

Emotion as a Thermostat: Representing Emotion Regulation Using a Damped Oscillator Model

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The authors present in this study a damped oscillator model that provides a direct mathematical basis for testing the notion of emotion as a self-regulatory thermostat. Parameters from this model reflect individual differences in emotional lability and the ability to regulate emotion. The authors discuss concepts such as intensity, rate of change, and acceleration in the context of emotion, and they illustrate the strengths of this approach in comparison with spectral analysis and growth curve models. The utility of this modeling approach is illustrated using daily emotion ratings from 179 college students over 52 consecutive days. Overall, the damped oscillator model provides a meaningful way of representing emotion regulation as a dynamic process and helps identify the dominant periodicities in individuals' emotions.

Keywords: differential equation, oscillator, dynamic, emotion regulation, spectral analysis

Building on popular belief in the “blue Monday” phenomenon, Larsen and Kasimatis (1990; see also Huttenlocher, 1992; Reid, 2000) presented early evidence advocating the existence of a weekly cycle in individuals' daily mood fluctuations. Using spectral analysis, these researchers found a strong weekly rhythm in a group of college students' average hedonic level. In broader contexts, rhythmicity has also been examined in relation to diurnal changes in mood and circadian activity (e.g., Larsen, 1985a; Murray, Allen, Trinder, & Burgess, 2002; Rusting & Larsen, 1998) and seasonal affective disorder (e.g., Johansson et al., 2003). As more emerging evidence has shown clear associations between these cyclicities and other key aspects of life (e.g., Brown, 2000; Pettengill, 1993), a closer examination of the nature and determinants of these dynamic processes is imperative.

Despite ample research suggesting the existence of different physiological and affective cycles in everyday life (e.g., Brown, 2000; Stone, 1985), methodologies amenable to the modeling of

cyclic change are still lacking. Even though researchers have begun to incorporate longitudinal designs into the study of affect (e.g., Diener, Fujita, & Smith, 1995; Eid & Diener, 1999), the focus on a linear notion of change is still prominent in these studies. With the exceptions of spectral analysis (e.g., Larsen & Kasimatis, 1990) and other related frequency-domain and time-series analyses, most of the dominant longitudinal methodologies (e.g., growth curves and hierarchical linear models; Bryk & Raudenbush, 1987; McArdle & Epstein, 1987; Meredith & Tisak, 1990) have been used to represent linear change, even though several nonlinear extensions of these methodologies have been proposed in the psychometric literature (e.g., Browne & du Toit, 1991).

In a recent theoretical model of mood regulation, Larsen (2000; see also Carver & Scheier, 1982, 1990) proposed that mood regulation is, by nature, a dynamic process. A weekly mood cycle, for instance, imparts that emotions are dynamic—they exhibit specific patterns of change over the course of the week. Following Larsen's (2000) lead, we present in this article a damped oscillator model (Boker & Graham, 1998; Nesselrode & Boker, 1994) that provides a direct mathematical basis for testing this mood regulation model. In our presentation, we discuss the basic elements of the model, including intensity, rate of change, and change in the rate of change as they pertain to the study of emotion. We also highlight some of the similarities and differences between this differential equation modeling approach and other more widely known approaches, such as growth curve model and spectral analysis. Finally, we illustrate through an empirical example the potential utility of the approach as a tool for understanding emotion processes (e.g., interindividual differences in patterns of day-to-day emotional variability; see Fleeson, Malanos, & Achille, 2002; Nesselrode & Baltes, 1979, for the rationales for such an examination).

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Intensity, Rate of Change, and Acceleration in Emotion

The temporal aspect of emotion and its relatedness to individual differences in emotion regulation have been discussed by several researchers (e.g., Davidson, 1998; Larsen, 2000). The rate of change and acceleration represent different temporal characteristics of affective processes. Before presenting the mathematical foundations of the damped oscillator model, we begin by clarifying some basic terms and concepts pertinent to the differential equation model, including intensity (or level), rate of change, and change in the rate of change (or acceleration).

Level or Intensity

Intensity (denoted as Y in the model equation to come), in the context of the differential equation model presented here, represents the magnitude of displacement or deviation in an individual's emotion compared with some baseline affect level.¹ As certain environmental cues or objects elicit a particular kind of emotion (e.g., sadness) from an individual, the extent to which the individual's level of emotion deviates from his or her baseline level represents the intensity of that emotion. High intensities represent large distances from baseline (regardless of direction), and low intensities represent small distances from baseline. Here, intensity is conceptualized and used explicitly to characterize states, rather than as a personality trait. Stable individual differences in emotion intensity observed across different situations and contexts (e.g., Moskowitz, 1982) may, in contrast, reflect a trait-like disposition of affect intensity (Larsen, 1985b; Larsen, Diener, & Emmons, 1986).

Rate of Change

The *rate of change* (Y') represents the magnitude of change in intensity over one unit of time (e.g., day to day, pretest vs. posttest, year to year, etc.). In other words, rate of change describes the change in emotion intensity in relation to time (i.e., first derivative of emotion intensity with respect to time). In studies of affect, individual differences in the rate of change have been found to emerge independently of initial affect intensity (Hemenover, 2003) and have been linked to attributes such as emotional clarity (Salovey, Mayer, Goldman, Turvey, & Palfai, 1995) and trait hostility (Fredrickson et al., 2000). Most of the findings with regard to rate of change apply primarily to long-term developmental changes (e.g., Kim, Conger, Lorenz, & Elder, 2001; see McArdle & Nesselroade, 2003, for a review). This concept of change, however is applicable to short-term, within-person variability in emotion as well.

Acceleration or Change in Rate of Change

Whereas the rate of change describes how much an attribute changes over time (i.e., change in intensity), acceleration indicates how fast the attribute changes over the same amount of time—thus, *change in the rate of change* (i.e., the second derivative of intensity with respect to time). Upon hearing depressing news, for example, an individual is likely to show a higher level (i.e., intensity) of sadness than his or her usual set point. The amount of change in sadness (i.e., rate of change) may differ as a result of the individual's degree of neuroticism. The speed with which this change takes place, by contrast, taps into the acceleration aspect of

the individual's emotion. The nature of a process as conveyed by the amount of acceleration is somewhat analogous to Davidson's (1998) concept of the "rise time" of an emotion, and he theorized that abnormalities in the rise time (or other temporal characteristics) of an individual's emotion may be indicative of emotion dysfunctions.

The concept of acceleration in emotion can be illustrated more concretely using the following example. The speed shown on a car speedometer (representing the rate of change) indicates the projected distance traveled by the car in one unit of time (e.g., 60 mph). However, the car may be speeding up or slowing down. This change in the rate of change (i.e., acceleration or braking) is indicated by how quickly the needle on the speedometer is moving. In the context of emotion, suppose one must give an important presentation the next day. It is possible, perhaps probable, that one's anxiety would increase. Assuming that this change in anxiety level is assessed on an hourly basis, the amount of increase in anxiety per hour would constitute the rate of change in anxiety. Initially, there may be a steady increase in anxiety level that persists for hours. However, as the presentation hour approaches, one's anxiety might really kick in, increasing more rapidly (accelerating) as the moment of truth arrives. Such changes in the rate of change are explicitly included in the forthcoming model.

To further illustrate the relationships between acceleration, rate of change, and intensity, as they might apply to emotion processes, we ask the reader to consider the scenario depicted in Figure 1. This figure, representing the progression of sadness, was generated using the damped oscillator model (the corresponding mathematical equation will be presented shortly). Initially (i.e., at Time 0), the individual's level of sadness (represented with a solid line) is perturbed and deviates momentarily from his or her usual set point (represented using a dashed line).² Over time, this heightened level of sadness dissipates, and the individual slowly returns to his or her usual baseline ($Y = 0$), overshooting a few times before settling into an equilibrium state. Instead of visualizing merely the relationship between level of sadness and time (as in Figure 1), we can, alternatively, examine the relationship between level (denoted as Y) and rate of change (denoted as Y' ; see Figure 2A), and between level and acceleration in sadness (denoted as Y'' ; see Figure 2B).

Examining Figure 2A, one can see that as the level of sadness decreases from an intensity level of 5.0 to about 1.0 (see also the corresponding change in level in Figure 1 with respect to time), the rate of change (Y') drops from 0.6 to about -6.0 (signifying a greater decrease in sadness). After going past an intensity level of 1.0, the magnitude of negative change slowly decreases and after

¹ Individuals may differ from one another in their baseline affect level. For instance, individuals high on neuroticism might have a higher disposition for unpleasant affect at baseline. However, such differences in individuals' trait-like baseline (also referred to as set point by Lykken & Tellegen, 1996) are not the focus of the oscillator model presented here. Rather, our focus is on how and when individuals deviate from and return to their baseline affect level, and their ability to minimize the discrepancies between their current emotional state and their equilibrium level.

² A set point is always located at $Y(t) = 0$. The focus of the oscillator model, as stated previously, is not on individual differences in trait-like baseline. Therefore, individual differences in trait-level baseline will have to be removed (thus putting everyone's baseline at 0) before a state-based model like the oscillator model is fitted.

