

FOLKGAMES AND GAME THEORY

This article contains essentially two parts: a basic introduction to the method of mathematical game theory (on the most elementary level), and a discussion of the possible value of game theory to the student of folkgames and recreation. This discussion is intended for non-mathematically oriented folklore students, who will find fuller expositions of the method, and techniques for solving more complex games in J.D. Williams' The Compleat Strategyst from which the first part of this paper was plagiarized.

Game theorists seem to agree that Game Theory is an unfortunate name for their mathematical approach to social behavior, and Williams feels The Theory of Games of Strategy would be more appropriate. Game Theory was first introduced by von Neumann and Morgenstern in Theory of Games and Economic Behavior in 1944. It is essentially a method for the study of decision making in situations of conflict.

Game may be defined as any conflict situation. A strategy may be defined as any complete plan of action. It is a plan that is so complete that it cannot be upset by enemy action or by nature as all possible contingencies have been accounted for in your strategy. Games may be classified according to the number of strategies available to each player. If Blue has five strategies and Red has three it is a 3 x 5 game. We will generally be concerned with the 2 x 2 game, in which only two players are involved. The number of players in any game is equal to the number of opposing interests. The player may be defined as an individual decision maker. (A game like bridge is thus a two person game.)

A payoff is what happens at the end of a game. If at the end of the game Blue wins \$10 and Red loses \$10 then:

$$\begin{aligned}\text{Blue's winnings} &= \text{Red's losses} \\ \text{Blue's winnings} - \text{Red's losses} &= 0 \\ \text{Blue's payoff} + \text{Red's payoff} &= 10 - 10 = 0\end{aligned}$$

Winning is conventionally represented by plus and losing by minus. The sum of the payoffs need not be zero, e.g. if the winner is required to donate 10% of his winnings to charity then:

$$\text{Blue payoff} - \text{Red payoff} = 9 - 10 = -1$$

When the sum of the payoff's is zero the game is called a zero-sum game. When the sum of the payoff's is not zero it is called a non-zero-sum game. War would be an example of a non-zero-sum game since utilities may be destroyed in ways that reflect a gain to neither side. Most work in game theory thus far have been on zero-sum games.

The rules specify the variables that each player controls and how resources may be utilized. We must be able to determine what happens at the end of the game from the rules. This information can then be noted in a game matrix. A game matrix is an array of boxes containing a payoff number:

		Red			
		1	2	3	4
Blue	1				
	2		-8		
	3				
	4			6	

When Blue plays strategy four and Red plays strategy three Blue wins six. When Blue plays strategy two and Red plays two Blue loses eight. The convention is that positive numbers mean a gain for blue and negative numbers a gain for Red. This is the Game Theory model of the conflict.

Game Theory assumes that "the sensible object of the player is to gain as much from the game as he can safely, in the face of a skillful opponent who is pursuing an antithetical goal. The individual must consider how to achieve as much as possible taking into account that his actions have an effect on all.

Let us consider a game that is in favor of Blue, viz. all the payoffs are positive. Game Theory assumes that "Blue wishes to act in such a manner that the least number he can win is as great as possible irrespective of what Red does. Red's desire is to make the greatest number of valubles that he must relinquish as small as possible, irrespective of Blue's action." This philosophy is sufficient to specify choices of strategy.

A range of mountains exist with sixteen peaks. Four roads enter the range from the north and the west and they intersect at each of the peaks. Mr. X likes high altitudes but his wife does not, so they decide that Mrs. X will choose one of the northern roads and Mr. X will choose one of the western roads; they agree to camp on the peak where their roads intersect. Let us represent the game in a matrix. Each strategy represents one of the roads, each box is the point of intersection of these roads, and the payoff is none other than the height of the peaks expressed in thousands of feet:

		Mrs. X.			
		1	2	3	4
Mr. X.	1	7	2	5	1
	2	2	2	3	4
	3	5	3	4	4
	4	3	2	1	6

Mr. X examines his strategies and notes that if he chooses strategy three the lowest he can possibly go is 3,000 feet and he will get even higher if the Mrs. is somewhat careless. The Mrs. notes that if she chooses strategy two the highest she will have to go is three thousand feet and she may be able to camp somewhat lower if her husband is somewhat careless.

