Generation of Synthetic Seismograms for an Acoustic Layer over an Acoustic Half Space

By L. Neil Frazer and Albert J. Rudman

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GEOPHYSICAL COMPUTER PROGRAM 8

DEPARTMENT OF NATURAL RESOURCES
GEOLOGICAL SURVEY OCCASIONAL PAPER 35

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To the Geophysics Community

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Responsibility for distribution of the program cards or furnished tapes will be assumed by the Indiana Geological Survey.

—Albert J. Rudman and Robert F. Blakely, editors

*Norman S. Neidell, Geoquest International; Sigmund Hammer, University of Wisconsin; Judson Mead, Indiana University; Franklin P. Prooer, Indiana University; and Joseph E. Robinson, Syracuse University.
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Generation of Synthetic Seismograms for an Acoustic Layer over an Acoustic Half Space

By L. NEIL FRAZER and ALBERT J. RUDMAN

Abstract
The problem of sound propagation in an acoustic layer over an acoustic half space (liquid over liquid) is solved by quadrature based on the generalized Filon method. Synthetic seismograms are computed by program NFAR1 for nearly arbitrary source-receiver distances and ranges of frequency. The user may optionally plot pressure and horizontal or vertical motion at specified distances. Test cases presented in the text demonstrate that the seismograms agree in arrival time and waveform for all anticipated events, such as direct, reflected, and refracted arrivals, including normal modes.

Introduction
Although modern developments in theoretical seismology make it possible to compute the response of elastic and acoustic models with many layers, such programs are difficult to use. They are long both in running time and in storage requirements, and calculations cannot be made for great distances or for arbitrarily high frequencies and wave numbers. These restrictions can only be dealt with at the cost of truncation of phases or inaccurate amplitudes. Therefore seismologists sometimes use the simplest models that exhibit phenomena being studied. In this regard the acoustic layer over an acoustic half space model (liquid over liquid) corresponds closely to reality for ocean-bottom studies, as ocean-bottom sediments do not transmit shear waves effectively. In continental crustal studies synthetic seismograms for an acoustic model allow one to estimate the relative amplitudes and travel times of P waves in the complicated wave train. The purpose of this paper is to introduce the reader to a computer program that generates a synthetic seismogram for an acoustic layer over an acoustic half space at nearly arbitrary distances and ranges of frequency. We hope this program will be useful to seismologists, but we also hope that its size, speed, and simplicity in use will interest other geophysicists who wish to teach themselves or others more about seismology.

Figure 1. The physical model: an acoustic layer over an acoustic half space. Source and receiver may also be in the half space.

The solution of sound propagation in an acoustic layer over an acoustic half space was set forth in detail by Pekeris (1948). The problem is still of interest because of the many features it shares with more complicated problems whose solutions cannot be written in closed form. The limited computing facilities of 1948 led Pekeris to calculate the solution for only the first few normal modes when the source and receiver were many wave lengths apart. In this paper we reformulate the solution by using a numerical technique, called the generalized Filon method, which allows the computation of the complete motion at much greater source-receiver distances than have hitherto been practicable.

The physical model is an acoustic layer overlying an acoustic half space with an explosion source in the layer and the upper surface of the layer free (fig. 1). We use cylindrical coordinates \((x, \theta, z)\) with the source on the axis at depth \(z = D_s\), the
SYNTHETIC SEISMOGRAMS FOR AN ACoustic LAYER OVER AN ACoustic HALF SPACE

receiver at depth \( z = DL \), the free surface at depth \( z = 0 \), and the interface between layer and half space at depth \( z = H \).

**Solution for a Whole Space**

To obtain the equations of motion we begin with the linearized momentum equation

\[
\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \mathbf{\nabla} \mathbf{\sigma} + \mathbf{f}
\]

in which \( \mathbf{u} \) is displacement, \( \mathbf{\sigma} \) is stress, \( \rho \) is density, and \( \mathbf{f} \) is equivalent body-force density (Burridge and Knopoff, 1964). We assume

\[
\mathbf{f}(\mathbf{x},t) = -\delta(t) \mathbf{\nabla} \delta(\mathbf{r} - \mathbf{r}_0)
\]

in which \( \delta \) is the Dirac delta function.

If we temporarily restrict ourselves to a homogeneous infinite whole space, we find

\[
\rho \frac{1}{c^2} \frac{\partial^2 \mathbf{u}}{\partial t^2} - \nabla^2 \phi = \lambda^{-1} \mathbf{p}
\]

where \( \mathbf{u} = \mathbf{\nabla} \phi \) and \( \mathbf{p} \) are the source and displacement potentials.

Setting \( \mathbf{p} = -\delta(t) \delta(t - t_0) \) and taking the Fourier Transform of both sides, we obtain

\[
(\nabla^2 + \frac{\omega^2}{a^2}) \delta(\mathbf{r}, \omega) = \lambda^{-1} \delta(\mathbf{r} - \mathbf{r}_0)
\]

Specializing to cylindrical coordinates \((x, \theta, z)\) and letting the source be on the \( z \) axis at depth \( DS \), we use the Hankel Transform to obtain

\[
(\frac{\alpha^2}{a^2} - k^2) \hat{\phi}(k, z, \omega) = \lambda^{-1} \delta(z - DS)
\]

We assume that \( \phi \) contains only energy moving outward from the source and obtain a solution

\[
\hat{\phi} = \frac{e^{\gamma z - DS}}{2\gamma \lambda}
\]

where

\[
\gamma = \sqrt{(\omega/a)^2 - k^2}
\]

Inversion of the Hankel and Fourier Transforms yields the complete solution for the infinite whole space

\[
\phi(x, z, t) = (2\pi)^{-1} \int_0^\infty \frac{dk}{2\gamma \lambda} \frac{J_1(kx)}{e^{\gamma |z - DS|}}
\]

If \( k = \omega/a \), \( \gamma = 0 \) and the integrand above is singular. However, in an attenuating medium

\[
\text{Im}(a^{-1}) > 0
\]

so that the zero of \( \gamma \) is actually in the first quadrant of the complex \( k \)-plane (fig. 3). This means that the contour of integration \( C_1 \) passes below the branch point \( \omega/a \) and \( \gamma \) is well defined along the \( \text{Re}(k) \) axis.
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

Solution for a Layer over a Half Space

Consider the case where source and receiver are in the layer. No upgoing waves can exist in the half space; pressure must vanish at the free surface and the vertical component of displacement must be continuous across the boundary \(z = H\). The displacement potential \(\hat{\phi}\) then has the form

\[
\hat{\phi}(z < DS < H) = \frac{-\sin(\gamma_1 z)}{\gamma_1 \lambda_1 \delta} \left[ \gamma_1 \cos \left( \gamma_1 (H-D) \right) - 1 \left( \rho_1 / \rho_2 \right) \gamma_2 \sin \left( \gamma_2 (H-D) \right) \right] \tag{10}
\]

\[
\hat{\phi}(DS < z < H) = \frac{-\sin(\gamma_1 DS)}{\gamma_1 \lambda_1 \delta} \left[ \gamma_1 \cos \left( \gamma_1 (H-z) \right) - 1 \left( \rho_1 / \rho_2 \right) \gamma_2 \sin \left( \gamma_2 (H-z) \right) \right] \tag{11}
\]

\[
\hat{\phi}(DS < H < z) = \frac{-\sin(\gamma_1 DS)}{\lambda_1 \delta} e \tag{12}
\]

where

\[
\Delta = \gamma_1 \cos(\gamma_1 H) - 1 \left( \rho_1 / \rho_2 \right) \gamma_2 \sin(\gamma_2 H) \tag{13}
\]

When the source is in the half space, we obtain instead

\[
\hat{\phi}(z < H < DS) = \frac{-\sin(\gamma_1 z)}{\lambda_2 \delta} e \tag{14}
\]

\[
\hat{\phi}(H < z < DS) = \frac{-e \gamma_2 (DS-H)}{\gamma_2 \lambda_2 \delta} \left[ \gamma_2 \cos \left( \gamma_2 (z-H) \right) \sin(\gamma_1 H) \right. \\
\left. + \gamma_1 \sin \left( \gamma_2 (z-H) \right) \cos(\gamma_1 H) \right] \tag{15}
\]
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

\[
\hat{Q}(H < DS < s) = \frac{-e^{i\gamma_1(s-H)}}{\gamma_2 k_0 \omega} \left\{ \gamma_2 \cos \left[ \frac{\gamma_2 (DS-H)}{\omega} \right] \sin(\gamma_1 H) \right. \\
+ \left. \gamma_1 \sin \left[ \frac{\gamma_2 (DS-H)}{\omega} \right] \cos(\gamma_1 H) \right\}
\]  

(16)

**Attenuation**

Following O'Connell and Budiansky (1978) we define the Q of our medium in terms of \(\chi^a(\omega)\) by

\[
Q = \frac{\text{Re}(i-1)}{\text{Im}(i-1)}
\]

(17)

Since \(\alpha^2 = (p/\gamma)^{1/2}\), specifying the frequency dependence of Q is equivalent to specifying the ratio of the real and imaginary parts of \(\alpha^2(\omega)\).

The Fourier Transform of a plane wave propagating in the positive z direction is proportional to \(\exp(iwz/\alpha)\). To find a form of \(\alpha^2\) that is consistent with causality, we let

\[
\alpha^2(u) = a + \zeta(u) + \eta(u)
\]

(18)

in which \(a > 0\) is a constant and \(\eta\) and \(\zeta\) are power-law parameterization used by Strick (1970)

\[
\eta = k_0 \text{sgn}(u) |u|^{s-1}
\]

(19)

in which \(k_0 > 0\) and \(s \approx .9\) is consistent with most data. For \(\xi\) we set

\[
\xi = k_0 \tan \left[ \frac{\pi}{s} \right] |u|^{s-1}
\]

(20)

It turns out that equations 19 and 20 have a simple functional form

\[
\zeta + \eta = k_0 \sec \left[ \frac{\pi}{s} \right] |u|^{s-1}
\]

(21)

\(-\pi/2 < \arg(u) < 3\pi/2\)

Analysis of equations 17-20 shows that for large \(\omega\), seismic Q is given by

\[
Q_\omega = \frac{a + \xi}{2 \eta}
\]

(22)

Therefore for values of \(\omega\) larger than 1, Q is independent of frequency if we choose \(s = .9\).

For values of \(\omega < 1\), \(Q_\omega \approx \xi/2\beta\) and so

\[
Q_\omega(\omega) = \frac{k_0 \tan \left[ \frac{\pi}{s} \right]}{2 \eta}
\]

(23)

SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

Contours of Integration
Up to this point we have assumed that the inverse transform of equations 10-12 and

$$\hat{g}(x,z,u) = (2\pi)^{-\frac{3}{2}} \int_{\Gamma_1} dk \, k \, J_0(kx) \hat{g}(k,z,u)$$  \hspace{1cm} (24)

where $\Gamma_1$ is the contour from 0 to $\infty$ along the Re($k$) axis in the complex plane (fig. 2). To manipulate equation 24 into a more suitable form, we assume $x > 0$ and make use of the following relations

$$\hat{g}(k,z,u) = \hat{g}(-k,z,u)$$  \hspace{1cm} (25)

$$J_{\eta_1}(z) = \frac{1}{2} \left[ J_{\eta_1}^{(0)}(z) + J_{\eta_1}^{(2)}(z) \right]$$  \hspace{1cm} (26)

$$H_{\eta_1}(z) = -\pi J_{\eta_1}(z)$$  \hspace{1cm} (27)

Using these we rewrite equation 24 as

$$\hat{g}(x,z,u) = \int_{\Gamma_2} dk \, k \, J_{\eta_1}^{(0)}(kx) \hat{g}(k,z,u)$$  \hspace{1cm} (28)

where $\Gamma_2$ is the contour (fig. 3) from $\infty$ along and above the negative Re($k$) axis, over the branch point for $H_{\eta_1}^{(1)}(kx)$ at the origin, and thence to $\infty$ just below the positive Re($k$) axis. The contours $\Gamma_2$ and $\Gamma$ are mathematically equivalent, but equation 28 will generally converge more rapidly on $\Gamma$ and most rapidly when the angle between the ends of $\Gamma$ and Re($k$) axis is close to

$$\phi = \tan^{-1} \left( \frac{x}{z-lz} \right)$$  \hspace{1cm} (29)

The horizontal and vertical motion and pressure are obtainable from equation 28. For numerical integration it is convenient to first rewrite them as integrals in the complex $p$-plane where $p$, the horizontal component of the wave-front vector, is related to $k$ by...
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

\( k = \omega p \)  

\[ u(x,z,w) = -\left( i \omega \right) \frac{1}{4 \pi} \int_{C} p \sum_{l=1}^{\infty} \frac{\hat{h}_{l}^{(1)}(\omega p \lambda_{l})}{\lambda_{l}} \delta(\omega p, z, \omega) dp \]  

\[ v(x,z,w) = \left( i \omega \right) \frac{1}{4 \pi} \int_{C} p \sum_{l=1}^{\infty} \frac{\hat{h}_{l}^{(1)}(\omega p \lambda_{l})}{\lambda_{l}^{2}} \delta(\omega p, z, \omega) dp \]  

\[ f(x,z,w) = \left( i \omega \right) \frac{1}{4 \pi} \int_{C} p \sum_{l=1}^{\infty} \frac{\hat{h}_{l}^{(1)}(\omega p \lambda_{l})}{\lambda_{l}} \delta(\omega p, z, \omega) dp \]  

The behavior of the integrands in the complex \( p \)-plane is the same as their behavior in the complex \( k \)-plane. (The branch point at \( \omega \alpha_{i} \) in the \( k \)-plane is at \( 1/\alpha_{i} \) in the \( p \)-plane.) The \( p \)-plane is preferred because the singularities of \( \delta \) occur at values of \( p \) that are nearly constant; therefore the same contour of integration can be used for all frequencies.