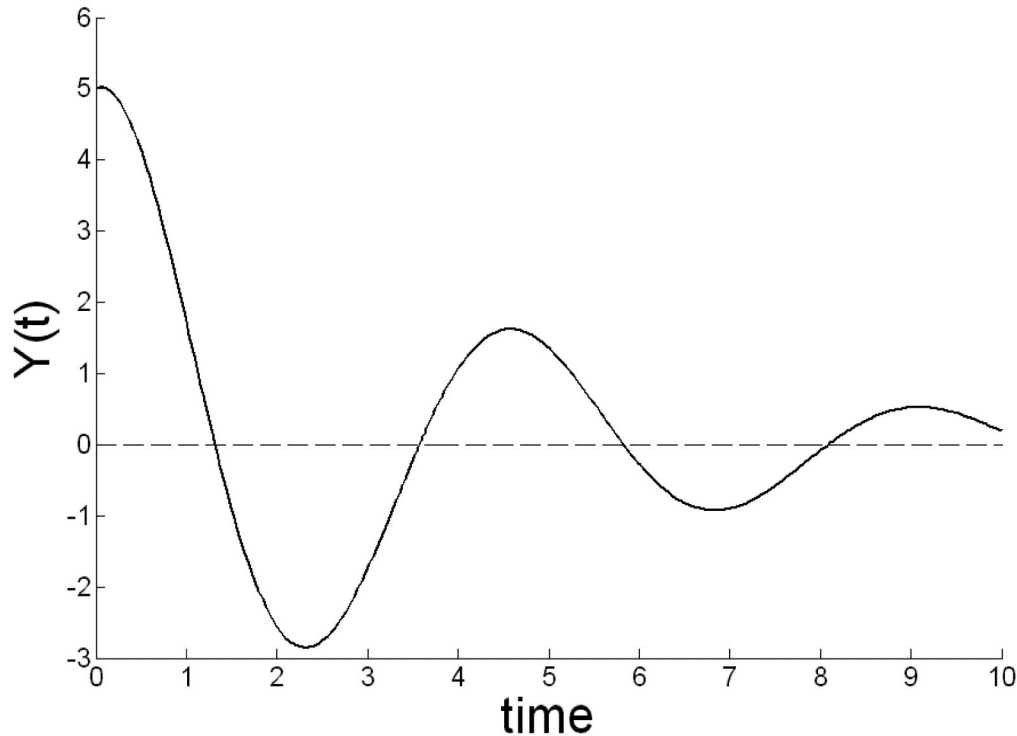


Figure 1. Trajectory of a construct as it returns to its set point (i.e., zero) over time. $Y(t)$ = level at time t .

hitting an uncharacteristically low sadness level (around -3.0), the individual begins to show elevations in sadness again (rate of change becomes positive). This dynamic interplay between level and rate of change continues until the individual returns to his or her set point, in which case level, rate of change, and acceleration would all be zero (where the two dashed lines cross in Figure 2A).

A similar inward spiral pattern is seen in the relationship between level and acceleration (see Figure 2B). As the individual's level of sadness decreases from 5.0 to 1.0 , there is a deceleration (i.e., Y'' is negative) in sadness. In other words, the decrease in sadness unfolds at a progressively slower pace. Then, acceleration in sadness (i.e., Y'' becomes positive) takes place as the individual's sadness shows a steeper descent to $Y = -3$, and this acceleration subsequently propels the individual's sadness level up to the set point again.

In short, intensity rate of change and acceleration all manifest changes but maintain a lawful relationship with one another as the individual experiences the ebb and flow of daily emotions. Furthermore, the damped oscillator model we present in this study allows researchers to effectively separate individual differences in emotion intensity from individual differences in frequency. In the context of the damped oscillator model, frequency describes how rapidly individuals experience ups and downs in their emotions, and it is similar to the concept of emotional lability (Harvey, Greenberg, & Serper, 1989; Larsen & Diener, 1985). As noted by many researchers, the magnitude of intraindividual variability is often operationalized by using an individual's within-person, across-occasion standard deviation (Eaton & Funder, 2001; Eid & Diener, 1999). However, this index does not distinguish linear change from cyclic change (Larsen, 1985a), nor does it identify

individuals who manifest gradual change in high amplitudes from those who exhibit rapid ups and downs in small magnitudes (Eaton & Funder, 2001). The latter can be pinpointed using the damped oscillator model presented here. Perhaps of most interest to affect researchers is the concept of damping portrayed by this model. This will be elaborated in greater detail next.

Emotion Regulation Based on Homeostatic Principles

Several researchers (Bisconti, Bergeman, & Boker, 2004; Carver & Scheier, 1982; Gross, Sutton, & Ketelaar, 1998; Headey & Wearing, 1989; Larsen, 2000; Lykken & Tellegen, 1996) have compared mood regulation and other regulatory behaviors with the characteristics of a thermostat—as discrepancies arise between one's ideal set point and the current (e.g., emotion) state, a kind of natural homeostasis kicks in and these discrepancies are successively minimized until one returns to the ideal set point. The speed with which individuals self-regulate corresponds directly to the idea of damping represented by the damped oscillator model. For example, highly neurotic individuals, because of their heightened sensitivity to stimuli that generate negative affect (Larsen & Ketelaar, 1991), may not show any damping in their negative emotions in specific settings, for example, when they are constantly perturbed by external cues. Other possible sources of individual differences in ability to minimize discrepancies, such as the effectiveness of different mood regulation strategies and differential sensitivity to affect-relevant stimuli, have also been outlined by Larsen (2000). In response to Larsen's (2000) mood regulation model, several researchers have suggested that these homeostatic principles can arguably be applied to the case of pleasant emotions

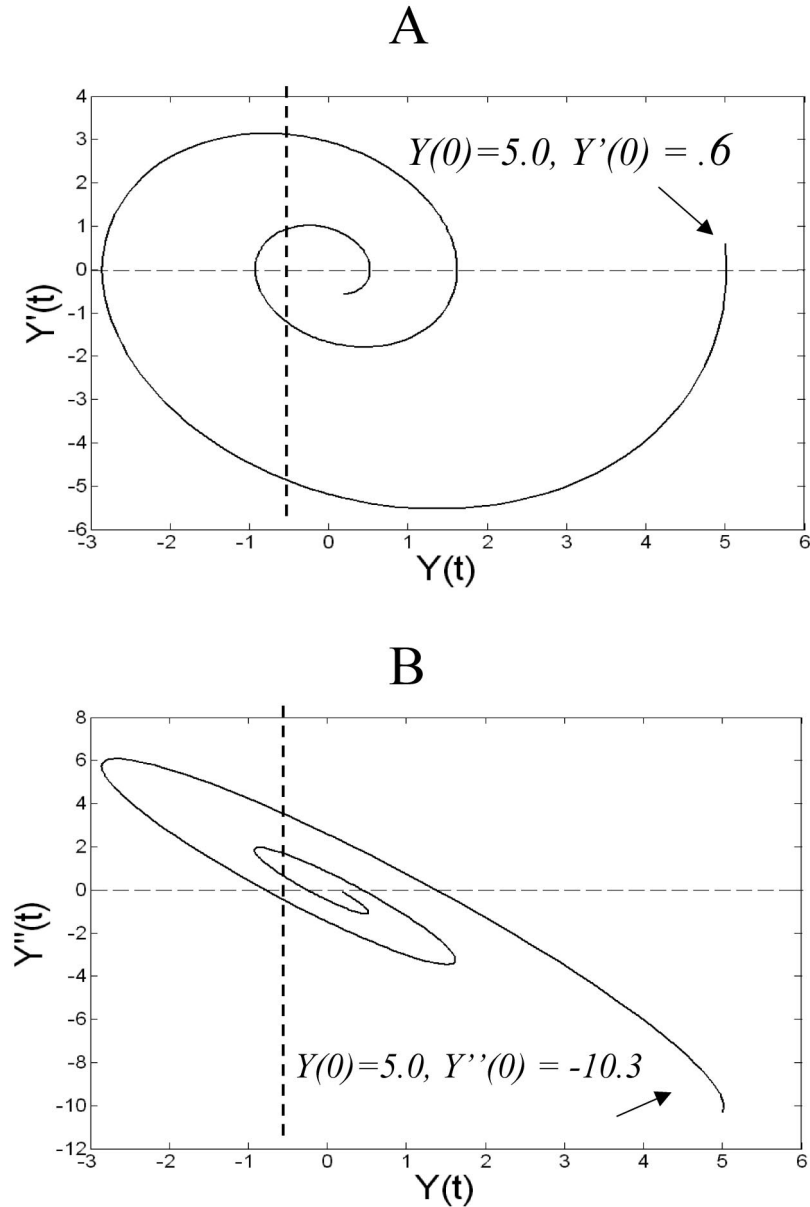


Figure 2. Plots reflecting the fluctuations in intensity over time as depicted in Figure 1 but with rate of change against intensity of emotion (A) and acceleration against intensity of emotion (B). $Y(t)$ = level at time t ; $Y'(t)$ = rate of change at time t ; $Y''(t)$ = acceleration in sadness at time t .

as well—that is, the case of a “happy thermostat” (Erber & Erber, 2000; see also other commentaries in the same issue, e.g., Freitas & Salovey, 2000; Isen, 2000; Watson, 2000). In the current study, we illustrate the possibility of fitting the thermostat model to both pleasant and unpleasant emotions. The differential equation that gives rise to the damped oscillatory behavior presented earlier will be reviewed next.

Damped Oscillator Model

Given a hypothetical construct, arbitrarily denoted here as Y (representing, e.g., sadness), the independent oscillator model

specifies the relationship among the intensity, rate of change, and acceleration of sadness as

$$Y''_i(t) = \eta Y_i(t) + \zeta Y'_i(t), \quad (1)$$

where $Y''_i(t)$ represents the acceleration in sadness at time t for person i , $Y'_i(t)$ represents the rate of change in sadness at time t for person i , and $Y_i(t)$ represents the intensity of sadness at time t for person i . Thus, person i 's sadness can be said to evolve continuously over time as a self-regulatory thermostat. The parameter η

describes the frequency of oscillation.³ The parameter ζ describes how promptly person i returns to his or her set point after perturbation.

Higher absolute values of η indicate more frequent fluctuations (i.e., oscillations), or in other words, more rapid ups and downs in sadness. Note that the η parameter is analogous to the frequency parameter used by Larsen and Diener (1987) in that both parameters capture how rapidly a construct changes, rather than how frequently a certain emotion or behavior is observed in the absolute sense (Diener, Larsen, Levine, & Emmons, 1985; Zelinski & Larsen, 2000). Thus, this parameter provides a direct representation for the concept of emotional lability (Harvey et al., 1989; Larsen & Diener, 1985).

