The results are shown in columns 2 to 5 of Table I of our latest "consistent DD" fits\textsuperscript{1} to our $^{90}$Zr($p,p'$) cross sections, and for two transitions in $^{89}$Y with a Density Dependent (DD) force used in both the folded optical potentials (OP) and the ($p,p'$) form factors. These microscopic "consistent DD" calculations, at 121.5 and 159.5 MeV, use the Local Density Approximation (LDA) in a Schroedinger approach with:

1. the latest Hamburg DD force,\textsuperscript{2} (2) point proton transition densities $p_P(r)$ unfolded from phenomenological ($e,e'$) transition charge densities,\textsuperscript{3,4} which were kept fixed, (3) the magnitude of each neutron transition density.

\begin{align*}
\rho_n^P(r) &= a_{DD}(E) \cdot (N/Z) \cdot \rho_P^P(r) \\
\text{which was adjusted with the factor } a_{DD}(E) \text{ until the theoretical cross section matched the peak of the } (p,p') \text{ cross section data, and}
\end{align*}

(4) exchange amplitudes which were treated approximately in a pseudo-potential approach\textsuperscript{5} to permit the insertion of these point proton densities from $(e,e')$ data.\textsuperscript{3,4}

The values of $a_{DD}(E)$ extracted from these fits to the first maximum of each measured cross section are displayed rather than the ratio $\rho_n^P / \rho_P^P$, because (a) the value $a = 1.00$ corresponds to the "scaling" $(\rho_n^P / \rho_P^P) = N/Z$, often assumed for a pure mass vibration, and (b) comparing $a_{DD}(E)$ with the value 1.00 readily permits, the recognition of which

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
\textbf{Ep (MeV)} & \textbf{121.5} & \textbf{159.5} & \textbf{140.5} & \textbf{800 MeV} & \textbf{aF(800)} & \textbf{aF(800) - aDD(141)} \\
\hline
$^{90}$Zr state & \textbf{aDD(122)} & \textbf{aDD(160)} & \textbf{aDD(141)} & \textbf{Diff. from aDD(141) = 1.00} & \textbf{Diff. from aF(800) = 1.00} & \textbf{aY(160) - aZ(160)} \\
\hline
$^{89}$Y state & \textbf{aDD(160)} & \textbf{aY(160)} & \textbf{aZ(160)} & \textbf{Diff. from aD(141)} & \textbf{Diff. from aY(160)} & \textbf{Diff. from aZ(160)} \\
\hline
8$^+$ & 1.034 & 1.034 & 1.034 & scaling (3\%) & 0.98 & scaling (1.6\%) \\
\hline
6$^+$ & 0.750 & 0.966 & 0.858 & p dom. (14\%) & 0.85 & p dom. (15\%) \\
\hline
4$^+$ & 0.763 & 1.030 & 0.896 & p dom. (10\%) & 0.90 & p dom. (10\%) \\
\hline
2$^+$ & 0.688 & 0.680 & 0.684 & p dom. (32\%) & 0.93 & p dom. (7\%) \\
\hline
2$^+$ & 1.254 & 1.344 & 1.274 & n dom. (27\%) & 1.34 & n dom. (34\%) \\
\hline
3$^+$ & 0.809 & 0.864 & 0.838 & p dom. (16\%) & 1.03 & p dom. (3\%) \\
\hline
5/2$^-$ & 0.66 & & & p dom. (34\%) & & -3.0\% \\
\hline
3/2$^-$ & 0.66 & & & p dom. (34\%) & & -3.0\% \\
\hline
\end{tabular}
\end{table}
transitions are dominated by proton or neutron contributions (over and above the factor of N/Z with \( a_{DD}(E) = 1.00 \)). Our use of point transition densities \( p^P(r) \) unfolded from transition charge densities \( p_{tr}(e,e') \) extracted phenomenologically from \((e,e')\) data allows us (1) to include valence and core excitation contributions phenomenologically and thus (2) avoid the lack of complete proton wave functions recently demonstrated by shell model and broken pair model calculations of \(^{90}\text{Zr}(e,e')\) cross sections\(^4\) and by a large basis shell model analysis\(^6\) of \(^{90}\text{Zr}(p,p')\) cross sections.