If quadrature methods are used to evaluate equations 31-33, many small integration steps will be required to track the oscillations of the integrand.

One way around this difficulty is the generalized Filon method (GFM) of Frasier (1978). The version of GFM that we use is the one designed for rapid evaluation of integrals of the form

\[ \int_{C} f(p)e^{ig(p)} dp \]  

in which \( f \) and \( g \) are complex functions of \( p, \omega \) is a large complex constant, and \( \Psi \) is some contour in the complex \( p \)-plane. We use the simplest possible GFM formula:

\[ \int_{C} f(p)e^{ig(p)} dp = \int_{p_{j}} f(p)e^{ig(p)} dp + \int_{p_{j-1}}^{p_{j}} f(p)e^{ig(p)} dp \]

\[ = \begin{cases} \frac{(4p)}{\delta(\delta e)} & \delta(\delta e) \neq 0 \text{ and } \delta(\delta e) \neq 0 \\ \frac{(4p)}{2} \left( f(p_{j-1}) + f(p_{j}) \right) e^{ig(p_{j})} & \text{if } \delta(\delta e) = 0 \end{cases} \]

in which

\( (4p) = p_{j} - p_{j-1}, \delta(e) = f(p_{j}) - f(p_{j-1}), \delta(e) = g(p_{j}) - g(p_{j-1}) \)

\( \delta(e) = e^{ig(p_{j})} \delta(e)^{2}e^{g(p_{j-1})}, \text{ and } \delta(\delta e) = f(p_{j})e^{ig(p_{j})} - f(p_{j-1})e^{ig(p_{j-1})} \)

\( \delta \) is the derivative of the integrand.
CONTOURS OF INTEGRATION

To apply equation 35 to the integral (equation 31), we let

\[ f(p) = p^2 \rho_1^{(2)} \left( u_p x \right) e^{-l_u p x} \phi_p (u_p z, \omega) \]

(36)

and

\[ g(p) = p \]

(37)

with \( \sigma = 1 \omega \chi \)

(38)

For the integral (32) we let

\[ f(p) = p \phi_0^{(1)} \left( u_p x \right) e^{-l_u p x} \phi_p (u_p z, \omega) \]

(39)

with \( g \) and \( \sigma \) given by (37) and (38). For the integral (33) we let

\[ f(p) = p \phi_0^{(1)} \left( u_p x \right) e^{-l_u p x} \phi_p (u_p z, \omega) \]

(40)

with \( g \) and \( \sigma \) again given by equation 37 and equation 38.

Transformation of \( u, v, \) and \( \rho \) back into the

\[ v(x, \omega, t) = (2\pi)^{-1} \int_{-\infty}^{\infty} d\omega e^{-iut} v(x, \omega, t) \hat{W}(\omega) \]

(41)

where the window function \( \hat{W}(\omega) \) is chosen so that its transform \( \hat{W}(t) \) has small side lobes and behaves like \( \delta(t) \). One such window is

\[ \hat{W}(\omega) = \begin{cases} \cos(\pi \omega) & -\Omega \leq \omega \leq \Omega \\ 0 & |\omega| > \Omega \end{cases} \]

(42)

Numerical computation of \( \gamma_1 \) and \( \gamma_2 \) as functions of \( \omega \) and \( p \) is accomplished by introducing the variables \( q_1 \) and \( q_2 \), where

\[ q_1 = \sqrt{1/u_1^2 - p^2} = \gamma_1/\omega \]

(43)

and

\[ q_2 = \sqrt{1/u_2^2 - p^2} = \gamma_2/\omega \]

(44)

It is easy to verify that in the \( p \)-plane along the contour \( \Gamma \) (Fig. 3) from point 1 to point 4

\[ q_1 = S(q_1^2) + 1 \text{ sgn} \left[ \text{Im} \left( q_1^2 \right) \right] T(q_1^2) \]

(45)

and

\[ q_2 = S(q_2^2) + 1 \text{ sgn} \left[ \text{Im} \left( q_2^2 \right) \right] T(q_2^2) \]

(46)

and from point 4 to point 5

\[ q_1 = \text{sgn} \left[ \text{Im} \left( q_1^2 \right) \right] T(q_1^2) + 17(q_1^2) \]

(47)
\[
q_2 = \text{sgn} \left[ \ln(q_2^2) \right] \frac{3(q_2^2) + 17(q_2^2)}{2}
\]
(48)

where
\[
S = \sqrt{\frac{|s| + \text{Re}(s)}{2}}
\]
(49)
and
\[
T = \sqrt{\frac{|s| - \text{Re}(s)}{2}}
\]
(50)

**Summary**

Computation of the synthetic seismogram is done by program NFAR1 (Appendix 2). Comment cards are included to relate the program steps to key equations developed in the theoretical discussion (p. 2-8). Of necessity there are an abundance of constants and variables in the program, and these are briefly described in the glossary (Appendix 1). The structure of the algorithm is given in the abbreviated form of a flow chart (Appendix 3). Four models were used to test and demonstrate the effectiveness of program NFAR1. These models are described in Appendix 4, which also includes the input data used to generate the seismograms. Appendix 5 reproduces the actual output for the four models (seismograms and line-printer output). Discussion in Appendix 5 includes a comparison of the waveforms observed on the seismogram with those waveforms predicted by theory.

Program NFAR1 is the first in a series of programs planned to solve the canonical problems of seismology. While the generalized Filon method (Frazer, 1978) has removed some difficulties in computing models with many layers, many problems connected with computing vertical wave functions remain; programs for many layered models are expensive and difficult to use. On the other hand, for the simple canonical problems whose vertical wave functions can be computed without the aid of matrix methods, these difficulties are essentially removed by the generalized Filon method (even in the case where elastic moduli, and therefore velocities, are complex functions of frequency).

Therefore it is now possible at last to write programs, like the present one, that compute the solution to physically interesting problems at nearly arbitrary distances and ranges of frequencies. Future programs will include the problem of an acoustic layer over an elastic half space and the problem of an elastic half space over an elastic half space.

**Literature Cited**

Burridge, R., and Koopoff, L.

Frazer, L. N.

Helmsberger, D. V.

O'Connell, R. J., and Bodiansky, B.

Officer, C. B.

Pekeris, C. L.
1948 - Theory of propagation of explosive sound in shallow water: Geol. Soc. Amer. Mem. 27.

Strick, E.
<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(= CRO*G2)</td>
</tr>
<tr>
<td>ADP</td>
<td>Default value of integration step. Related to V1, V2, F0, H.</td>
</tr>
<tr>
<td>AIG1</td>
<td>Imaginary part of G1.</td>
</tr>
<tr>
<td>AIG2</td>
<td>Imaginary part of G2.</td>
</tr>
<tr>
<td>AKAP1</td>
<td>( \Delta K/\Delta P1 ). Related to F0, S1, TAU1, Q10.</td>
</tr>
<tr>
<td>AKAP2</td>
<td>( \Delta K/\Delta P2 ). Related to F0, S2, TAU2, Q20.</td>
</tr>
<tr>
<td>ALAM1</td>
<td>Complex ratio of ( (\text{RH01}/A1^2) ).</td>
</tr>
<tr>
<td>ALAM2</td>
<td>Complex ratio of ( (\text{RH02}/A2^2) ).</td>
</tr>
<tr>
<td>AM</td>
<td>Absolute value of displacement (UT).</td>
</tr>
<tr>
<td>AMT</td>
<td>Minimum value of ratio involving TAU1 or TAU2, W, receiver distances.</td>
</tr>
<tr>
<td>APM</td>
<td>Constant involving EPS12, W, receiver distances and depths.</td>
</tr>
<tr>
<td>A1</td>
<td>Complex ratio of ( (\text{RA1}/\text{ETA1}) ).</td>
</tr>
<tr>
<td>A2</td>
<td>Complex ratio of ( (\text{RA2}/\text{ETA2}) ).</td>
</tr>
<tr>
<td>AZ</td>
<td>Complex square root of GSQ.</td>
</tr>
<tr>
<td>B</td>
<td>Equivalent of DEL (= G1 + A).</td>
</tr>
<tr>
<td>BEX</td>
<td>Default constant (= 20.) involving geometry of shot, receiver, layer.</td>
</tr>
<tr>
<td>BEX2</td>
<td>Default constant (= 10.) involving geometry of shot, receiver, layer.</td>
</tr>
<tr>
<td>BGST</td>
<td>Plotting parameter. Default = 2.0.</td>
</tr>
<tr>
<td>BIG</td>
<td>Plotting parameter. Default = -10. or AM.</td>
</tr>
<tr>
<td>C</td>
<td>Ratio of A/B.</td>
</tr>
<tr>
<td>CI</td>
<td>The imaginary unit (0, 1).</td>
</tr>
<tr>
<td>CLOSE</td>
<td>Parameter to select DP default.</td>
</tr>
<tr>
<td>CRO</td>
<td>Complex density ratio.</td>
</tr>
<tr>
<td>CSIG</td>
<td>Imaginary frequency-related function.</td>
</tr>
<tr>
<td>C0</td>
<td>Complex zero.</td>
</tr>
</tbody>
</table>
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

Glossary of Constants and Variables Used in Program NFAR1—Continued

C1  The real unit (1, 0).
D   Equivalent of C1 or C1 + a complex exponent.
DBR Distance between receivers.
DEL Equivalent of B or B*D.
DELTA Plotting parameter. Default = 0.10 or 10.0.
DORG Plotting parameter. Default = BGST.
DP   Related to incremental step.
DP12 Incremental step in integration along leg 1-2. Default = ADP.
DP23 Incremental step in integration along leg 2-3. Default = ADP.
DP34 Incremental step in integration along leg 3-4. Default = ADP.
DP45 Incremental step in integration along leg 4-5. Default = ADP.
DR   Depth of receiver.
DRMDS Receiver depth minus shot depth.
DRMH Receiver depth minus layer thickness.
DS   Depth of source.
DSMDR Shot depth minus receiver depth.
DSMH Shot depth minus layer thickness.
DT   Incremental period step (~ T/NT).
DTP  Plotting parameter. Default = T/6.
DU   Incremental step related to U2-U1.
DW   Angular frequency step (2π/T).
EMP14 Complex exponential of π/4.
EPS12 Integration factor along leg 1-2. Default = .001.
EPS45 Integration factor along leg 4-5. Default = .001.
ETA1 (= ΔK1*WS1-1).
APPENDIX

Glossary of Constants and Variables Used in Program NFAR1—Continued

ETA2 \(= \Delta K2*W^{82-1}\).

E1 Complex array.

E2 Complex array.

FC Frequency.

FID Float (ID).

FMAX Maximum frequency.

FMIN Minimum frequency.

FRL Distance to first receiver.

F0 Reference frequency. Default = 1.0 Hertz.

GSQ \(= A1*A1 - PSQ\).

G1 \(\sqrt{1/V1^2 - p^2}\)

G2 \(\sqrt{1/V2^2 - p^2}\)

H Thickness of layer.

HANN Cosine function of frequency.

HMDR Layer thickness minus receiver depth.

HMDS Layer thickness minus shot depth.

ID Identification of model number.

II Index.

IOP Integer identifying six possible geometries of shot and receiver.

IP Index.

I PLOT Plotting parameter (flag). 1 = plot, 0 = no plot.

IR Index.

IR ESP Integer identifying three possible response functions: 1 = pressure,
  2 = horizontal displacement, 3 = vertical displacement.

IT Index.