The parameter ζ , by contrast, governs the speed with which person i 's sadness returns to or moves away from an ideal set point. When ζ is less than zero, there is evidence of damping, that is, there is a decrease in oscillation magnitude over time. When ζ is greater than zero, there is evidence of amplification (i.e., movement away from equilibrium). Finally, a ζ estimate of zero indicates that the pattern of fluctuations (i.e., oscillations) is constant over the entire duration that the phenomenon was observed. In sum, the model is parameterized such that it can represent a particular construct's frequency of oscillations and rate of return to equilibrium (or rate of divergence from equilibrium) over time.

Dynamic Interplay Between Frequency and Damping

The dynamic interplay between frequency and damping determines how fast a dynamic process (e.g., emotion) returns to a target equilibrium level (see, e.g., Nesselroade & Boker, 1994). In fact, this can help shed light on the distinctions between mood and emotion. Figures 3A–F represent trajectories generated using the damped oscillator model with different frequency and damping parameters, and one of two initial emotion intensities (either 5.0 or –5.0). Compared with Figure 2, the process depicted in Figures 3A and 3B unfolds at a slightly slower frequency, coupled with a greater magnitude of damping. Thus, this may represent individuals who have developed a heightened sensitivity to physiological cues or other affect-relevant cues and are thus able to start regulating their emotion very promptly upon perturbation (see other examples in Larsen, 2000).

As the damping parameter, ζ , is increased to –4.0 in Figures 3C and 3D, the discrepancies between current emotion state and one's set point are minimized even more effectively, and no signs of oversuppression are shown. The resultant shape may represent, for instance, the case of mood—instead of showing rapid ups and downs, there is now a gradual but stable decline (or increase) in the construct under study. This scenario can be used to represent other more gradual changes in stable individual traits. Note that the time scales used in this illustration are completely arbitrary and can reflect, contingent upon one's research question, processes that unfold over minutes, days, years, or decades.

Finally, when damping is omitted in Figures 3E and 3F and the frequency parameter is set to –0.8, the resultant trajectories correspond to two 7-day cycles. As we will elaborate in further detail, when there is no damping in the system and η is negative, the integral solution to Equation 1 is identical to the sinusoidal model—a cyclic model that serves as the core of spectral analysis (for specific details on this integral solution see, e.g., Zill, 1993).

Under this specific constraint, the sinusoidal model can in fact be viewed as a nested version of Equation 1. The respective strengths of spectral analysis and the damped oscillator approach presented in this study are discussed more thoroughly in Appendix A. Next, we present an overview of how Equation 1 can be fitted as a structural equation model using available software such as LISREL (Jöreskog & Sörbom, 1993), Mx (Neale, Boker, Xie, & Maes, 1999) and Mplus (Muthén & Muthén, 2001).

Fitting the Damped Oscillator Model as a Structural Model

In this study, a fourth-order latent differential structural approach is used to fit the damped oscillator model as a structural equation model (Boker, 2003; see similar but alternative approaches in Boker & Bisconti, in press; Boker & Graham, 1998; Boker, Neale, & Rausch, 2004). This approach is functionally similar to the approach used in growth curve modeling (e.g., McArdle & Epstein, 1987; Meredith & Tisak, 1990) and polynomial regression (Cohen, 1968; Wishart, 1938). More specifically, relationships among level, rate of change, and acceleration are specified using a set of fixed factor loadings in similar fashion to how fixed loadings are used to specify a construct's patterns of change (including, e.g., linear, quadratic, and other components of change) in a growth curve model.

Given the novelty of this approach, we begin by first introducing a simplified second-order estimation approach (Boker & Bisconti, in press; Boker et al., 2004). We will then expand this to a more complex fourth-order approach. A path diagram of the damped oscillator model estimated using the second-order estimation approach is shown in Figure 4. We consider a hypothetical scenario in which a particular emotion, sadness, is measured using three indicators: unhappiness, depression, and loneliness. In the corresponding path diagram, latent constructs and observed variables are represented using circles and squares, respectively. Variances and covariances among different variables are represented using two-headed arrows, whereas factor loadings and regression paths are represented using one-headed arrows. The latent factors SAD, DSAD, and D2SAD represent the intensity of sadness and its corresponding first and second derivatives (representing rate of change and acceleration, respectively).

The second-order latent structural approach is essentially used to specify the curve of a latent factor, SAD. Because the different components of this model can have important implications for the modeling of emotion, we will provide a general overview of the elements depicted in Figure 4. The full model encompasses two basic parts: a measurement model that specifies the relationship between factors and their associated indicators (see Figure 4B) and a dynamic model that imposes a certain functional curve on the factors (see Figure 4A). A state-space embedding technique (Boker & Bisconti, in press; Boker et al., 2004) is first used to lag each individual's time series against itself to create a matrix containing the measurements at time t , $t - 1$, $t - 2$, and so on. This technique is typically used in approaches wherein the data being

³ Formally, the frequency, ω , is equal to $\frac{1}{2\pi}\sqrt{-\eta}$. Note that only negative values of η are interpretable from a mathematical standpoint.

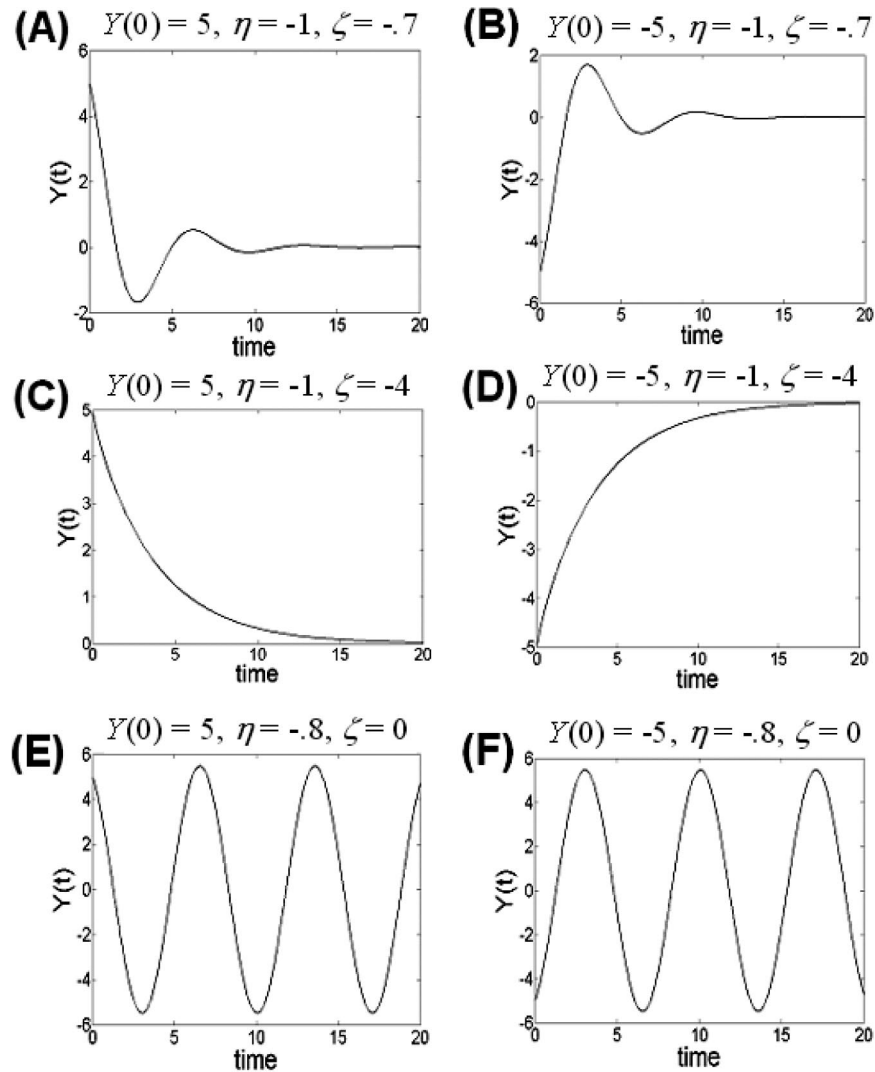


Figure 3. Plots generated using the damped oscillator model with two initial emotion intensities (5.0 and -5.0 , respectively) and different frequency (i.e., η) and damping (i.e., ζ) parameters. A and B: The hypothesized process shows gradual damping in magnitude over time under two initial conditions. C and D: The process approaches its equilibrium state very quickly without overshooting. E and F: There is no damping in the system. $Y(t)$ = level at time t .

analyzed involve a large number of measurement occasions (e.g., *P*-technique model; Cattell, 1963; Cattell, Cattell, & Rhymer, 1947; and dynamic factor analysis model; Molenaar, 1985; Nesselrode, McArdle, Aggen, & Meyers, 2002).⁴

Measurement model and the role of shocks. The measurement model in Figure 4B is just a usual factor analytic model, in which three indicators (unhappiness, depression, and loneliness) are used to identify the latent factor SAD, and one of the factor loadings is fixed at 1.0 for identification purposes. The terms S_t, S_{t-1}, S_{t-2} , and S_{t-3} are shocks or state components associated with sadness at each of the four particular time points. These shock terms have potentially interesting meanings from a substantive perspective—they capture a certain amount of common variance among the three indicators at each particular time point and yet they do not

show systematic patterns of variation over time. Browne and Nesselrode (in press) used the example of daily hassles to illustrate the role of these shock terms. More specifically, today's hassles can influence a person's unhappiness, depression, and

⁴ For example, one may choose to reformat a time series (say, for the indicator depression) with 100 measurement occasions into four blocks of data points, $dep_t, dep_{t-1}, dep_{t-2}$, and dep_{t-3} . The vector of depression scores at time t , dep_t , would contain data from time $t = 4$ to 100, dep_{t-1} would contain data from time $t = 3$ to 99, dep_{t-2} would contain data from $t = 2$ to 98, and dep_{t-3} would contain data from time $t = 1$ to 97. Thus, the number of manifest indicators included in the structural model in Figure 4 is significantly reduced, and yet patterns of intraindividual variability and any covariations in intraindividual variability among items are preserved.

