightarrow 99 : 83$$

Fret 4

$$F_4 = \frac{F_3 + F_5}{2} = \frac{\frac{16m}{99} + \frac{m}{9}}{2} = \frac{\frac{64m}{396} + \frac{99m}{396}}{2} = \frac{\frac{163m}{396}}{2} = \frac{163m}{792}$$

$$V_4 = m - F_4 = m - \frac{163m}{792} = \frac{629m}{792} \rightarrow 792 : 629$$

C.2 John Dowland

All frets are identical to Gerle's ratios except:

Fret 3

$$\begin{aligned}F_{3Dowland} &= (3 + 4 + \frac{1}{2}) * (\frac{F_{1Dowland}}{3}) \\&= (7 + \frac{1}{2}) * (\frac{\frac{2m}{33}}{3}) = (7 + \frac{1}{2}) * (\frac{2m}{99}) \\&= \frac{14m}{99} + \frac{2m}{198} = \frac{28m}{198} + \frac{2m}{198} = \frac{30m}{198} \\V_{3Dowland} &= m - F_{3Dowland} = m - \frac{30m}{198} = \frac{168m}{198} \rightarrow 198 : 168\end{aligned}$$

Fret 4

$$\begin{aligned}F_{4Dowland} &= \frac{F_{2Dowland} + F_{5Dowland}}{2} \\&= \frac{\frac{30m}{198} + \frac{m}{4}}{2} = \frac{\frac{120m}{792} + \frac{198m}{792}}{2} = \frac{\frac{318m}{792}}{2} = \frac{318m}{1584} \\V_{4Dowland} &= m - F_{4Dowland} = m - \frac{318m}{1584} = \frac{1266m}{1584} \rightarrow 1584 : 1266\end{aligned}$$

Frets 8, 9 and 10

$$\begin{aligned}F_{8Dowland} &= \frac{m - F_{1Dowland}}{3} = \frac{m - \frac{2m}{33}}{3} = \frac{\frac{31m}{33}}{3} = \frac{31m}{99} \\F_{9Dowland} &= \frac{m - F_{2Dowland}}{3} = \frac{m - \frac{m}{9}}{3} = \frac{\frac{8m}{9}}{3} = \frac{8m}{27} \\F_{10Dowland} &= \frac{m - F_{3Dowland}}{3} = \frac{m - \frac{30m}{198}}{3} = \frac{\frac{168m}{198}}{3} = \frac{168m}{594} \\V_{8Dowland} &= m - F_{8Dowland} = m - \frac{31m}{99} = \frac{68m}{99} \rightarrow 99 : 68 \\V_{9Dowland} &= m - F_{9Dowland} = m - \frac{8m}{27} = \frac{19m}{27} \rightarrow 27 : 19 \\V_{10Dowland} &= m - F_{10Dowland} = m - \frac{168m}{594} = \frac{426m}{594} \rightarrow 594 : 426\end{aligned}$$

C.3 Silvestro Ganassi

Fret 1

$$F_{1Ganassi} = \frac{F_2}{2} = \frac{\frac{m}{9}}{2} = \frac{m}{18}$$

$$V_{1Ganassi} = m - F_1 = m - \frac{m}{18} = \frac{17}{18} \rightarrow 18 : 17$$

Fret 3

$$F_{3Ganassi} = F_{1Ganassi} + F_{2Ganassi} = \frac{m}{18} + \frac{m}{9} = \frac{m}{18} + \frac{2m}{18} = \frac{3m}{18} = \frac{m}{6}$$

$$V_{3Ganassi} = m - F_{3Ganassi} = m - \frac{m}{6} = \frac{5m}{6} \rightarrow 6 : 5$$

Fret 4

$$F_{4Ganassi} = \frac{F_{3Ganassi} + F_{5Ganassi}}{2} = \frac{\frac{m}{6} + \frac{m}{4}}{2} = \frac{\frac{4m}{24} + \frac{6m}{24}}{2} = \frac{\frac{10m}{24}}{2} = \frac{10m}{48}$$

$$V_{4Ganassi} = m - F_{4Ganassi} = m - \frac{10m}{48} = \frac{38m}{48} \rightarrow 48 : 38$$

Fret 8

$$\begin{aligned} F_{8Ganassi} &= F_{7Ganassi} + (F_{6Ganassi} - F_{5Ganassi}) \\ &= \frac{m}{3} + \frac{7m}{24} - \frac{m}{4} = \frac{m}{3} + \frac{7m}{24} - \frac{6m}{24} = \frac{m}{3} + \frac{m}{24} \\ &= \frac{8m}{24} + \frac{m}{24} = \frac{9m}{24} = \frac{3m}{8} \end{aligned}$$

$$V_{8Ganassi} = m - F_{8Ganassi} = m - \frac{3m}{8} = \frac{5m}{8} \rightarrow 8 : 5$$

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