I W Index (frequency).
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LINT</td>
<td>Logical switch read in to choose integration-plot option.</td>
</tr>
<tr>
<td>L1</td>
<td>Logical switch.</td>
</tr>
<tr>
<td>L45</td>
<td>Logical switch.</td>
</tr>
<tr>
<td>NB12</td>
<td>Number of high gradients along leg 1-2.</td>
</tr>
<tr>
<td>NB23</td>
<td>Number of high gradients along leg 2-3.</td>
</tr>
<tr>
<td>NB34</td>
<td>Number of high gradients along leg 3-4.</td>
</tr>
<tr>
<td>NB45</td>
<td>Number of high gradients along leg 4-5.</td>
</tr>
<tr>
<td>NPI</td>
<td>(=\ NT/TP).</td>
</tr>
<tr>
<td>NP12</td>
<td>Number of points used in integration of leg 1-2 for given frequency. Default = 10.</td>
</tr>
<tr>
<td>NP23</td>
<td>Number of points used in integration of leg 2-3 for given frequency. Default = 11.</td>
</tr>
<tr>
<td>NP34</td>
<td>Number of points used in integration of leg 3-4 for given frequency. Default = 30.</td>
</tr>
<tr>
<td>NP45</td>
<td>Number of points used in integration of leg 4-5 for given frequency. Default = 10.</td>
</tr>
<tr>
<td>NR</td>
<td>Number of receivers.</td>
</tr>
<tr>
<td>NT</td>
<td>NW*2.</td>
</tr>
<tr>
<td>NW</td>
<td>Maximum frequency in spectra of the synthetics.</td>
</tr>
<tr>
<td>NWM</td>
<td>NW-1.</td>
</tr>
<tr>
<td>PF</td>
<td>Complex number related to APM, PS, CLOSE, EMPI4.</td>
</tr>
<tr>
<td>PI</td>
<td>3.1415...</td>
</tr>
<tr>
<td>PI2</td>
<td>PI/2.</td>
</tr>
<tr>
<td>PI4</td>
<td>PI/4.</td>
</tr>
<tr>
<td>PS</td>
<td>Complex number related to AMT, CLOSE.</td>
</tr>
<tr>
<td>PSQ</td>
<td>P2*P2.</td>
</tr>
<tr>
<td>P2</td>
<td>Related to PS plus other constants.</td>
</tr>
</tbody>
</table>
APPENDIX

Glossary of Constants and Variables Used in Program NFAR1—Continued

Q10. Q of layer. Default = 20000.


RA1  TAU1 + XI1.

RA2  TAU2 + XI2.

RH01  Density of layer.

RH02  Density of half space.

RL  Distance to receiver.

RZ  Real part of SQRT (GSQ).

S2  A “twiddle” factor related to Q20. Default = 0.9.

S1  A “twiddle” factor related to Q10. Default = 0.9.

T  Maximum period (length of seismogram). Default = 360 secs.

TAU1  Wave slowness of layer (= 1/V1).

TAU2  Wave slowness of half space (= 1/V2).

TC  Period.

TIM  Array of times.

TMAX  Maximum period.

TMIN  Minimum period.

TORG  Plot parameter. Default = 2.5.

TP  Number of period increments (= T/DTP).

TP1  2*PI.

TSHIFT  Plotting parameter. Time before onset of first arrival. Default = DTP.

TS1  Tangent (2*S1).

TS2  Tangent (2*S2).

TT  Reduced time for plot of synthetic.

U  Related to UU.
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE
Glossary of Constants and Variables Used in Program NFAR1—Continued

UORG  Plotting parameter. Default = 2.0.
UORG1 Plotting parameter.
UORG2 Plotting parameter.
UORG4 Plotting parameter.
UORG5 Plotting parameter.
UT  Real part of complex spectral displacement.
UU  Complex function of spectral displacement.
UZ  Spectral equivalent of particle displacement.
U1  Temporary storage location during quadrature.
U2  Temporary storage location during quadrature.
VR  Reducing velocity for plot. Default is maximum (V1, V2).
V1  Velocity of layer.
V2  Velocity of half space.
W  Angular frequency.
WSQ  Angular frequency squared.
W0  Reference angular frequency.
XI1 (= ETA1*TS2).
XI2 (= ETA2*TS2).
ZLAG  Complex function of HANN, EMPI4, W.
ZLAM  Equivalent of ALAM1 or ALAM2 depending on geometry.
ZR  Complex (RH01,0) or (RH02,0).
Z1
Z2
Z3
Z4

Miscellaneous complex variables for temporary storage.
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

Appendix 2. Fortran IV Program NFAR1

Program NFAR1 contains comment cards identifying the purpose of each major section. The main calling program (flow diagram shown in Appendix 3) uses two subroutines and five Calcomp subroutines. Running time of 140 seconds and 28,736 words of memory (single precision 60 bits) are required on a CDC 6600 for the case of a half space. (See Test Case 1, Appendix 4.)

```fortran
PROGRAM NFAR1(INPUT, TAPE15=INPUT, OUTPUT, TAPE17=OUTPUT, * PLOT, TAPE7=PLOT)
  +
CC
CC
  THIS PROGRAM IS CALLED NFAR1. THERE ARE TWO
  SUBROUTINES: RESP AND FFT.

CC
CC
  IT COMPUTES THE MOTION IN A HOMOGENEOUS ACOUSTIC LAYER
CC
  OVER A HOMOGENEOUS ACOUSTIC HALFSPACE. TIMES, DISTANCES AND
CC
  FREQUENCIES USED IN THIS PROGRAM OR PRINTED OUT MAY BE REGARDED
CC
  AS BEING IN UNITS OF SEC, KM, Hz OR IN MILLISECS, M, Khz.

CC
CC
  COMPLEX A1,A2,ALAM1,2LAG,ZRO,ZLAM
  COMPLEX P2,PS,PF,DP
  COMPLEX CI,C0,C1,C00
  COMPLEX CSIG(11),EMPIW,E1(11),E2(11)
  COMPLEX DU,DL,UL,UU(11)
  COMPLEX U(256,11), UI(512)
  DIMENSION UT(515), ITM(515)
  LOGICAL L1,LINT, L05
  EQUIVALENCE (UT(11), UI(11))
  COMMON P2,PSQ,PI2,PI4,M,FRD, R, DBR, DBR0, DBR1, DBR2, C1, CO,
  1 CSIG, EMPIW, CI, A1, A2, E2, E1P, O2,
  2 DR, DS, 1, H, R0, R0D, R0B, R0B0, R0B1, R0B2, R0B3,
  3 LG0(C1, CO) = CABS(C1-C0)/AMAX1(CABS(C1), CABS(C0), 1.E-30), GT..2
  CALL IDENT(7)

CC
CC
  READ INPUT FILE

CC
CC
  READ(15,111) DR
  READ(15,111) DS
  READ(15,111) H
  READ(15,111) V1
  READ(15,111) RH01
  READ(15,111) Q10
  READ(15,111) S1
  READ(15,111) V2
  READ(15,111) RHO2
```
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

Fortran IV Program NFAR1—Continued

READ(15,111) Q0
READ(15,111) S2
READ(15,112) NR
READ(15,111) FRL
READ(15,111) DRR
READ(15,111) EPS12
READ(15,111) EPS45
READ(15,111) F0
READ(15,111) DP12
READ(15,111) DP25
READ(15,111) DP45
READ(15,111) DP45
READ(15,112) INESP
READ(15,113) LINT
READ(15,112) ID
READ(15,111) T
READ(15,112) NW
READ(15,111) VR
READ(15,111) TSHIFT
READ(15,111) DTP
READ(15,111) UORG
READ(15,111) DORG
READ(15,111) BST
READ(15,112) IAXIS

SET PARAMETERS READ IN AS 0.0 TO DEFAULT VALUES

IF(Q0.EQ.0.) Q0 = 1.0
IF(Q10.EQ.0.) Q10 = 0.0002.
IF(Q20.EQ.0.) Q20 = 2000.
IF(S1.EQ.0.) S1 = 0.9
IF(S2.EQ.0.) S2 = 0.9
IF(T.EQ.0.) T = 360.
IF(VR.EQ.0.) VR = AMAX1(V1,V2)
IF(DTP.EQ.0.) DTP = T/6.
IF(TSHIFT.EQ.0.) TSHIFT = DTP
IF(BST.EQ.0.) BST = 2.
IF(UORG.EQ.0.) UORG = 2.
APPENDIX

Fortran IV Program NFAR1—Continued

IF(DOR.G.EQ.0.) DORG = BGST
IF(TORG.G.EQ.0.) TORG = 2.5
IF(EPS12.G.EQ.0.) EPS12 = .001
IF(EPS45.G.EQ.0.) EPS45 = .001

STOP PROGRAM IF CONSTRAINTS ARE EXCEEDED FOR THE MODEL

IF(DR.EQ.0. AND. IRESP.LE.2) STOP 1
IF(DS.EQ.0.) STOP 2
IF(FRL/(Y1*FL) .LE. .001) STOP 3
IF(H.EQ.0. OR. H.EQ.DS OR. DS.EQ.DR) STOP 4

COMPUTE DEFAULT VALUES FOR INTEGRATION INTERVALS (HP)

TAU1 = 1./Y1
TAU2 = 1./Y2
CLOSE = AMAX1(TAU1,TAU2)/20.
ADP = CLOSE/(FD*(10.*CLOSE**4)*12.)

IF(DP12.EQ.0.) DP12 = 6.*ADP
IF(DP23.EQ.0.) DP23 = 6.*ADP
IF(DP34.EQ.0.) DP34 = 2.*ADP
IF(DP45.EQ.0.) DP45 = 6.*ADP

INITIALIZE INTERNAL VARIABLES

NT = 2*MW
TP1 = 6.28318530717958
PI = TP1/2.
BEX = 20.
BEX2 = 10.
PI2 = PI/2.
PI4 = PI/4.
TS1 = TAN(PI2)
TS2 = TAN(2*PI2)
WO = TP1*FO
DW = TP1/T

RELATED TO EQUATIONS 19 AND 22 OF TEXT

AKAP1 = (W0**((1.-%1))*TAU1/(2.*Q10))
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

Fortran IV Program NFAR1—Continued

AKAP2 = (W0**2*(1.-S2)*TAU2/(2.*Q20))

RELATED TO EQUATIONS 10-16 OF TEXT

HMDS = H - DS
HMDR = H - DR
DMH = DR - H
DSM = DS - H
CRO = CMPLX(RH01,RH02,0.)
DSMCR = DS - DR
DMDS = DR - DS
ZRO = CMPLX(RH01,0.)
IF(DR.GT.H) ZRO = CMPLX(RH02,0.)
CI = CMPLX(0.,1.)
C0 = CMPLX(0.,0.)
C1 = CMPLX(1.,0.)
EMPI4 = CEXP(CMPLX(0.,-PI4))
RMN = NW - 1
TMIN = T/FLOAT(NWM)
TMAX = T
FMIN = T/T
FMAX = FLOAT(RWN)/T

IOP DETERMINES WHICH OF EQUATIONS 10-16 WILL BE USED
(SEE SUBROUTINE RESP FOR ‘IOP’ APPLICATIONS)

IF(DS.LT.H.AND.DR.LT.H.AND.DR.LT.DS) IOP=1
IF(DS.LT.H.AND.DR.LE.H.AND.DS.LE.DR) IOP=2
IF(DS.LT.H.AND.DR.GT.H) IOP=3
IF(DS.GT.H.AND.DR.LT.H) IOP=4
IF(DS.GT.H.AND.DR.GT.H.AND.DR.GT.DS) IOP=5
IF(DS.GT.H.AND.DR.GT.H.AND.DR.LE.DS) IOP=6

WRITE INPUT PARAMETERS OR THE DEFAULTED VALUES

WRITE(17,101)ID,V1,V2,RH01,RH02,S1,S2,Q10,Q20,
+ N,VR,DS,TSRIFT,DR,DTP,NN,UORG,FLR,ISOR,DR,ISOR,
+ ID,RIST,EPI12,DP15,EPS45,DP25,PO,DP34,T,DP46
WRITE(17,110)NW,LINT,IRESP,TMIN,TMAX,FMIN,FMAX
APPENDIX

Fortran IV Program NFAR1—Continued

```
ENTER FREQUENCY LOOP
DO 80D IW=1,NW
     W = DW*FLOAT(IW)
     AMT = AMN11(A(1),TAU1,TAU2)/80.,12./(W*(FRL+DBR*FLOAT(NR-1)))
80 CONTINUE
DO 25 IR=1, NR
     CSIG(ER) = CMPLX(0.,W*(FRL+FLOAT(IR-1)*DBR))
UI(ER) = CO
     CONTINUE
W5Q = W*W

CALCULATE LAME PARAMETER LAMBDA

ETA1 = AKAP1*W**((S1-1.))
ETA2 = AKAP2*W**((S2-1.))
XI1 = ETA1*TS1
XI2 = ETA2*TS2
RA1 = TAU1 + XI1
RA2 = TAU2 + XI2
A1 = CMPLX(ETA1,ETA1)
A2 = CMPLX(ETA2,ETA2)
ALAM1 = CMPLX(THO1,0.)/(A1*A1)
ALAM2 = CMPLX(THO2,0.)/(A2*A2)
ZLAM = ALAM1
IF(DS.GT.H) ZLAM = ALAM2
NB12 = 0
NB22 = 0
NB32 = 0
NB42 = 0
NP12 = 0
NP22 = 0
NP32 = 0
NP42 = 0

QUADRATURE FOR LEG 1-2 (EQUATIONS 31 OR 32 OR 33 AND 35):

LQ5 = .FALSE.
PS = CMPLX(-AMT,CLOSE*AMT)
P2 = PS
```
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE
Fortran IV Program NFAR1—Continued

APM = -1.14*Aalog(EP512)/((W*(FRL*ABS(DR-DS))))
PF = EMPI4*CMPLX(APM,0.) - PS
NP12 = INT(CARS(FP-PS)*(W/WO)**.75)/DP12
IF(NP12.LT.10) NP12 = 10
DP = (PF - PS)/CMPLX(FLOAT(NP12),0.)