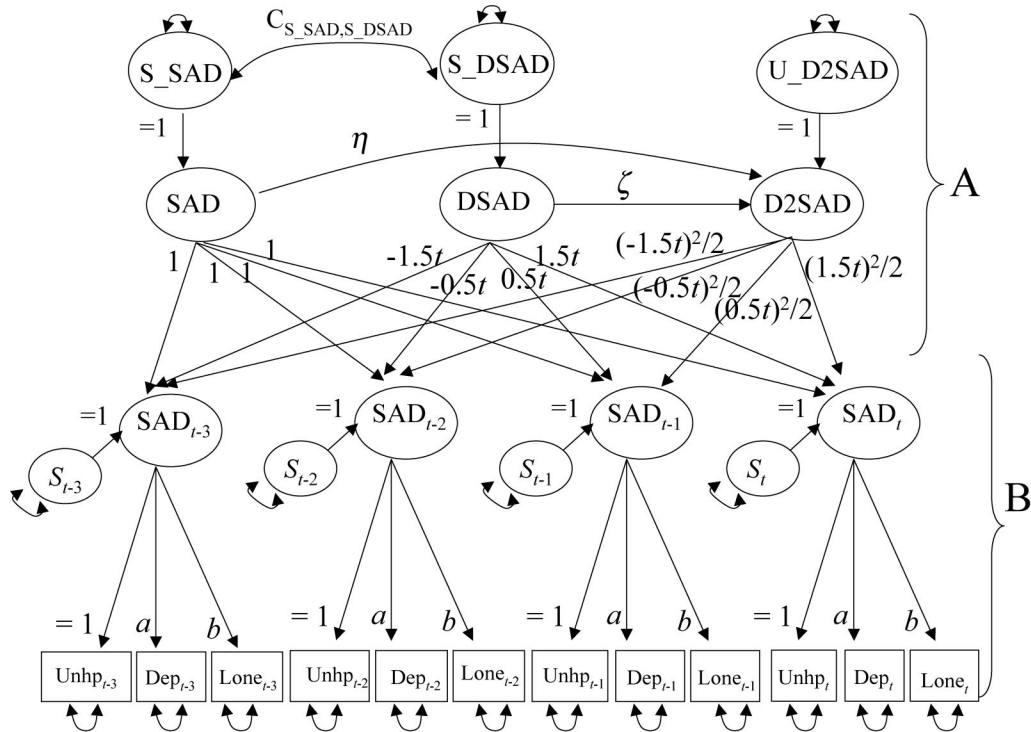


Figure 4. A path diagram showing the relationships among level of sadness (SAD) and its corresponding rate of change (DSAD) and acceleration (D2SAD) as hypothesized in the damped oscillator model for the dynamic model (A) and the measurement model (B). Unhp = unhappiness; Dep = depression; Lone = loneliness; t = time; a = factor loading of SAD on depression constrained to be invariant over time; b = factor loading of SAD on loneliness constrained to be invariant over time. S_SAD and S_DSAD = shocks to an individual’s sadness and to its corresponding first derivative, respectively; C_{S_SAD, S_DSAD} = covariance between the shock terms associated with SAD and DSAD; U_D2SAD = residuals in D2SAD not accounted for by the model. $S_t, S_{t-1}, S_{t-2},$ and S_{t-3} are shocks to an individual’s sadness unique to a particular measurement occasion. This is in contrast to S_SAD , which represents systematic shocks to an individual’s sadness that persist over all measurement occasions.

loneliness all at the same time (thus contributing to some amount of shared variance among these three items). However, the impact of these daily hassles does not persist in a systematic manner beyond today and hence they do not covary over time. Therefore, these state components can be conceived as shocks to one’s emotion status at a particular time point.

Dynamic model: The curve of a factor. The dynamic model in Figure 4A is used to specify the trajectory of sadness over time. By using the specific loadings in Figure 4, the latent components SAD, DSAD, and D2SAD represent the intensity, rate of change, and acceleration in sadness, respectively. The element t is a user-specified scaling value that determines the time interval between two successive measurement occasions. Therefore, if two indicators (e.g., items or tests) are measured over different intervals, different values of t can be specified for each of these indicators to incorporate unequal measurement intervals.

The parameters η and ζ are estimated as regressions of acceleration (D2SAD) on intensity (SAD) and rate of change (DSAD). If the model depicted in Figure 4 is fitted to data from a single individual over many measurement occasions, variances of the components S_SAD , S_DSAD , and $D2SAD$ (where $S_$ represents

shock to sadness) capture the magnitudes of systematic within-person variability in level and rate of change of sadness over time. The covariance between these two components represents the amount of covariations between these two sources of intraindividual variability. If this model is fitted to data from multiple individuals, variances of S_SAD and S_DSAD encompass both intraindividual variability and interindividual differences in level and rate of change. If that is the case, these two sources of variance are, to some extent, confounded.

Finally, the component U_D2SAD is the residual (or uncertainty) in D2SAD not accounted for by the damped oscillator model. This modeling uncertainty is not attributable to measurement errors in the indicators and essentially reflects the discrepancy between one’s hypothesized model and the true mechanism that underlies the dynamics of sadness. In general, all the variance and covariance components in the dynamic model (i.e., Figure 4A) capture systematic patterns of variation (or covariation) over time that are quite distinct from the instantaneous shock components in the measurement model. Researchers may choose to estimate or omit some of these components to test their specific hypotheses of interest. In this study, we omit the shock components and focus

instead on a more parsimonious model that captures only the systematic variability over time. This will be elaborated further in a moment.

In short, combining the measurement and dynamic models in Figure 4 yields a growth curve model for the factor SAD that conforms to the damped oscillator model. However, the current approach is different from conventional growth curve models in some subtle ways. These differences are detailed in Appendix B. Similarities between the current approach and time series models (or more specifically, autoregressive moving average models) will also be highlighted briefly.

A fourth-order approach with no shock components. The damped oscillator model defined in Equation 1 is formulated on the basis of information up to the second derivative (i.e., acceleration). Recently, Boker (2003) demonstrated that when one incorporates the third and fourth derivatives (i.e., D3SAD and D4SAD) into the estimation process, the redundancy in the relationships between successive derivatives can help yield more accurate parameter estimates. The idea is simply to capitalize on the fact that the regression estimates of η linking SAD to D2SAD, DSAD to D3SAD, and D2SAD to D4SAD are all mathematically equivalent and can thus be constrained to be equal to one another. The same procedures are used to constrain the ζ estimates linking DSAD to D2SAD, D2SAD to D3SAD, and D3SAD to D4SAD to be equal to one another. Thus, one estimate for η and one estimate for ζ are obtained on the basis of information from the first to fourth derivatives.

In this alternative approach, the measurement model and dynamic model are also expressed as one single model. This is accomplished by defining a matrix of factor loadings, L , that combines the relationships between factors and indicators (see Figure 4B) with the growth curve loadings (see Figure 4A). The resultant model is shown in Figure 5. By doing this, the instantaneous shock components (S_t , S_{t-1} , S_{t-2} , and S_{t-3}) are not estimated. Because these shock components do not show any continuity over time, they are pushed down and are estimated instead as part of the measurement errors. We emphasize, however, that these shocks may represent an integral and important part of emotion processes—we opted for a more constrained model because we wish to focus on the dynamics of emotion over time.

The structural model shown in Figure 4 can be fitted to empirical data using conventional structural equation modeling programs (e.g., LISREL, Mplus, and Mx). Currently, the condensed version depicted in Figure 5 can only be fitted using Mx because of the constraints one needs to place on the factor loadings matrix. However, if the fourth-order model in Figure 5 is parameterized as an expanded model depicting the curve of a factor (i.e., analogous to the model in Figure 4 but with the third and fourth derivatives included as well), then it can be fitted using any structural equation modeling software. In the present context, we fitted first the condensed model using Mx, and later, fitted the expanded version using Mplus to utilize the program's specific option for handling incomplete data (i.e., the option for performing full information maximum likelihood estimation in the presence of incomplete

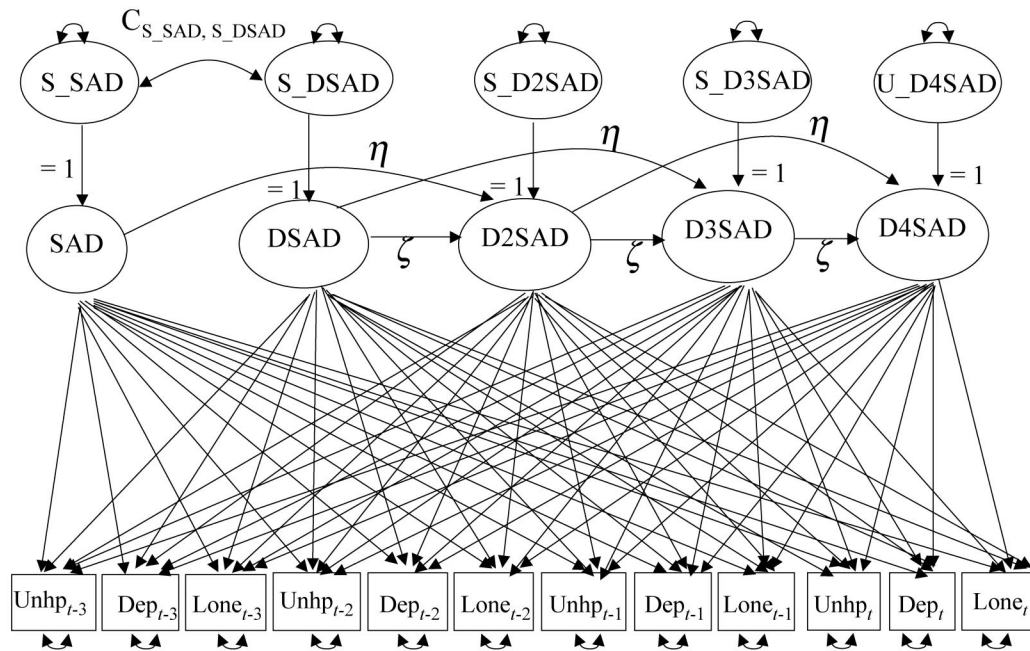


Figure 5. A fourth-order differential structural approach used in this study to fit the damped oscillator model. The dynamic and measurement portions of the model are not defined jointly by a factor loading matrix L shown in Table 1. t = time; Unhp = unhappiness; Dep = depression; Lone = loneliness; SAD = intensity of sadness; DSAD, D2SAD, D3SAD, and D4SAD = first, second, third, and fourth derivatives of sadness, respectively; S_SAD, S_DSAD, S_D2SAD, and S_D3SAD = shocks to an individual's sadness and to its corresponding first, second, and third derivatives, respectively; C_{S_SAD, S_DSAD} = covariance between the shock terms associated with SAD and DSAD; U_D4SAD = residuals in D4SAD not accounted for by the model.