The primary advantage of our use of these proton point transition densities \( p^P(r) \) from phenomenological \( e,e' \) transition charge densities in our ALLWORLD \((p,p')\) calculations is that we are able to extract phenomenological neutron transition densities, \( p^n(r) \), which for the present calculations, and only as a first approximation, we have assumed have the same radial shapes as the point proton \( p^P(r) \).

As a test of the quality of the these "consistent DD" \((p,p')\) calculations, for the values of \( a_{DD}(160) \) in Table I extracted from fits to the first maximum of our cross section data at 159.5 MeV, we then examined the quality of the shapes predicted for \((p,p')\) analyzing powers \( (A_y) \) in comparison with our measured \( A_y \) data. These "consistent DD" predictions for the shapes of \((p,p')\) \( A_y \) are clearly superior to calculations with phenomenological optical potentials with the same DD force in only the \((p,p')\) form factors and superior to calculations with a free NN potential.

The present "consistent DD" calculations probably do not yet yield the correct magnitude for the neutron transition densities, even within the present Schroedinger approach, because we have assumed the radial shapes of these neutron densities are the same as the corresponding proton point transition densities. Our ongoing calculations are investigating the effects on theoretical predictions for \( A_y \) shapes, and on cross section shapes and magnitudes, of changing the radial shapes of these neutron point transition densities. We are also looking into the different effects on shapes of analyzing powers and cross sections of (1) different radial overlaps in the \((p,p')\) form factors (at different projectile energies \( E_p \)) of proton transition densities \( p^P \) and \( p^n \) with \( a_{DD} \) dependent DD potentials \( v_{DD}^{pp} \) and \( v_{DD}^{pn} \), and (b) with the proton distorted waves generated from a folded OP using the same DD forces; and (2) in our \( 800 \) MeV calculations\(^7\) we are looking at the radial overlaps of the \( p^P \) and \( p^n \) \( tr \) \( tr \) phenomenological neutron transition densities \( p^n(r) \) with \( a_{DD}(800) \) \( tr \) \( tr \) different free NN potentials \( V(800) \) and \( V(800) \) in the \( pp \) \( pp \) \( (p,p')\) form factors, and (b) with the proton distorted waves generated from a folded potential using these same free NN forces.

The values of \( a_F(800) \) shown in columns 6 to 8 of Table I are the results of microscopic fits\(^7\) to two sets of cross section data\(^8,9\) at \( 800 \) MeV, with a procedure identical to that for the DD calculations described above (with \( p^P(r) \) from \((e,e')\) charge densities), except that at \( 800 \) MeV the Love-Franey force was used in a folded OP and in the \((p,p')\) form factors. Since it is difficult to estimate overall errors in the values of \( a \) extracted from "consistent DD" fits at each projectile energy \( (E_p) \), and only two transitions show significant changes of \( a_{DD} \) as \( E_p \) changes from 121.5 to 159.5 MeV, these values at \( 800 \) MeV \( (a_F(800)) \) are compared with average values \( a_{DD}(AV) = a_{DD}(141) \) \( (\text{in column } 4) \) formed at 140.5 MeV from the values extracted from our "consistent DD" fits.
to our cross section data at 121.5 and 159.5 MeV.

The most striking patterns which emerge from the comparisons of these values $a_{DD}(141)$ and $a_F(800)$ ($F$ reminds us of the free NN force used at 800 MeV) are:

1. In the $(p,p')$ reaction at both 141 and 800 MeV the transition to the $8^+$ state has essentially the same large ratio of neutron to proton contributions, $(\rho_{n}^{\text{tr}})/\rho_{p}^{\text{tr}} = N/Z$ and $a = 1.00$, whereas a neutron contribution to this transition goes entirely undetected in the analyses of a recent $(e,e')$ experiment\(^4\) and this state is established ((for $(e,e')$ scattering) as a pure proton state involving only a recoupling of the $(l_g/g_2/2)$ component of the ground state; and