COMPUTE(U,V OR P) ACCORDING TO EQUATIONS 10-16 FOR FIRST VALUE OF P ON LEG 1-2
CALL RESP
DO 710 IF=1,NP12
U1 = U2
LOOP OVER RECEIVERS
DO 706 IR=1,NR
E1(IR) = E2(IR)
706 CONTINUE
P2 = PS + (PF-PS)*CMPLX(FLOAT(IP)/FLOAT(NP12),0.)

COMPUTE(U,V OR P) ACCORDING TO EQUATIONS 10-16 FOR EACH VALUE OF P ON LEG 1-2.
CALL RESP
LOOP OVER RECEIVERS
DO 708 IR=1,NR
DU = U2*E2(IR)-U1*(E1(IR)-(E2(IR)-E1(IR))*(U2-U1))/(CSIG(IR)*DP)
DU = DU/CSIG(IR)
UU(IR) = UU(IR) - DU
708 CONTINUE
IF(UBIG(U2,U1)) NB12 = NB12 + 1
710 CONTINUE

QUADRATURE FOR LEG 2-3(EQUATIONS 31 OR 32 OR 33 AND 35):
PS = CMPLX(-AMT,CLOSE*AMT)
APPENDIX

Fortran IV Program NFAR1—Continued

P2 = PS
PP = CMPLX(CLOSE+AMT,-AMT)
NP3$ = INT(CABS(PP-PS)*((W/W0)**.75)/DP23)
NP3 = 2*(NP3$+2)**11
IF(NP3$ LT 11) NP3$ = 11
DP = (PP-PS)/CMPLX(FLOAT(NP3$),0.)

CC
COMPUTE (U,V OR P) ACCORDING TO EQUATIONS 10-16 FOR FIRST
VALUE OF P ON LEG 2-3.

CALL RESP

DO 720 IP=1,NP23
U1 = U2

LOOP OVER RECEIVERS.

DO 726 IR=1,NR
E1(IR) = E2(IR)
726 CONTINUE
P2 = PS + (PP-PS)*CMPLX(FLOAT(IP)/FLOAT(NP23),0.)

CC
COMPUTE (U,V OR P) ACCORDING TO EQUATIONS 10-16 FOR EACH
VALUE OF P ON LEG 2-3.

CALL RESP

LOOP OVER RECEIVERS

DO 728 IR=1,NR
DU = U2*E2(IR)-U1*E1(IR)-(E2(IR)-E1(IR))*U2-U1)/(CSIG(IR)*DP)
DU = DU/CSIG(IR)
UU(IR) = UH(IR) + DU
728 CONTINUE
IF(LBIG(U2,U1)) NB23 = NB23 + 1
720 CONTINUE

QUADRATURE FOR LEG 3-4 (EQUATIONS 31 OR 32 OR 33 AND 35):
PS = CMPLX(CLOSE+AMT,-AMT)
SYNTHETIC SEISMOGRAMES FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE
Fortran IV Program NFA1—Continued

P2 = PS
PF = CMPLX(AMAX1(RA1,RA2)+1.*AMT,-AMT)
NP34 = INT(1.5*CMPLX(PF-PS)*((W/W0)**2.5)/DP34)
TP(NP34,L7,30) NP34 = 30
DP = (PF-PS)/CMPLX(FLOAT(NP34),0.)

COMPUTE (U,V, OR P) ACCORDING TO EQUATIONS 10-16 FOR EACH
VALUE OF P ON LEG 3-4

CALL RESP
DO 730 IP=1,NP34
U1 = U2
LOOP OVER RECEIVERS
DO 736 IR=1,NR
E1(IR) = E2(IR)
CONTINUE
P2 = PS + (PF-PS)*CMPLX((FLOAT(IP)/FLOAT(NP34)),0.)

CALL RESP
DO 738 IR=1,NR
DU = U2*E2(IR)-U1*E1(IR)-(E2(IR)-E1(IR))*(U2-U1)/(CSIG(IR)*DP)
DU = DU/CSIG(IR)
UD(IR) = UD(IR) + DU
CONTINUE
IF(LRG(U2,U1)) NB34 = NB34 + 1

QUADRATURE FOR LEG 4-5(EQUATIONS 31 OR 32 OR 33 AND 35):
L45 = .TRUE.
PS = CMPLX(AMAX1(RA1,RA2)+3.*AMT,-AMT)
P2 = PS
APM = -1.14*ALOG(PSABS)/(W*(FRL+ABS(DR-D5)))
APM = APM + 2.*AMT
APPENDIX

Fortran IV Program NFAR1—Continued

   PF = CONJG(EMPI4)*CMPLX(AM,0.) + PS
   NF45 = INT(CABS(PF-PS)*(W/W0)/DP45)
   IF(NP45.LT.10) NP45 = 10
   DP = (PF-PS)/CMPLX(FLOAT(NP45),0.)
   COMPUTE(U,V, OR P) ACCORDING TO EQUATIONS 10-16 FOR FIRST
   VALUE OF P ON LEG 4-5.
   CALL RESP
   DO 740 IP=1,NP45
        U1 = U2
   C
   LOOP OVER RECEIVERS
   DO 746 IR=1,NR
        E1(IR) = E2(IR)
   746 CONTINUE
        P2 = PS + (PF-PS)*CMPLX(FLOAT(IP)/FLOAT(NP45),0.)
   C
   COMPUTE (U,V, OR P) ACCORDING TO 10-16 FOR EACH VALUE
   OF P ON LEG 4-5.
   CALL RESP
   LOOP OVER RECEIVERS
   DO 748 IR=1,NR
        DU = U2*E2(IR) - U1*E1(IR) - (E2(IR) - E1(IR))*(U2-U1)/(CSIG(IR)*DP)
        DU = DU/CSIG(IR)
        UU(IR) = UU(IR) + DU
   748 CONTINUE
   IF(LBIG(U2,U1)) NB45 = NB45 + 1
   740 CONTINUE
   END OF QUADRATURE FOR THIS W.
   770 CONTINUE
   C
   MULTIPLY BY P-INDEPENDENT VARIABLES. APPLY COSINE WINDOW
IN FREQUENCY DOMAIN.

HANN = 100.*COS(PI2*FLOAT(IW-1)/FLOAT(NW-1))
ZLAG = CMPLX(HANN,0.)*CMPLX(WSQ/SQRT(W*PI2),0.)
IF(IRES.P.EQ.1) ZLAG = -ZLAG*CMPLX(WSQ,0.)*280
IF(IRES.P.EQ.2) ZLAG = -ZLAG*CMPLX(0.,W)
IF(LINT) ZLAG = ZLAG/CMPLX(0.,-W)
DO 780 IR=1,NR
   RL = PNL*FLOAT(IR-1)*DRR
   U(IW,IR) = U(IW,IR)*2LAG*CEXP(CMPLX(0.,W*(THRESH-RL/YR)))
780 CONTINUE
FC = 1.0/TC
WRITE(11,119) IW,TC,NP12,NP23,NP34,NP45,
   * NB10,NB23,NB34,NB45,FC,UU(I)
119 CONTINUE
PREPARE PARAMETERS TO PLOT SYNTHETICS
L1 = .FALSE.
DT = T/FLOAT(NT)
TP = T/OTP
NPI = NT/TP
SS = 9999.
WRITE HEADINGS AT TOP OF PLOT
CALL SYMBOL(0.,40,0.5,0.2,6HMODEL ,90.,0.6)
FID = FLOAT (ID)
CALL NUMBER(0.,4.1,6.0,2.,FID,90.,0.)
CALL SYMBOL(0.,4.2,25.0,2.,16HACOUSTIC LAYER ,90.,0.16)
CALL NUMBER(0.,4.4,5.1,0.2,4.,90.,0.2)
CALL SYMBOL(0.,4.6,1.0,2.,19RM) OVER HALF SPACE,90.,0.19)
IF(IRES.P.EQ.1)CALL SYMBOL(.65,3.0,0.2,15H PRESSURE ,
   * 90.,0.15)
IF(IRES.P.EQ.2)CALL SYMBOL(.65,3.0,0.2,17HORIZONTAL MOTION,
   * 90.,0.17)
IF(IRES.P.EQ.3)CALL SYMBOL(.65,3.0,0.2,15VERTICAL MOTION,
   * 90.,0.15)
APPENDIX

Fortran IV Program NFAR1—Continued

IF(LINT) CALL SYMBOL(.95,3.2,0.2,12H(INTEGRATED),90.0, +12)
CALL SYMBOL(1.25,7.75,0.2,8HDISTANCE,90.0,8)
CALL SYMBOL(1.55,8.12,0.2,8H KMS),90.0,8)
CALL SYMBOL(1.25,0.0,0.2,18H ORIGIN TIME,90.0,13)
CALL SYMBOL(1.55,0.75,0.2,7H(GEO),90.0,7)

BEGIN LOOP TO PLOT A SERIES OF SYNTHETICS

DO 1000 IR=1,NR
FL = FRL * FLOAT(IR-1)*DBR
DO 1001 IW=1,NFM
UZ(IW+1) = U(IW,IR)
UZ(IW+1) = CONJG(U(IW,IR))
1001 CONTINUE
UZ(IW) = C0
UZ(IW+1) = C0
CALL FFT(NF,UE,-1.)
DO 1002 IT=1,NT
UT(1) = REAL(UZ(IT))
1002 CONTINUE

SET MAXIMUM AMPLITUDE OF WAVEFORM
GO THROUGH LOOP ONLY ONCE

IF(L1) GO TO 1040
BIG = -10.
DO 1020 II=1,NT
AM = ABS(UT(II))
IF(AM.GT.BIG) BIG = AM
1020 CONTINUE
BIG = BGST/BIG
L1 = .TRUE.
1040 CONTINUE

ADJUST AMPLITUDE FOR 1/R DECAY WITH DISTANCE OF DIRECT WAVE

RL = FRL * FLOAT(IR-1)*DBR
AM = BIG*((RL*RL + (DS-DR)**2)**.50)/SQRT(RL)
AM = AM*SQRT(RL)/((FRL*FRL + (DS-DR)**2)**.50)
PREPARE X (xTIM) AND Y (xUT) DATA FOR PLOTS

TT = BL/WR - TSHIFT
DO 1030 II=1,NT
UT(II) = AM*UT(II)
1030 TIM(II) = FLOAT(II)*DT + TT

PLOT LABELS FOR SYNTHETICS

UORG1 = UORG + 0.55
CALL NUMBER(UORG1,1.10,0.2,TT,90.0,1)
UORG2 = UORG + 0.75
CALL NUMBER(UORG2,8.25,0.2,RL,90.0,1)
CALL SYMBOL(UORG2,1.5,0.2,6,270.0,-1)

SETScale FOR X(xUT) AND Y(xTIM) AXES

UT(NT+1)=-1.0
UT(NT+2)=2.0
UT(NT+3)=0.5
TIM(NT+1)=TT
TIM(NT+2)=7/6
IF(T.LE.20.) DELTA = 0.10
IF(T.GT.20.) DELTA = 10.0
TIM(NT+3)=DELTA

PLOT SYNTHETIC SEISMOGRAM

UORG4 = UORG + 1.35
CALL IULINE(TIM,UT,NT,1,UORG4,1.50,0,0,90.0)

PLOT VARIABLE SCALES IF IAXIS = 1

IF(IAXIS.EQ.1)GO TO 920

SET NEW VERTICAL POSITION OF SYNTHETIC AND LOOP

GO TO 950
920 TIM(NT+1)=TT
APPENDIX

Fortran IV Program NFAR1—Continued

UORG = UORG + 1.35
CALL IAXIS(90.0,TIM(NT+1),TIM(NT+2),TIM(NT+3),
           NH ,-,UORG5,1.50,6.0,0,0.0,0.08)

950 UORG = UORG + DORG
1000 CONTINUE

PLOT FINAL AXIS AND CLOSE PLOT

UORG = UORG + 1.0
TIM(NT+1)=0.0
CALL IAXIS(90.0,TIM(NT+1),TIM(NT+2),TIM(NT+3),
           12,TIMEIN SECS,-12,UORG4,1.50,6.0,0,0.0,0.10)
CALL CLOSEFF
STOP