data). The matrices of parameters to be estimated include the matrices L , A , and S shown in Table 1. The matrix L carries the associated factor loadings that define the relationships among level and its higher-order derivatives, the matrix A contains the regressions among latent derivatives, and the covariance matrix S carries the variances and covariances among sources of intraindividual variability in level and rate of change and residuals in acceleration.

An Empirical Example

We used the damped oscillator approach presented herein to replicate Larsen and Kasimatis’s (1990) finding of a weekly cycle in average hedonic level using data that have been published elsewhere (e.g., Diener et al., 1995; Eid & Diener, 1999). The sample consisted of 179 college students (98 men and 81 women, average age = 20.24, $SD = 1.81$) at the University of Illinois at Urbana–Champaign. Participants completed a set of affect ratings on 52 consecutive days. In addition to testing the generalizability

of Larsen and Kasimatis’s (1990) earlier findings, we also present some interemotion differences in oscillation frequency and inter-individual differences in frequency and damping.

Because emotion was measured in days and the dominant cycle in most participants’ data was a weekly cycle, damping is not very meaningful in this particular context. As a result, this data set does not demonstrate fully the strengths of the damped oscillator approach as a homeostatic emotion regulation model. However, this data set is ideal for illustration purposes because most participants in the study are characterized by a clear 7-day affect cycle.

Affect Measures

Participants completed daily self-reports of 24 emotions. They were asked to rate how often they felt each of the emotions on a 7-point scale, ranging from 1 (*none*) to 7 (*always*). Using the damped oscillator approach, we examined patterns of intraindividual variability in six emotion factors that have been identified

Table 1
Matrices Involved in Estimating the Damped Oscillator Model Using a Latent Differential Structural Approach

Variable	SAD	DSAD	D2SAD	D3SAD	D4SAD
$L =$ matrix of factor loadings					
Unhp _{<i>t-3</i>}	1	$-1.5t$	$(-1.5t)^2/2$	$(-1.5t)^3/6$	$(-1.5t)^4/24$
Unhp _{<i>t-2</i>}	1	$-0.5t$	$(-0.5t)^2/2$	$(-0.5t)^3/6$	$(-0.5t)^4/24$
Unhp _{<i>t-1</i>}	1	$0.5t$	$(0.5t)^2/2$	$(0.5t)^3/6$	$(0.5t)^4/24$
Unhp _{<i>t</i>}	1	$1.5t$	$(1.5t)^2/2$	$(1.5t)^3/6$	$(1.5t)^4/24$
Dep _{<i>t-3</i>}	a	$a(-1.5t)$	$a(-1.5t)^2/2$	$a(-1.5t)^3/6$	$a(-1.5t)^4/24$
Dep _{<i>t-2</i>}	a	$a(-0.5t)$	$a(-0.5t)^2/2$	$a(-0.5t)^3/6$	$a(-0.5t)^4/24$
Dep _{<i>t-1</i>}	a	$a(0.5t)$	$a(0.5t)^2/2$	$a(0.5t)^3/6$	$a(0.5t)^4/24$
Dep _{<i>t</i>}	a	$a(1.5t)$	$a(1.5t)^2/2$	$a(1.5t)^3/6$	$a(1.5t)^4/24$
Lone _{<i>t-3</i>}	b	$b(-1.5t)$	$b(-1.5t)^2/2$	$b(-1.5t)^3/6$	$b(-1.5t)^4/24$
Lone _{<i>t-2</i>}	b	$b(-0.5t)$	$b(-0.5t)^2/2$	$b(-0.5t)^3/6$	$b(-0.5t)^4/24$
Lone _{<i>t-1</i>}	b	$b(0.5t)$	$b(0.5t)^2/2$	$b(0.5t)^3/6$	$b(0.5t)^4/24$
Lone _{<i>t</i>}	b	$b(1.5t)$	$b(1.5t)^2/2$	$b(1.5t)^3/6$	$b(1.5t)^4/24$
$A =$ matrix with regressions among latent derivatives					
SAD	0	0	0	0	0
DSAD	0	0	0	0	0
D2SAD	η	ζ	0	0	0
D3SAD	0	η	ζ	0	0
D4SAD	0	0	η	ζ	0
$S =$ covariance matrix of intraindividual variability and residuals					
	S_SAD	S_DSAD	S_D2SAD	S_D3SAD	U_D4SAD
S_SAD	V_{S_SAD}	—	—	—	—
S_DSAD	$C_{SAD, DSAD}$	V_{S_DSAD}	—	—	—
S_D2SAD	0	0	V_{S_D2SAD}	—	—
S_D3SAD	0	0	0	V_{S_D3SAD}	—
U_D4SAD	0	0	0	0	V_{U_D4SAD}

Note. SAD = intensity of sadness; DSAD, D2SAD, D3SAD, and D4SAD = the first, second, third, and fourth derivatives of sadness, respectively; a = factor loading of SAD on depression; b = factor loading of SAD on loneliness; Unhp = unhappiness; Dep = depression; Lone = loneliness; S_SAD, S_DSAD, S_D2SAD, and S_D3SAD = shocks to an individual’s sadness and to its corresponding first, second, and third derivatives, respectively; V_{S_SAD} , V_{S_DSAD} , V_{S_D2SAD} , and V_{S_D3SAD} = the variances of the shock terms; $C_{SAD, DSAD}$ = covariance between the shock terms associated with SAD and DSAD; U_D4SAD = residuals in D4SAD not accounted for by the model, and V_{U_D4SAD} is its associated variance.

elsewhere (Diener et al., 1995). These factors are love, joy, sadness, fear, anger, and shame. Each factor was measured using four items (see Table 2).

Data Analysis

Each participant's time series for each of the six factors was detrended prior to model fitting to eliminate spurious correlations among the six emotions due to any common linear trend (McCleary & Hay, 1980). In addition, this procedure removes individual differences in equilibrium level. In other words, all participants' equilibrium level on each variable is shifted to a zero point. We organize our results into three sections to (a) replicate Larsen and Kasimatis's (1990) finding on weekly cycle in individuals' aggregate hedonic level, (b) present interemotion differences in periodicity, and (c) demonstrate individual differences in frequency and damping.

First, to illustrate the damped oscillator approach's utility in recovering systematic oscillation frequency, we computed each participant's daily hedonic level based on the definition used in Larsen and Kasimatis (1990),⁵ and we aggregated these daily measures across all participants to yield a single time series of hedonic level. We then fitted the damped oscillator model to this single time series with six estimation occasions to obtain a frequency estimate and a damping estimate for the aggregate hedonic level. On the basis of preliminary analysis, we were best able to recover the 7-day cycle evident in the participants' aggregate hedonic level by using six estimation occasions. We therefore chose six estimation occasions for all subsequent analyses. Note that because the damped oscillator model involves an oscillatory (i.e., nonlinear) function, the estimates yielded from this aggregate curve will not be the same as the estimates obtained from averaging across different individual curves. In other words, this aggregate curve may, in fact, characterize no one individual's curve. However, this step was performed primarily for illustration purposes because there is a clear 7-day affect cycle in the aggregate data.

Second, we examined interemotion differences in frequency and damping. Daily emotion scores were aggregated across participants to yield a 52×4 data matrix for each emotion (4 manifest indicators measured over 52 days). This matrix was lagged against itself to yield a 24×24 covariance matrix (with 4 manifest indicators \times 6 estimation occasions) for model fitting. This procedure was performed separately for each of the six emotions. Six frequency estimates and six damping estimates were obtained by fitting the damped oscillator model separately to each of the six emotion measures. Finally, individual differences in affective pe-

riodicity and damping were examined by fitting the independent oscillator model to each individual's data separately. We only focused in this case on three emotions: love, joy, and sadness.

Results

Replicating a 7-Day Cycle in Aggregate Hedonic Level

Consistent with Larsen and Kasimatis's (1990) finding, a 7-day cycle is evident in these participants' aggregate hedonic level (see Figure 6). Examination of the plot in Figure 6 indicates that the aggregate average hedonic level peaks on the 7th day of each week, in this case a Saturday. Results from fitting the damped oscillator model to the aggregate average hedonic level data yielded an η estimate of $-.77$. In day units, this equals an oscillation period of 7.16 days, thus showing close correspondence to a weekly cycle. Discrepancy from a precise 7-day estimate was evaluated by fitting a second model wherein η was fixed at $-.80$ (a 7-day period of oscillation). The change in fit was very small, $\Delta\chi^2(1) = 1.9$, $p = .05$, indicating that the 7.16-day estimate was not significantly different from a 7-day estimate. Thus, the damped oscillator model was able to recover the 7-day frequency in aggregate hedonic level accurately.