2. there is a very strong energy dependence of the neutron sensitivity of our proton probe for the group (a) of more collective states ($2^+$ and $3^-$) but there is almost no change of the value of $a$ (from 122 to 800 MeV) for the other group (b) of states ($4^+,$ $6^+,$ $8^+$, and $2^+$) believed\(^4\) to have simpler structures (at least in regard to proton configurations). This energy dependence, of $(\rho_{n}^{\text{tr}})/\rho_{p}^{\text{tr}} = a (N/Z)$ for the $2^+$ transition, extracted from our microscopic $(p,p')$ calculations with point proton $p_{P}^{\text{tr}}$ unfolded from the phenomenological $(e,e')$ transition charge density, agrees with the results of a recent investigation\(^1\) of the neutron sensitivity of the proton probe in which the collective form factor was assumed for $(p,p')$ transitions and phenomenological optical potentials (OP) were used; in our calculations we used folded optical potentials at all energies $E_p$, with a DD force at 122 and 160 MeV and free NN force at 800 MeV. This similar tracking of the energy dependent neutron sensitivity of the proton probe in our more microscopic calculations [with radial shapes of $p_{P}^{\text{tr}}$ and $p_{n}^{\text{tr}}$ here defined by $(e,e')$ data] suggests that the effective form factors at 122 and 160 MeV (with interior contributions reduced by the Pauli Blocking included in the present DD force) when overlapped with the distorted waves at 122 and 160 MeV (also with interior contributions reduced by the use of the DD force in the folded OP) lead to similar overall transition amplitudes as with a collective form factor and phenomenological OP.

The new and surprising result from our “consistent DD” calculations at 122 and 160 MeV and free NN force calculations at 800 MeV is the lack of energy-dependent neutron sensitivity of our proton probe for the transitions to the states with simpler structures\(^4\) ($4^+$, $6^+$, $8^+$, and $2^+$). This difference in neutron sensitivity of the $(p,p')$ reaction in transitions to collective and simpler states in $^{90}$Zr would not have been very convincing from our calculations alone, but the similar tracking of our values of $(\rho_{n}^{\text{tr}})/\rho_{p}^{\text{tr}}$ with projectile energy with those from Ref. 10 for the more collective $2^+$ state support our energy dependent sensitivity for collective states; our additional result of a strong energy-dependent neutron sensitivity also for the collective $3^-$ state more strongly suggests that this striking difference in neutron sensitivity is sensitive to differences in nuclear structure. This striking pattern of different energy-dependent neutron sensitivity for collective and simpler states is obscured if a conventional OP (of Single Woods-Saxon (SWS) type) is used in microscopic calculations with the same DD force still used in the $(p,p')$ form factor (see Table II and discussion below). In the “consistent DD” calculations leading to Table I results (at 122 and 160 MeV), the DD force reduces the magnitude of the distorted waves derived from the folded OP for both protons and neutrons (compared with
Table II

| $E_p$(MeV) | $^{90}$Zr state (N/Z=1.25) | $^{99}$Y state (N/Z=1.28) | 
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|            | $\alpha_{PH}(122)$ | $\alpha_{PH}(160)$ | $\alpha_{PH}(141)$ | $\alpha_{PH}(160)$ | $\alpha_{PH}(141)$ | $\alpha_{PH}(141)$ |
| 121.5      | 0.08            | 0.256           | 0.17            | p dom. (83%)       | 1.00            | 0.43            | p dom. (57%)       |
| 159.5      | 0.24            | 0.384           | 0.31            | p dom. (69%)       |                | 0.43            | p dom. (57%)       |
| 140.5      | 0.36            | 0.568           | 0.41            | p dom. (54%)       |                |                |                  |
|            | 0.44            | 0.44            | 0.44            | p dom. (56%)       |                |                |                  |
| 8$^+$      | 0.80            | No data at max. |                | p dom. (56%)       | 1.00            | 0.43            | p dom. (52%)       |
| 2$^+$      | 0.48            | No Calc. yet    |                | 0.43              |                | 0.43            | p dom. (57%)       |
| 5/2$^-$    | 0.43            |                |                | 0.43              |                | 0.43            | p dom. (57%)       |
| 3/2$^-$    | 0.43            |                |                | 0.43              |                | 0.43            | p dom. (57%)       |

the similar calculations with the SWS phenomenological OP); if the present DD force has reduced these DD distorted waves too much then our procedure has resulted in erroneously high values of $\rho_n^0/\rho_p^0$ because we increase the magnitude of this neutron $\rho_n^0$ magnitude fixed at the value calibrated by $(e,e')$ scattering. There is a possibility that more correct values of improved "consistent DD" $\alpha_{DD}(122)$ and $\alpha_{DD}(160)$ could later be obtained, with lower values of the average $\alpha_{DD}(141)$, but such lower values (for less reduction of the DD distorted waves) would only make the strong energy dependence of the neutron sensitivity for the $2^+$ and $3^+$ transitions even more striking.