In this problem the minimum payoff and the maximum payoff are equal so the game is said to have a saddle-point. This means that the worst Blue can safely do is equal to best Red can safely do. If there is a saddle-point players should always use the strategies that correspond to it. If either departs from it he will suffer unnecessarily, if both

depart the situation is fluid and someone will suffer. If a saddle-point is present security measures are totally unnecessary. Mr. or Mrs. X can announce a strategy choice and the other will be unable to exploit the information and drive the other above three thousand feet.

Let us consider a 2 x 2 strategy game:

		Red		
		1	2	
Blue	1	6	5	Row min. 5*
	2	5	4	4
		6	5*	Column max.

Examine the rows (horizontal) and note down the worst that Blue could possibly do. In row one the worst Blue can do is five; in row two four. Now examine the columns (vertical). Note down for each column the worst that Red can do. In column one the worst that Red can do is six; in column two five. If Blue chooses strategy one he will never get less than five; if Red chooses strategy two he will never lose more than five. The game has a saddle-point. When the larger of the row minimum is equal to the smaller of the column maximum the game has a saddle-point. Players should always choose strategies that correspond to the saddle-point.

Let us consider another 2 x 2 game:

		Red		
		1	2	
Blue	1	3	6	Row min. 3
	2	5	4	4*
		5*	6	Col. max.

We will note that the maximum of the row minimum does not equal the minimum of the column maximums so there is no saddle-point. Blue can guarantee himself at least 4 units if he plays strategy two and Red will never have to pay more than 5 units if he plays strategy one.

If Blue chooses strategy one then Red will discover it and drive his earnings down to 3. If Blue chooses strategy two Red will keep the payoff at 4. Yet Blue can win between 4 and 5 if he uses both his strategies. In any one play of the game the players are limited to one strategy by definition. So Blue must sometimes use strategy one and sometimes strategy two. Blue must use a grand strategy which contains the original strategies as elements. Elements, that will tell him when to use each individual strategy. In Game Theory a grand strategy is known as a mixed strategy. Blue must keep his decision as to which particular strategy he will use in a specific play of the game secret from his opponent, but he may allow his opponent to gain complete information of his past actions as well as his grand strategy. If Blue's grand strategy is good, Red cannot prevent him from winning all that the game affords him, and although Red may know Blue's grand strategy he will be unable to determine his choice in a particular game if Blue leaves his decision to a chance device. Decisions regarding choice of strategy should always be left to a chance device unless the game contains a saddle-point.

If in our game Red tosses a coin to decide whether to play strategy one or strategy two - it means that half the time Red will have to pay 3 and half the time 6 when Blue plays strategy one. On the average

then, Red will have to pay:

$$\frac{1}{2}(3+6) = 4\frac{1}{2}$$

and when Blue uses strategy two:

$$\frac{1}{2}(5+4) = 4\frac{1}{2}$$

So by tossing a coin Red can make the payoff average $4\frac{1}{2}$ instead the 5 he would have had to pay and chosen the strategy which made him pay the minimum of the maximums, which was strategy one. Mixed strategies are always expressed in terms of odds. That means Blue may be called upon to mix his strategies according to the odds of 3:1 or some other mixture.

There is a mechanical method for finding the odds in any 2 x 2 game. For finding the odds Red is to use simply subtract the numbers in the second row from those in the first row. The numbers that result are the odds for the adjacent strategy. The same method can be used to determine the odds Blue should use; subtract the numbers in the second column from those in the first, the result is the odds for the adjacent strategy. e. g.

		Red	
		3	6
Blue		5	4

$$\begin{array}{l} \text{Odds for Red one} \quad 6-4 = 2 \\ \text{Red two} \quad 3-5 = -2 = 2 \end{array} \quad 2 : 2 = 1 : 1$$

$$\begin{array}{l} \text{Odds for Blue one} \quad 3-6 = -3 = 3 \\ \text{Blue two} \quad 5-4 = 1 \end{array} \quad 3 : 1$$

We can see that for Blue to get the most out of the game he must use strategy one three times to every time he uses strategy two. For Red to lose the least he must mix his strategies equally. Saddle-points must always be looked for first, since the method for finding the odds will give false results when the game has a saddle-point.