Interemotion Differences in Periodicity

The damped oscillator model was used to estimate the periodicity present in the six aggregate emotions using Mplus (Muthén & Muthén, 2001) with full information maximum likelihood estimation. In all cases, incomplete data were treated as missing at random (Little & Rubin, 1987). The corresponding parameter estimates are summarized in Table 3.

The frequency estimates indicated strong weekly cycles in the pleasant emotions, joy and love, and in the unpleasant emotions, sadness, fear, and shame. Only anger seemed to diverge slightly from a weekly cycle, as it was characterized by a higher frequency compared with other emotions. However, despite all six emotions exhibiting weekly cycles, an inspection of the plots associated with each emotion revealed that emotions of different valences tended to peak on different days of the week. Love and joy were observed to peak over weekends, whereas unpleasant emotions (sadness, fear, anger, and shame) typically manifested surges in magnitude in the middle of the week.

Damping was not significant for most of the emotions. Even when damping was statistically different from zero (i.e., for love and sadness), the parameters were still small ($-.01$ and $-.02$, respectively). This indicates that the weekly cycle continues unabated through the entire 52 days of study. In sum, interemotion differences in weekly cycle indicate that the well-known speculation of a blue Monday phenomenon is, for the most part, attributable to a decline in pleasant emotions on Mondays. Furthermore, because of the lack of damping, the blue Monday decline in pleasant emotions persisted throughout the duration of this study.

Table 2
List of Manifest Variables Used to Identify Six Latent Emotions

Factor	Manifest indicators
Love	Love, affection, caring, and fondness.
Joy	Joy, happiness, contentment, and satisfaction.
Sadness	Sadness, unhappiness, depression, and loneliness.
Fear	Fear, worry, anxiety, and nervousness.
Anger	Anger, irritation, disgust, and rage.
Shame	Shame, guilt, regret, and embarrassment.

⁵ We computed each participant's average hedonic level on a particular day as the difference between his or her pleasant emotion (averaged across two items: love and joy) and unpleasant emotion (averaged across four items: sadness, fear, anger, and shame).

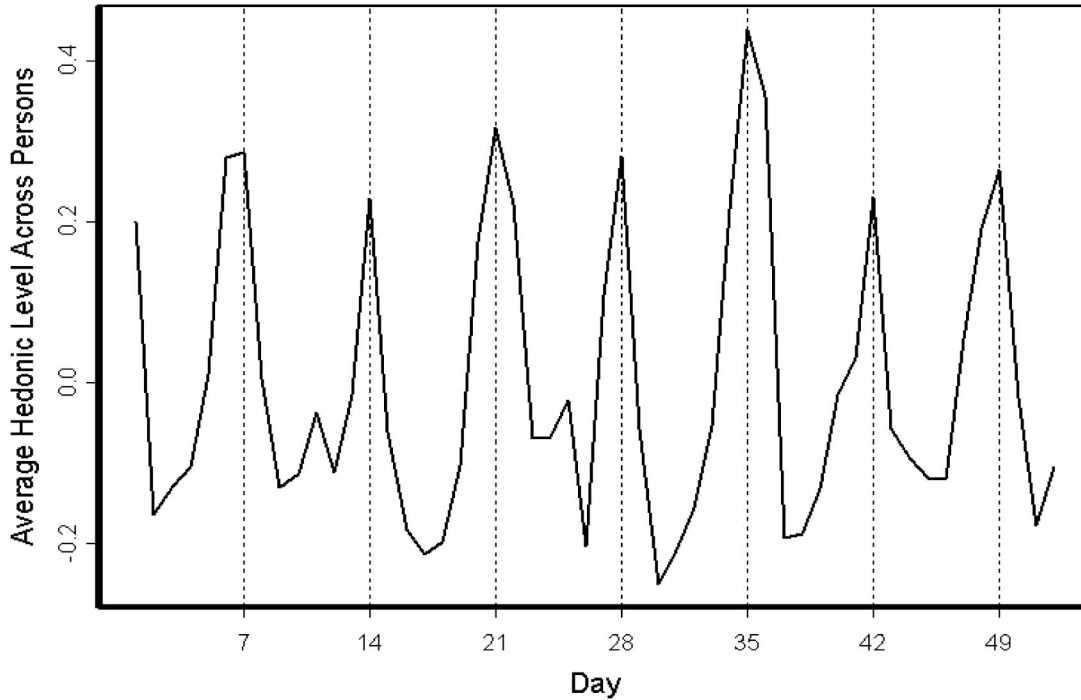


Figure 6. A plot of the participants' changes in average hedonic level as a group. A weekly trend was apparent in the group's surges in hedonic level over weekends (on Days 7, 14, 21, etc., which are all marked with dotted lines).

Interindividual Differences in Frequency and Damping

The damped oscillator model was fitted separately to each individual's data on love, joy, and sadness. We obtained a total of three frequency estimates and three damping estimates for each person. The ranges and averages of these parameters are shown in Table 4. Individual differences were quite apparent in both the frequency of oscillation and the rate of damping. A subset of individuals' estimation results did not meet the statistical criteria for convergence (possibly because of high degrees of incompleteness in their data). Their estimates were therefore excluded from the interindividual analyses.

We then examined the individual differences in frequency and damping estimates by using gender, affect intensity, extraversion, and neuroticism as predictors in a series of multiple regression analyses. Significant gender differences were found only in the periodicity of sadness $F(1, 125) = 4.27, p = .041$. On average,

women exhibited a higher frequency of fluctuation in sadness ($M = -1.02, SD = 0.41$; i.e., average period of 6.22 days) compared with men ($M = -0.87, SD = 0.42$; i.e., average period of 6.74 days; $p = .036$). This indicates that men, on average, were slightly more entrained to a weekly cycle (i.e., their average η estimate was closer to the ideal η estimate of $-.80$ corresponding to a period of 7 days) than were women. In addition, there was also a marginally significant effect of affect intensity (as measured by the Affect Intensity Measure; Larsen, 1985b) on the frequency of love, $F(1, 137) = 3.76, p = .055$. In particular, participants who were higher on affect intensity also experienced fluctuations in love at a higher frequency. Individual differences in other frequency and damping estimates were not significantly related to differences in gender, affect intensity, neuroticism, and extraversion ($p > .05$).

Discussion

The purpose of this study was to present a damped oscillator model and to demonstrate how it can be fitted as a structural equation model to empirical data. The damped oscillator model provides a direct representation of the concept of emotion as a thermostat. The specific parameters of this model, including frequency and damping, offer a practical way for modeling individuals' emotional lability and the effectiveness of their regulatory behaviors within a process-oriented framework. Furthermore, the particular estimation approach used to fit the damped oscillator model in this study is highly flexible and can be used to fit other dynamic models in the form of differential or difference equation models.

Table 3
Estimated Values of η and ζ Based on All Participants' Data

Emotion	η	SE	Period in days	ζ	SE
Love	-.83	.005***	6.90	-.01	.005*
Joy	-.83	.007***	6.90	-.01	.006
Sadness	-.84	.006***	6.86	-.02	.005*
Fear	-.84	.006***	6.86	-.01	.005
Shame	-.84	.008***	6.86	-.01	.007
Anger	-.93	.007***	6.52	-.01	.006

* $p < .05$. *** $p < .001$.

Table 4
Summary Statistics of Parameter Values Estimated Individually for Each Participant

Factor	<i>M</i>	<i>SD</i>	Range	<i>N</i>
η				
Average hedonic level	-0.96 (6.41)	.63	-2.91 to -0.11 (3.68 to 18.94)	150
Love	-0.92 (6.55)	.42	-2.70 to -0.11 (3.82 to 18.94)	142
Joy	-0.89 (6.66)	.41	-2.09 to -0.07 (4.34 to 23.74)	135
Sadness	-0.94 (6.48)	.42	-2.04 to -0.13 (4.40 to 17.43)	130
ζ				
Average hedonic level	-0.01	.19	-0.96 to 0.86	150
Love	-0.02	.07	-0.43 to 0.20	142
Joy	-0.01	.09	-0.76 to 0.27	135
Sadness	-0.02	.09	-0.61 to 0.21	130

Note. Values in parentheses represent period in days.

Using the damped oscillator approach, we replicated Larsen and Kasimatis's (1990) earlier finding on a 7-day cycle in college students' aggregate hedonic level. Consistent with Stone's (1985) earlier findings, results from model fitting also reveal that the blue Monday phenomenon is more attributable to postweekend declines in pleasant emotions rather than increases in unpleasant emotions. Generally, even though all six emotions manifested similar periodicity (i.e., close to a period of 7 days), emotions of opposite valence (i.e., pleasant or unpleasant) tended not to occur together in high intensity. This difference in affective dynamics is consistent with previous findings regarding the independence of pleasant and unpleasant emotions (e.g., Diener & Emmons, 1984; Diener & Iran-Nejad, 1986).

In addition to using the damped oscillator model to examine interemotion differences, we also used it to examine interindividual differences in emotion oscillations. Using each individual's frequency estimates for love, joy, and sadness as indicators of the individual's periodicities in these three emotions, we found significant gender differences in the periodicity of sadness. Male participants' sadness was slightly more entrained to a weekly cycle, whereas female participants exhibited changes in sadness at a slightly higher frequency. In addition, there was also a marginally significant relationship between affect intensity and the frequency of love—participants who were higher on affect intensity also manifested fluctuations in love at a higher frequency. However, we did not find other personality differences in the periodicity of average hedonic level, love, joy, or sadness. It is possible that interindividual differences in personality may relate, in the context of a weekly cycle, more to damping than frequency.