FORMAT FOR INPUT PARAMETERS

109 FORMAT(X,T30,*PARAMETERS EITHER READ IN BY USER OR OBTAINED BY*,
           *DEFAULT*,/,*T57,MODEL *,/,*T36,LABER *,
           *PARAMETERS*,T68,HALF SPACE PARAMETERS*,/,,
           T30,V1 IN KM/S,T45,*,*,T10.3,T55,*,*,V2 IN KM/S *,T80,*,*,F10.3/,
           T30,DH0 IN CM/T45,*,*,T10.3,T65,*DH2 IN CM/T45,T80,*,*,
           F10.3/T30,*,*,T45,*,*,F10.3,T65,*,*,F10.3,*,*,T400,MODEL GEOMETRY*,T70,PLOT PARAMETERS*,/,,T30,KM IN KM*,T45,*,
           *,F10.3,T65,*,VR IN KM/S*,T80,*,*,F10.3,*,T30,*,DKS IN KM*,T45,*,*,
           F10.3,T55,*,$,\
           TSHIFT IN SECS*,T80,*,*,F10.3,/,T30,*,DB IN KM*,T45,*,*,F10.3,T65,
           TSTOP IN SEC*,INCH*,T80,*,*,F10.3,/,T30,*,Nh*,*,T45,*,*,18,T65,*,UORG IN,
           INCHES*,T80,*,*,F10.3,/,T30,*,FH* IN KM*,T45,*,*,F10.3,*,*,T30,*,TORG IN
           INCHES*,T80,*,*,F10.3,/,T30,*,DBR IN KM*,T45,*,*,F10.3,*,*,DORG IN
           INCHES*,T80,*,*,F10.3,/,T30,*,DTR*,*,T45,*,*,18,T65,*,UORG IN
           T80,*,*,F10.3,/,T55,*,COMPUTATIONAL PARAMETERS*,/,,T30,*,EP512*,T45,
           *,*,F10.3,T65,*DP12*,T80,*,*,F10.4,/,T30,*,EP545*,*,T45,*,*,F10.4,
           T65,*DP34*,T80,*,*,F10.4,/,T30,*,T IN SEC*,T45,*,*,F10.3,
           T65,*DP45*,T80,*,*,F10.4)

OUTPUT FORMAT FOR HEADINGS OF INTEGRATION TABLE
SYNTHETIC SEISMOGRAMS FOR AN ACoustic LAYER OVER AN ACoustic HALF SPACE
Fortran IV Program NFAR1—Continued

C 110 FORMAT (1X, T29, *NW*, 7H(2**N), T45, *=/ ,18.765,
+ *LINt (LOGICAL)*, T90, **= ,16./ ,T30, *IRESP* T45, *=/ ,18// ,
+ T12, *THE FOLLOWING TABLE SUMMARIZES THE NOS OF POINTS (NP)*,
+ * AND THE NOS OF HIGH GRADIENTS (NB ) */ , T12, *WHICH OCCURRED* ,
+ * IN THE* ,
+ * INTEGRATION PROCESS AT GIVEN PERIODS (FREQUENCIES), THE RANGE* ,
+ T12, *OF PERIODS IS FROM* , PB, 3, 3X, *TO* , PB, 3, 3X, *SECONDS. (OR* ,
+ PB, 3, 3X, *TO* , PB, 3, 3X, *CPS* ) */ , T12, *THE SPECTRUM (U )*,
+ * IS GIVEN FOR THE SYNTHETIC AT THE NEAREST RECEIVER* , */ ,
+ T12, *IN* , T22, *(SEC)* , T31, *NP12* , T44, *NP23* , T51, *NP34* , T61,
+ *NP45* , T83, *CPS* , T93, *REP (U*) , T103, *IM (U*) */ , T31, *NB12* , T41,
+ *NB23* , T51, *NB34* , T61, *NB45* )

C C FORMAT FOR INPUT PARAMETERS

C 111 FORMAT (30X, F10.0)
112 FORMAT (30X, F10.0)
113 FORMAT (30X, L10)

C C FORMAT FOR OUTPUT (SHOWING INTEGRATION VALUES AT VARIOUS FREQUENCIES)
C C CONVErtion
C 119 FORMAT (* , T10, 14, T18, F10.3, T30, 15, T40, 15, T50, 15, T60, 15,
+ T30, 15, T40, 15, T50, 15, T60, 15,
+ T79, 9.4, T90, 2E10.3)
END

SUBROUTINE RESP
COMPUTES DISPLACEMENTS AND PRESSURE AS PER EQUATIONS 10-16.

SUBROUTINE RESP
COMPLEX U2, P2, G1, G2, Q10, Q11, EMP14
COMPLEX A1, A2, CSIG(11), Z1, CRO, E2(11)
APPENDIX

Fortran IV Program NFAR1—Continued

COMPLEX Z2,Z3,Z4,A,B,C,D,DEL,C1,C0
LOGICAL LHS
COMMON P2,WGQ,P12,W14,W,FHL,NR,DRR,IRESP,CH0,BEX,BEX2,C1,C0,
1 CSIG,EMPI4,C1,A1,A2,B2,IOP,U2,
2 DR,DS,H,HRD5,HMRD,HRM,DSMR,LS,DSMX,DRMS

C
C COMPUTATIONS FOR EQUATION 47.

C
PSQ = P2*P2
GSQ = A1*A1 - PSQ
AZ = CABS(GSQ)*.5
RZ = REAL(GSQ)*.5
IF(L45) GO TO 3
G1 = CMPLX(SQRT(AZ+RZ),SIGN(SQRT(AZ-RZ),AIMAG(GSQ)))*CMPLX(W,0.)
GO TO 4
3 CONTINUE
G1 = CMPLX(SIGN(SQRT(AZ+RZ),AIMAG(GSQ)),SQRT(AZ-RZ))*CMPLX(W,0.)
GO TO 4
4 CONTINUE
GSQ = AZ*A2 - PSQ
AZ = CABS(GSQ)*.5
RZ = REAL(GSQ)*.5
IF(L45) GO TO 6
G2 = CMPLX(SQRT(AZ+RZ),SIGN(SQRT(AZ-RZ),AIMAG(GSQ)))*CMPLX(W,0.)
GO TO 7
6 CONTINUE
G2 = CMPLX(SIGN(SQRT(AZ+RZ),AIMAG(GSQ)),SQRT(AZ-RZ))*CMPLX(W,0.)
7 CONTINUE
A = CH0*C2
B = G1 + A
A = G1 - A
C = A/B
AG1 = AIMAG(G1)
AG2 = AIMAG(G2)
D = C1
DEL = B
IF(AG1>.GT.BEX2) GO TO 8
D = C1 - C*CEXP(CMPLX(0.,1.*H)*G1)
DEL = B*D
8 CONTINUE
GO TO (10,20,30,40,50,60), IOP
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE
Fortran IV Program NFAR1—Continued

IOP=1: DR.LT.DS.LT.H (RECEIVER ABOVE SHOT ABOVE HALF SPACE.)
(EQUATION 10)

10 CONTINUE
U2 = 0
IF(AIG*DBMDR.GT.BEX) GO TO 70
U2 = CEXP(G1*CMPLX(0.,DBMDR))/D
IF(AIG*HMDS.GT.BEX) GO TO 101
Z1 = C1 + C*CEXP(G1*CMPLX(0.,2.*HMDS))
U2 = U2*Z1
101 CONTINUE
Z1 = 0
IF(AIG*DR.GT.BEX) GO TO 102
Z1 = CEXP(G1*CMPLX(0.,2.*DR))
102 CONTINUE
GO TO (11,12,13), IRESP
11 U2 = U2*(C1-Z1)*CMPLX(0.,5.)/G1
GO TO 70
12 U2 = U2*(C1-Z1)*CMPLX(0.,5.)/G1
GO TO 70
13 U2 = U2*(C1-Z1)*CMPLX(5.,0.)
GO TO 70

IOP=2: DS.LT.DR.LT.H (RECEIVER BELOW SHOT; BOTH IN LAYER.)
(EQUATION 11)

20 CONTINUE
U2 = 0
IF(AIG*DRMDR.GT.BEX) GO TO 70
U2 = CEXP(G1*CMPLX(0.,DRMDR))/D
IF(AIG*DRS.GT.BEX) GO TO 201
U2 = U2*(C1-CEXP(G1*CMPLX(0.,2.*DS)))
201 CONTINUE
Z1 = 0
IF(AIG*HMDS.GT.BEX) GO TO 202
Z1 = C*CEXP(G1*CMPLX(0.,2.*HMDS))
202 CONTINUE
GO TO (21,22,23), IRESP
21 U2 = U2*CMPLX(0.,5.)*(C1+Z1)/G1
APPENDIX

Fortran IV Program NFAR1—Continued

GO TO 70
22 U2 = U2*CMPLX(0.,5.)*(C1-Z1)*P2/G1
GO TO 70
23 U2 = U2*CMPLX(-5.0.)*(C1-Z1)
GO TO 70
C
C IOPE=3: DS.LT.H.LT.DR (SHOT IN LAYER; RECEIVER IN HALF SPACE.)
(EQUATION 12)
C
30 CONTINUE
U2 = 0.
IF(AIG1*HMDS + AIG2*DMHS.GT.BEX) GO TO 70
U2 = CEXP(G1*CMPLX(0.,HMDS) + G2*CMPLX(0.,DMHS))/DEL
IF(AIG1*DS.GT.BEK2) GO TO 301
U2 = U2*(G1 - CEXP(G1*CMPLX(0.,Z2.*DS)))
301 CONTINUE
GO TO (31,32,33), IRESP
31 U2 = U2*C1
GO TO 70
32 U2 = U2*C1*P2
GO TO 70
33 U2 = -U2*C2
GO TO 70
C
C IOPE=4: DR.LT.H.LT.DS (RECEIVER IN LAYER; SHOT IN HALF SPACE.)
(EQUATION 14)
C
40 CONTINUE
U2 = 0.
IF(AIG1*HMDR + AIG2*DSMH.GT.BEX) GO TO 70
U2 = CEXP(G1*CMPLX(0.,HMDR) + G2*CMPLX(0.,DSMH))/DEL
Z1 = 0.
IF(AIG1*DR.GT.BEK2) GO TO 401
Z1 = CEXP(G1*CMPLX(0.,Z2.*DR))
401 CONTINUE
GO TO (41,42,43), IRESP
41 U2 = U2*C1*(C1-Z1)
GO TO 70
42 U2 = U2*C1*(C1-Z1)*P2
GO TO 70
Fortran IV Program NFAR1—Continued

SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

\[ A3 \quad U_2 = U_2^* (C_1 + Z_1)*g_1 \]

GO TO 70

IOP = 5:  H.L.T.DR.LT.DS (RECEIVER ABOVE SHOT; BOTH IN HALF SPACE)

(EQUATION 15)

50 CONTINUE

U_2 = C_0

IF (AIG2 * DSMR.GT.BEX) GO TO 70

U_2 = CEXP(G2 * CMPLX(0., DSMR))/DEL

Z_1 = C_0

IF (AIG2 * DRMH.GT.BEX2) GO TO 501

Z_1 = CEXP(G2 * CMPLX(0., 2.*DRMH))

501 CONTINUE

Z_2 = C_0

IF (AIGH.GT.BEX2) GO TO 502

Z_2 = CEXP(G1 * CMPLX(0., 2.*H))

502 CONTINUE

Z_3 = G1 * (C_1 + Z_2)

Z_4 = G2 * (C_1 - Z_2)

GO TO (51, 52, 53), IRESP

51 U_2 = U_2^* CMPLX(0., 0.)*(Z_4 * (C_1 + Z_1) - Z_3 * (C_1 - Z_1))/G2

GO TO 70

52 U_2 = U_2^* CMPLX(0., 0.)*(Z_4 * (C_1 + Z_1) + Z_3 * (C_1 - Z_1))/P2/G2

GO TO 70

53 U_2 = U_2^* CMPLX(0., 0.)*(Z_4 * (C_1 - Z_1) + Z_3 * (C_1 - Z_1))

GO TO 70

IOP = 6:  H.L.T.DS.LT.DR (RECEIVER BELOW SHOT; BOTH IN HALF SPACE)

(EQUATION 16)

60 CONTINUE

U_2 = C_0

IF (AIG2 * DRMDS.GT.BEX) GO TO 70

U_2 = CEXP(G2 * CMPLX(0., DRMDS))/DEL

Z_1 = C_0

IF (AIG2 * DSMH.GT.BEX2) GO TO 601

Z_1 = CEXP(G2 * CMPLX(0., 2.*DSMH))