There is a quantity of payoff in each game that good play will win, and this is called the value of the game. On the average, you cannot win more than the value of the game unless your opponent is playing poorly. If the game is fair to both players its value will be zero. If the game has a saddle-point the value of the game is the value of the saddle. In a 2 x 2 game requiring a mixed strategy, the value of the game is the average payoff which results from the use of the best mixture of one player's strategies against either strategy of the other.

In a 2 x 2 game if there is a saddle-point and one player does not utilize this strategy he will suffer. In a mixed strategy 2 x 2 game if one player plays poorly, the payoff will still average at the value of the game if the other player utilizes the optimum mixture. In other words, in a 2 x 2 game so that the payoff will average at the value of the game if the other player utilizes the optimum mixture. In other

words, in a 2 x 2 game, one player playing well is sufficient to stabilize the game so that the payoff will average at the game's value. This is not true for games of 3 x 3 and larger.

Let us examine one sample problem:

Blue has two installations. He is capable of successfully defending either of them, but not both; and Red is capable of attacking either but not both. Further, one of the installations is three times as valuable as the other. What strategies should they adopt?

The game matrix should look like this:

		Red		
		1	2	
Blue	1	4	1	[The defense or attack of the lesser installation be designated as Strategy one.]
	2	3	4	

There is no saddle-point, so we must compute the best mixed strategies for both Red and Blue:

Blue odds for strategy one $3-4 = -1 = 1$
 strategy two $4-3 = 1$

Red odds for strategy one $1-4 = -3 = 3$
 strategy two $4-3 = 1$

Blue should define his more valuable position with the odds of 3:1. Red should favor attacking the lesser installation with the odds of 3:1. The value of the game for Blue is:

$$\frac{1(4) + (3)}{4} = \frac{13}{4} = 3\frac{1}{4}$$

Mathematical abstractions or models in the behavioral sciences have met with both the extremes of oppositions and support. Those opposed to such models claim that they are sterile and restrictive and they oversimplify the phenomena being studied. They ignore subtleties and much of the vital content of the subject matter is often destroyed for the sake of the mathematical analysis. Those who strongly support the mathematical point of view claim that if the problem being studied is really understood it can be mathematized.

Mathematical models are indeed restrictive and many of the nuances of the problem are lost. Yet mathematical models allow a greater depth of analysis of a problem than can normally be achieved if it is approached in the language of sociology, economics or of the other behavioral sciences. The models will allow for an analysis of some aspects of the essential features of behavioral problems.

The real game i.e. the real conflict situation includes rules, regulations, motives of players, geographical features and other insignificant aspects to the actual play of the game. What we are analyzing is a written abstraction of the real game, and all we can expect is that

there is a significant relation between the two that allow us to transpose our conclusions from the model to the real situation. The significance of the relationship between the two depends upon how well the model is formulated.

Some questions might already have arisen in your minds concerning the problems that have already been presented. How are the payoff boxes filled in? The Game Theorist can only answer that the assumption is that the payoff can be measured numerically and that the payoffs can all be expressed in the same units. That means if money and prestige are involved, there is an exchange ratio between the two so they may both be expressed numerically in the same matrix. Another obvious problem is defining the problem sufficiently so that all alternatives have been enumerated.

But our crucial problem is that concerning the utility of Game Theory to the folklorist interested in folkgames and recreation. We may intimate from the preceding discussion that Game Theory is not an essential methodological tool for the folklorist. But we should not dismiss Game Theory too readily as it appears useful for analyzing certain kinds of problems. Through Game Theory we can establish criteria for evaluating the fairness of folkgames. Are all folkgames fair? Do unfair folkgames remain in the folk repertoire of games? Are unfair folkgames only found among certain groups, or are they equally distributed throughout a culture? Game Theory also provides us with objective criteria for evaluating good and poor play.

I propose to analyze several guessing games all of which are merely variations on one particular theme and which in themselves are not particularly interesting; however they are amenable to the elementary mathematical analysis of which we are capable.