Given the individual differences in class and/or work schedules among the college students in this study, the blue Monday phenomenon as reflected in the aggregate data might be potentially stronger if data from individuals with a more homogeneous work schedule (e.g., individuals working regularly on a Monday–Friday schedule) are analyzed. Because different individuals' rhythms are likely to be slightly off phased (i.e., each individual's pleasant and unpleasant emotions are likely to peak on different days of the week, even though they may all conform to a weekly cycle), these individual-level dynamics can only be extracted if model fitting or analysis is done at the individual level. Although we did fit the

oscillator model to each participant's data, we did not examine the individuals' idiosyncratic reactions to Mondays. In other words, we did not investigate whether the postweekend declines in pleasant emotions observed at the aggregate level reflect similar reactions at the individual level. Spectral analysis may be a particularly useful descriptive tool in this case as it provides a quick and convenient way to extract peaks in each individual's emotions. If researchers are interested in extracting the different cycles embedded in an individual's data, spectral analysis again offers a convenient way of answering this research question.

We have demonstrated the potential utility of assessing the frequency of emotion fluctuations in addition to the intensity (i.e., amplitude) of mood. We have also pinpointed the correspondence between the damped oscillator model and a homeostatic emotion regulation model suggested by various researchers (e.g., Gross, Sutton, & Ketelaar, 1998; Headey & Wearing, 1989; Larsen, 2000; Lykken & Tellegen, 1996). As we have stated previously, the current data set does not illustrate fully the potentials of the damped oscillator model as an emotion regulation model. To do this, we would need data sampled at much closer intervals. For example, we would need to start measuring an individual's emotion status upon being exposed to an affect-relevant cue and record his or her fluctuations in emotion closely as the individual's emotion returns to its equilibrium set point (e.g., by using approaches such as the experience sampling method; Csikszentmihalyi & Larson, 1987). Interindividual differences in this damping rate can provide interesting insights into sources of individual difference in emotion regulation.

Another possible extension to the approach presented herein is to expand the damped oscillator model to examine how different processes might be dynamically coupled to one another. One example of such models is the coupled oscillators model presented by Boker and Graham (1998). When one uses this approach, two or more processes can be modeled as multiple oscillators that are coupled to one another. For instance, by coupling an individual's unpleasant emotion to his or her pleasant emotion, the two processes may fluctuate in perfect synchrony with one another. This may occur under specific environmental influences (e.g., under stress; Zautra, Potter, & Reich, 1998). By incorporating this coupling term, an individual's failure to suppress (i.e., to damp) his or

her unpleasant emotion can also lead to more rapid oscillations in pleasant emotion. These dynamic models thus offer ample opportunities for researchers to examine the linkages or divergences between emotions of different valences or activation poles (Feldman Barrett, 1998; Green & Citrin, 1994; Trierweiler, Eid, & Lischetzke, 2002; Watson, Wiese, Vaidya, & Tellegen, 1999). This notion of dynamic linkages between different emotions also adds interesting perspectives to other prevalent models of affect (e.g., circumplex models; Browne, 1992; Fabrigar, Visser, & Browne, 1997).

In sum, this study examined weekly periodicity in a sample of college students using a damped oscillator approach. We replicated previous findings on the existence of a weekly cycle in average hedonic level (e.g., Larsen & Kasimatis, 1990) and found that the blue Monday phenomenon seems primarily to be a result of declines in pleasant emotion. Furthermore, although aggregated data showed clear 7-day cycles, analysis at the individual level revealed substantial individual differences in entrainment to this cycle. Implications are that dynamics of emotional experience are much more complex than what snapshots at a particular time point could convey. However, as we hope we have illustrated here, the damped oscillator approach provides important methodological and theoretical advantages as a tool for representing emotion regulation as a dynamic process—or in the present context, as a self-regulatory thermostat.

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Appendix A

Spectral Analysis and the Damped Oscillator Approach

Spectral analysis is a descriptive approach that decomposes a time series into a set of sine and cosine functions (i.e., oscillatory functions) in the frequency domain (see Chatfield, 1996; Gottman, 1979; Warner, 1998). In brief, this data-analytic technique identifies the weights or densities of all possible frequencies of sine and cosine waves that exist within a time series. By examining the magnitudes of these weights, one can identify hidden cycles that may not be apparent by inspection. For example, if a set of time-series data shows high spectral density–weight for a 7-day frequency and low weights for other frequencies, one might conclude that the time series is characterized by a strong weekly rhythm (e.g., Larsen & Kasimatis, 1990). That is, much of the variance in the time series is accounted for by a particular sinusoidal cycle.

Currently, spectral analysis is one of the most popular tools for detecting the existence of cycles in time-series data (Gottman, 1979; Warner, 1998). The sinusoidal model that forms the basis of spectral analysis (see Warner, 1998) is identical to the integral solution of the damped oscillator model (see Zill, 1993) when there is no damping and η is negative.^{A1} In this section, we highlight some of the similarities and differences between the spectral analysis approach adopted by Larsen and Kasimatis (1990) and the damped oscillator approach used in this study. We demonstrate that although both of these approaches yield similar information concerning the periodicity of a construct, these approaches have their own strengths and weaknesses in helping to address different research questions.

Distinctions Between Spectral Analysis and the Latent Structural Approach

Typically, spectral analysis is used when researchers often do not have preconceived notions or expectations of the periodicity of a construct or hypotheses on how this periodicity is related to other constructs. Most often, spectral analysis is used as a tool to identify and describe the periodicity or seasonality in the data, rather than to model this periodicity in relation to other constructs (Warner, 1998; for an exception, see Larsen & Kasimatis, 1990).

Because the sinusoidal model is identical to the damped oscillator model with no damping, all the estimates available from spectral analysis can also be obtained from estimates of the damped oscillator model. However, because spectral analysis is available in most statistical packages (e.g., SAS, S-plus, R, and SPSS), it is conveniently equipped with options and estimates that these programs execute and output automatically. These estimates will, however, have to be computed in some additional steps when the oscillator approach is used. For instance, in bivariate spectral analysis, wherein two time series are subject to spectral analysis simultaneously, most software packages output the cross-phase between the two series automatically. This parameter represents the difference between the first peaks of the two series in radian units and provides an indication of the amount of time one construct is lagging the other by (e.g., pleasant emotion may precede unpleasant emotion by one day). To obtain the same infor-

mation using the damped oscillator approach, one will have to fit the oscillator model separately to the two time series and subsequently compute the cross-phase in an additional step.

The damped oscillator model does have an important feature that spectral analysis does not offer—it incorporates a damping parameter that is not part of the sinusoidal model assumed in spectral analysis. This gives the former some added flexibility in shaping the corresponding trajectory of change into different functional forms (see Figure 3) and conveys important meanings in the context of the homeostatic emotion regulation model discussed earlier. As an illustration, we analyzed the time series in Figures 3A and 3C using spectral analysis. The resultant periodograms are shown in Figure A1. Even though the true frequency ($\eta = -1$, corresponding to a period of 6.823 marked with a dashed line)^{A2} can still be recovered in the first case, the damping shown in Figure 3A is manifested as a nonstationary trend in the periodogram (see Panel A in Figure A1).^{A3} When the magnitude of damping is increased further to $\zeta = -4$, in which case the resultant trajectory no longer appears cyclic, spectral analysis fails to recover the true frequency (see Panel B in Figure A1). This, however, is not a problem if the damped oscillator approach is used.

^{A1} Under a special condition where $[(-\zeta/2)^2 - |\eta|]$ is less than zero, the integral solution for the damped oscillator model in Equation 1 (Zill, 1993) is expressed as

$$Y(t) = e^{-\frac{|\zeta|}{2}t} \left[c_1 \cos \sqrt{|\eta| - \left(\frac{|\zeta|}{2}\right)^2} t + c_2 \sin \sqrt{|\eta| - \left(\frac{|\zeta|}{2}\right)^2} t \right], \quad (A1)$$

where c_1 and c_2 are arbitrary constants derived from one's initial level and rate of change. In the case of $\zeta = 0$, it simplifies to

$$Y(t) = c_1 \cos \sqrt{|\eta|}t + c_2 \sin \sqrt{|\eta|}t. \quad (A2)$$

Equation A2 is identical to the sinusoidal model used in harmonic analysis and is reexpressed in an alternative but equivalent form in spectral analysis (see Warner, 1998).

^{A2} Given a known oscillation period of T , the theoretical value of η can be computed as $\eta = -\left(\frac{2\pi}{T}\right)^2$, where T represents the period of oscillation. For example, a 7-day cycle would yield an η estimate of $\eta = -\left(\frac{2\pi}{7}\right)^2 = -.80$.

^{A3} When spectral analysis is used to fit a series of different frequencies to data, the particular frequencies fitted are a function of the number of occasions in the data. Panel A in Figure A1 is an example of commonly observed phenomenon often termed the *leakage effect* (Warner, 1998). In this case, the dominant frequency leaks into the nearest fitted frequency. Thus, the estimated period is close but does not coincide perfectly with the true period.