601 CONTINUE

Z_2 = C_0
IF(AIG1).GT.BEX2) GO TO 602
Z2 = CEXP(G1)*CMLX(0., 2.*H)
602 CONTINUE
23 = G1*(C1+Z2)*(C1-Z1)
23 = 23 + G2*(C1+Z2)*(C1+Z1)
GO TO (61, 62, 63), IRESP
61 U2 = U2*CMLX(0., .5)*Z3/G2
GO TO 70
62 U2 = U2*CMLX(0., .5)*Z3*P2/G2
GO TO 70
63 U2 = U2*CMLX(-.5, 0.)*Z3
70 CONTINUE
U2 = U2*CQSRT(P2)
DO 80 IR=1, N,
E2(IR) = CMLX(0., 0.)
IF(AIMAG(CSIG(IR))*.AIMAG(P2), .GT. .30.) GO TO 80
E2(IR) = CEXP(CSIG(IR) P2)
80 CONTINUE
RETURN
END
SUBROUTINE FFT(LX, CX, SIGNI)
C
C FAST FOURIER TRANSFORM ALGORITHM FROM CLAIBROT'S FUNDAMENTALS
C OF GEOPHYSICAL DATA PROCESSING, 1976, McGRAW-HILL, P.12.
C
C COMPLEX CX(LX), CTEMP, CW
J = 1
SC = SORT(1., FLOAT(LX))
DO 1 IR=1, LX
IF(I .GT. J) GO TO 1
CTEMP = CX(J) * SC
CX(J) = CX(I) * SC
CX(I) = CTEMP
1 M = LX/2
2 IF(J .LE. M) GO TO 3
J = J - M
M = M/2
IF(M .GE. 1) GO TO 2
3 J = J + M
L = 1
ISTEP = 2*L
DO 5 M=1,L
AA = 3.14159265358979*(M-1)/L
CM = CMPLX(COS(AA),SIN(AA))
DO 5 I=M,L,ISTEP
CTEMP = CM*CX(I*L)
CX(I*L) = CX(I) - CTEMP
5 CX(I) = CX(I) + CTEMP
L = ISTEP
IF(L.LT.LK) GO TO 4
RETURN
END
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

Appendix 3. Generalized Flow Diagram of Program NFAR1
<table>
<thead>
<tr>
<th>Card</th>
<th>Description</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Depth of receiver</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>2</td>
<td>Depth of source</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>3</td>
<td>Thickness of layer</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>4</td>
<td>Velocity of layer</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>5</td>
<td>Density of layer</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>6</td>
<td>Q of layer</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>7</td>
<td>&quot;S&quot; factor of layer</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>8</td>
<td>Velocity of half space</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>9</td>
<td>Density of half space</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>10</td>
<td>Q of half space</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>11</td>
<td>&quot;S&quot; factor of half space</td>
<td>(30X, F10.0)</td>
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<tr>
<td>12</td>
<td>Number of receivers</td>
<td>(30X, I10)</td>
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<td>13</td>
<td>Distance to first receiver</td>
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</tr>
<tr>
<td>14</td>
<td>Distance between receivers</td>
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<tr>
<td>15</td>
<td>Integration factor</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>16</td>
<td>Integration factor</td>
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</tr>
<tr>
<td>17</td>
<td>Reference frequency</td>
<td>(30X, F10.0)</td>
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<tr>
<td>18</td>
<td>Integration step leg 1-2</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>19</td>
<td>Integration step leg 2-3</td>
<td>(30X, F10.0)</td>
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<tr>
<td>20</td>
<td>Integration step leg 3-4</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>21</td>
<td>Integration step leg 4-5</td>
<td>(30X, F10.0)</td>
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<tr>
<td>22</td>
<td>Response option</td>
<td>(30X, I10)</td>
</tr>
<tr>
<td>23</td>
<td>Integration option</td>
<td>(30X, L10)</td>
</tr>
<tr>
<td>24</td>
<td>Identification of model number</td>
<td>(30X, I10)</td>
</tr>
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</table>
APPENDIX

Format for Input Cards for Program NFAR1—Continued

<table>
<thead>
<tr>
<th>Card</th>
<th>Description</th>
<th>Format</th>
</tr>
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<tbody>
<tr>
<td>25</td>
<td>T</td>
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</tr>
<tr>
<td>26</td>
<td>NW</td>
<td>(30X, I10)</td>
</tr>
<tr>
<td>27</td>
<td>VR</td>
<td>(30X, F10.0)</td>
</tr>
<tr>
<td>28</td>
<td>TSHIFT</td>
<td>(30X, F10.0)</td>
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<tr>
<td>29</td>
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<td>(30X, F10.0)</td>
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<td>UORG</td>
<td>(30X, F10.0)</td>
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<td>31</td>
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<td>(30X, F10.0)</td>
</tr>
<tr>
<td>34</td>
<td>IPLOT</td>
<td>(30X, I10)</td>
</tr>
</tbody>
</table>

The first 30 spaces are used to annotate the parameters.

Four models were run for program NFAR1 to illustrate the variety of options available. Input records are listed below. Corresponding output and discussion are given in Appendix 5. Amplitudes and pressures plotted in this study are relative to amplitude and pressure of the incident-source function and are therefore dimensionless.

Model 1. Study of Arrival Times and Amplitudes in a Half Space: Near Source

The records listed below were used to generate synthetic seismograms of horizontal, integrated particle motion for a simulated infinite half space by setting layer and half-space parameters equal to each other: $(V_1 = V_2 = 1.5 \text{ km/sec}; RH01 = RH02 = 1 \text{ gm/cm}^2; Q_1 = Q_2 = 100,000; S_1 = S_2 = .90)$. The depths of the source and the receiver are $24.999 \text{ km and } 24.998 \text{ km (fig. 4).}$ Synthetic seismograms were generated at distances of 10, 30, and 50 km (fig. 5). These geometries adequately separated the direct arrivals from the surface reflection for easy identification on the seismogram. One hundred and twenty-eight frequencies were used in the synthesis over a record length of 120 seconds. All parameters were specified in this model except for the DP integration values; these were set to zero to obtain default values. (The reason for this choice is given in the discussion of the output (Appendix 5).)
Figure 4. Selected ray paths for a layer over a half space. # signs indicate polarity of motion relative to ray-propagation direction (positive if particle motion is in the same direction as the ray). Arrow at the receiver identifies the direction of vertical motion on the seismogram. "x" and "y" identify parameters used in computing formulas in Table 8.
Figure 5. Synthetic seismograms of horizontal, integrated motion for an infinite half-space: Model 1. Seismograms are generated by program NPARI at distances of 10, 30, and 50 km. The half space is simulated by setting all parameters in the layer and the half space equal to each other. (See Appendices 4 and 5 for discussions of input and output.)
## Synthetics Seismograms for an Acoustic Layer over an Acoustic Half Space

**Input Records Used in Generating Output of Model 1**

<table>
<thead>
<tr>
<th>Record</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Depth of Receiver-DR</td>
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</tr>
<tr>
<td>2</td>
<td>Depth of Source-DS</td>
<td>24.999</td>
</tr>
<tr>
<td>3</td>
<td>Thickness of Layer-H</td>
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<tr>
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<td>Density of Layer-Rho1</td>
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<td></td>
<td>Q of Layer-Q10</td>
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<td>S Factor of Layer-S1</td>
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<td></td>
<td>Velocity of Half Space-V2</td>
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<td></td>
<td>Density of Half Space-Rho2</td>
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<td>Q of Half Space-Q20</td>
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<td>S Factor of Half Space-S2</td>
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<td>Distance Between Receivers-Dbn</td>
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<td>Integration Factor-Eps45</td>
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<td>Integration Step-Dp45</td>
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<td>Response Option-Iresp</td>
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<td>Integration Option-Lint</td>
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<td>Plotting Parameter-Dtp</td>
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<td>Plotting Parameter-Uorg</td>
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<td></td>
<td>Plotting Parameter-Torg</td>
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<td>34</td>
<td>Plotting Parameter-Iaxis</td>
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</tbody>
</table>

### Models 2A-2G. Variation of Input Parameters

The synthetic seismograms generated by program NFA1 for a particular geometry may differ widely depending on the user's choice of the input parameters. Model 2 consists of four runs of Model 1; each illustrates the effects of some of these input parameters.

### 2A. Test of Large Integration Steps (DP)

Model 1 was run with the four integration steps DP12, DP23, DP34, and DP45 set equal to "zero" and thereby invoking default values of .002, .002, .008, and .002. For Model 2, however, DP values 10 times larger were used (see below):
(All records are the same as that used for Model 1 except for Records 18-21.)

<table>
<thead>
<tr>
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<th>Integration step</th>
<th>Value</th>
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<td>Integration step</td>
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</table>
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

2B. Test of Default Options

Program NFAR1 will assign or calculate default values for certain input parameters if the user sets those equal to zero or leaves them blank. Model 1 was rerun by setting all possible parameters to zero.

Input Records Used in Generating Output of Model 2

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<th>Record</th>
<th>Description</th>
<th>Value</th>
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<td>24.998</td>
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<tr>
<td>2</td>
<td>DEPTH OF SOURCE-DS</td>
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<tr>
<td>3</td>
<td>THICKNESS OF LAYER-H</td>
<td>25.000</td>
</tr>
<tr>
<td></td>
<td>VELOCITY OF LAYER-V1</td>
<td>1.500</td>
</tr>
<tr>
<td></td>
<td>DENSITY OF LAYER-RHO1</td>
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<td>Q OF LAYER-Q10</td>
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<td>S FACTOR OF LAYER-S1</td>
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<td>VELOCITY OF HALF SPACE-V2</td>
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<td>DENSITY OF HALF SPACE-RHO2</td>
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</tr>
<tr>
<td></td>
<td>Q OF HALF SPACE-Q20</td>
<td>0.000</td>
</tr>
<tr>
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<td>S FACTOR OF HALF SPACE-S2</td>
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<td>NUMBER OF RECEIVERS-NR</td>
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<td>DISTANCE BETWEEN RECEIVERS-DBR</td>
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<td>PLOTTING PARAMETER-IAXIS</td>
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APPENDIX

2C-2G. Test of Seismogram Output Options

Program NFAR1 allows the user to choose the nature of the output motion as pressure or as vertical or horizontal particle motion. The user may also choose to integrate the output waveform. Models 2C through 2G illustrate all of these options.

2C. Test of Horizontal Motion (Nonintegrated)

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2D. Test of Vertical Motion (Integrated)

<table>
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<td>22 Response option — IRESP</td>
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<td>23 Integration option — LINT</td>
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<tr>
<td></td>
<td>34</td>
<td></td>
</tr>
</tbody>
</table>
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

2E. Test of Vertical Motion (Nonintegrated)

<table>
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<tr>
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</thead>
<tbody>
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34

2F. Test of Pressure (Integrated)

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<td>22 Response option — IRESP 1</td>
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<td>23 Integration option True</td>
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34
APPENDIX

2G. Test of Pressure (Nonintegrated)

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<tr>
<td>Integration option</td>
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</tr>
</tbody>
</table>

Model 3. Study of Times and Amplitudes for a Layer over a Half Space: Near Source

Model 3 investigates the behavior of program NFAR1 for a layer over a half space for a near source. (Maximum receiver distance of 50 km is only two times the layer thickness of 25 km.) Source and receiver depths (22.5 km) are sufficiently near the layer-half space interface to separate the arrival of the head wave from the direct and surface-reflected waves. Velocity and density values were chosen to provide a strong contrast (V1 = 1.5 km/sec vs V2 = 3.0 km/sec and RH01 = 1.0 gm/cc vs RH02 = 2.0 gm/cc). The records listed below were used to generate three synthetic seismograms at 20-km intervals. Most parameters were entered as “zero” values to further demonstrate the effectiveness of the default system.
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

Input Records Used in Generating Output of Model 3

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<th>Record</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<td>DEPTH OF RECEIVER-DR</td>
<td>22.501</td>
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<tr>
<td>2</td>
<td>DEPTH OF SOURCE-DS</td>
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<tr>
<td>3</td>
<td>THICKNESS OF LAYER-H</td>
<td>25.00</td>
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<tr>
<td></td>
<td>VELOCITY OF LAYER-V1</td>
<td>1.500</td>
</tr>
<tr>
<td></td>
<td>DENSITY OF LAYER-RHO1</td>
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<td>Q OF LAYER-Q10</td>
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<td>DISTANCE BETWEEN RECEIVERS-DBR</td>
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<td>PLOTTING PARAMETER-TSHIFT</td>
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<td>PLOTTING PARAMETER-DTP</td>
<td>40.00</td>
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<td>PLOTTING PARAMETER-TORG</td>
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</tr>
<tr>
<td></td>
<td>PLOTTING PARAMETER-DORG</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>PLOTTING PARAMETER-BGST</td>
<td>0.000</td>
</tr>
<tr>
<td>34</td>
<td>PLOTTING PARAMETER-IAXIS</td>
<td>1</td>
</tr>
</tbody>
</table>

Model 4. Study of Normal Modes for a Layer over a Half Space: Far Source

For large ranges (as much as 1,000 times the depth of the layer), the source can be assumed to be a point generating plane waves that are multiply reflected both at the surface and at the bottom of a layer. Superposition of these upgoing and downgoing waves traveling at specified angles with the vertical gives rise to bundles of energies of specified frequencies: the so-called normal modes. An approximate mathematical solution of this problem was introduced by Pekeris (1948). Model 4 simulates the Pekeris problem and is a measure of the versatility of program NFAR1 in generating the normal modes for a far source.