Other interesting features from Table I are that the values $\alpha_{DD}(141)$ (see columns 4 and 5) suggest that protons dominate the transitions to the $2^+$, $4^+$, $6^+$, and $3^-$ states (by 10% to 32% more than the factor N/Z for $\alpha_{DD}(141)$ very close to 1.00). At 800 MeV, however, the proton probe senses the relative neutron contributions in both the $8^+$ and $3^-$ transitions with the "scaling" value $(N/Z$ and $\alpha = 1.00)$, the $2^+$ transition remains heavily neutron-dominated (34% above the scaling value of $\alpha = 1.00$); but the $2^+$ transition is observed with a very much reduced proton dominance (7% below the value of $\alpha = 1.00$ compared with 32% below the scaling value for $E_p = 141$ MeV), in other words the neutron sensitivity of our proton probe has increased as $E_p$ increased from 141 to 800 MeV.

We show the much lower values of $\alpha_{PH}(122)$ and $\alpha_{PH}(160)$ in Table II which we had to use avoid
overestimating the \((p, p')\) cross sections of our data at 121.5 and 159.5 MeV, when the same DD force is used in the \((p, p')\) form factors, but a phenomenological OP (conventional SWS OP) is used which fits our elastic data. A comparison of columns 4 and 5 of Tables I and II shows the enormous effect of failing to use the DD force (which includes Pauli Blocking effects) consistently in a folded OP as well as in the \((p, p')\) form factors; very different conclusions are drawn from columns 5, e.g. the \(3^+\) transition (with \(a_{PH}(141) = 0.17\)) would appear very heavily dominated by protons from Table II, whereas Table I shows this transition with \(a_{DD}(141)\) essentially at the scaling value with much larger contributions from neutrons; and Table II would suggest the \(2^+\) transition is dominated by protons, but Table I shows this transition heavily dominated by neutrons contributions.

We note that for \(^{90}\text{Zr}\) \((N/Z = 1.25)\) a transition with \(a = 1.00\) has a neutron contribution 25% greater than an isoscalar transition (\(p_n = p_p\)).

As a test of which type of calculation at 159.5 MeV is more valid, we compared the shapes of \((p, p')\) analyzing powers \(A_y\) predicted, for the values of \(a_{DD}(160)\) and \(a_{PH}(160)\) of Tables I and II needed to fit the first maximum of each measured cross section. As examples, we show the cross section fits and predictions for \(A_y\) in Figures 1 through 4 for the \(^{90}\text{Zr}\) transitions to the \(6^+, 4^+, 2^+, \text{and} 2^+\) states, respectively. Figures 1 to 4 show: (1) that for all transitions the full-line curves for analyzing powers \(A_y\) predicted by our “consistent DD” calculations (with values of \(a_{DD}(160)\) in Table I) are much superior to the short-dash curves for \(A_y\) predicted by our calculations (with values of \(a_{PH}(160)\) in Table II) with a phenomenological OP and DD in the \((p, p')\) form factor, (2) the “consistent DD” prediction is most successful