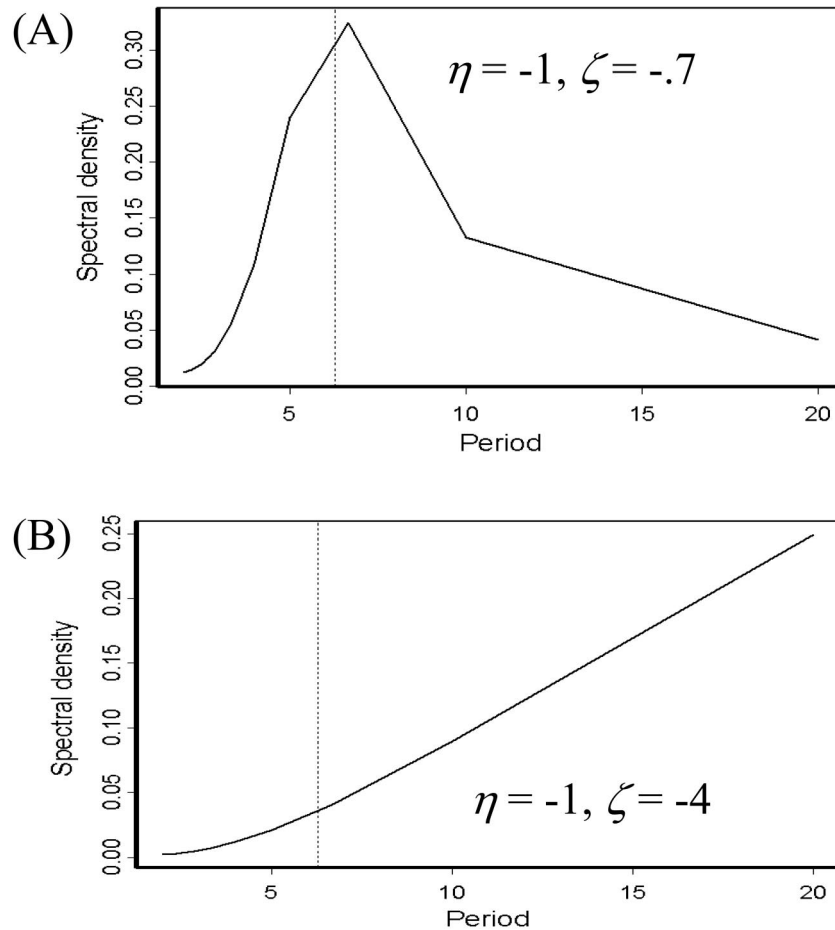


Figure A1. Periodograms of simulated data with the same parameters as in Figure 3A, with $\eta = -1.0$ and $\zeta = -0.7$ (A) and of simulated data with the same parameters as in Figure 3C, with $\eta = -1.0$ and $\zeta = -4.0$ (B). A: The peak in spectral density signifies the dominant period recovered by spectral analysis, and the true period is marked with a dashed line. B: No single period was identified by spectral analysis as the dominant period, and the true period is marked with a dashed line.

Although spectral analysis is a well-known analytic tool that is available in most software packages, the associated functions for spectral analysis do not handle incomplete data. The damped oscillator approach, however, can be implemented within a structural equation modeling framework and thus offers several options that have not been implemented in spectral analysis (e.g., full information maximum likelihood and multiple imputation). This is due, however, to limitations imposed by the software packages, rather than spectral analysis itself. In addition, multiple indicators can be used as markers of a latent construct, and this multivariate measurement model can be combined with the cyclic dynamic model in one single step, rather than in a two-step procedure (as in spectral analysis). If a researcher chooses, the oscillator model can also be fitted simultaneously to multivariate data from multiple individuals. In spectral analysis, however, one must first

derive composite or factor scores and then conduct the spectral analyses separately for each participant.

In addition to the benefits of convenience and accessibility outlined earlier, spectral analysis does have another important strength in helping to answer a specific type of research question. When a data set is characterized by multiple cycles (e.g., daily cycles, weekly cycles, and menstrual cycles all embedded in the same data set), and a researcher is interested in extracting all of these cycles, spectral analysis offers a quick and easy way to accomplish this task. The damped oscillator approach, however, only extracts the most dominant cycle in a data set. To accomplish the same purpose, one will have to fit the oscillator model repeatedly—each time extracting the most dominant cycle and then reanalyzing the residuals.

(Appendixes continue)

Appendix B

A Comparison Between the Fourth-Order Differential Structural Approach and Growth Curve Models

Contemporary growth curve or hierarchical linear models (Bryk & Raudenbush, 1987; McArdle & Epstein, 1987; Meredith & Tisak, 1990) allow researchers to form testable hypotheses regarding level (or intensity), rate of change, and associated interindividual differences. Although second-order change (or acceleration) can readily be incorporated into growth curve models, few researchers have focused on capturing second-order changes.

In a typical growth curve analysis, the latent process of interest is usually identified using a single indicator. A path diagram depicting how linear and quadratic slopes are typically defined in growth curve models with one indicator is shown in Figure B1. This quadratic growth curve model only captures part of the dynamic model defined in Figure 4A. Furthermore, the multivariate measurement model in Figure 4B is not part of the hypotheses tested. Because of this, the shock components shown in Figure 4 are not explicitly modeled in Figure B1, unless multivariate information is incorporated. Therefore, the model in Figure 4 can be interpreted as a curve of the factor, sadness, whereas the model in Figure B1 is used to represent the curve of only one indicator, unhappiness.

Four other important distinctions exist between the differential structural approach and the growth curve approach. First, the state-space embedding technique used in the differential structural approach to capture systematic patterns of covariation for long time series is not usually used in growth curve modeling. The typical use of growth curve models to represent long-term developmental changes also precludes the need for using this technique. Secondly, the scaling value for time t used in the matrix of

factor loadings L (see Table 1) to define measurement intervals is unique to this approach and can be used to accommodate unequal measurement intervals among tests or items.

Third, the loadings of the acceleration factor on manifest indicators differ slightly from typical loadings used to define a quadratic factor (see, e.g., Biesanz, Deeb-Sossa, Papadakis, Bollen, & Curran, 2004; Willett & Sayer, 1994). Basically, the loadings for rate of change are identical to the loadings for slope in a growth curve model (except for an additional scaling value t in the former). However, instead of simply squaring these linear loadings to define the acceleration factor, the squared loadings are divided by two to indicate the derivative relationships among level, rate of change, and acceleration. More specifically, one will have to take the derivative of these acceleration loadings with respect to time to obtain the loadings for the rate of change. Therefore, acceleration is represented as the change in the rate of change with respect to time. This approach was used by Wishart (1938) in the context of polynomial regression to define average growth rate and rate of change of growth rate in bacon pigs.

Finally, because the specific loadings used to define the latent derivatives establish their roles as level, rate of change, acceleration, and other higher-order changes, the fourth-order (or any higher order) differential structural approach can be modified slightly to fit other kinds of differential equation models. More important, the differential structural approach opens the opportunity for formulating specific testable hypotheses of the dynamics of factors (e.g., as conformed to the damped oscillator model). In fact, the damped oscillator model imposes an alternative autoregressive

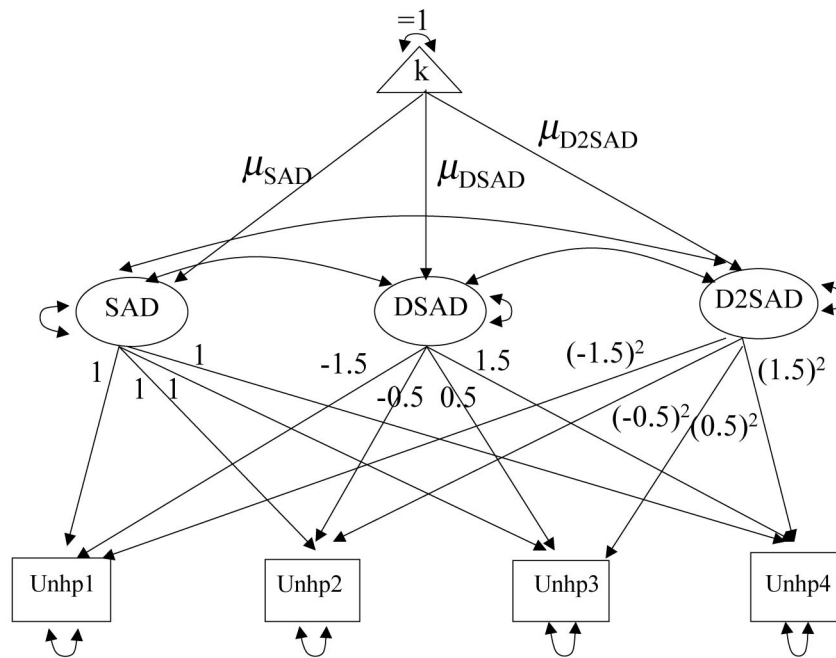


Figure B1. Path diagram of a typical univariate growth curve model with linear and quadratic slopes. The triangle k represents a constant term with its variance fixed at 1.0. Regression estimates from k to SAD, DSAD, and D2SAD (i.e., μ_{SAD} , μ_{DSAD} , and μ_{D2SAD} , respectively) correspond to the means of the three latent components. SAD = intensity of sadness; DSAD and D2SAD = first and second derivatives of sadness, respectively; Unhp1, Unhp2, Unhp3, and Unhp4 = unhappiness measured at Times 1, 2, 3, and 4, respectively.

moving average model process (more specifically an AMRA[2,1] model)^{B1} process on the latent derivatives. Essentially, the η and ζ regression weights represent lag-2 and lag-1 autoregressive weights in a second-order autoregressive process (i.e., AR[2]), respectively. Sources of intraindividual variability in level and rate of change (i.e., S_SAD and S_DSAD in Figure 4) replace the random shocks in a moving average process, and the lag-1 moving average weight is just manifested as covariance between these two sources of intraindividual variability (for details, see Browne & Nesselrode, in press). Yule (1927) pointed out that a stationary AR(2) process with a positive lag-1 autoregressive weight (denoted as α_1) between 0 and 2 and a negative lag-2 autoregressive weight (denoted as α_2) between -1 and $-\alpha_1^2/4$ yields an autocorrelation function that follows a damped sine wave, which can be used to model a pendulum that is subjected to random shocks (for details, see Box & Jenkins, 1976; Browne & Nesselrode, in press; Wei, 1990). Therefore, the dynamic process characterizing the

damped oscillator model is analogous to an autoregressive moving average model (2,1) process. Using the differential structural approach, however, one can effectively capitalize on information from high-order derivatives to yield more accurate estimates for ζ and η (or the lag-1 and lag-2 autoregressive weights). This approach also provides a more general way of fitting other kinds of differential or difference equation models.

^{B1} That is, the hypothesized model can be decomposed into a lag-2 autoregressive process and a lag-1 moving average structure.

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