William Wells Newell in his book Games and Songs of American Children cites a game called "Odd or Even":

A small number of beans or other counters are held in the hand, and the question is, Odd or Even? If the guess is even, and the true number is odd, it is said "Give me one to make it odd," and vice versa. The game is continued until all the counters belong to one or other of the two players. (p. 147)

Though his particular version does not say what happens when the player guesses right we may assume that he wins the counters his opponent is holding. Let us assume that the player who holds the beans in his hand will never use more than one bean to symbolize odds and two beans to symbolize evens as anything greater would prove an unnecessary risk. Let Blue be the holder and Red the guesser. Our matrix would be as follows:

		Red		
		odd	even	
Blue	odd	-1	1	3
	even	1	-2	2
				:
				2
				3 : 2

There is no saddle-point and each player must mix his 'odds' and 'evens' according to the ratio of 3:2. The value of the game to blue is:

$$\frac{3(-1) + 2(1)}{5} = \frac{-1}{5}$$

The game is unfair to Blue.

Let's examine another game, this one contained in the first volume of Frank C. Brown's Collection of North Carolina Folklore (the section on games is edited by Paul Brewster). A game is cited called "Hul Gul" in which a child holds out a handful of corn and another must guess the amount contained. If the guess is correct the guesser gets all, if it is incorrect he must give the number he guessed. For the purpose of analysis we will limit the maximum number of grains one can hold to three. We must do this here since our knowledge of theory is too elementary. We are faced with the following 3 x 3 game:

		1	2	3	
		-1	2	3	Blue strategy mix
Blue	1	1	-2	3	5
(holder)	2	1	2	-3	8
	3	1	2	-3	9
					Red strategy 6 : 3 : 2

As you are not familiar with the solution of a 3 x 3 game you will have to depend solely upon my analysis. Blue (the holder) should vary holding one, two or three grains of corn according to the odds of 5:8:9. Red (the guesser) should guess one, two and three according to the mixture of 6:3:2. The value of the game for Blue is:

$$\frac{5(-1) + 8(1) + 9(1)}{22} = \frac{12}{22} = \frac{6}{11}$$

The game is unfair to Red, the guesser.

A variant of this game is called "Jack in the Bush". The game is the same except that if the guesser is wrong he must only pay the difference. The matrix is thus:

		1	2	3
		-1	1	2
Blue(holder)	1	1	-2	1
	2	2	1	-3
	3	2	1	-3

Blue the holder must vary strategies one, two and three according to the ratio of 15:7:9. Red must do likewise. The value of the game to Blue is:

$$\frac{15(-1) + 7(1) + 9(2)}{31} = \frac{10}{31}$$

The game is also unfair to Red the guesser.

Before we conclude that all games are unfair let's examine a game called rock, paper, scissors and which is also analyzed by Williams.

The two players involved symbolize with one of their hands one of three objects, a rock, paper or scissors. Scissors win over paper since they cut paper, rock wins over scissors since a scissors can't cut rock, paper wins over rock since a paper can cover the rock. The loser in each particular play of the game received a blow on the fist which we can evaluate as -1. A draw results in no action and may be valued at zero.

	Red		
	r	p	s
r	0	-1	1
p	1	0	-1
s	-1	1	0

The game is symmetrical with respect to the diagonal (which consists of zeros) one side being opposite in sign to the other. Game Theory teaches us that such a game has a value of zero (it is fair to both players) and both players will employ the same grand strategy. The grand strategy in this case happens to be a mixture of 1:1:1.

We can see by the above examples that not all folkgames are fair. Using the techniques of Game Theory the folklorist can go on to discover which games are fair, which are not, and whether this seems to have any effect on their popularity. Are unfair games more prevalent among children than adults? Do unfair games tend to drop out of the repertoire? Do games of complete information (which therefore must have a saddle-point) drop out of the repertoire once the saddle-point is discovered? At what age group does the solution to the game become known? (Tic-Tac-Toe is a case in point.)

Game Theory also provides a method for evaluating good and poor play. To what extent is the optimum strategy in a particular game approached by the folk players? Are there certain kinds of games in which optimum strategies are more often approached than in others?

Game Theory was not created especially for the folklorist and it appears that it is not a crucial tool in his analysis of folkgames. It however provides the method for analysis of certain problems relating to folkgames which should be of interest to the student of folklore.

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