Pekeris's model assumes that the source and the receiver are on the bottom of the layer (0.0185 km). The range at which the synthetic seismogram was calculated is 460 X depth of the layer (= 8.4 km). Velocity in the half space (1.65 km/sec) is 1.1 times the velocity in the layer (1.50 km/sec). The
density in the half space (RH02 = 2.10 gm/cc)
is 2.0 times the density in the layer (RH01 = 1.05 gm/cc).

Several test runs were made for Model 4 to select the optimum input parameters. The
following summary of these trial runs may be of value to the user in his own applications.
The first test was with default DP values for the four legs (2.7, 2.7, 1.1, 2.7), but these
generated so much numerical noise that the seismograms beyond 1.0 km blew up. DP
values of .02, .02, .008, and .02 yielded an excellent seismogram at 8.4 km but required a
run time of 2,421 seconds. (This input was used for Model 4.) DP values of .08, .08, .08,
and .08 reduced the run time to 270 seconds. This run, however, yielded a seismogram
reduced 20 percent in amplitude from the correct values.
Proper choice of the seismogram length (T) also presents special problems. If T is too
large, the modal group is squeezed together too closely to identify waveforms. If T is too
small (T = 0.6), wraparound phenomena associated with Frequency Domain Methods
are observed. A seismogram length of T = 1.2
seconds is satisfactory, provided ample
frequencies are used (NW = 256). If NW =
128, the seismogram exhibits a uniform
oscillation instead of the expected dispersive
character associated with arriving modes.
The number of points NW selected by the
user must be 2N. This gives a maximum
frequency in the synthesis of

\[ \Omega = \frac{NW}{2T} \]

If \( \Omega \) becomes very large, numerical
problems will eventually result. But these will
be obvious in the output synthetics, and a
smaller value of NW (or larger value of T) can
then be used.
<table>
<thead>
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<th>Value</th>
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<td>2</td>
<td>Depth of Source-DS</td>
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<td>Thickness of Layer-H</td>
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<td>Density of Layer-RHO1</td>
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<td>Q of Layer-Q10</td>
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<td></td>
<td>S Factor of Layer-S1</td>
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<td>Q of Half Space-Q20</td>
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<td>Number of Receivers-NR</td>
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<td></td>
<td>Distance to First Receiver-FRL</td>
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<td></td>
<td>Distance Between Receivers-DBR</td>
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<td></td>
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<td></td>
<td>Integration Step-DP34</td>
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<td>4</td>
<td>Integration Option-LINT</td>
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<td>Model Identification-ID</td>
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<td>Length of Seismogram-T</td>
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<tr>
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<td>Plotting Parameter-DTP</td>
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<tr>
<td></td>
<td>Plotting Parameter-BGST</td>
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</tr>
<tr>
<td>34</td>
<td>Plotting Parameter-IAXIS</td>
<td>1</td>
</tr>
</tbody>
</table>
Appendix 5. Output of Four Models

The output for the four models described in Appendix 4 is listed below. Output consists of (a) synthetic seismograms and (b) a line-printer table of input parameters and a table summarizing the number of points and high gradients used in the integration process at given periods (frequencies) at the first receiver. The spectrum of the seismogram recorded at the first receiver is also given.

The user may choose to plot seismograms of pressure, vertical particle motion, or horizontal particle motion at selected distances. Amplitudes may be optionally integrated. A choice of two scales is available.

Model 1. Study of Arrival Times and Amplitudes in a Half Space: Near Source

The input parameters for the half space (Appendix 4) are reprinted in table 1. Although zero values were input for the integration-step size (DP) and for the reducing velocity (VR), program NFAR1 calculates default values that are computationally acceptable. Larger DP values (Test Case 2A) reduce run time but also reduce record quality.

The computer summary of integration points and gradients (table 2) shows that the number of steps (NP) used to compute legs 1-2, 3-4, and 4-5 exceeds the default values at all frequencies. The number of high gradients (NB) seldom exceeds a small percentage of the total integration steps, except for leg 2-3. We see, also, that this near-source record is dominated by the very lowest frequencies (.0083 to .0583 cps).

Three synthetic seismograms were plotted at distances of 10, 30, and 50 km (fig. 5). As expected for a half space, only the direct and surface-reflected waves can be seen.

Note that no events can be seen from the half space; the identical velocity and density parameters in the layer and the half space effectively eliminate the layer-half space interface. The reflected waveform is inverted as expected for a reflection from a free surface. The simple pulse form is consistent with the integration of a doublet used as a source function, since the far field displacement associated with the source in equation 2 is $S(t - R/v)$.

Program NFAR1 adjusts the time scale by a reducing velocity (VR) that effectively aligns the direct arrivals on all seismograms. On the small scale beneath each seismogram, the shot occurs at time zero.

Table 3 lists the seismogram and predicted times of the direct arrival. Predicted times (T) are computed from the relation $T = R/V$ and correlate well with the observed values, where $R$ is the source to receiver distance. The amplitude of the direct wave falls off as $1/R$.

Program NFAR1 multiplies all amplitudes by $R$ to achieve a uniform size for the direct arrival. The normalized amplitudes are, therefore, all unit size as expected.

Table 4 lists the seismogram and predicted times and amplitudes for the horizontal component of the surface-reflected wave. Predicted times (see annotation for table 4) agree closely with that observed on the seismograms. Amplitudes also compare closely after the observed values are compensated for (a) geometric spreading, (b) increase in the horizontal component of the amplitude with distance, and (c) adjustment for multiplication in program NFAR1 of all amplitudes by the distance $R$. Again, the computational equations are annotated in table 4.
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

Table 1. Output from program NF01 listing input parameters read in by the user or calculated for Model 1 (Appendix 4)

PARAMETERS EITHER READ IN BY USER OR OBTAINED BY DEFAULT

MODEL  1

<table>
<thead>
<tr>
<th>LAYER PARAMETERS</th>
<th>HALF SPACE PARAMETERS</th>
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<tbody>
<tr>
<td>$V_1$ IN KM/S</td>
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</tr>
<tr>
<td>$\rho_1$ IN GM/CC</td>
<td>$1.000$</td>
</tr>
<tr>
<td>$S_1$</td>
<td>$.900$</td>
</tr>
<tr>
<td>$Q_{10}$</td>
<td>$100000.000$</td>
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<tr>
<td></td>
<td>$V_2$ IN KM/S</td>
</tr>
<tr>
<td></td>
<td>$\rho_2$ IN GM/CC</td>
</tr>
<tr>
<td></td>
<td>$S_2$</td>
</tr>
<tr>
<td></td>
<td>$Q_{20}$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>MODEL GEOMETRY</th>
<th>PLOT PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ IN KM</td>
<td>$25.000$</td>
</tr>
<tr>
<td>$D_1$ IN KM</td>
<td>$24.999$</td>
</tr>
<tr>
<td>$D_2$ IN KM</td>
<td>$24.998$</td>
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<tr>
<td>$N_R$</td>
<td>$3$</td>
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<tr>
<td>$F_R$ IN KM</td>
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<td>$D_B$ IN KM</td>
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<td>$I_D$</td>
<td>$1$</td>
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<td></td>
<td>$V_R$ IN KM/S</td>
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<td>$T_{SHIFT}$ IN SECS</td>
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<td>$U_{ORG}$ IN INCHES</td>
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<td>$T_{ORG}$ IN INCHES</td>
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<td></td>
<td>$D_{ORG}$ IN INCHES</td>
</tr>
<tr>
<td></td>
<td>$B_{GST}$ IN INCHES</td>
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</table>

<table>
<thead>
<tr>
<th>COMPUTATIONAL PARAMETERS</th>
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<td>$E_P S_{12}$</td>
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<tr>
<td>$E_P S_{45}$</td>
</tr>
<tr>
<td>$F_0$ IN CPS</td>
</tr>
<tr>
<td>$T$ IN SECS</td>
</tr>
<tr>
<td>$N_M (=2^{*N})$</td>
</tr>
<tr>
<td>$I_{RESP}$</td>
</tr>
<tr>
<td>$D_P S_{12}$</td>
</tr>
<tr>
<td>$D_P S_{23}$</td>
</tr>
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</tr>
<tr>
<td>$D_P S_{45}$</td>
</tr>
<tr>
<td>$L_INT$ (LOGICAL)</td>
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</table>
Table 2. Output from program NFAR1 listing integration data and spectrum for Model 1 (Appendix 5)

THE FOLLOWING TABLE SUMMARIZES THE NOS OF POINTS (NP) AND THE NOS OF HIGH GRADIENTS (NB ) WHICH OCCURRED IN THE INTEGRATION PROCESS AT GIVEN PERIODS (FREQUENCIES). THE RANGE OF PERIODS IS FROM .945 TO 120.000 SECONDS, FOR .008 TO 1.058 CPS.

THE SPECTRUM (UU) IS GIVEN FOR THE SYNTHETIC AT THE NEAREST RECEIVER

<table>
<thead>
<tr>
<th>IW</th>
<th>T(SEC)</th>
<th>NP12</th>
<th>NP23</th>
<th>NP34</th>
<th>NP45</th>
<th>CPS</th>
<th>RE(UU)</th>
<th>IM(UU)</th>
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<td>.341E+02</td>
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<td>2</td>
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<td>30</td>
<td>62</td>
<td>.0167</td>
<td>-.616E+01</td>
<td>.804E+01</td>
</tr>
<tr>
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<td>40.000</td>
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<td>11</td>
<td>36</td>
<td>62</td>
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<td>-.371E+01</td>
<td>.351E+01</td>
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<td>62</td>
<td>.0333</td>
<td>-.270E+01</td>
<td>.138E+01</td>
</tr>
<tr>
<td>5</td>
<td>24.000</td>
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<td>11</td>
<td>59</td>
<td>63</td>
<td>.0417</td>
<td>-.193E+01</td>
<td>.470E+00</td>
</tr>
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<td>6</td>
<td>20.000</td>
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<td>11</td>
<td>70</td>
<td>63</td>
<td>.0500</td>
<td>-.154E+01</td>
<td>-.694E-03</td>
</tr>
<tr>
<td>7</td>
<td>17.143</td>
<td>127</td>
<td>11</td>
<td>81</td>
<td>63</td>
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<td>-.119E+01</td>
<td>-.420E+00</td>
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<tr>
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<td>15.000</td>
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<td>93</td>
<td>63</td>
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<td>-.766E+00</td>
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<tr>
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<td>11</td>
<td>115</td>
<td>63</td>
<td>.0833</td>
<td>-.197E+00</td>
<td>-.664E+00</td>
</tr>
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</table>

. .

| 124 | .968 | 62  | 37   | 1257 | 71  | 1.0333| .159E-01| .103E-02|
| 125 | .960 | 62  | 37   | 1266 | 71  | 1.0417| .148E-01| .699E-02|
| 126 | .952 | 61  | 37   | 1276 | 71  | 1.0500| .106E-01| .117E-01|
| 127 | .945 | 61  | 37   | 1286 | 71  | 1.0583| .581E-02| .137E-01|
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

Table 3. Predicted and seismogram times and amplitudes for the direct wave of Model 1.

<table>
<thead>
<tr>
<th>Distance (km)</th>
<th>Seismogram time(^1) (sec)</th>
<th>Predicted time(^2) (sec)</th>
<th>Seismogram amplitude(^3) (normalized units)</th>
<th>Predicted amplitude(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7.0</td>
<td>6.7</td>
<td>1.00</td>
<td>1.00</td>
</tr>
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<tr>
<td>50</td>
<td>34.0</td>
<td>33.3</td>
<td>1.00</td>
<td>1.00</td>
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</tbody>
</table>

\(^1\) Measured to peak value in figure 5.
\(^2\) \(T = R/V1\).
\(^3\) Program NFA1 compensates for geometric spreading of the direct wave by multiplying each trace by \(R\), where \(R\) is the path length of the direct wave from the source to the receiver at that receiver offset.

Table 4. Predicted and seismogram times and amplitudes for the surface-reflected wave of Model 1.

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<th>Distance (km)</th>
<th>Seismogram time(^4) (sec)</th>
<th>Predicted time(^4) (sec)</th>
<th>Seismogram amplitude(^4) (normalized units)</th>
<th>Predicted amplitude(^4)</th>
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</thead>
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<td>10</td>
<td>34.0</td>
<td>34.0</td>
<td>-.03</td>
<td>-.04</td>
</tr>
<tr>
<td>30</td>
<td>39.0</td>
<td>38.9</td>
<td>-.26</td>
<td>-.26</td>
</tr>
<tr>
<td>50</td>
<td>47.5</td>
<td>47.1</td>
<td>-.50</td>
<td>-.50</td>
</tr>
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</table>

\(^4\) Measured to peak value in figure 5.
\(^5\) \(T = (2 \sqrt{R^2/4 + DR^2})/V1\).
\(^6\) (See fig. 4.) Amplitude decreases due to geometric spreading \(1/(2 \sqrt{R^2/4 + DR^2})\). The horizontal component increases with \(R\) as a cosine function \((R/3)/(2 \sqrt{R^2/4 + DR^2})\). Program NFA1 multiplies all amplitudes by \(R\). Therefore, predicted amplitude (relative to direct wave) is given by \((R^2/4) (2 \sqrt{R^2/4 + DR^2})\). The sign of a wave reflected from a free surface is shifted 180°.