![Figure 1](image_url)  
**Figure 1.** Fits to cross section data for the first \(6^+\) state in \(^{90}\text{Zr}\) measured at a proton bombarding energy of 159.5 MeV, with “consistent DD” calculations (full-line curve with \(a_{DD}(160) = 0.97\), DD in OP; long-dash curve without neutrons, \(a_{DD}(160) = 0.0\), DD in OP), from a calculation with a phenomenological OP (short-dash curve and \(a_{PH} = 0.38\), DD only in \(p, p'\)) and with free Paris potential in folded OP and in \((p, p')\) (the dot-dash curve with \(a_{FREE-P}(160) = 0.97\)). All calculations in this and subsequent figures use proton transition densities from electron scattering. \(A_y\) curves in lower part of figure are predictions from these fits to cross section data.
Figure 2. Fits to cross section data for the first $4^+$ state in $^{92}$Zr with "consistent DD" calculations (full-line curve with $a_{DD}(160) = 1.03$, DD in OP; long-dash curve without neutrons, $a_{DD}(160) = 0.0$, DD in OP), a calculation with a phenomenological OP (short-dash curve with $a_{PH}(160) = 0.57$, DD only in $(p,p')$ form factor). The $A_y$ curves in lower part of figure are predictions from these fits to the cross section data.

Figure 3. Fits to cross section data for the first $2^+$ state in $^{92}$Zr from "consistent DD" calculations with DD in OP (full-line curve with $a_{DD}(160) = 0.68$, and long-dash curve without neutrons, $a_{DD}(160) = 0.0$, DD in OP), and a calculation with a phenomenological OP (short-dash curve, with $a_{PH}(160) = 0.44$). The $A_y$ curves in lower part of figure are predictions from these fits to cross section data.
for the $A_y$ shape for the $6^+$ state, a good fit to the
$A_y$ shape for the $4^+$ state, but much worse for the $A_y$
shape for the more collective $2^+_2$ state, (3) the
"consistent DD" prediction for the $A_y$ shape for the $2^+_2$
state known to have a simpler proton structure$^4$) is
significantly better than for the collective $2^+_1$
(4) the dot-dash curves in Figs. 1 and 4 are examples
of the much poorer $A_y$ shapes predicted by our
calculations$^1$ with the free Paris NN force ($k_F = 0.0$)
in both the folded OP and in the $(p,p')$ form factor,
(5) the long-dash curves from "consistent DD"
calculations (with $p^n_{tr}$ = 0.0, with only proton
contributions) fit the minima of the $A_y$ data much
better than when the neutron $p^n_{tr}$ are included to fit
the cross section magnitudes, but the predicted shapes

of the full-line curves (with $a_{DD}(160)$ from Table I)
agree better with the $A_y$ data at forward angles when
neutron $p^n_{tr}$ are included (except for the failure near 20°
for the $2^+_1$ state). This last result suggests that the
radial shapes of $p^n_{tr}$ should not be the same as that of
the $p^p_{tr}$ as we have assumed in the present calculations,
as a first approximation to the effects of neutron
contributions on $A_y$ shapes. New calculations show
significant sensitivity of the $A_y$ shape and the cross
section to changes in the radial shape near the maximum
of the neutron $p^n_{tr}$.

The predictions for the $A_y$ shape for the
transition to the $8^+$ state (not shown here) are very
good with both the "consistent DD" calculation
($a_{DD}(160) = 1.034$) and with the phenomenological OP
calculations ($a_{PH}(160) = 0.256$).

The values of $a_{DD}(160)$ needed in "consistent DD"
calculations to fit the first maximum of the cross for
each $L = 2$ transition to the $3/2^-$ and $5/2^+$ states in
$^{90}$Zr are given at the bottom of Table I. These values
are only 3.0 percent lower than the value for the $2^+_1$
state in $^{90}$Zr. In Fig. 5 we show (a) the very
interesting differences in the shapes of the measured
cross sections, which (b) are approximately described
by the full-line curves from the "consistent DD"
calculations especially at smaller and middle angles,
and we (c) note that much of the correct shape is
already present in the long-dash curves for the proton
amplitudes alone (with $p^n_{tr} = 0.0$). In Fig. 5, the
small-dash curves are for calculations with the
phenomenological OP and the DD force only in the $(p,p')$
form factors (and $a_{DD}(160)$ the same as in the
"consistent DD" calculations); poorer fits are found
(from 15° to 30°) but at larger $q$ (near 45°) this
small-dash curve for the $3/2^-$ state gives a better fit
than does the "consistent DD" calculation (full-line