Models 2A-2G. Comparison of Seismograms for Various Input Parameters for a Half Space

Besides the geometry, velocity, and density assigned to any given model, the user also faces numerous choices for input parameters. Model 2 repeats the geometry and parameters of Model 1 (the infinite half space) while varying selected options one at a time for seven seismograms (2A through 2G). Seismograms recorded at a fixed distance (50 km) are composited in figure 6 for comparison.
Figure 6. Synthetic seismograms for an infinite half space: Model 2. Each seismogram displays some of the options available with program NFACT. (See Appendices 4 and 5 for discussion of Models 2A-2G.) Horizontal scale is in seconds.
2A. Test of Integration-Step Size (DP)

Integration-step sizes (DP) 10 times larger than those obtained by default values for Model 1 yielded a seismogram with some low-frequency noise (fig. 64), and the amplitudes of waves decreased to 80 percent of the predicted values. On the positive side, running times for these large DP values decreased from 140 seconds to 40 seconds for the half-space model.

2B. Test of Default Values

If the user is uncertain of the proper or optimum value for input parameters, he may enter zero values, and program NFAR1 will substitute fixed or computed values. In this test all possible parameters were set to zero, and the following default values were obtained (table 5). The major differences between Test Case 1 input and the default values are in Q1, Q2, and T (the seismogram length). Minor differences are observed in plotting parameters, TSHIFT and DTP.

Table 5. Output from program NFAR1 listing input parameters read in by the user or calculated for Model 2 (Appendix 4)

<table>
<thead>
<tr>
<th>PARAMETERS EITHER READ IN BY USER OR OBTAINED BY DEFAULT</th>
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<td>MODEL 2</td>
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<tr>
<th>LAYER PARAMETERS</th>
<th>HALF SPACE PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1 IN KM/S</td>
<td>V2 IN KM/S</td>
</tr>
<tr>
<td>1,500</td>
<td>1,500</td>
</tr>
<tr>
<td>RHO1 IN GM/CC</td>
<td>RHO2 IN GM/CC</td>
</tr>
<tr>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>S1</td>
<td>S2</td>
</tr>
<tr>
<td>0.900</td>
<td>0.900</td>
</tr>
<tr>
<td>Q10</td>
<td>Q20</td>
</tr>
<tr>
<td>20,000,000</td>
<td>2,000,000</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>MODEL GEOMETRY</th>
<th>PLOT PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>H IN KM</td>
<td>VR IN KM/S</td>
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<tr>
<td>25,000</td>
<td>1,500</td>
</tr>
<tr>
<td>D5 IN KM</td>
<td>TSHIFT IN SECS</td>
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<tr>
<td>24.999</td>
<td>60,000</td>
</tr>
<tr>
<td>DR IN KM</td>
<td>DTP IN SEC/INCH</td>
</tr>
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<td>50,000</td>
</tr>
<tr>
<td>NR</td>
<td>UORG IN INCHES</td>
</tr>
<tr>
<td>3</td>
<td>2.000</td>
</tr>
<tr>
<td>FRL IN KM</td>
<td>TORG IN INCHES</td>
</tr>
<tr>
<td>10,000</td>
<td>2.500</td>
</tr>
<tr>
<td>DBR IN KM</td>
<td>DORG IN INCHES</td>
</tr>
<tr>
<td>20,000</td>
<td>2.000</td>
</tr>
<tr>
<td>ID</td>
<td>BGST IN INCHES</td>
</tr>
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2C. Test of Horizontal Motion (Nonintegrated)

The waveform of the direct wave (fig. 6C) is a doublet with the reflected wave inverted.

2D. Test of Vertical Motion (Integrated)

Particle motion of a direct wave originating from a source on the same horizontal plane as the receiver is entirely horizontal. A vertical-motion seismogram (fig. 6D), therefore, will not detect the direct arrival. The reflected arrival, however, is detected. The amplitude size is controlled by the plotting technique, where the maximum pulse height on the first seismogram (at 10 km) is adjusted to 1 inch. Changes in amplitude with distance are related to the fact that the vertical motion for a given ray is proportional to the sine of the angle of incidence.

2E. Test of Vertical Motion (Nonintegrated)

The seismogram (not included in figure 6) shows a doublet pulse of large magnitude related to the reflected wave.

2F. Test of Pressure Motion (Integrated)

The amplitude and waveform of the pressure pulse (integrated) resemble the horizontal (nonintegrated) waveform (fig. 6F).

2G. Test of Pressure Motion (Nonintegrated)

The nonintegrated waveform has (as expected) three peaks (fig. 6G). The high-frequency oscillations seen in figure 6G are due to the fact that pressure is proportional to the second derivative of integrated displacement. Differentiation has amplified the side lobes of the time-domain version of the pulse shape given by equation 42.

Model 3. Study of Times and Amplitudes for a Layer over a Half Space: Near Source

Output for Model 3 is given in tables 6 and 7 and figure 7. Table 8 lists the seismogram and predicted times for the direct, refracted, reflected, and multiply reflected waves. The following discussion of arrival times (table 8) presumes the reader has examined the source-receiver geometry and nomenclature illustrated in figure 4.

Close agreement between predicted and seismogram arrival times and polarities occurs for the refracted (head) wave, surface reflection, and the second and fourth multiply reflected waves. Predicted times are computed from formulas listed in the right-hand column of table 8. Examination of Model 3 seismograms (fig. 7) shows that the direct wave, the first multiple reflection, and all odd-numbered multiples are either not present or not readily identifiable.

These observations may be explained as follows:

(a) Because the source and receiver are at the same depth, the direct wave motion is horizontal and not observable on a vertical seismogram.

(b) The first multiple reflection from the bottom and the first multiple from the surface have equal path lengths. These multiples have opposite polarities and so cancel each other. Similarly, all odd-numbered multiples may be expected to cancel each other.
Table 6. Output from program NFAR1 listing input parameters read in by the user or calculated for Model 3 (Appendix 4)

PARAMETERS EITHER READ IN BY USER OR OBTAINED BY DEFAULT

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Table 7. Output from program NFAR1 listing integration data and spectrum for Model 3 (Appendix 5)

The following table summarizes the nos of points (NP) and the nos of high gradients (NB) which occurred in the integration process at given periods (frequencies). The range of periods is from 1.890 to 240,000 seconds, or .004 to .529 CPS.

The spectrum (UU) is given for the synthetic at the nearest receiver.

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Note: The table continues in a similar fashion with the values for other IWs.

**Synthetic Seismograms for an Acoustic Layer over an Acoustic Half Space**
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<th>Ray path</th>
<th>10 km</th>
<th>30 km</th>
<th>80 km</th>
<th>Computation formulas for predicted times</th>
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<td>Seismogram</td>
<td>Predicted</td>
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<td>36.1</td>
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</table>

1 All times are in seconds. Signs indicate expected polarity.

*Signifies event not observed on the seismogram (fig. 7).
SYNTHETIC SEISMOGRAMS FOR AN ACOUSTIC LAYER OVER AN ACOUSTIC HALF SPACE

(c) The arrival times predicted for the bottom reflection closely match the times observed on the seismogram but differ from the expected polarity as shown in figure 4. Ordinarily, a reflection from a lower to a higher velocity shows no change. If the incident angle is more grazing than the critical angle, however, there is a phase change with a marked change in waveform (Officer, 1968, fig. 3-27). The event identified as a bottom reflection in figure 7 has its waveform modified by interaction with the refracted arrival. It is of interest that the refracted waveform closely approximates that computed by Helmberger (1968) for an acoustic layer over an elastic half space. Event A appears as a prominent arrival at 50 km. Study of the event at 10 and 30 km demonstrates that it only becomes prominent with distance. Such characteristics suggest it is a phase change associated with the second bottom multiple as the incident angle changes.

The observed amplitudes of all events identified on the Model 3 seismogram agree with the predictions of geometric-ray theory to within ±5 percent, except those phases (the refracted wave and the critical reflections) for which geometric-ray theory is invalid.

Model 4. Study of Normal Modes for a Layer over a Half Space: Far Source

In 1948, Pekeris calculated the amplitude and frequency characteristics of a seismogram originating from a point source in shallow water (fig. 8). At far distances, multiple-reflected wave groups called normal modes arrive at definite times. Properties of normal modes are treated in Pekeris (1948) and Officer (1958). If we follow Pekeris's discussion, we shall observe this sequence of events:

(a) The first mode begins as a nearly sinusoidal wave, called the ground wave, of gradually increasing frequency and amplitude. A comparison of the Pekeris ground wave (fig. 8) and that produced by program NFAR1 (fig. 9) shows a close visual agreement at a distance of 8.4 km. The first mode arrives at $t = R/V_2$ and is characterized by a limiting frequency $f_L$.

$$f_L = \frac{V_2}{4H} \sqrt{1 - \left(\frac{V_1^2}{V_2^2}\right)}$$

Table 9 demonstrates the close comparison of the predicted times and frequencies and the observed values in figure 9. Differences between figures 8 and 9 are due to the fact that NFAR1 included the contribution of higher modes.

(b) A high-frequency wave, the water wave, arrives at a time $t = R/V_1$. Predicted and seismogram times compare favorably (table 9). As time progresses, the water wave is seen

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<tr>
<th>Wave type</th>
<th>Predicted times (sec)</th>
<th>Seismogram times (sec)</th>
<th>Predicted frequencies (CPS)</th>
<th>Seismogram frequencies (CPS)</th>
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</tr>
<tr>
<td>Airy Phase</td>
<td>5.7$^2$</td>
<td>5.7</td>
<td>73</td>
<td>71</td>
</tr>
</tbody>
</table>

$^1$ Formulas for predicting times and frequencies are given in Appendix 5.

$^2$ The maximum amplitude (Airy Phase) is observed on the Pekeris seismogram (fig. 8). 62 seconds after the arrival of the ground wave.
to decrease in frequency and increase in amplitude on both seismograms (fig. 8 vs fig. 9).

(c) As the water wave decreases in frequency and the ground wave increases in frequency, the two frequencies eventually coincide. At this time, corresponding to a minimum group velocity \( u_g \), a maximum amplitude is reached called the Airy Phase. As time progresses, the frequency of the Airy Phase is constant while the amplitude decreases continuously.

The Pekeris and NFAR1 seismograms show the arrival of the Airy Phase at 5.7 seconds (corresponding to a \( u_g \) value of 1.47 km/sec). Use of a group Velocity Dispersion Plot (Pekeris, 1948, p. 93) yields an Airy frequency of 73 cps. Excellent agreement is observed for the seismogram and predicted frequencies (table 9).

The seismogram for Model 4 was of short time duration because the receiver was only 8.4 km from the source. At that distance the signals were not dispersed sufficiently to justify a record length (T) greater than 1.2 seconds.
Figure 9. Synthetic seismograms for a layer over a half space: Model 4. The record at 8.4 km simulates the Pekeris (1948) model.
INDIANA GEOLOGICAL SURVEY GEOPHYSICAL COMPUTER PROGRAMS
ERRATA

Geophysical Computer Program 1 (Occasional Paper 10)

Page 9, 19 lines from the bottom of the page:
Second line of R(M,N,A) now reads 1+P(1,J+1)+P(1,J-1)+P(1,J+1)+P(1,J-1))/8.0
Second line of R(M,N,A) should read 1+P(1,J+2)+P(1,J-2)+P(1,J+2)+P(1,J-2))/8.0

Page 9, 4 lines from the bottom of the page:

Page 14, line 6, which reads C(6,12)=~0.0400T, may be deleted.

Geophysical Computer Program 2 (Occasional Paper 13)

Page 11, line 18:
Now reads: (1,170)ITYPE,Z(I),X(I)
Should read: (2,230)ITYPE,Z(I),X(I)

Page 12, after line 18:
Insert: 230 FORMAT (1I,F4.0,F4.1)

Geophysical Computer Program 3 (Occasional Paper 14)

Page 12, line 11:
Now reads: 10 A(I+MN)=A(I)
Should read: 10 A(M+K-I)=A(N+K-I)

Geophysical Computer Program 7 (Occasional Paper 29)

Subroutine MYLINE2 has been removed from the program. Delete all references to this subroutine and read all references to "11 subroutines" as "10 subroutines."

Page 38:
Now reads: 30 X 27 km region
Should read: 31 X 27 km region

Page 39:
Now reads: 6 X 40 km region
Should read: 10 X 40 km region

Page 44:
Now reads: distance 200 km
Should read: distance 20 km

Page 52:
Now reads: as a function time
Should read: as a function of time