![Figure 4. Analyzing power predictions compared with $A_y$
data for the second $2^+$ state in $^{90}$Zr, from fits to
cross section data, from "consistent DD" calculations
with DD in the folded OP (full-line curve with
$a_{DD}(160) = 1.34$, and the long-dash curve without
neutrons, $a_{DD}(160) = 0.0$), and from a calculation with
the free Paris potential in a folded OP and in $(p,p')$
form factors (the dot-dash curve is with
$a_{FREE-P}(160) = 1.60$).](image-url)
Figure 5. Fits to cross section data for the $3/2^-$ and $5/2^-$ states in $^{89}$Y measured at a proton bombarding energy of 159 MeV ($5/2^-$ state in upper part of Figure, $3/2^-$ state in lower part), with “consistent DD” calculations (full-line curves for $a_{DD}(160) = 0.66$, DD in OP; long-dash curves without neutrons, $a_{DD}(160) = 0.0$, DD in OP), and a calculation with a phenomenological OP and DD in the $(p,p')$ form factor (short-dash curves with $a_{PH}(160) = 0.43$).

To summarize, we have discovered that “consistent DD” calculations (with $pP(r)$ unfolded from $(e,e')$ charge densities) reveal (1) large neutron contributions to transitions in $^{90}$Zr and $^{89}$Y involving multipoles from 2 to 8, (2) a strong energy-dependent sensitivity of our proton probe for neutrons for collective states but none for transitions to simple states ($4^+_1, 6^+_1, 8^+_2$, and $2^+_1$), (3) these interesting patterns are obscured if the DD force is not used in a folded OP as well as in the $(p,p')$ form factors, and (4) neutron densities for $L = 2$ transitions to the $3/2^-$ and $5/2^-$ states in $^{89}$Y have neutron contributions very close to that found for the $2^+_1$ state $^{90}$Zr, but the cross section shapes for the $3/2^-$ states and $2^+_1$ states are very similar and that for the $3/2^-$ state is quite different, (5) “consistent DD” calculations provide much better predictions of $(p,p') A_y$ shapes than calculations with a phenomenological optical potential. We conclude that measurements of $(p,p')$ analyzing powers ($A_y$), coupled with “consistent DD” calculations at lower intermediate energies and Dirac impulse approximation (IA) calculations at higher energies, will provide valuable phenomenological neutron transition densities from which nuclear spectroscopists can derive new information on the neutron structures of excited states.

In an attempt to trace the source of this very interesting difference in the energy-dependent neutron curve). The shape of the data, and the "consistent DD" calculation (full-line curve), for the $3/2^-$ state are very similar to those for the $2^+_1$ state in $^{90}$Zr at the same projectile energy, but the shape for this $3/2^-$ state is quite different. These $^{89}$Y data, and microscopic calculations, suggest quite different structures of these two states, as deduced for the proton structures from $(e,e')$ scattering analyses.4/
sensitivity for collective and simpler transitions, we are looking at (1) the effects of different radial overlaps of proton and neutron densities with different forces ($V_{pp}$ and $V_{pn}$) and with the DD distorted waves, and (2) whether the effects of these radial overlaps are different in transitions for which the transition densities are known to be different (as the proton densities are known to be for our $^{90}$Zr and $^{89}$Y transitions).

In collaboration with our new faculty member at the University of Georgia, E.R. Siciliano, we will shortly begin Dirac IA calculations for the $^{90}$Zr(p,p') reaction, first at 800 MeV and later at lower energies to compare with "consistent DD" calculations. We plan further (p,p') analyzing power measurements at different projectile energies, because our most recent "consistent DD" calculations show major changes in predicted A shapes, which will give greater sensitivity to neutron transition density shapes than is possible from spot measurements at only one projectile energy.

Even if our planned $A_{y}$ measurements show that the "consistent DD" calculations cannot fully describe all the data, even when neutron transition density shapes are changed, these $A_{y}$ data (and cross sections) will provide valuable tests of the relative merits of the Schroedinger "consistent DD" approach (with Pauli Blocking included approximately) and the current Dirac approach (without Pauli Blocking); these tests will probably point to the need for a blending of these two approaches.

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1) V. Penumetcha and Alan Scott, with advice from W.G. Love; the Florida State program ALLWORLD, modified by M.A. Franey was used. Calculations with earlier DD forces were done at the University of Hamburg and at Georgia.


11) Alan Scott, unpublished.